Implementation of Block Bootstrap for Portfolios of Returns

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Introduction

Risk managers use Value at Risk (VaR) as an important tool to measure and control the level of risk that the firm undertakes as frequently as investors value portfolio returns. What made us curious about are the methods used by financial institutions to find the global equity portfolio VaR and the logics behind them. Therefore we chose the block bootstrap of portfolio returns as our group project's subject. In this paper, we are going through a scenario-by-scenario analysis that was mainly divided into three parts. To begin with, we compare the 10-day portfolio lognormal returns using both block bootstrap and IID bootstrap method under different assumptions. In the second part, we extend out analysis from a 5-trading day portfolio to a 1-year portfolio incrementing weekly. Finally, we draw a conclusion about diversification of global equity portfolio risk management in practice.

Part I: Analyzing Basic 10-day portfolio return

We first constructed a global equity portfolio composed of three equally weighted categories of assets, which are from US market, non-us developed market and emerging market respectively. Then, we got the total return of this portfolio by adding up each asset return according to their weights.

Based on different assumptions, four measures are used to value the portfolio return's VaR.

1. <u>Maintaining the cross correlation among the assets,</u> we ran a 100,000 time simulation, built a 10-day portfolio and bootstrapped the blocked returns. Thus, we got a 5% 10-day value at risk number and a 1% 10-day value at risk number.
2. <u>Similarly, maintaining the cross correlation among the assets</u>, we ran a 100,000 time simulation, built a 10-day portfolio and bootstrapped the IID (Independent and Identically Distributed) returns. Thus, we got a 5% 10-day value at risk number and a 1% 10-day value at risk number.
3. <u>Destroying the cross correlation among the assets</u>, we bootstrapped the 10-day blocked return.
4. <u>Similarly, destroying the cross correlation among the assets</u>, we bootstrapped the 10-day IID return.

The results are shown on the following **Exhibit 1**:

Exhibit 1. 5% and 1% VaR obtained from Block bootstrap and IID bootstrap

Assumptions	Maintain Correlation		Destory Correlation	
Methods	Block bootstrap	IID bootstrap	Block bootstrap	IID bootstrap
5% VaR	5.0060	4.3381	3.1945	2.9828
1% VaR	9.3099	6.7119	5.2355	4.6401

Comparing the results ran by two methods, it’s clear that the result from block bootstrap is larger than that from IID bootstrap. We can come to the same conclusion when running the bootstrap without maintaining cross correlation among the assets.

However, what does the different results mean in real finance world? Will the result maintain the same if the portfolio has a different maturity? In order to confirm our results, we decided to test on different day’s periods and explore the relationship between blocked bootstrap and IID in the next part.

Part II: Analyzing Difference between blocked bootstrap and IID bootstrap

In the second scenario, we mainly analyzed:

(1) the difference between the 5% VaR using blocked bootstrap and 5% VaR using IID bootstrap.

(2) the difference between the 1% VaR using blocked bootstrap and 1% VaR using IID bootstrap.

We maintain portfolio weights of 1/3 each and maintain correlation. We assume that regularly there are 5 trading days per week and 250 trading days per year.

Initially, we calculate 5 days return for the portfolio using blocked bootstrap. And then we increase by 5 days to 250 days return, thus we get 50 statistics of q-day returns, where q = (5, 10, 15, 20 …….250).

For each q-day returns distribution, we get the VaR (0.05) and VaR (0.01) for blocked bootstrap.

Then we do the same with IID bootstrap. We calculate 5 days return to 250 days return increasing by 5 days separately and get VaR (0.05) and VaR (0.01) for each q-day returns.

Finally we calculate differences between the VaR (0.05)/VaR (0.01) using blocked bootstrap and VaR (0.05) /VaR (0.01) using IID with the following codes:

```
var005diff (qq) = var005block-var005iid;
var001diff (qq) = var001block-var001iid; qq=(1, 2, 3…..50)
```

With 50 differences of VaR (0.05) and VaR (0.01), we plot the VaR005 difference between blocked and IID measures and VaR001 difference between blocked and IID measures shown in the following **Exhibit 2 and 3** which are derived from results attached by the end of the report (**Exhibit 6 and 7**).

Exhibit 2. 5% VaR difference between block and IID measure

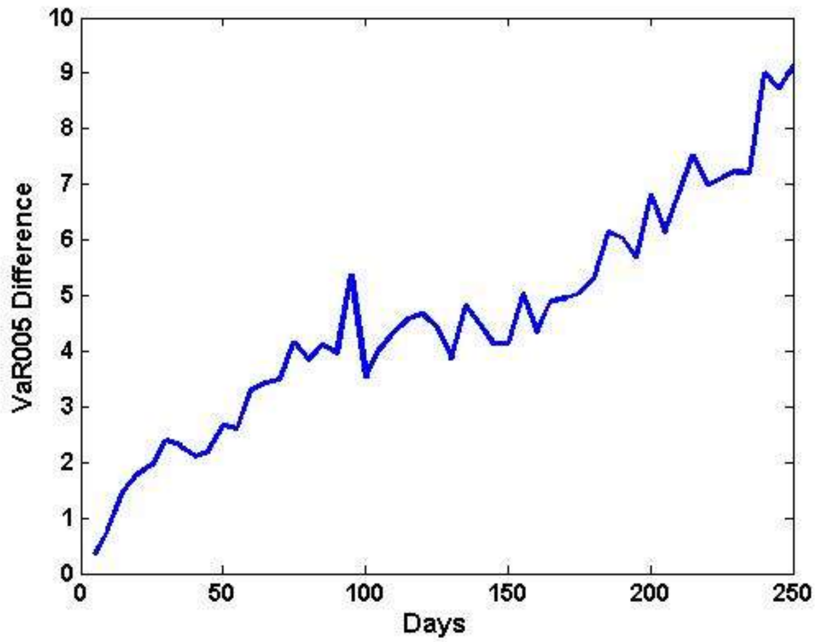
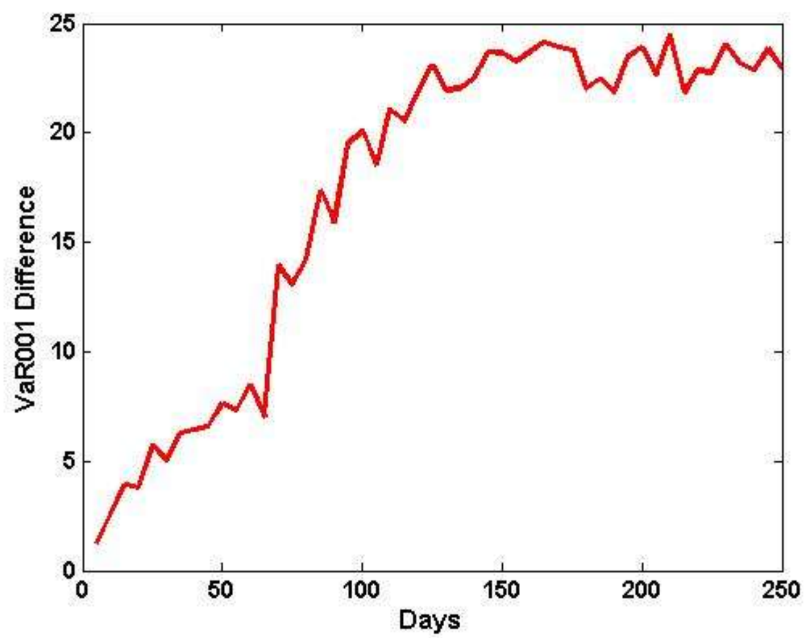


Exhibit 3. 1% VaR difference between block and IID measure



As shown on the graphs above, it is obvious that both VaR (0.05) difference between block and IID measures and VaR(0.01) difference between block and IID measures increase when the days of return being analyzed increases,. This indicates that VaR of block bootstrap becomes larger and larger when compared with VaR of IID as days increase.

This can be explained by the continuance of existing trends in the market. When the price of a security increases, the rising trend of price tends to maintain for a while followed by additional gains. However, when the prices fall, the bear market tends to keep for a while too. Thus, due to momentum effects, the VaR of block bootstrap tends to be larger because its samples consist of continuous days.

In real business world, we can get the conclusion that blocked bootstrap measure is more conservative than IID measure. And with the time increasing, the gap between the results of two measures enlarges. For example, if a risk management team in the commercial bank wants to measure the potential losses of products investing in global equity markets more cautiously, it will choose blocked bootstrap to get VaR to measure the potential loss. Moreover, risk mangers dealing with longer periods products should be more cautiously than those with shorter periods ones selecting between the two measures.

Part III: Analyzing Portfolio Management (Three-dimensional Surface Graphs)

In the former sections, we all assume that the three assets have the same weight; in other words, we maintain each asset's weight with 1/3, 1/3, and 1/3 in all portfolios.

In this section, we now try to find out that how VaR be affected when the three assets have different weights. We have three different types of assets (MSCI USA Index, MSCI Developed Market Index, and MSCI Emerging Market Index) and each asset has different level of risk. This could help us evaluate how different weights of each asset in a portfolio with different level of risk could affect VaR.

To find out the result, we firstly set our assumptions as follows:

- Using blocked bootstrap
- Maintaining cross correlation
- Keeping bootstrap 10-day returns

After running the simulation, we put all the outcomes in the following two surface diagrams (**Exhibit 4 and 5**) which are derived from scatter diagrams attached by the end of the report (**Exhibit 8 and 9**). The two graphs are all three-dimensional. The x axis represents the weight of asset 1 (MSCI USA Index); the y axis represents the weight of asset 2 (MSCI Developed Market Index) and the z axis represents VaR. The reason for not showing the weight of asset 3 (MSCI Emerging Market Index) is that we could easily know what is the weight of asset 3 by showing the weights of the other two assets because the sum of weights of three assets must be 1.

Exhibit 4. 5% VaR with Different Weights of 3 Categorized Assets (Surface Graph)

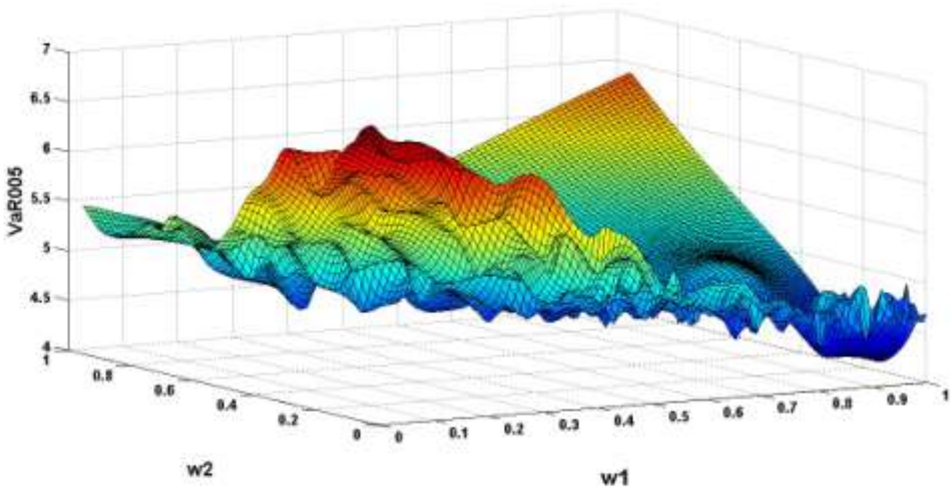
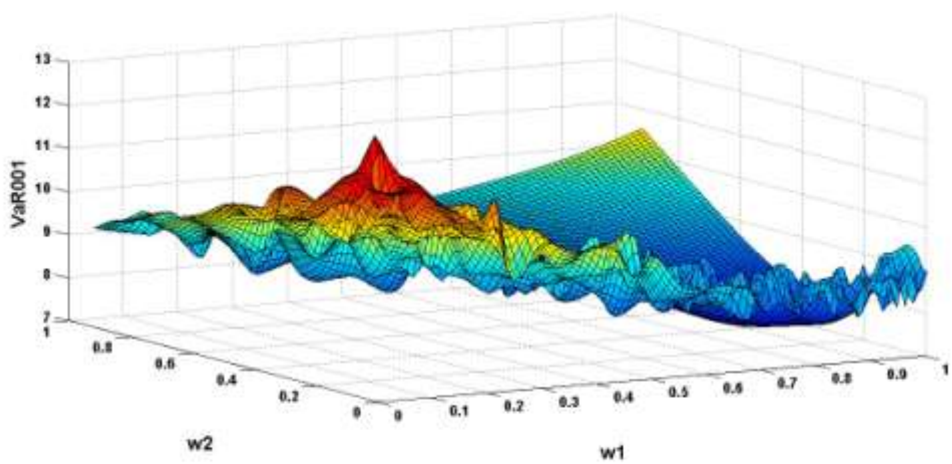


Exhibit 5. 1% VaR with Different Weights of 3 Categorized Assets (Surface Graph)



(Note of the surface graphs: Color from Red to Blue, Altitude from High to Low accordingly)

From the graph, we have two major findings. The first one is that based on the graph, we could find out VaR reaches its lowest level when the weights of both assets are all around 0.3; in other words, VaR reaches its lowest level when the portfolio puts equal amount in each three asset. This could complement the concept of "portfolio diversification". Portfolio diversification means reducing risk by investing in a variety of assets. Any risk-averse investor will diversify to at least some extent. Diversification relies on the lack of a tight positive relationship among the assets' returns, and works even when correlations are near zero. In our case, these three assets are not a combination with perfectly zero correlation; however, the diversification still successfully leads the VaR of portfolio to the lowest level.

The second finding is that we could find out which asset is the riskiest one and which assets is the least risky one through the graph. There are three different scenarios here:

- When the x axis and y axis all equal to 0, which means that the portfolio is not diversified and just has asset 3 (MSCI Emerging Market Index) in it.
- When the x axis equal to 0 and y axis all equal to 1, which means that the portfolio is not diversified and just has asset 2 (MSCI Developed Market Index) in it.
- When the x axis equal to 1 and y axis all equal to 0, which means that the portfolio is not diversified and just has asset 1 (MSCI USA Index) in it.

Compared these three scenarios to each other, we could find out that the first scenario (MSCI Emerging Market Index) has the highest VaR. The third scenario (MSCI USA Index) has the lowest VaR. This means that the emerging market has the highest risk than other two assets.

As an added explanation, the smooth part in the picture shown in three-dimensional graph has no meaning because the sum of weights of three assets is over 1 which is not possible based on our weights totaled in 1 assumption of the portfolio.

Conclusion

According to the three step-by-step analysis using both block bootstrap and IID bootstrap, we found that VaR obtained from block bootstrap is larger than the one obtained from IID bootstrap and the difference tend to be larger when the time horizon extends. Therefore, we made the conclusion that block bootstrap is a more conservative way of measuring risk than IID bootstrap. In addition, by comparing different weight composition of the global equity portfolios, we found that it will be better to diversify one's portfolio in order to maintain the risk under control.

Exhibit 6. 5% VaR: difference between blocked and IID

# of Weeks	Differences	# of Weeks	Differences	# of Weeks	Differences	# of Weeks	Differences	# of Weeks	Differences
1	0.6592	11	2.9021	21	4.2091	31	4.7828	41	6.6189
2	0.5618	12	2.8460	22	4.1461	32	5.1112	42	6.7107
3	1.3739	13	3.0123	23	4.3265	33	4.8728	43	7.1792
4	1.6857	14	3.9066	24	4.7588	34	5.2469	44	7.0788
5	2.0538	15	4.1650	25	3.9591	35	5.0649	45	7.5505
6	2.4536	16	3.8345	26	4.2614	36	5.1272	46	6.7234
7	2.7507	17	3.8926	27	4.1242	37	5.4337	47	7.1461
8	2.6686	18	4.9478	28	3.5716	38	5.3125	48	7.9740
9	2.1004	19	3.9809	29	4.2316	39	5.8232	49	7.8273
10	2.7730	20	5.1786	30	4.4194	40	6.1564	50	9.6886

Exhibit 7. 1% VaR: difference between blocked and IID

# of Weeks	Differences	# of Weeks	Differences	# of Weeks	Differences	# of Weeks	Differences	# of Weeks	Differences
1	1.4029	11	7.3419	21	20.5549	31	24.1750	41	23.2842
2	2.7597	12	7.9203	22	20.9567	32	24.3998	42	23.3825
3	2.8202	13	6.1843	23	21.0069	33	23.9649	43	22.7742
4	3.3565	14	6.9605	24	21.9986	34	23.9975	44	23.0361
5	4.2144	15	9.6405	25	21.9611	35	23.6285	45	23.3570
6	4.9376	16	14.0292	26	22.4210	36	23.0198	46	21.9824
7	5.6976	17	17.2113	27	22.9513	37	22.4517	47	22.2985
8	7.3959	18	19.0726	28	22.3950	38	22.1782	48	24.0397
9	6.4948	19	17.0473	29	22.3950	39	21.6643	49	22.9750
10	7.1449	20	19.1804	30	23.5096	40	22.3213	50	23.4762

Exhibit 8. 5% VaR with Different Weights of 3 Categorized Assets (Scatter Graph)

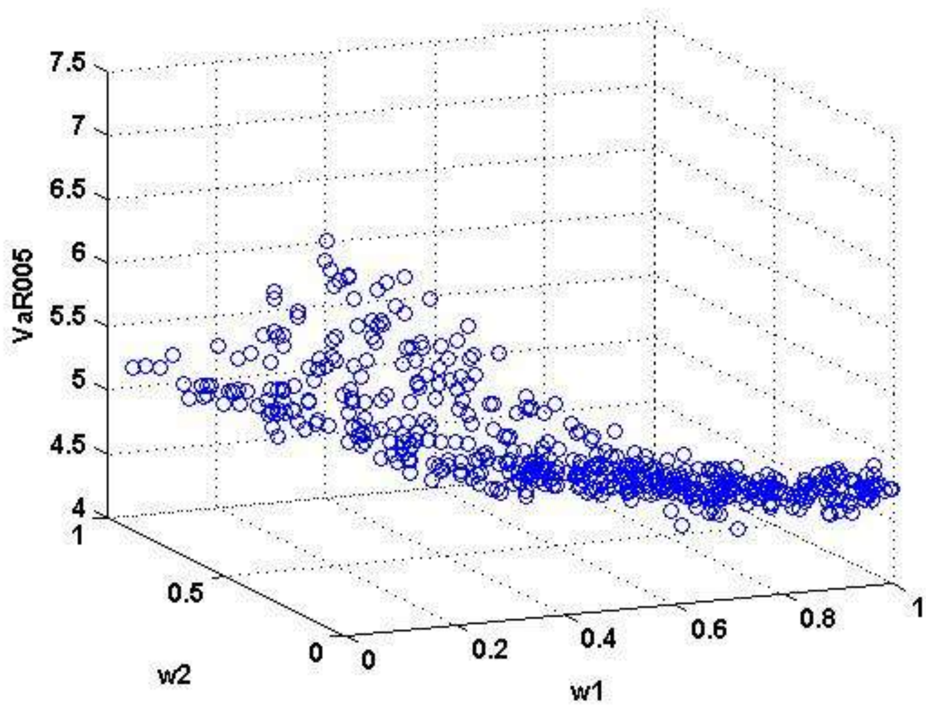
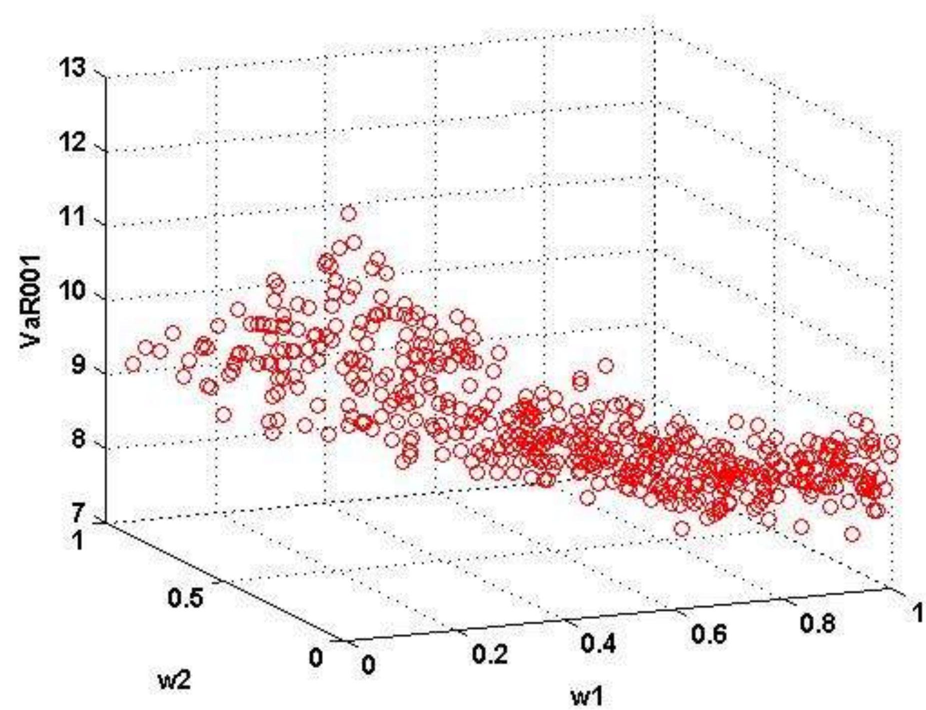


Exhibit 9. 1% VaR with Different Weights of 3 Categorized Assets (Scatter Graph)



Appendix: Codes

```
% Whole project
% Find Portfolio VAR global equity portfolio
% using blocked bootstrap and iid bootstrap
clc
clear all
% Load global equity data (MSCI indices)
load msciusnousem
% move portfolio matrix
wldeqp = msci3;
% find equity 1 day returns
% Lognormal returns: 1 Day
eqret = log(wldeqp(2:end,2:end))-log(wldeqp(1:end-1,2:end));



w = [1/3 1/3 1/3];
% construct global portfolio
portret = eqret * w';

% /////First Analysis/////
% Four measures to value the VaR using 10 days length

% First measure

nsamp=length(eqret(:,2));
index = 1:nsamp-9;
for i = 1:100000
    % start points for 10 day returns
    startt = sample(index,1);
    % 10 day blocks
    ret10daybs = eqret(startt:startt+9,:);
    % build 10 day portfolio
    port10daybs(i) = prod(exp(ret10daybs))*w'*100;
end
% bootstrap VaR's
disp('5% VaR: maintain cov Block bootstrap')
```

```
100-quantile(port10daybs,0.05)
disp('1% VaR: maintain cov Block bootstrap')
100-quantile(port10daybs,0.01)

% Second measure

for i = 1:100000
    % sample 1 day returns for 10 days
    ret10daybsiid = sample(eqret,10);
    % build 10 day portfolio
    port10daybsiid(i) = prod(exp(ret10daybsiid))*w'*100;
end
% bootstrap VaR's
disp('5% VaR: maintain cov IID bootstrap')
100-quantile(port10daybsiid,0.05)
disp('1% VaR: maintain cov IID bootstrap')
100-quantile(port10daybsiid,0.01)

% Third measure

nsamp=length(eqret(:,2));
index = 1:nsamp-9;
for i = 1:100000
    % start points for 10 day returns
    for j=1:3
        startt = sample(index,1);
        ret10daybs(:,j) = eqret(startt:startt+9,j);
    end
    % build 10 day portfolio
    port10daybs(i) = prod(exp(ret10daybs))*w'*100;
end
% bootstrap VaR's
disp('5% VaR: destory cov Block bootstrap')
100-quantile(port10daybs,0.05)
disp('1% VaR: destory cov Block bootstrap')
100-quantile(port10daybs,0.01)

% Fourth measure
```

```
for i = 1:100000
    % sample 1 day returns for 10 days
    for j=1:3
        ret10daybsiid(:,j) = sample(eqret(:,j),10);
    end
    % build 10 day portfolio
    port10daybsiid(i) = prod(exp(ret10daybsiid))*w'*100;
end
% bootstrap VaR's
disp('5% VaR: destory cov IID bootstrap')
100-quantile(port10daybsiid,0.05)
disp('1% VaR: destory cov IID bootstrap')
100-quantile(port10daybsiid,0.01)


% /////Second Analysis/////
% Analyze difference between blocked and IID measures




for qq=1:50
    q=qq*5;
  % bootstrap q day blocked returns
  nsamp=length(eqret(:,2));
  index = 1:nsamp-q+1;
  for i = 1:100000
    % start points for q day returns
    startt = sample(index,1);
    % q day blocks
    ret10daybs = eqret(startt:startt+q-1,:);
    % build q day portfolio
    port10daybs(i) = prod(exp(ret10daybs))*w'*100;
  end
  % bootstrap VaR's
  var005block=100-quantile(port10daybs,0.05);
  var001block=100-quantile(port10daybs,0.01);
```

```
  % bootstrap q day returns (iid assumptions)
  for i = 1:100000
    % sample 1 day returns for q days
    ret10daybsiid = sample(eqret,q);
    % build q day portfolio
    port10daybsiid(i) = prod(exp(ret10daybsiid))*w'*100;
  end
  % bootstrap VaR's
  var005iid=100-quantile(port10daybsiid,0.05);
  var001iid=100-quantile(port10daybsiid,0.01);

  %Calculate differences
  var005diff(qq)=var005block-var005iid;
  var001diff(qq)=var001block-var001iid;
end

%show difference between blocked measure and IID measure
disp('5% VaR: difference between blocked and IID')
var005diff
disp('1% VaR: difference between blocked and IID')
var001diff

%plot days and VaR005 diff between blocked and IID measures
plot((1:50)*5,var005diff)
xlabel('Days');
ylabel('VaR005 Difference')
%plot days and VaR001 diff between blocked and IID measures
plot((1:50)*5,var001diff)
xlabel('Days');
ylabel('VaR001 Difference')


% /////Third Analysis/////
% Analyze VaR using different weights in first and second column assets
```

```
for j=1:500
    w1(j)=rand;
    w2(j)=(1-w1(j))*rand;
    w3(j)=1-w1(j)-w2(j);
    w=[w1(j),w2(j),w3(j)];

  nsamp=length(eqret(:,2));
  % bootstrap 10 day blocked returns
  index = 1:nsamp-9;
  for i = 1:1000
    % start points for 10 day returns
    startt = sample(index,1);
    % 10 day blocks
    ret10daybs = eqret(startt:startt+9,:);
    % build 10 day portfolio
    port10daybs(i) = prod(exp(ret10daybs))*w'*100;
  end
  % bootstrap VaR's
  var005(j)=100-quantile(port10daybs,0.05);
  var001(j)=100-quantile(port10daybs,0.01);
end

% VaR 5% plot scatter graph
scatter3(w1,w2,var005)
xlabel('w1');
ylabel('w2');
zlabel('VaR005');


[X,Y,Z]=griddata(w1,w2,var005,linspace(min(w1),max(w1))',linspace(min(w2),max(
w2)),'v4');
figure,surf(X,Y,Z)
xlabel('w1');
ylabel('w2');
zlabel('VaR005');

% VaR 1% plot scatter graph
scatter3(w1,w2,var001)
```

```
xlabel('w1');
ylabel('w2');
zlabel('VaR001');


[X,Y,Z]=griddata(w1,w2,var001,linspace(min(w1),max(w1))',linspace(min(w2),max(
w2)),'v4');
figure,surf(X,Y,Z)
xlabel('w1');
ylabel('w2');
zlabel('VaR001');
```