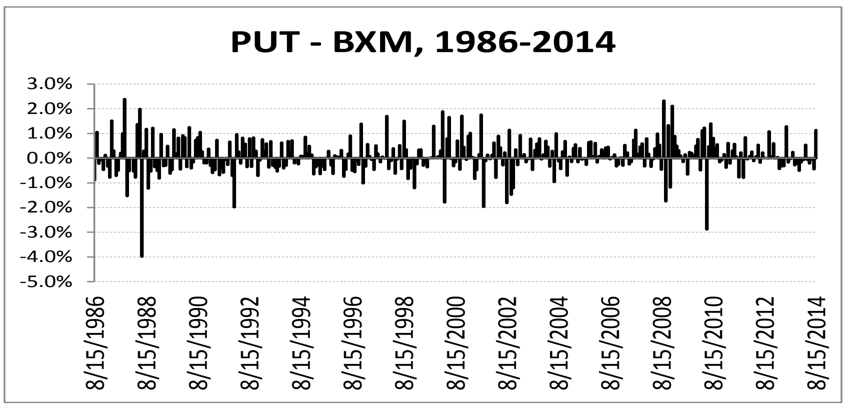
The BXM and PUT Conundrum

Over the years, the CBOE has fielded the following question multiple times: Why is the CBOE S&P 500 PutWrite Index (PUT[SM] index) outperforming the CBOE S&P 500 BuyWrite Index (BXM[SM] index)? In truth, the PUT index does not always outperform the BXM index. This is apparent in Figure 1, a chart of the spread between the PUT and BXM over roll cycles, where a roll cycle extends from the point of sale of the options embedded in the indexes to the point of sale in the next month.

Figure 1. The PUT-BXM Spread over Roll Cycles, 1986-2014.



Source: CBOE

This note tries to elucidate why the BXM and PUT indexes have different rates of return, a feature which was not expected by some investors, and what factors determine which index has the best return over a particular period of time. We review the different compositions of the BXM and PUT indexes, and test different hypotheses proposed to resolve the puzzle. Our sample period extends from 1986 to 2014, and is divided into "pre-VWAP" and "post-VWAP" periods, from 1986 to June 2004, and July 2004 to 2014. In June 2004, the CBOE changed the calculation of the BXM: the call was deemed sold at its volume-weighted average price (VWAP) over a period of time after the settlement of the previous call. Prior to June 2004, the call had been deemed sold at its bid price at the open (1986 to October 1992) or close of the day (November 1992 to 2014).

We find that the spread between PUT and BXM returns is not statistically significant, and that it is primarily explained by two factors: first, the differential effect of the spread between the SOQ and S&P 500 VWAP on roll-date returns past 2004, and second, structural differences between the designs of the indexes.

1. Composition of BXM and PUT portfolios and Rates of Return

BXM = Long S&P 500®, short at-the-money monthly SPX call

PUT= Treasury investments at 1 month and 3 month rates, short N monthly at-the-money SPX puts, where N is a number of puts selected to ensure that the PUT index is collateralized, i.e. the value of the PUT index at the next roll date is non-negative. The SPX options expire on the third Friday of the month and are replaced by new options, a process called the roll. On non-roll dates, the rates of return are calculated as

<u>Rates of return on non-roll dates</u>

$$BXM = \frac{S_t + D_t - C_t}{S_{t-1} - C_{t-1}}; PUT = \frac{M_t - NP_t}{M_{t-1} - NP_{t-1}}$$

where S_t is the value of the S&P 500 at the close of date t, D_t is the S&P 500 dividend distributed on date t, M_t is the value of Treasury accounts at the close of date t, and C_t and P_t are the mid-quotes of the BXM call and PUT put at the close of date.

Since 2004, the BXM rate of return on roll dates has been compounded from three rates, the rate from the previous close to the time of final settlement, the rate from final settlement to the sale of the next call option at the volume weighted average price ("VWAP price"), and the rate from the point of sale to the close. During the second so-called "VWAP" period, the BXM portfolio carries the S&P 500, and the rate of return on this second leg is the ratio of the S&P 500 VWAP value to the Special Opening Quotation of the S&P 500 ("SOQ", as defined below). The BXM VWAP period now extends for two hours. The VWAP period for the calculation of the PUT on roll dates was introduced in 2007 and it extends for half an hour. The only impact it has on the rate of return is that the puts are sold at their VWAP prices. The rate of return of the PUT index is always the percentage change in the value of the Treasury account less the mark of the puts. The difference between roll and non-roll dates is the debits and credits to the Treasury account coming from settlement of the old puts and sale of new puts. Formally:

<u>Rates of return on roll dates.</u>

$$(1) BXM = \frac{SOQ_t + D_t - \max[0, SOQ - K]}{S_{t-1} - C_{t-1}} * \frac{S_{VWAP}}{SOQ_t} * \frac{S_t - C_t}{S_{VWAP} - C_{VWAP}}$$

$$(2)) PUT = \frac{M_{t-1}(1+r) - N_{old} Max[0, K - SOQ] + N_{new}(P_{VWAP} - P_t)}{M_{t-1} - NP_{t-1}}$$

SOQ is the Special Opening Quotation of the S&P 500 index used to settle SPX options. K is the strike of the option. $S_{VWAP,}$ C_{VWAP} and P_{VWAP} are the volume-weighted prices of the S&P 500 call option and put options respectively, and r is the effective rate of interest from date t-1 to t.

2. **Is the PUT-BXM Spread Significantly Biased?**

Figure 2 presents means, standard deviations and t statistics for the PUT-BXM spread during three periods, from July 1986 to July 2014, from July 1986 to June 2004, when the VWAP was introduced in the calculation of the BXM, and from July 2004 to July 2014.

Figure 2 . PUT-BXM Spread Descriptive Statistics

PUT - BXM Spread	Jul 86 to July 14	Jul 86 to June 04	Jul 04 to July 14	
Mean	0.09%	0.05%	0.15%	
Std. Deviation	0.82%	0.77%	0.90%	
t-Test	0.11	0.07	0.17	

Source: CBOE

According to t- statistics shown in Figure 2 the PUT-BXM spread is not significantly different from 0 over the full sample period or over its pre-VWAP and post-VWAP sub-periods.

3. Factors of the PUT-BXM Spread

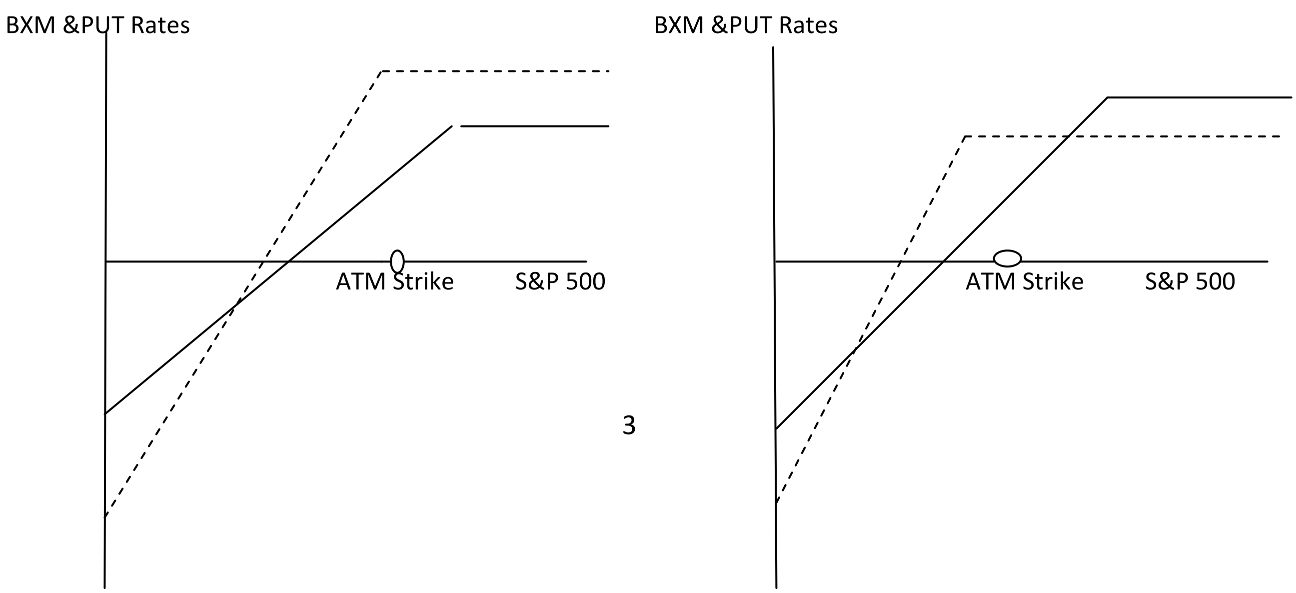
Four hypotheses have been proposed to explain the PUT-BXM spread.

(1) VWAP Hypothesis: The SOQ is often greater than the subsequent VWAP value of the S&P 500. As seen from Equation (1), the ratio of the SOQ to the VWAP drags down the rate of return of the BXM when the BXM call expires in the-money, which has occurred 70% of the time since 2004. In addition, the rate of return of the PUT index is independent of the S&P 500 VWAP, but benefits from a high SOQ value. The VWAP hypothesis is that the combination of both effects biases the PUT-BXM spread.

(2) Interest Rate Hypothesis: The interest rate hypothesis focuses on the different terms of the rates of interest in the calculation of the benchmarks. The PUT index explicitly invests at one-and three month Treasury rates, with the bulk of the investment at the three-month rate. The rate implicit in the BXM is a one-month rate of interest.

(3) **Design Hypothesis:** Aside from the different rates of interest implicit in their calculation, the PUT and BXM are different portfolios. This can be seen from Figure 3, a graph of their rates of return at settlement expressed as a function of the value of the S&P 500. The PUT index is the dotted line. There are two possible configurations. In each case, the PUT-BXM spread flips from negative to positive values as the rate of return of the S&P 500 increases from -100% to 0% (at the ATM strike). Past this range, the spread either remains positive (left panel) or reverts to negative (right panel). In the second case, the PUT-BXM spread is a non-linear function of the rate of return of the S&P 500.

Figure 3. Rates of return of PUT and BXM at Settlement



(1) Skew Hypothesis: The conjunction of negative skewness and fat tails of the distribution of S&P 500 returns lifts the prices of out-of-the-money SPX puts, and this generates a leftward skew of their implied volatilities. The PUT index might benefit from the potentially "richer" price of the put option.

To test these hypotheses we run regressions of the PUT-BXM spread over the roll cycle on four factors: the percentage difference between the SOQ and VWAP value of the S&P 500, the rate of return of the S&P 500, the difference between the three and one-month Treasury bill rates, and the CBOE S&P 500 SKEW Index ("SKEW^{SM}" index). The VWAP calculation started in 2004, and the SKEW index time series started in 1990, hence we run separate regressions from 1986 to 2004, 2004 to 2014 and 1990 to 2014.

Figure 4. Regression of PUT-BXM Spread on Potential Factors, 1986 - 2014

Regressions of PUT-BXM on SOQ/VWAP, S&P 500 RR,Tbill Spread &SKEW				
	1986-2014	**1986-2004**	**2004-2014**	**1990-2014**
R Square	0.41	0.24	0.66	0.44
Intercept	0.00	0.00	0.00	0.00
t-stat	-0.43	2.31	-0.72	0.14
SOQ/VWAP	1.07		1.05	-8.18
t-stat	**12.50**		**13.74**	**12.69**
S&P 500 RR	-0.07	-0.08	-0.05	-0.06
t-stat	**-8.95**	**-7.68**	**-5.51**	**-8.18**
TBill 3M-1M Spread	2.04	2.31	-3.24	-0.3
t-stat	**2.56**	**2.41**	-1.83	-0.06
SKEW				-3.00E-06
t-stat				-0.06

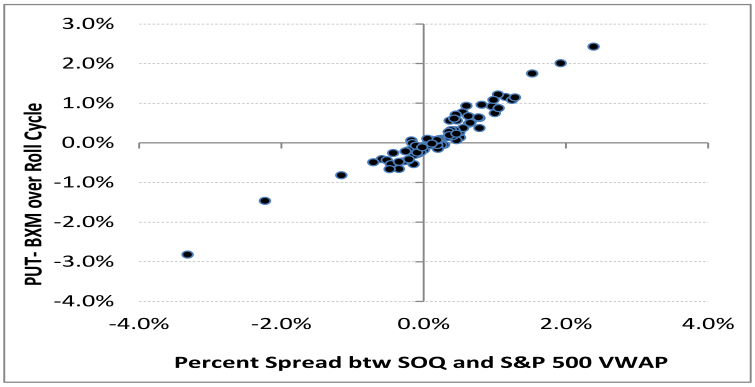
Source: CBOE

The t-statistics shown in Figure 4 indicate that the SOQ/VWAP ratio first, and second the structural difference between the PUT and BXM – as proxied by the impact of the S&P 500 return- are significant factors of the PUT-BXM spread. The T-Bill rate spread is borderline significant in the period from 1986 to 2004. The SKEW index has no significance.

The following charts add further color on the determinants of the PUT-BXM spread.

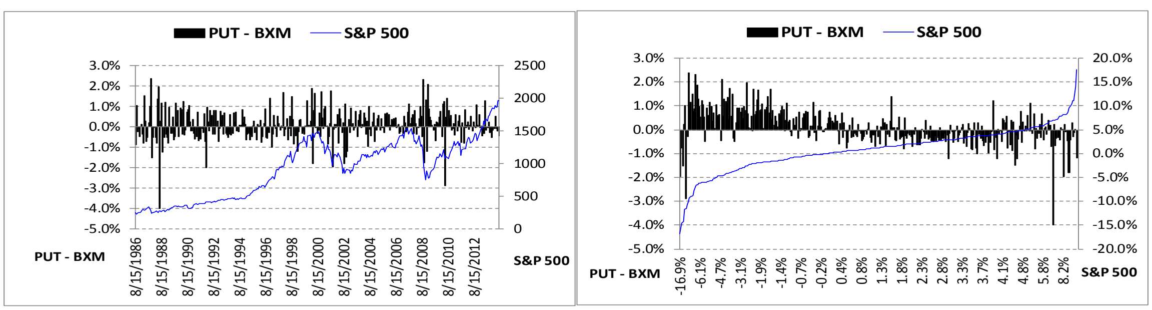
4. Additional Graphic Evidence

Figure 5. PUT-BXM Spread vs. SOQ –VWAP Spread, 2004 to 2014



Source: CBOE

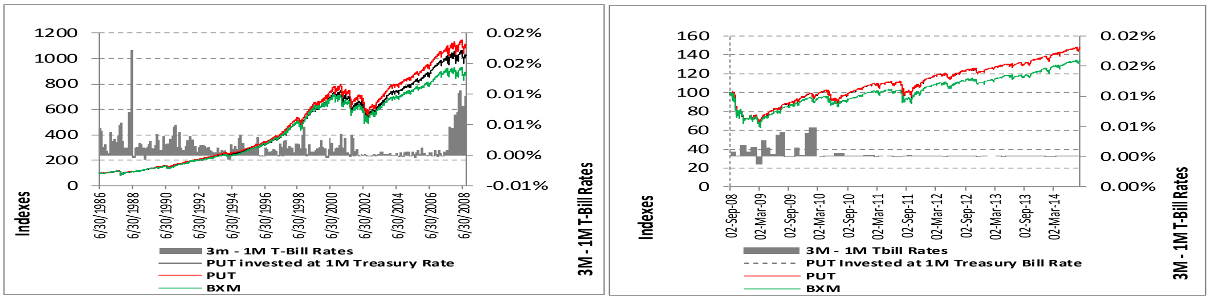
Figure 6. PUT-BXM Spread vs. S&P 500 Rate of Return, 2004 -2014



Source: CBOE

Under the design hypothesis, and as illustrated in Figure 3, the difference between PUT and BXM returns depends on the value of the S&P 500. The left panel of Figure 6 shows the PUT-BXM spread and the S&P 500 in chronological order, while the right panel sorts the S&P 500 returns in ascending order. With this sorting, the pattern of the PUT-BXM spread is consistent with the patterns shown in Figure 3. The spread is negative for extremely negative S&P 500 returns, positive for moderate to small positive S&P 500 returns, and a mix of positive and negative but predominantly negative for higher S&P 500 returns.

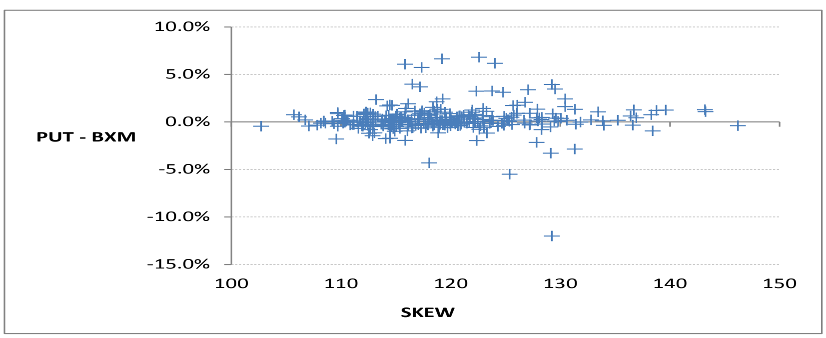
Figure 7. PUT, One-Month PUT and BXM, 1986-2014



Source: CBOE

Figure 7 shows the PUT index, the BXM index and a variation of the PUT index invested at a one-month Treasury bill rate ("one-month PUT") over two periods. The first period extends from 1986 to 2008, and the second from 2008 to 2014. The difference between the 3-month and one-month rates is charted as the gray columns. From 2003 to 2008, the one-month PUT trailed the three-month PUT but still did better than the BXM. From 2008 to 2014, as rates of interest practically dropped to zero, there was little visible difference between the one and three-month PUTs, yet the PUT continued to gradually overtake the BXM as time passed. The interest rate differential partially explains the PUT-BXM spread.

Figure 8 PUT – BXM vs SKEW



Source: CBOE

Figure 8 reveals little correlation between the SKEW and the PUT-BXM spread.

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