Brandeis University INTERNATIONAL BUSINESS SCHOOL

Expected Shortfall and VaR estimation of Barrier Option on changing Barrier Value

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Introduction

We choose barrier option as the topic of our group project because barrier options change distribution of profit and loss from continues distribution to partially discrete distribution; therefore impact VaR and Expected Shortfall. We are all very interested to explore the changes in VaR, especially Expected Shortfall by using barrier options. In this group project, we focused our study on Expected Shortfalls as well as VaR with difference barrier values of a barrier put option. We also compared Expected Shortfalls (ES) and VaR from barrier option with those from vanilla put option and un-hedged portfolio.

How to Do this

We adjusted the barrieroptvar.m codes and added our own codes into it to solve above problem. First, We created a big for-loop to change the barrier value from 99 to 80 and got prices, VaRs and ESs for every barrier value.

In order to calculate the ES of barrier hedged portfolio, we estimated the following three different scenarios using different ways (as in line 92 to line 115 of the codes). We used 'record' to keep track on which one of these three scenarios was happening in certain case.:

1. If the critical value lies in the left tail (record = 0): using mean function could easily get the ES in this case
2. If the critical value lies in the middle hedged peak (0<record<5%*total amount) : we should take the value lies exactly on critical value into account using sum function and then dividing by 5% of total amount.
3. If there is no value lies in the left side (record = 5%*total amount): in this case, no stock had ever hit the barrier which means barrier option act like a vanilla option. ES should equal to that of a vanilla option.

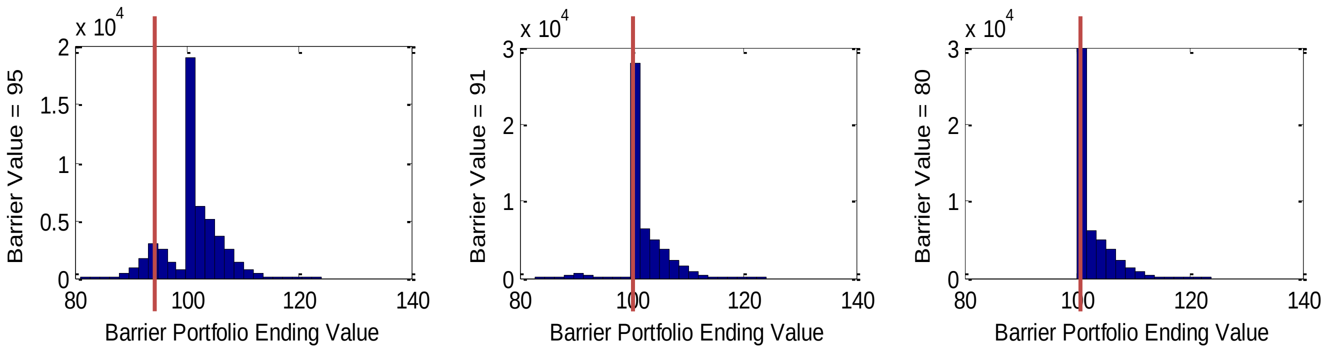


Figure 1 three scenarios of barrier hedged portfolio's ending value and its' critical value line

We attached the codes and result in Appendix I & II. In order to give more accurate results, we ran a 5,000,000 time simulation. For your convenience, the simulation times are 50,000 in the codes we attached.

What did We get

Output Explanation

vanoptval	– Price of vanilla put option
barrieroptval	– Price of barrier put option
var	– VaR of unhedged portfolio
varvan	– VaR of hedged portfolio using vanilla put option
varbarrier	– VaR of hedged portfolio using barrier put option on different barrier value
esunhedge	– Expected shortfall of unhedged portfolio
esvan	– Expected shortfall of hedged portfolio using vanilla put option
esbarrier	– Expected shortfall of hedged portfolio using barrier put option on different barrier value
record	– Indicate how many ending value is exactly equal to the critical value, by following equation: record = niteral*varp - count(barrierport<barriercrit)

Cost of Options

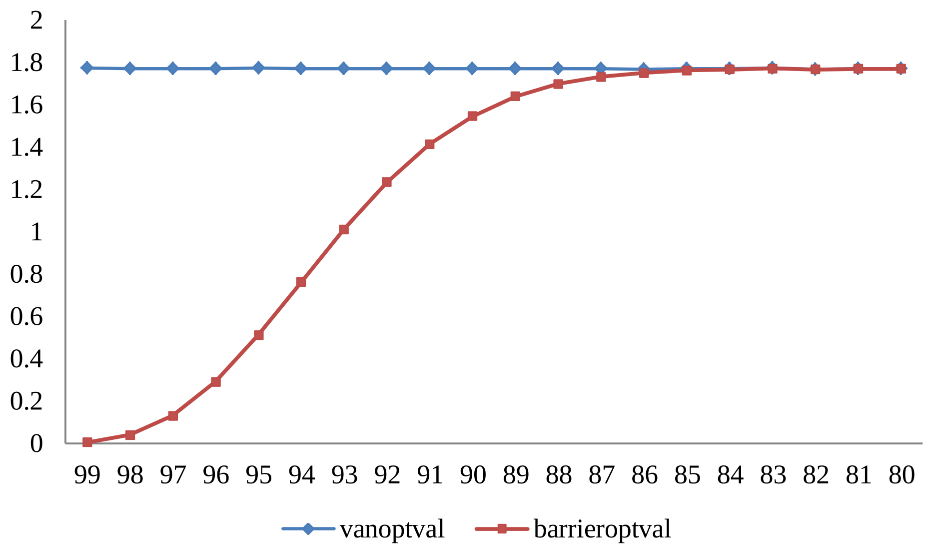


Figure 2 Cost of vanilla put option and barrier option

Figure 2 reflects how the price of barrier put option and vanilla option change in terms of different barrier prices. From the 5,000,000 times simulation result, the price of vanilla put option is bouncing around 1.77 and does not change by different barrier values. On these 20 barrier values, the price of a barrier put option is always lower than that of a vanilla option. Since the barrier option will not be effective as long as the price hits barrier during a period, barrier option requires a stricter condition for the execution of option. Therefore, barrier put option is always cheaper than vanilla put option.

Another trend shown from the graph is that prices of barrier and vanilla put option converge as barrier becomes lower than certain level. The horizon we used in our model is 20 days, and we assumed the log return follows normal distribution. Hence, there will be certain case where all simulated portfolios have not hit the barrier value during such a short period. The lower the barrier value is, the lower the probability of price hitting the barrier. In our case, it is almost impossible for the stock price to hit the barrier lower than 82. Therefore, we can find the convergence trend of barrier and vanilla option prices.

Expected Shortfall

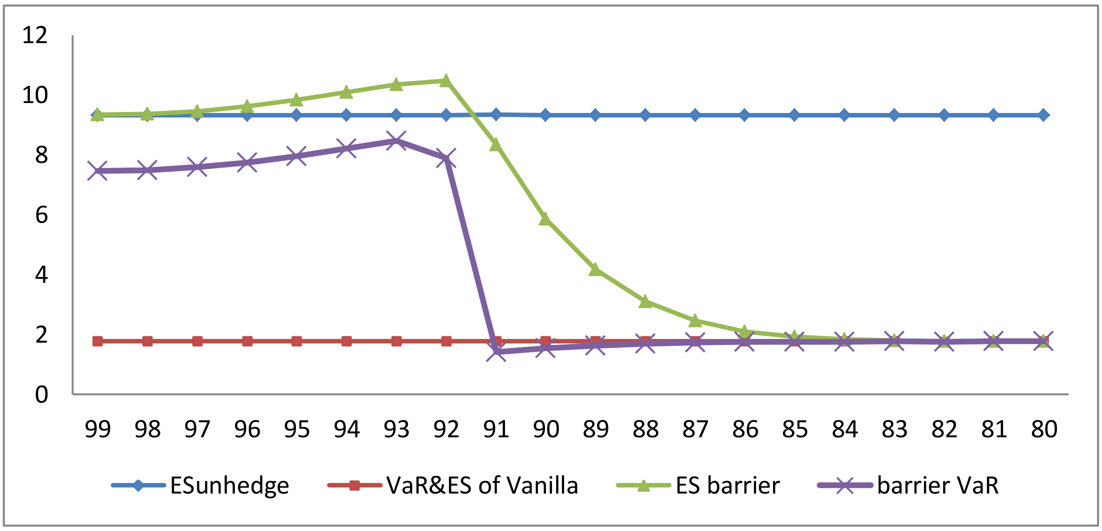


Figure 3 Expected Shortfall & VaR of Barrier hedged, Vanilla put option hedged and Unhedged portfolio

1. VaR of Vanilla = ES of Vanilla = the cost of Vanilla option

This is because the striking price of this put option is the same with the beginning stock price. Vanilla put option won't be ineffective by the moving of stock price. Therefore, it can lock the stock price and the investors will only have to pay for the option itself. If the price is going up, the cost of option will be deducted from the profit of selling stocks. If the price is going down, the investor will execute vanilla option and make sure the loss will not exceed the cost of option itself.

However, barrier put option is different. With the barrier going down, there will be a balance between the loss that option could save and the cost of option itself.

2. ES of barrier > VaR of barrier

ES took extreme left tail into account.

3. VaR of barrier > VaR of Vanilla when barrier value >= 92

More than 5% of portfolio's barrier is ineffective and therefore left tail appears.

4. VaR of barrier < VaR of Vanilla when barrier value <= 91

Less than 5% of the portfolio's barrier is ineffective and therefore even though there is value lying on left tail, the VaR lies in the hedged side. And because the costs of barrier option are always less than that of vanilla option, the VaR of barrier would be lower than that of vanilla.

5. ES of barrier > ES of unhedged and is increasing while barrier value >= 92

In the first eight trials, when barrier decreases from 99 to 92, the 'record' vector equals zero. In these cases, the barriers are relatively high. There will be a bigger probability for the stock price to hit the and thus the option expires. If the option is easy to expire, our portfolio is likely to be unprotected as in the unhedged case. We calculated the ES difference between barrier option case and unhedged case and the result is in the figure 4 below. When the barrier ranges from 92 – 99, the ES difference is just the cost on the barrier put option. In this case, the barrier is so high that the portfolio is unprotected. The ES of barrier option case is just the sum of ES in the unprotected case and the cost of barrier option.

6. ES of barrier < ES of unhedged and is decresing to converge with ES of Vanilla

From barrier of 92, the protection on stock price from option begins to offset the cost of barrier option, and ES in barrier portfolio falls quickly and approaches the ES in vanilla portfolio. When the barrier hits 83 or lower, barrier ES is almost the same with vanilla ES. In this case, the barrier is so low that there is quite a small probability for the stock price to hit the barrier in a 20-day period. Therefore, the barrier option can be treated as a vanilla option.

Barrier (A)	esunhedge (B)	esbarrier (C)	barrieroptval (D)	esbarrier–esunhedge (C - B)	barriervar (E)	Record ** (F)
99	9.3333	9.3387	0.0054	0.0054	7.4615	0
98	9.3244	9.3641	0.0397	0.0397	7.4928	0
97	9.3247	9.4554	0.1307	0.1307	7.5892	0
96	9.3333	9.6248	0.2915	0.2915	7.7499	0
95	9.3307	9.8423	0.5116	0.5116	7.9663	0
94	9.33	10.0917	0.7617	0.7617	8.2208	0
93	9.3319	10.3441	1.0122	1.0122	8.4702	0
92	9.3286	10.487	1.2335	1.1584	7.8913	0
91	9.3374	8.3317	1.4141	-1.0057	1.4141	67531
90	9.3337	5.8577	1.5461	-3.476	1.5461	147054
89	9.3313	4.1739	1.64	-5.1574	1.6400	194823
88	9.3289	3.0989	1.6983	-6.23	1.6983	221931
87	9.3307	2.4594	1.7326	-6.8713	1.7326	236525
86	9.3249	2.0991	1.7503	-7.2258	1.7503	243968
85	9.3327	1.926	1.7625	-7.4067	1.7625	247356
84	9.3325	1.8369	1.7675	-7.4956	1.7675	248951
83	9.3251	1.8005	1.7718	-7.5246	1.7718	249592
82	9.3295	1.7762	1.7673	-7.5533	1.7673	249879
81	9.3272	1.7733	1.7699	-7.5539	1.7699	249957
80	9.3312	1.7707	1.7696	-7.5605	1.7696	249986
Vannilla Put Option		ES	Vanoptval			
		1.7711	1.7711			250000

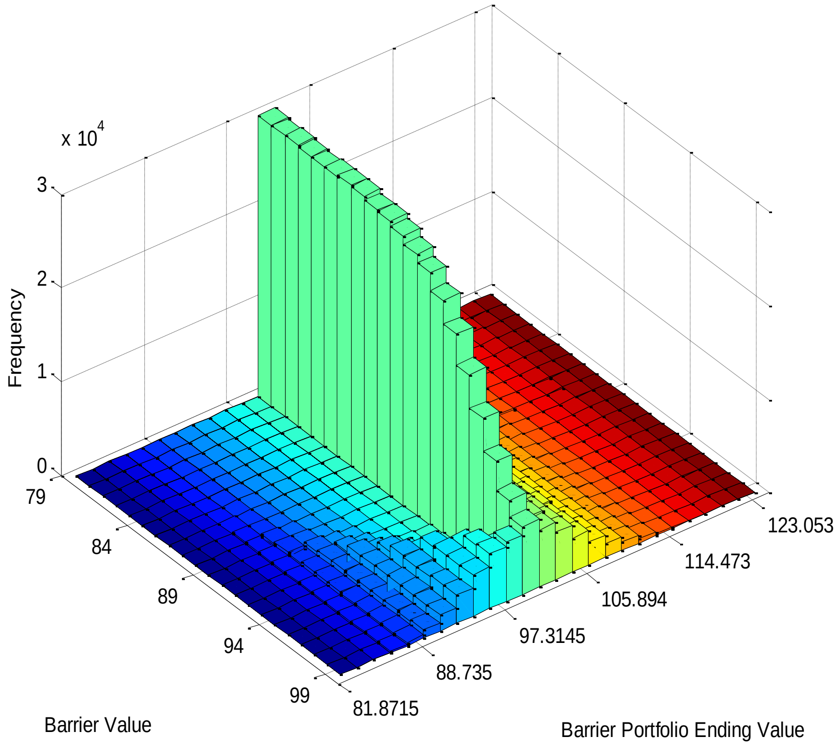
Conclusion

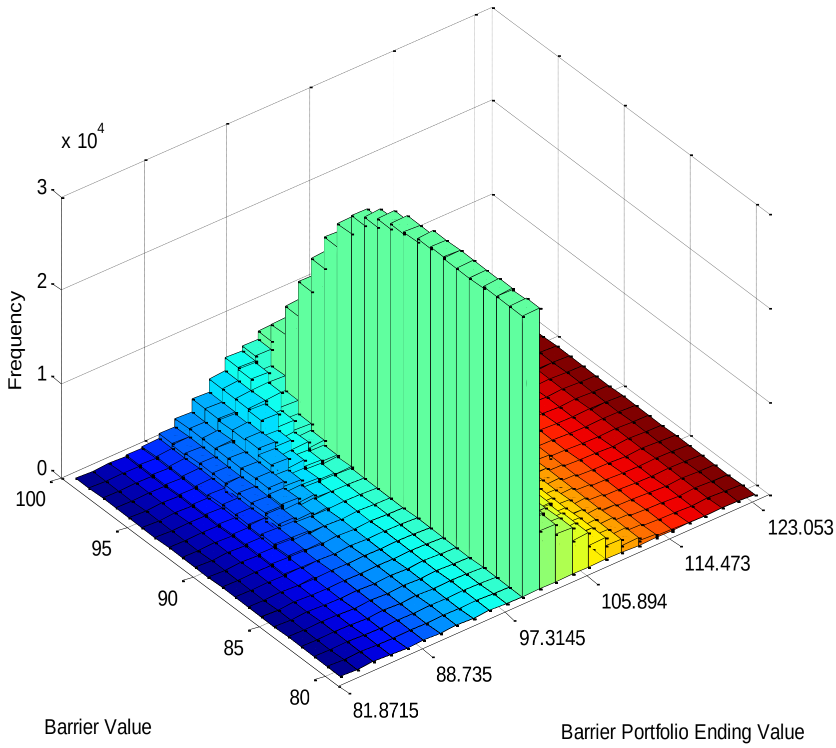
All in all, ES of barrier option provides us a different aspect to estimate the risks on barrier hedged portfolio. As we compare ES to VaR of barrier as well as VaR of unhedged portfolio and Vanilla Option hedged portfolio, we found out certain routine and explained the different between these indicators. We think that Expected Shortfall could provide a more conservative evaluation of the risk since ES can show more details on extreme fat tail than VaR.

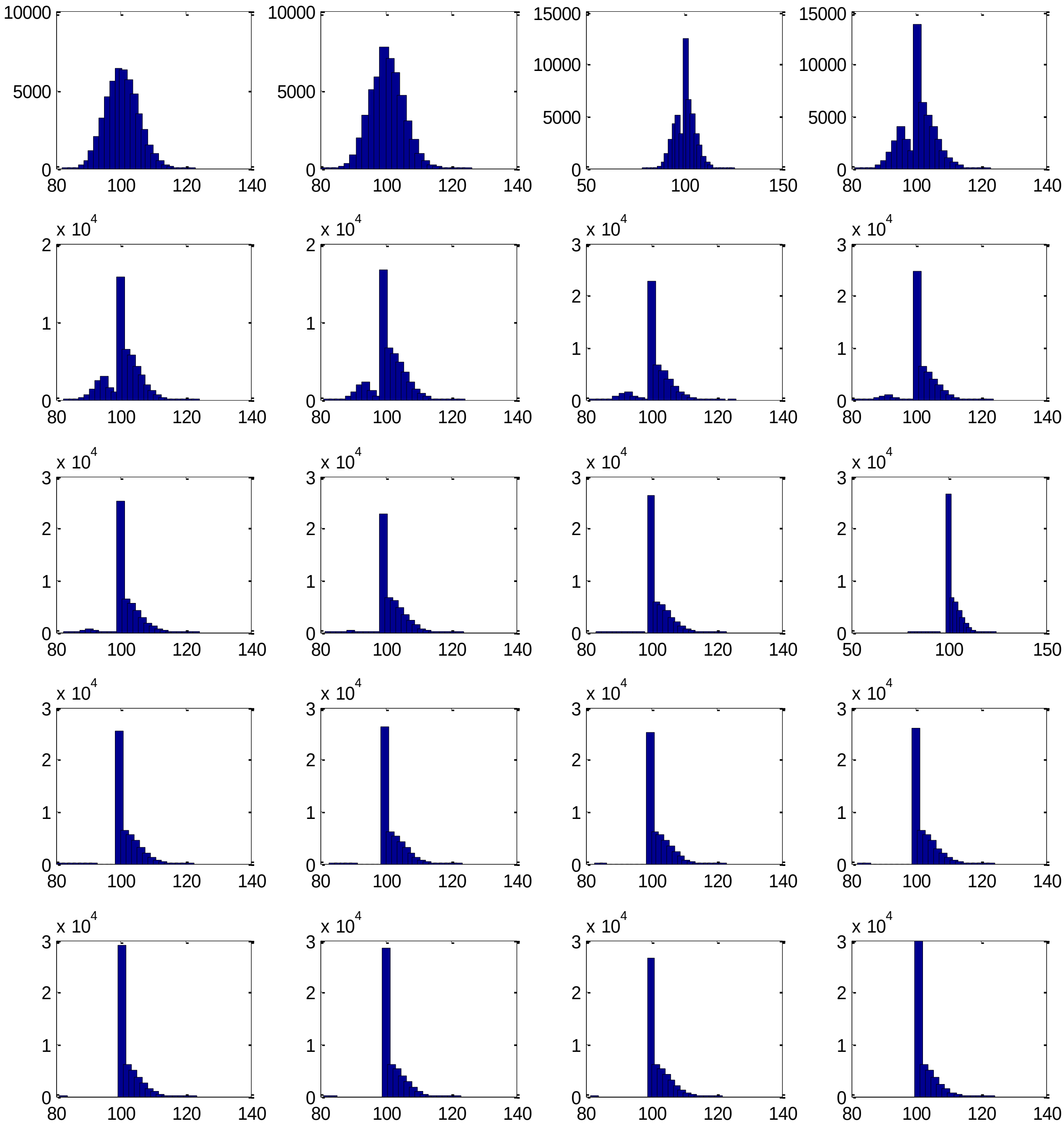
Appendix I: results

Barrier	vanoptval	barrieroptval	unhedged VaR	vanilla hedged VaR	barrier hedged VaR	ESunhedge	Esvan	ESbarrier	esbarrier - esunhedge	Record
99	1.7726	0.0054	7.4561	1.7726	7.4615	9.3333	1.7726	9.3387	0.0054	0
98	1.7718	0.0397	7.4531	1.7718	7.4928	9.3244	1.7718	9.3641	0.0397	0
97	1.7701	0.1307	7.4585	1.7701	7.5892	9.3247	1.7701	9.4554	0.1307	0
96	1.7718	0.2915	7.4583	1.7718	7.7499	9.3333	1.7718	9.6248	0.2915	0
95	1.7734	0.5116	7.4547	1.7734	7.9663	9.3307	1.7734	9.8423	0.5116	0
94	1.7723	0.7617	7.4591	1.7723	8.2208	9.33	1.7723	10.0917	0.7617	0
93	1.7718	1.0122	7.458	1.7718	8.4702	9.3319	1.7718	10.3441	1.0122	0
92	1.7702	1.2335	7.4581	1.7702	7.8913	9.3286	1.7702	10.487	1.1584	0
91	1.7717	1.4141	7.4621	1.7717	1.4141	9.3374	1.7717	8.3317	-1.0057	67531
90	1.7706	1.5461	7.4612	1.7706	1.5461	9.3337	1.7706	5.8577	-3.476	147054
89	1.7715	1.64	7.4534	1.7715	1.64	9.3313	1.7715	4.1739	-5.1574	194823
88	1.7714	1.6983	7.4544	1.7714	1.6983	9.3289	1.7714	3.0989	-6.23	221931
87	1.7704	1.7326	7.4529	1.7704	1.7326	9.3307	1.7704	2.4594	-6.8713	236525
86	1.769	1.7503	7.4559	1.769	1.7503	9.3249	1.769	2.0991	-7.2258	243968
85	1.7709	1.7625	7.4581	1.7709	1.7625	9.3327	1.7709	1.926	-7.4067	247356
84	1.7713	1.7675	7.4526	1.7713	1.7675	9.3325	1.7713	1.8369	-7.4956	248951
83	1.7733	1.7718	7.4523	1.7733	1.7718	9.3251	1.7733	1.8005	-7.5246	249592
82	1.7678	1.7673	7.4561	1.7678	1.7673	9.3295	1.7678	1.7762	-7.5533	249879
81	1.7701	1.7699	7.4591	1.7701	1.7699	9.3272	1.7701	1.7733	-7.5539	249957
80	1.7696	1.7696	7.4576	1.7696	1.7696	9.3312	1.7696	1.7707	-7.5605	249986

Appendix II: Graph






10000
5000
0
80 100 120 140
15000
10000
5000
0
50 100 150
x 10^4
3
2
1
0
80 100 120 140
50 100 150

Appendix III: Codes

```
clear all
load dow.dat
p = dow(:,2);
ret1 = log(p(2:end)./p(1:end-1));

% set parameters
price = 100;
strike = 100;
barrierastart = 100;
options = 1;
shares  = 1;
horizon = 20;
rf = 0.05;
varp = 0.05;

dowstd = std(ret1);
dowmean = mean(ret1);

% Risk neutral : return = risk free rate
% risk neutral mean
% x = lognormal(m,v)
% Expected growth in log(x) = m + v/2

rndowmean = rf/250 - dowstd^2/2;
% Expected price growth = 0.05/250 - v/2 + v/2 = 0.05/250

% stage one: Risk neutral option pricing
% Actualy, we use 5000000 instead of 50000 to come up with our results
niterat = 50000;
vanillaput = zeros(1,niterat);
barrierput   = zeros(1,niterat);
record = zeros(1,20);
barrierport = zeros(20,niterat);
for j=1:20
    % set barrier to 99:-1:80
    barrier = barrierastart-j;
for i = 1:niterat
    rets = normal(horizon,rndowmean,dowstd^2);
    pricepath = [price; exp( cumsum(rets) ) *price];
    vanillaput(i) = max(strike-pricepath(end),0);
    pathmin = min(pricepath);
    if(pathmin>barrier)
```

```
        barrierput(i) = max(strike-pricepath(end),0);
    else
        barrierput(i) = 0;
    end
end

% estimate the price of both two option
vanoptval(j)  = exp(-rf*(horizon/250))*mean(vanillaput);
barrieroptval(j)= exp( -0.05*(horizon/250))*mean(barrierput);

% stage two:  Get value at risk using true means
vanport = zeros(1,niterat);

unhedge = zeros(1,niterat);
for i = 1:niterat;
    rets = normal(horizon,dowmean,dowstd^2);
    pricepath = [price; exp( cumsum(rets) ) *price];
    vanillaput(i) = max(strike-pricepath(end),0);
    pathmin = min(pricepath);
    if(pathmin>barrier)
         barrierput(i)   =  max(strike-pricepath(end),0);
    else
        barrierput(i) = 0;
    end
    vanport(i) = shares*pricepath(end) + options*vanillaput(i);
    barrierport(j,i) = shares*pricepath(end) + options*barrierput(i);
    unhedge(i) = shares*pricepath(end);
end

% Find the starting costs of the portfolios
costvan = shares*price+options*vanoptval(j);
costbarrier = shares*price+options*barrieroptval(j);
costunhedge = shares*price;

% estimate Expected Shortfall - not discounting since only 20 days future

%critical value
vancrit = quantile(vanport,varp);
bp = barrierport(j,:);
barriercrit(j) = quantile((bp), varp);
unhedgecrit = quantile(unhedge,varp);
esvan(j) = costvan - vancrit;

% plot the barrier portfolio value
```

```
subplot(5,4,j)
[y, x] = hist(unhedge,25);
y2 = hist(bp,x);
bar([x'],[y2'], 1)

% if there is no value lies to the left of critical value, then the
% expected shortfall will be the same as the cost of the option
if count(bp<barriercrit(j))>0
    % if there are not more than 5% value lying to the left of critical
    % value, then we should take remaining value right on the critical
    % value into account.
    %'+1' is taking the value right on critical into account.
    if niterat*varp > count(bp<barriercrit(j))+1
        % use 'record' to keep track on how many value under certain criti-
        % -cal level (0.05 in this case)is lying excatly on critical value
        record(j)=niterat*varp - count(bp<barriercrit(j));
        esbarrier(j) = ...
            costbarrier - (sum(bp(bp<barriercrit(j)))+...
            (niterat*varp-count(bp<barriercrit(j)))*...
            barriercrit(j))/(niterat*varp);
    else
        record(j)=niterat*varp - count(bp<barriercrit(j))-1;
        esbarrier(j) = costbarrier - mean(bp(bp<=...
            barriercrit(j)));
    end
else
    record(j)=123;
    esbarrier(j) = costbarrier - barriercrit(j);
end
% If the barrier drops down to below 82 then there is no portfolio with a
% value below crtical value
esunhedge(j) = costunhedge - mean(unhedge(unhedge<unhedgecrit));

% estimate VaR - not discounting since only 20 days future
varvan(j) = shares*price+options*vanoptval(j) - vancrit;
varbarrier(j) = shares*price+options*barrieroptval(j) - barriercrit(j);
var(j) = shares*price - unhedgecrit;

end

% display var and mean percentage change in value (risk, return)
'vanoptval'
vanoptval
'barrieroptval'
```

```
barrieroptval
'unhedged'
var
'vanilla hedge'
varvan
'barrier hedge'
varbarrier
'expected short-fall for unhedge portfolio'
esunhedge
'expected short-fall for vanilla-put portfolio'
esvan
'expected short-fall for barrier put portfolio'
esbarrier
'how many crit out of order'
record

figure;
[y, x] = hist(unhedge,25);
y2=hist(barrierport',x);
% flip y2 over~ just to show the graph in two sides
for m=1:20
    mm(:,m)=y2(:,21-m);
end
x1 = [x(1) x(5:5:25)];
subplot(2,1,1)
bar3(mm',1);
axis([0 26 0 21 0 30000]);
set(gca,'xticklabel',x1);
set(gca,'yticklabel',79:5:99);
xlabel('Barrier Portfolio Ending Value');
ylabel('Barrier Value');
zlabel('Frequency');
hold on
grid on
subplot(2,1,2)
bar3(y2',1)
axis([0 26 0 21 0 30000]);
set(gca,'yticklabel',100:-5:80);
set(gca,'xticklabel',x1);
xlabel('Barrier Portfolio Ending Value');
ylabel('Barrier Value')
zlabel('Frequency');

figure;
```

```
subplot(1,3,1)
[y, x] = hist(unhedge,25);
yy = hist(barrierport(5,:),x);
bar([x'],[yy'], 1);
xlabel('Barrier Portfolio Ending Value');
ylabel('Barrier Value = 95');
subplot(1,3,2)
[y, x] = hist(unhedge,25);
yy = hist(barrierport(9,:),x);
bar([x'],[yy'], 1);
xlabel('Barrier Portfolio Ending Value');
ylabel('Barrier Value = 91');
subplot(1,3,3)
[y, x] = hist(unhedge,25);
yy = hist(barrierport(15,:),x);
bar([x'],[yy'], 1);
xlabel('Barrier Portfolio Ending Value');
ylabel('Barrier Value = 80');
```