

Methodology of the CBOE S&P 500[®] PutWrite Index (PUTSM)
(with supplemental information regarding the CBOE S&P 500 PutWrite T-W Index (PWTSM))

The CBOE S&P 500[®] PutWrite Index (ticker symbol "PUT") tracks the value of a passive investment strategy ("CBOE S&P 500 Collateralized Put Strategy") which consists of overlaying S&P 500 (SPM) short put options over a money market account invested in one- and three-months Treasury bills. The SPX puts are struck at-the-money and are sold on a monthly basis, usually on the 3rd Friday of the month. This is called the "roll date" and it matches the date of S&P 500 option expirations. The base date of the PUT Index is June 1, 1988, when its value was 100. The daily historical data for the Put Index was recently extended back to June 30, 1986, on which date the PUT Index was at 89.91.¹

Daily historical data on the PUT Index is available at www.cboe.com/PUT and from Bloomberg and other price quote vendors.

Roll Date Transactions

At each roll date, any settlement loss from the expiring SPX puts is financed by the Treasury bill accounts and a new batch of at-the-money SPX puts is sold. The revenue from their sale is added to the Treasury bill account. The three-month Treasury bills are deemed to mature when SPX puts on the March quarterly cycle months are sold. The total cash available is then reinvested at the three-month Treasury bill rate. In other months, the revenue from the sale of puts is invested separately at the one-month Treasury bill rate.

Number of SPX Puts Sold

The number of SPX puts sold is chosen to ensure full collateralization. This means that at the expiration of the puts, the total value of the Treasury bill investment(s) must be equal to the maximum possible loss from final settlement of the put options, or $N \cdot K$ where N is the number of puts sold and K is the at-the-money strike price.

Selection of the "At-the-Money" Strike Price

The strike price of the new SPX puts that are sold is the strike price of listed SPX puts that is closest to but not greater than the last value of the S&P 500 Index reported before 11:00 a.m. ET. For example, if the last S&P 500 Index value reported before 11:00 a.m. ET is 1233.10 and the closest listed SPX put strike price below 1233.10 is 1230 then 1230 strike SPX puts are sold.

$X \leq \text{last value (SP)} \text{ before } 11:00 \text{ a.m.}$

① closest listed. 无法判断. 只能 int(spx price)

② 无法精确到该时段.

当 < before 11:00 am.

> 已解决.

近似到 1237 → 1235

但不确定当时有没有

1235.

而是1230

¹ The historical series starts on June 30, 1986, at which time the value of the portfolio was 89.91. This portfolio was initiated on the preceding 3rd Friday, June 19, 1986, when the first puts were sold and the proceeds from their sale was added to an initial cash injection.

3rd Fri. roll
6/19/1986 Friday
initiated
sell first puts
6/30/1986
historical series starts
89.91.
6/1/1988
base date
100.

3rd Fri every month: } S&P price
 11:00 am. } $X \leq$ last value before 11:00 am.

11/20/1992之前
 (4:00 pm.)
 (4:00 pm.)

July 2014

11:30 am } VWAP(option price) or last bid price before 12:00 pm.
 12:00 pm } ② 计算 option price 的 VWAP 用 CBOE MDR 系统

(4:00 pm.)
 该时段没有
 该时段 volume 也没有

Sale Price of Put Options²

The SPX puts are deemed to be sold at a price equal to the volume-weighted average of the traded prices ("VWAP") of put options with that strike during the half-hour period beginning at 11:30 a.m. ET. The CBOE calculates the VWAP in a two-step process: first, the CBOE excludes trades between 11:30 a.m. and 12:00 p.m. ET that are identified as having been executed as part of a "spread", and then the CBOE calculates the weighted average of all remaining transaction prices at that strike between 11:30 a.m. and 12:00 p.m. ET, with weights equal to the fraction of total non-spread volume transacted at each price during this period. The source of the transaction prices used in the calculation of the VWAP is CBOE's Market Data Retrieval ("MDR") System.³ If no transactions occur at the new put strike between 11:30 a.m. and 12:00 p.m. ET, then the new put options are deemed sold at the last bid price reported before 12:00 p.m. ET.

结算时
 ⑤ 用的不是 SPX 而是 S&P 500, Bloomberg 不可靠.
 特殊报价式

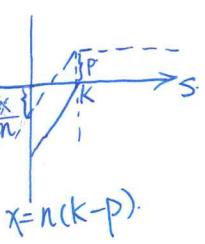
Final Settlement Price of Expiring SPX Puts

S&P 500 \Rightarrow SOQ
 $K-S \Rightarrow K-SOQ$

At expiration, the SPX puts are settled to a Special Opening Quotation (or SOQ, ticker "SET") of the S&P 500.⁴ The SOQ is a special calculation of the S&P 500 Index that is compiled from the opening prices of component stocks underlying the S&P 500 Index. The SOQ calculation is performed when all 500 stocks underlying the S&P 500 Index have opened for trading, and is usually determined before 11:00 a.m. ET.⁵ The final settlement price of the expiring SPX puts is equal to $\max[0, K - SOQ]$, i.e., the maximum of 0 and the difference between their strike price and the SOQ.

Index Calculation

The CBOE calculates the PUT in real-time every fifteen seconds during each trading day excluding roll days. On any given date, the index represents the mark-to-market value of the base date \$100 invested in the PUT strategy.



At the close of every business date, the value of the PUT is equal to the value of the Treasury bill account less the mark-to-market value of the puts:

day 1: X 自准备钱
 $n = \frac{X}{K-P}$

自己要准备钱 X
 某 put 得 NP

$X + nP$ 投入 treasury bill

² A slightly different roll procedure is used to calculate the historical series of the CBOE S&P 500 PutWrite Index. This is to take into account the changes in the timing of the expiration of S&P 500 options, and to mimic the changes made in the calculation of the BXM series over time. Up to November 20, 1992, the roll is deemed to take place at the close of the 3rd Friday, the strike price of the new put is determined at 4:00 p.m. ET, and the new puts are deemed sold at the last bid price before 4:00 p.m. ET. After this date, the index is rolled at 11:00 a.m. ET instead. And starting on March 17, 2006, the new puts were sold at the VWAP.

³ Time & Sales information from CBOE's MDR System is disseminated through the Options Price Reporting Authority (OPRA) and is publicly available through most price quote vendors.

⁴ If the third Friday is an exchange holiday, the put option will be settled against the SOQ on the previous business day and the new put option will be selected on that day as well.

⁵ If one or more stocks in the S&P 500 Index do not open on the day the SOQ is calculated, the final settlement price for SPX options is determined in accordance with the Rules and By-Laws of the Options Clearing Corporation.

day 1:
 $MV(\text{treasury bill}) = X + nP$
 $MV(\text{put}) = 0$
 at the money

Total MV = $X + nP$

write
 settle 日 5

⑥ 不一定周五, 会根据 holiday 调整

11/20/1992 之前
 3/17/2006 之后
 11:00 am 11:30 am 12:00 pm
 4:00 pm
 ④ 策略修正过 3 次, 并不是统一

$$PUT_t = M_t - N_{last} P_t$$

↑ treasury bill ↑ MV(put)

historical put price 不可得
用BS model 去计算出来:

$$C_0 = S_0 Nd_1 - Ke^{-rT} Nd_2$$

$$P_0 = Xe^{-rT} (1 - Nd_2) - S_0 (1 - Nd_1)$$

$$d_1 = \frac{\ln(\frac{S_0}{X}) + [R_f + 0.5\sigma^2]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

最早 avail. 是 2006.1.19

where M_t is the total Treasury bill balance at the close of date t , N_{last} is the number of put options sold at the last roll date, and P_t is the arithmetic average of the last bid and ask prices of the put option reported before 4:00 p.m. ET on date t .

On all but roll dates, the Treasury bill balance is obtained by compounding the one and three-month Treasury balances at the previous business close at their respective daily rates.

$$M_t^i = (1 + r_{t-1}^i) M_{t-1}^i$$

where $i = 1$ and 3 for one and three-month Treasury bills, and r_{t-1}^i is the corresponding Treasury bill rate from the previous to the current close. The Treasury bill rates between two roll dates are obtained by compounding the daily rates.

On every third roll date, the Treasury bills are deemed to mature, the cash is used to pay for final settlement of the puts if they expire in-the-money, and new puts are sold. The net cash balance available for reinvestment is:

$$M_t = \sum_i (1 + r_{t-1}^i) M_{t-1}^i - N_{last} \text{Max}[0, K_{old} - SOQ_t] + N_{new} P_{vwap}$$

where K_{old} is the strike price of the put options sold at the previous roll date, SOQ_t is the final settlement price on roll date t , N_{new} is the number of new puts sold and P_{vwap} is the volume-weighted average price at which the new options are sold. This balance is reinvested at the three-month Treasury bill rate. Hence in the month following a third roll date, the one-month Treasury balance is zero.

The number of new puts sold on any roll date t is set such that the Treasury balance at the next roll date covers the maximum put settlement loss:

Third roll date: $N_{new} = (\sum_i (1 + r_{t-1}^i) M_{t-1}^i - N_{last} \text{Max}[0, K_{old} - SOQ_t]) / (K_{new} / (1 + R_3) - P_{vwap})$

Other roll dates: $N_{new} = [M_{1_Roll} + M_{3_Roll}] / [K - P_{vwap} * (1 + R_1)]$

$$M_{1_Roll} = \text{Max}[0, (1 + r_{t-1}^1) M_{t-1}^1 - N_{last} \text{Max}[0, K_{old} - SOQ_t]] * (1 + R_1)$$

$$M_{3_Roll} = ((1 + r_{t-1}^3) M_{t-1}^3 + \text{Min}[0, (1 + r_{t-1}^1) M_{t-1}^1 - N_{last} \text{Max}[0, K_{old} - SOQ_t]]) * (1 + R_3)$$

where K_{new} is the strike price at which the new puts are sold, and R_1 and R_3 are the one- and three-month Treasury bill rates to the next roll date.

其中:
volatility 用BS model
上为 implied vol.
可以设定 binomial formula
time period
ATM 100%
time to maturity
a. time to maturity 也不精确. 是断点.
b. 只有 100% (ATM) 90% 110% 130% 等断点. 取无限精确
c. R_f 用 10 yr TB, not sure. 也不精确.



historical option price:

Real option price = expired option listed price earliest from 2013
BS option price (modeled) = implied vol. { real in market { specific to this option. → 2013:
simulated volatility surface { general to market
earliest from 2006. 1月. T=8. VIX 是其中个别特例. weighted avg. 全部 strike

[a] T 断点. 不连续.
[b] ATM K 断点.

SUPPLEMENTAL INFORMATION (added July 11, 2014)

CBOE S&P 500 PutWrite T-W Index (PWT)

CBOE introduced the CBOE S&P 500 PutWrite T-W Index (Ticker: PWT) on July 3, 2014. The PWT Index replicates the methodology used to calculate the PUT Index, with one exception. That is, on each roll date the SPX puts are deemed to be sold at the P_{twap} , a price equal to the time-weighted average of reported bid prices, of the selected SPX put option beginning at 11:30 a.m. ET and ending at 12:00 p.m. ET. Accordingly, P_{twap} is used in place of the P_{vwap} on PWT roll dates.

CBOE has not calculated a separate series of historical values for the PWT Index prior to July 3, 2014. Rather, historical values for the PWT Index prior to July 3, 2014, may be considered the same as PUT Index values.

treasury bill 的报价: yield on a discount basis.

$$\text{折价率} = d = (1 - p) \times \frac{360}{n}$$

$$P = 1 - d \times \frac{n}{360}$$

eg: 90-day Tbill 4% par=1000

$$1000 \times (1 - 4\% \times \frac{90}{360}) = 990$$

The CBOE S&P 500 PutWrite Index (PUT) and CBOE S&P 500 PutWrite T-W Index (PWT) are designed to represent a proposed hypothetical short put strategy. Like many passive indexes, the PUT and PWT Indexes do not take into account significant factors such as transaction costs and taxes and, because of factors such as these, many or most investors should be expected to underperform passive indexes. In the construction of the hypothetical PUT and PWT indexes, the SPX puts are assumed to be written at a certain price on the third Friday of the month. However, there is no guarantee that all investors will be able to sell at this price, and investors attempting to replicate the PUT and PWT Indexes should discuss with their brokers possible timing and liquidity issues. Transaction costs for a put writing strategy such as the PUT or PWT could be significantly higher than transaction costs for a passive strategy of investing in Treasury bills. Past performance does not guarantee future results. Standard & Poor's®, S&P®, and S&P 500® are registered trademarks of The McGraw-Hill Companies, Inc. and are licensed for use by the Chicago Board Options Exchange, Incorporated (CBOE). CBOE, not S&P, calculates and disseminates the PUT and PWT Indexes. CBOE® and Chicago Board Options Exchange® are registered trademarks of the CBOE, and SPXSM, BXMSM, PUTSM, and PWTSM are service marks of the CBOE. The methodology of the CBOE S&P 500 PutWrite Index is owned by CBOE and may be covered by one or more patents or pending patent applications.

不交易成本. 税

Options involve risk and are not suitable for all investors. Prior to buying or selling an option, a person must receive a copy of Characteristics and Risks of Standardized Options (ODD). Copies of the ODD are available from your broker, by calling 1-888-OPTIONS, or from The Options Clearing Corporation, One North Wacker Drive, Suite 500, Chicago, Illinois 60606. Supporting documentation for claims, comparisons, recommendations, statistics or other technical data is available by calling 1-888-OPTIONS, sending an e-mail to help@cboe.com, or by visiting www.cboe.com/put.

模型

不是加仓 rolling 以维持高

www.cboe.com/put

consistent rolling

① vix surface. ^{extrapolate} surface 太大. 看条件
confirm vix. 是特殊情况? 确认下

① 计算错误.

② cash investment. 出问题.

③ 真的可以投资吗.

② strike price: daily \Rightarrow intraday. ^{可以} 一天波动有百分之

为什么有这个波动差异

SR 1.19

T-bill 90-day Tbill 4%. par=1000

$$1000 * (1 - 4\% * \frac{90}{360}) = 990$$