Riemann-Based Information Extraction (RBIE)

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1. Motivation

Let $x(t) \in \mathbb{R}$ be a real-valued signal composed of both structure and noise. Traditional signal-processing methods rely on decomposition via Fourier or wavelet transforms, yet they fail to exploit the deeper geometric or number-theoretic structure in complex-valued domains. We propose a novel method of extracting information from noise by leveraging a zeta-like decomposition inspired by the distribution of nontrivial Riemann zeroes. We treat the imaginary axis as a structure-revealing coordinate and define a filtering procedure that collapses the information space back to the real axis.

2. Assumptions

- The signal $x(t)$ contains both structured and unstructured (noisy) components.
- Structured components exhibit latent frequency regularity (e.g., harmonics, periodicity, recurrence).
- These latent structures are best identified in a complex frequency domain, similar in nature to Riemann zeta function zeroes.

3. Complex Domain Encoding

We define a Zeta-like Transform of the signal:

$$\mathcal{Z}_x(s) := \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt, \quad \text{where } s = \sigma + i\omega \tag{1}$$

This transform, structurally analogous to a Laplace Transform, extends analysis into the complex frequency domain.

- $\sigma \in \mathbb{R}$: real decay/structure axis
- $\omega \in \mathbb{R}$: imaginary oscillatory axis

4. Spectral Structure Detection

We define pseudo-zeroes $s_i = \sigma_i + i\omega_i \in \mathbb{C}$ such that:

$$|\mathcal{Z}_x(s_i)| \approx \varepsilon \quad \text{(local minima in transformed space)} \tag{2}$$

These minima are interpreted as information-null frequencies, akin to interference points or resonance cancellations — analogous to zero-crossings in the Riemann zeta function.

5. Information Projection and Filtering

Let $\mathcal{F}(\omega)$ denote a structural density function over the imaginary domain:

$$\mathcal{F}(\omega) := \sum_i \delta(\omega - \omega_i) \tag{3}$$

This function acts as a resonance filter kernel. High concentrations of ω_i suggest structured information embedded at those oscillatory frequencies.

We define the Signal Projection Operator:

$$P_{\text{real}}[x(t)] := \operatorname{Re}\left[\int_{-\infty}^{\infty} \hat{x}(\sigma, \omega) \cdot (1 - \mathcal{F}(\omega)) \cdot e^{\sigma t} d\sigma\right] \tag{4}$$

Alternate form (simpler model):

$$x_{\text{signal}}(t) := \int_{\sigma \in \mathbb{R}} \operatorname{Re}[\mathcal{Z}_x(\sigma + i\omega_0)] \cdot e^{\sigma t} d\sigma \tag{5}$$

Here ω_0 is set to 0, effectively collapsing the information space to the real axis after identifying structure.

6. Entropy-Based Filtering

We may optionally define an Imaginary Entropy Functional:

$$\mathcal{H}_{\text{imag}} := -\sum_i p_i \log p_i, \quad \text{where } p_i = \frac{|\omega_i|}{\sum_j |\omega_j|} \tag{6}$$

Low imaginary entropy corresponds to strong concentration of structure. High entropy corresponds to dispersed noise. This entropy can be used as an adaptive threshold: collapse only high-entropy regions to real space, and preserve low-entropy structural frequencies.

7. Signal Recovery Procedure

The complete RBIE procedure is:

1. Transform $x(t) \to \mathcal{Z}_x(s)$
2. Detect pseudo-zeroes s_i
3. Construct $\mathcal{F}(\omega)$ from $\operatorname{Im}(s_i)$
4. Define filtering mask
5. Collapse transform to real axis $\to$ reconstruct $x_{\text{signal}}(t)$

8. Future Directions

- Construction of complex-valued autoencoders to learn $\mathcal{F}(\omega)$ dynamically
- Explore connections to Riemann prime-counting formula, viewing information as a prime-distribution analog
- Extend to Hilbert space signal manifolds using $\zeta(s)$ as a structural prior
- Incorporate topological constraints, e.g., zero distribution geometry over critical strip as a classifier