

# The Greek Letters

## Delta

$\Delta = \frac{\partial C}{\partial S} = N(d_1)$   
 $\Delta = \frac{\partial P}{\partial S} = N(d_1) - 1$   
 • delta is the slope  
 Call option price  
 Current stock price  
 Delta  
 OTM, ATM, ITM  
 X < 5% 提醒  
 时间越长的越接近 0.5  
 当 S < K, 更大的 Δ 与 longer T; 当 S > K, 更大的 Δ 与 shorter T  
 For a call option at the money, Δ = 0.5!

## Gamma

$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{\partial^2 P}{\partial S^2}$   
 $\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T}}$   
 Gamma is largest when an option is at-the-money  
 When gamma is large, delta will be changing rapidly  
 Can be used to minimize the hedging error  
 Gamma is the same for call and put options  
 Hedge against larger changes in stock price  
 $S \gg K, \Delta_c \rightarrow 1, \Delta_p \rightarrow 0, \Gamma \rightarrow 0$   
 $S = K, \Delta_c \rightarrow 1/2, \Delta_p \rightarrow -1/2, \Gamma \rightarrow \text{Max}$   
 $S \ll K, \Delta_c \rightarrow 0, \Delta_p \rightarrow -1, \Gamma \rightarrow 0$   
 题型: 让 delta 和 gamma 都为 C

## Theta

$\Theta = \frac{\partial C}{\partial t} = \frac{\partial P}{\partial t}$   
 For long position, theta < 0, means option lose value as time goes by  
 Short term at the money option has a greatest negative theta.  
 For American option, theta is always negative;  
 Affect the value of call and put in a similar way.  
 Vary with changes in stock prices and as time passes.  
 Most pronounced when the option is at the money, especially near expiration  
 Usually negative  
 Usually increase in absolute value as expiration approaches  
 European in the money put option is possible to have a positive theta

## Vega

$V = \frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = S e^{-\sigma^2 T} \sqrt{T} N'(d_1)$   
 Vega  
 Vega of a call is equal to the Vega of a put  
 Most sensitive to changes in volatility when they are at the money

## Rho

$\rho_c = \frac{\partial C}{\partial r} = K e^{-rT} N(d_2)$   
 $\rho_p = \frac{\partial P}{\partial r} = -K e^{-rT} N(-d_2)$   
 The equity options are not as sensitive to changes in the risk-free rate as they are to changes in the other variables.  
 In the money calls and puts are more sensitive to changes in rates than out-of-the-money options.  
 Increase in rates cause larger increases for in-the-money call prices and larger decreases for in-the-money puts

$$rV = \theta + r\Delta + 0.5\sigma^2 S^2 \Gamma$$

	Long call			Long put		
	OTM	ATM	ITM	OTM	ATM	ITM
Δ	→ 0	0.5	→ 1	→ -1	-0.5	→ 0
Γ	Low	high > 0 esp. short-term	Low	Low	high > 0 esp. short-term	Low
Θ	Low	high > 0 esp. long-term	Low	Low	high > 0 esp. long-term	Low
ρ	Low	high < 0 esp. short-term	Low	Low	high < 0 esp. short-term	Low

gamma和theta是完全下翻, 只是正负不一样  
 一个在x轴上面一个在下面  
 gamma和vega是完全一样, 只是T的顺序不一样  
 一个是时间短一点大, 一个时间长一点