2 Physics and Instrumentation in PET*

Dale L Bailey, Joel S Karp and Suleman Surti

Introduction

In 1928 Paul AM Dirac postulated that a subatomic particle existed which was equivalent in mass to an electron but carried a positive charge. Carl Anderson experimentally observed these particles, which he called *positrons*, in cosmic ray research using cloud chambers in 1932. Both received Nobel Prizes in physics for their contributions. The positrons observed by Anderson were produced naturally in the upper atmosphere by the conversion of high-energy cosmic radiation into an electron–positron pair. Soon after this it was shown that when positrons interact with matter they give rise to two photons which, in general, are emitted simultaneously in almost exactly opposed directions. This sequence of events touches on many of the momentous developments in physics that occurred in the first 50 years of the twentieth century: radioactivity, Einstein's special relativity (energy–mass equivalence famously described by $E = mc^2$), quantum mechanics, de Broglie's wave–particle duality, and the laws of conservation of physical properties.

Today we produce positron-emitting radionuclides under controlled laboratory conditions in particle accelerators in the hospital setting for use in positron emission tomography (PET). In this chapter we will examine the basic physics of radioactivity and positrons and their detection as it relates to PET.

Models of the Atom

We use models, or representations, constantly in our lives. A painting, for example, is one individual's representation of a particular scene or feeling. It is clearly not the scene itself, but it is a model, or an attempt, to capture some expression of the reality as perceived by the artist. Likewise, scientists use models to describe various concepts about very-large-scale phenomena such as the universe, and very-small-scale phenomena such as the constituent components of all matter. One important feature of a model is that it usually has a restricted range over which it applies. Thus, we employ different models to account for different observations of the same entity, the classical example being the wave–particle duality of radiation: sometimes it is convenient to picture radiation as small discrete "packets" of energy that we can count individually, and at other times radiation appears to behave like a continuous entity or wave. The latter is evidenced by phenomena such as the diffraction of coherent light sources in a double-slit experiment. This could present a problem if we were to confuse the model and reality, but we emphasize again that the model is a *representation* of the underlying reality that we observe.

Amongst the ancient Greeks, Aristotle favored a continuous matter model composed of air, earth, fire, and water, where one could go on dividing matter infinitely into smaller and smaller portions. Others, though, such as Democritus, preferred a model in which matter was

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corpuscular. By the nineteenth century it was clear that chemicals combined in set proportions, thus supporting a corpuscular, or discrete, model of matter. At the turn of the twentieth century evidence was mounting that there were basic building blocks of matter called atoms (*Greek*: *indivisible*), but the question remained as to what, if anything, the atoms themselves were composed of. It was shown by JJ Thomson and, later, Ernest (Lord) Rutherford, that atoms could be broken down into smaller units in experiments using cathode ray tubes. Thomson proposed a model of the atom that was composed of a large, uniform and positively charged sphere with smaller negative charges embedded in it to form an electrostatically neutral mixture. His model of the atom is known as the "plum pudding" atom. Rutherford showed, however, that alpha particles (doubly ionized helium nuclei emitted from some unstable atoms such as radium) could pass through sheets of aluminum, and that this was at odds with the Thomson model. He proposed a model similar to that used to describe the orbit of the planets of the solar system about the sun (the "planetary" model). The Rutherford model had a central positive core – the nucleus – about which a cloud of electrons circulated. It predicted that most of the space in matter was unoccupied (thus allowing particles and electromagnetic radiation to pass through). The Rutherford model, however, presented a problem because classical physics predicted that the revolving electrons would emit energy, resulting in a spiralling of the electrons into the nucleus. In 1913, Bohr introduced the constraint that electrons could only orbit at certain discrete radii, or energy levels, and that in turn only a small, finite number of electrons could exist in each energy level. Most of what was required to understand the subatomic behavior of particles was now known. This is the Bohr (planetary) model of the atom. Later, the neutron was proposed by Chadwick (1932) as a large particle roughly equivalent to the mass of a proton, but without any charge, that also existed in the nucleus of the atom.

We shall continue to use the planetary model of the atom for much of our discussion. The model breaks down in the realm of quantum mechanics, where Newtonian physics and the laws of motion no longer apply, and as particles approach relativistic speeds (i.e., approaching the speed of light). Also, there are times when we must invoke a non-particulate model of the atom where the particles need to be viewed as waves (or, more correctly, *wave functions*). Electrons, for example, can be considered at times to be waves. This helps to explain how an electron can pass through a "forbidden" zone between energy levels and appear in

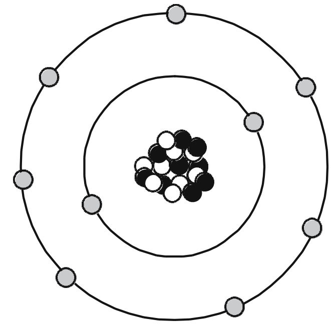

Figure 2.1. Atomic "planetary" model of radioactive fluorine-18 (^{18}F). The nucleus contains 9 protons (●) and 9 neutrons (○) and there are 9 electrons circulating in defined orbits. Stable fluorine would contain 10 neutrons.

the next level without apparently having passed through the forbidden area, defined as a region of space where there is zero probability of the existence of an electron. It can do so if its wave function is zero in this region. For a periodic wave with positive and negative components this occurs when the wave function takes a value of zero. Likewise, electromagnetic radiation can be viewed as particulate at times and as a wave function at other times. The planetary model of the atom is composed of nucleons (protons and neutrons in the nucleus of the atom) and circulating electrons. It is now known that these particles are not the fundamental building blocks of matter but are themselves composed of smaller particles called *quarks*. A deeper understanding of the elementary particles, and the frequently peculiar world of quantum physics, is beyond the scope of this book.

The simple planetary model of the atom is illustrated in Fig. 2.1 for the case of radioactive fluorine-18 ($^{18}_{9}F$). Nine orbital electrons circulate in defined energy levels about a central nucleus containing nine neutrons and nine protons. Stable fluorine is $^{19}_{9}F$ i.e., the nucleus contains one more neutron than protons and this produces a stable configuration. In all non-ionized atoms the number of electrons equals the number of protons, with the difference between the atomic number (Z) and mass number (A) being accounted for by the neutrons. In practice we usually omit the atomic number when writing radionuclide species (e.g., ^{18}F) as it is implicit in the element's symbol.

Mass and Energy

In 1900 Max Planck demonstrated that the energy (E) of electromagnetic radiation was simply related to the frequency of the radiation (υ) by a constant (Planck's constant, h):

$$E = h\upsilon \quad (1)$$

In addition, experiments indicated that the radiation was only released in discrete "bursts". This was a startling result as it departed from the classical assumption of continuous energy to one in which electromagnetic radiation could only exist in integral multiples of the product of $h\upsilon$. The radiation was said to be *quantized*, and the discrete quanta became known as *photons*. Each photon contained an amount of energy that was an integer multiple of $h\upsilon$. The unit for energy is the joule (J), and we can calculate the energy of the radiation contained in a photon of wavelength of, for example, 450 nm as:

$$E = h\upsilon = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34}\text{J.s} \times 3 \times 10^{8}\text{m.s}^{-1}}{450 \times 10^{-9}\text{m}} \quad (2)$$
$$= 4.42 \times 10^{-19}\text{J}$$

This radiation (450 nm) corresponds to the portion of the visible spectrum towards the ultraviolet end. Each photon of light at 450 nm contains the equivalent of 4.42×10^{-19} J of energy in a discrete burst. We shall see the significance of this result later in this chapter when we discuss the emission of photons from scintillators.

The joule is the Système International d'Unites (abbreviated SI) unit of energy, however, a derived unit used frequently in discussions of the energy of electromagnetic and particulate radiation is the *electron volt* (eV). The electron volt is defined as the energy acquired when a unit charge is moved through a potential difference of one volt. Energy in joules can be converted to energy in electron volts (eV) by dividing by the conversion factor 1.6×10^{-19} J.eV^{-1}. Thus, the energy in eV for photons of 450 nm would be:

$$E = 4.42 \times 10^{-19}\text{J} \equiv \frac{4.42 \times 10^{-19}\text{J}}{1.6 \times 10^{-19}\text{J.eV}^{-1}} \quad (3)$$
$$= 2.76 \text{ eV}$$

X rays and gamma rays have energies of thousands to millions of electron volts per photon (Fig. 2.2).

Einstein's Special Theory of Relativity, published in 1905 while he was working in the patent office in Zurich, turned the physical sciences on its head. It predicted, amongst other things, that the speed of light was constant for all observers independent of their frame of reference (and therefore that time was no longer constant), and that mass and energy were equivalent. This means that we can talk about the *rest-mass equivalent energy* of a particle, which is the energy that would be liberated if all of the mass were to be converted to energy. By *rest mass* we mean that the particle

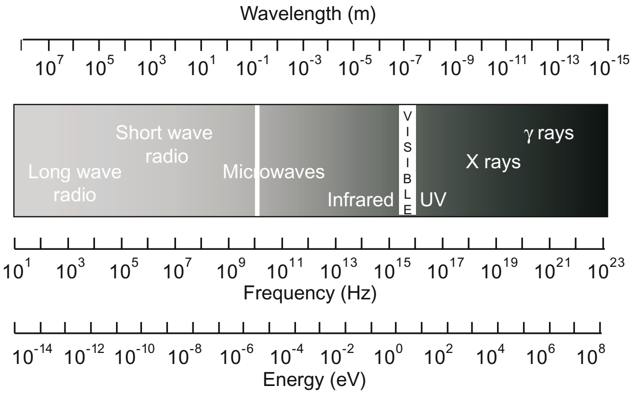

Figure 2.2. The electromagnetic spectrum showing the relationship between wavelength, frequency, and energy measured in electron volts (eV).

is considered to be at rest, i.e., it has no kinetic energy. Consider the electron, which has a rest mass of 9.11×10^{-31} kg; we can calculate the amount of energy this mass is equivalent to from:

$$\begin{aligned} E &= mc^2 \\ &= 9.11 \times 10^{-31}\text{kg} \times (3 \times 10^{8})^2 \text{ m.s}^{-1} \\ &= 8.2 \times 10^{-14}\text{ J} \\ &\equiv \frac{8.2 \times 10^{-14}\text{J}}{1.6 \times 10^{-19}\text{ J.eV}^{-1}} \\ &= 511 \text{ keV} \end{aligned} \quad (4)$$

The reader may recognize this as the energy of the photons emitted in positron–electron annihilation.

Conservation Laws

The principle of the conservation of fundamental properties comes from classical Newtonian physics. The concepts of conservation of mass and conservation of energy arose independently, but we now see that, because of the theory of relativity, they are merely two expressions of the same fundamental quantity. In the last 20–30 years the conservation laws have taken on slightly different interpretations from the classical ones: previously they were considered to be inviolate and equally applicable to all situations. Now, however, there are more conservation laws, and they have specific domains in which they apply as well as situations in which they break down. To classify these we must mention the four fundamental forces of nature. They are called the *gravitational*, *electromagnetic*, *strong*, and *weak* forces. It is believed that these forces are the only mechanisms which can act on the various

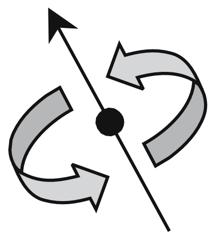

Figure 2.3. The spin quantum number for a particle can be pictured as a vector in the direction of the axis about which a particle is rotating. In this example, spin can be either "up" or "down".

properties of fundamental particles which make up all matter. These properties are electrostatic charge, energy and mass, momentum, spin and iso-spin, parity, strangeness and hypercharge (a quantity derived from strangeness and baryon numbers).

Charge is the electrostatic charge on a particle or atom and occurs in integer multiples of 1.6×10^{-19}.

Energy and *mass* conservation are well known from classical theory and are unified under special relativity.

Angular and *linear momentum* are the product of the mass (or moment of inertia) and the linear (or angular) velocity of a particle or atom.

Spin (s) and Isospin (i): Spin is the intrinsic angular momentum of a particle. It can be thought of by using the model of a ball rotating about its axis (Fig. 2.3). Associated with this rotation will be angular momentum which can take values in an arbitrary direction (labelled z) between –s to +s. The universe can be divided into two groups of particles on the basis of spin: those with spin $\frac{1}{2}$, and those with integer spin of 0, 1, or 2. The particles with spin $\frac{1}{2}$ are the mass-containing particles of the universe (fermions); the spin 0, 1, and 2 particles are the "force-carrying" particles (bosons). Some bosons, such as the pion, which serve as exchange particles for the strong nuclear force, are "virtual" particles that are very short-lived. Only spin $\frac{1}{2}$ particles are subject to the Pauli exclusion principle, which states that no two particles can have exactly the same angular momentum, spin, and other quantum mechanical physical properties. It was the concept of spin that led Dirac to suggest that the electron had an antimatter equivalent, the positron. Iso-spin is another quantum mechanical property used to describe the symmetry between different particles that behave almost identically under the influence of the strong force. In particular, the isospin relates the symmetry between a particle and its anti-particle as well as nucleons such as protons and neutrons that behave identically when subjected to the strong nuclear force. Similar to the spin, the isospin, i, can have half integer as well as integer values together with a special z direction which ranges in magnitude from $-i$ to $+i$. We shall see later that under certain conditions a high-energy photon (which has zero charge and isospin) can spontaneously materialize into an electron–positron pair. In this case both charge and isospin are conserved, as the electron has charge –1 and spin $+\frac{1}{2}$, and the positron has charge +1 and spin $-\frac{1}{2}$. Dirac possessed an over whelming sense of the symmetry in the universe, and this encouraged him to postulate the existence of the positron. Table 2.1 shows physical properties of some subatomic particles.

Parity is concerned with the symmetry properties of the particle. If all of the coordinates of a particle are reversed, the result may either be identical to the original particle, in which case it would be said to have *even* parity, or the mirror image of the original, in which case the parity is *odd*. Examples illustrating odd and even functions are shown in Fig. 2.4. Parity is conserved in all but weak interactions, such as beta decay.

The main interactions that we are concerned with are summarized in Table 2.2.

These are believed to be the only forces which exist in nature, and the search has been ongoing since the time of Einstein to unify these in to one all-encompassing law, often referred to as the Grand Unified Theory. To date, however, all attempts to find a grand unifying theory have been unsuccessful.

The fundamental properties and forces described here are referred to as the "Standard Model". This is the most widely accepted theory of elementary parti-

Table 2.1. Physical properties of some subatomic particles.

Particle	Symbol	Rest Mass (kg)	Charge	Spin	Isospin	Parity
Electron	e^-	9.11×10^{-31}	–1	$\frac{1}{2}$	$+\frac{1}{2}$	Even
Positron	e^+	9.11×10^{-31}	+1	$\frac{1}{2}$	$-\frac{1}{2}$	Even
Proton	p^+	1.673×10^{-27}	+1	$\frac{1}{2}$	$+\frac{1}{2}$	Even
Neutron	n^0	1.675×10^{-27}	0	$\frac{1}{2}$	$-\frac{1}{2}$	Even
Photon	Q	0	0	1	–	Odd
Neutrino	n	~0	0	$\frac{1}{2}$	$\frac{1}{2}$	Even

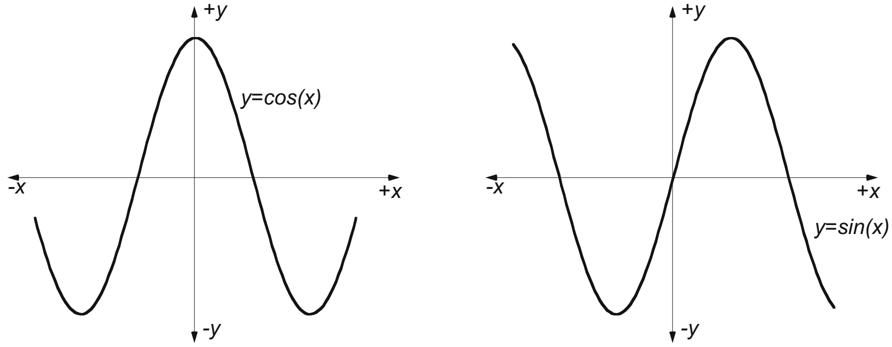

Figure 2.4. Examples of even (left) and odd (right) functions, to illustrate parity. In the even example ($y = \cos(x)$) the positive and negative values of x have the same y-values; for the odd function ($y = \sin(x)$) the negative x-values have opposite sign to the positive x-values.

Table 2.2. The table indicates whether the property listed is conserved under each of the fundamental interactions shown (gravity is omitted).

Property	Electromagnetic	Strong	Weak
Charge	Yes	Yes	Yes
Energy/mass	Yes	Yes	Yes
Angular momentum	Yes	Yes	Yes
Linear momentum	Yes	Yes	Yes
Iso-spin	No	Yes	No
Parity	Yes	Yes	No
Strangeness	Yes	Yes	No

cles and their interactions, which applies for all forces but gravity. The Standard Model remains a model though, and does not explain all observed phenomena, and work continues to find a grand unifying theory.

Radiation

Radiation can be classified into electromagnetic or particulate. *Ionising radiation* is radiation that has sufficient energy associated with it to remove electrons from atoms, thus causing ionisation. This is restricted to high-energy electromagnetic radiation (x and γ radiation) and charged particles (α, β^-, β^+). Examples of non-ionising electromagnetic radiation include light, radio, and microwaves. We will concern ourselves specifically with ionising radiation as this is of most interest in nuclear medicine and radiological imaging.

Electromagnetic Radiation

Electromagnetic radiation is pure energy. The amount of energy associated with each "bundle", or quantum, of energy is determined by the wavelength (λ) of the radiation. Human senses are capable of detecting some forms of electromagnetic radiation, for example, thermal radiation, or heat, ($\lambda \approx 10^{-5}$m), and visible light ($\lambda \approx 10^{-7}$m). The energy of the radiation can be absorbed to differing degrees by different materials: light can be stopped (absorbed) by paper, whereas radiation with longer wavelength (e.g., radio waves) or higher energy (γ rays) can penetrate the same paper.

We commenced our discussion at the beginning of this chapter with the comment that we are dealing with models of reality, rather than an accurate description of the reality itself; we likened this to dealing with paintings of landscapes rather than viewing the landscapes themselves. This is certainly the case when we discuss electromagnetic and particulate radiation. It had long been known that light acted like a wave, most notably because it caused interference patterns from which the wavelength of the light could be determined. Radiation was thought to emanate from its point of origin like ripples on the surface of a pond after a stone is dropped into it. This concept was not without its difficulties, most notably, the nature of the medium through which the energy was transmitted. This proposed medium was known as the "ether", and many experiments sought to produce evidence of its existence to no avail. Einstein, however, interpreted some experiments performed at the turn of the twentieth century where light shone on a photocathode could induce an electric current (known as the photoelectric effect) as showing that light acted as a particle. Einstein proposed that radiant energy was quantized into discrete packets, called *photons*. Thus, electromagnetic radiation could be viewed as having wave-like and particle-like properties. This view persists to this day and is known as the wave–particle duality. In 1924, Louis Victor, the Duc de Broglie, proposed that if wave–particle duality could apply to electromagnetic radiation, it could also apply to matter. It is now known that this is