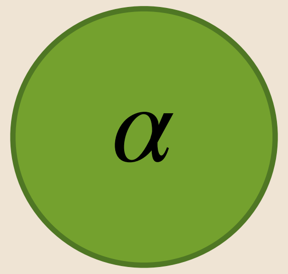
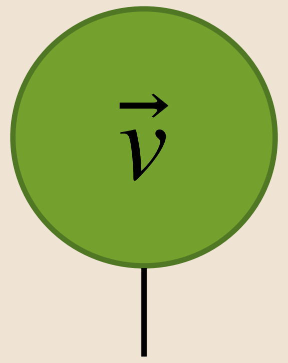
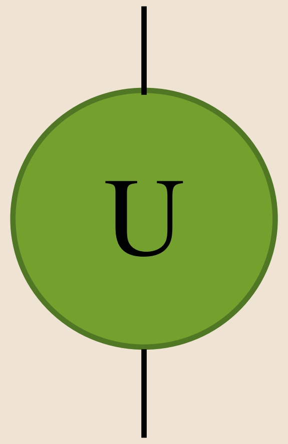
Tensors



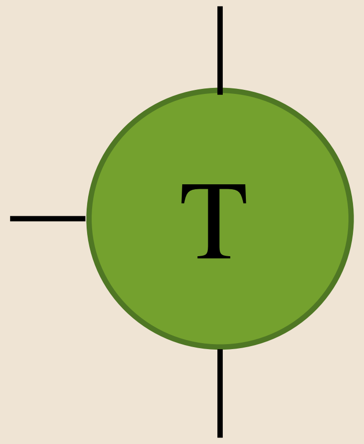
= order-0 tensor = **scalar**



= order-1 tensor = **vector**

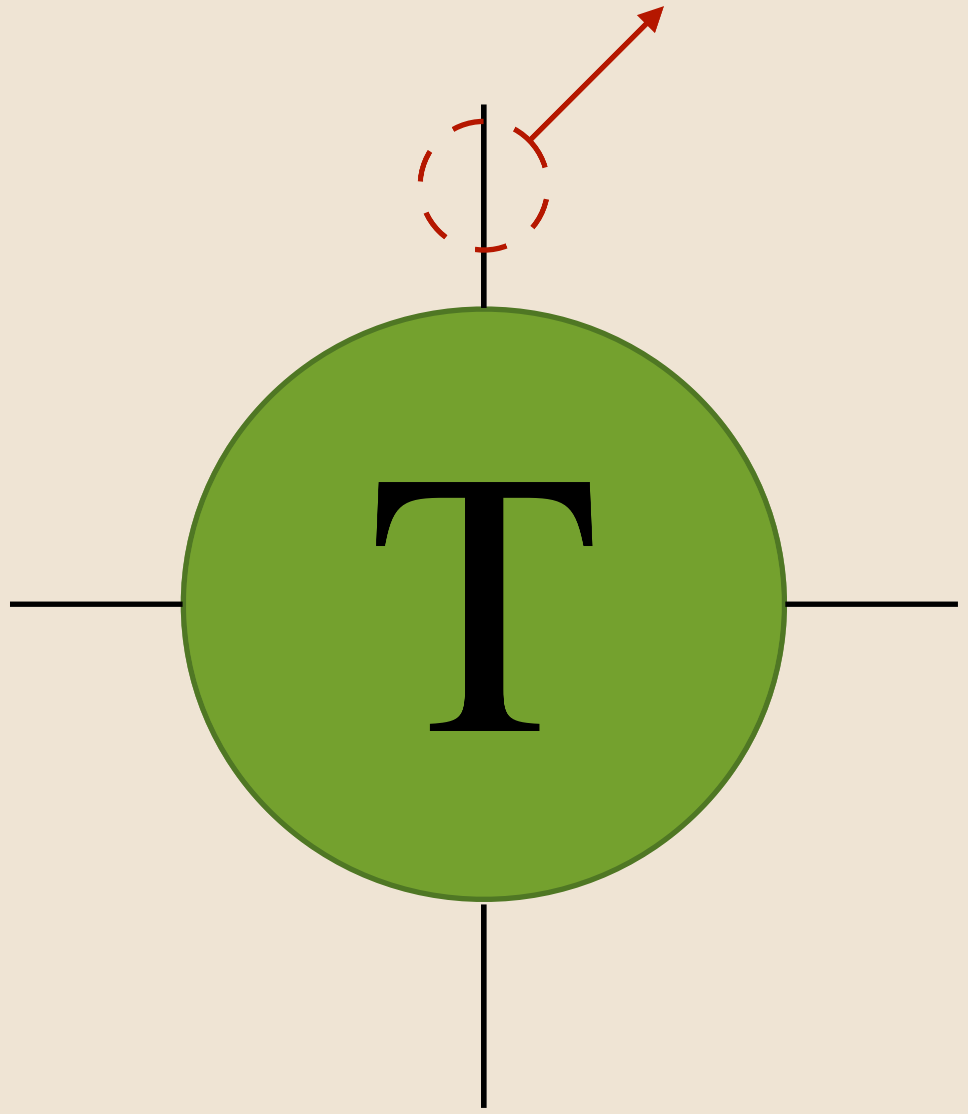


= order-2 tensor = **matrix**



= order-3 tensor = **tensor**

Tensor **leg**



Reshape a tensor

We can manipulate **Tensors** and **reshape** their indexes (legs) as we prefer:

$$v_\alpha = \begin{pmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{pmatrix} \xrightarrow{\textbf{Reshape}} v_{\alpha\beta} = \begin{pmatrix} v_0 & v_1 \\ v_2 & v_3 \end{pmatrix}$$

This means that a tensor of **any order** can be mapped to a **matrix**. So, we can use linear algebra to work with tensors.

$$T^{\delta\eta\nu}_{\alpha\beta\gamma} \xrightarrow{\textbf{Reshape}} T_{\alpha\beta}$$

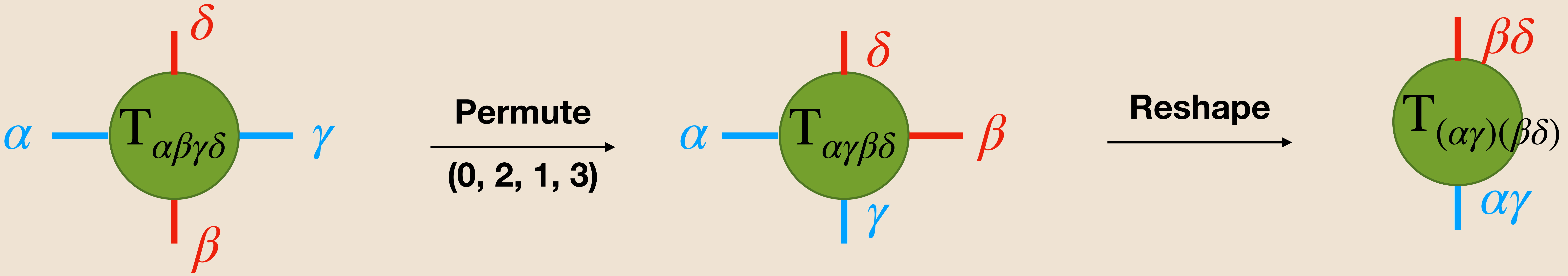
Permute a tensor

To reshape a tensor in the correct way we have to group together the legs involved in the following operation.
To do so, we have to know how to permute the legs of a tensor. In the case of a matrix it is a simple **transposition.**

$$v_{\alpha\beta} = \begin{pmatrix} v_0 & v_1 \\ v_2 & v_3 \end{pmatrix} \xrightarrow{\textbf{Transpose}} v_{\beta\alpha} = \begin{pmatrix} v_0 & v_2 \\ v_1 & v_3 \end{pmatrix}$$

We can generalise this to n-legged tensors. Suppose we want to reshape $T_{\alpha\beta\gamma\delta}$ in a matrix

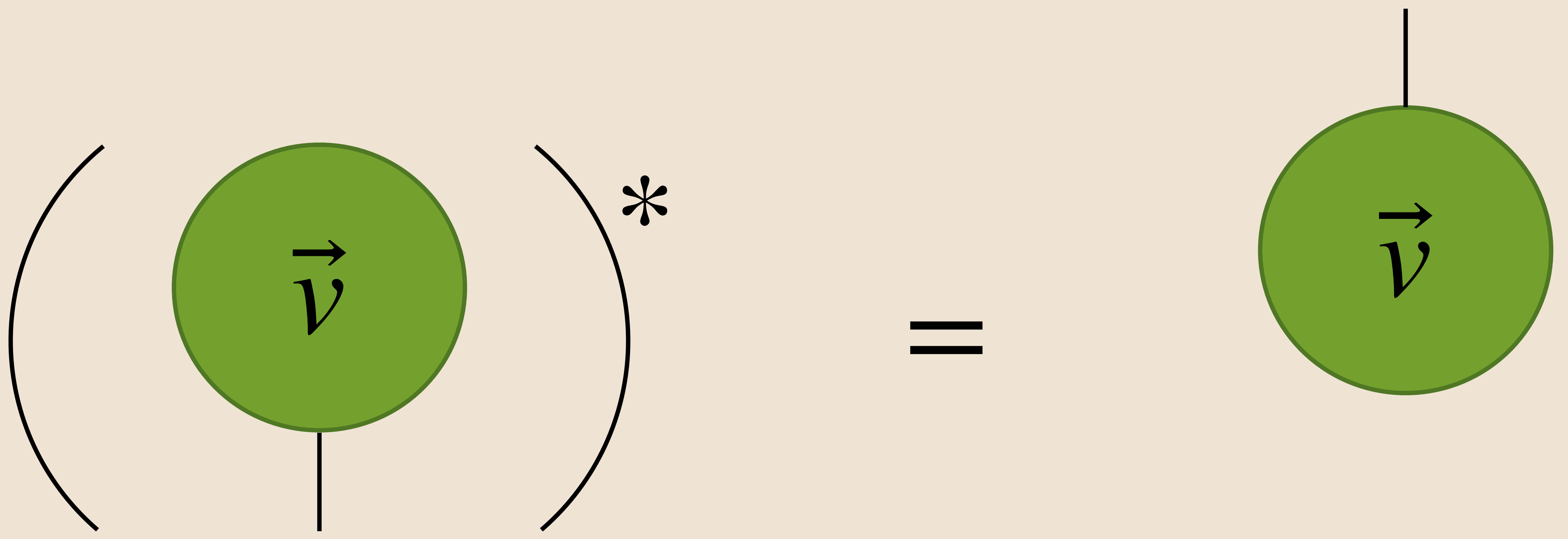
$T_{(\alpha\gamma),(\beta\delta)}$



$O(n)$

Complex conjugate of a tensor

We start by introducing the **complex conjugate** of a order-1 tensor:

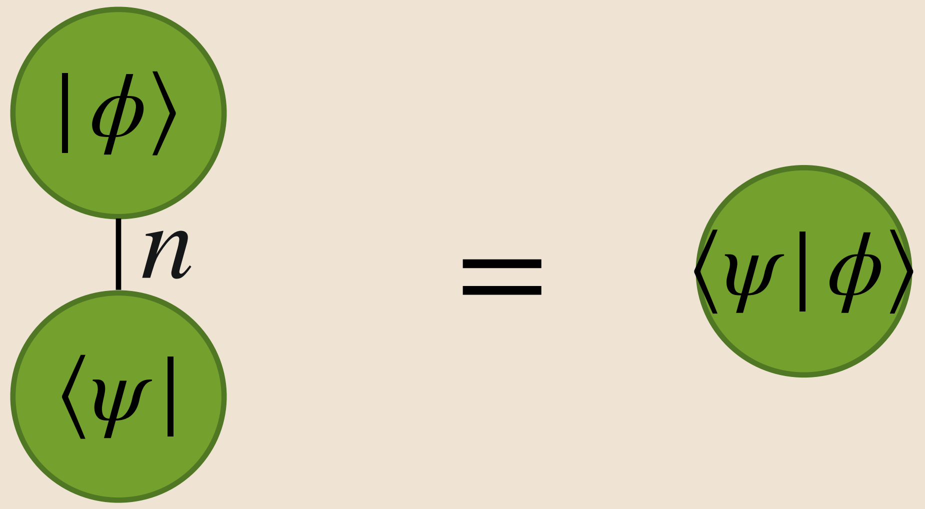


Tensor contraction

Then we introduce the **contraction** between two tensor along their **legs.**

- We start from two order-1 tensor, and it is equivalent to the scalar product between two vectors:

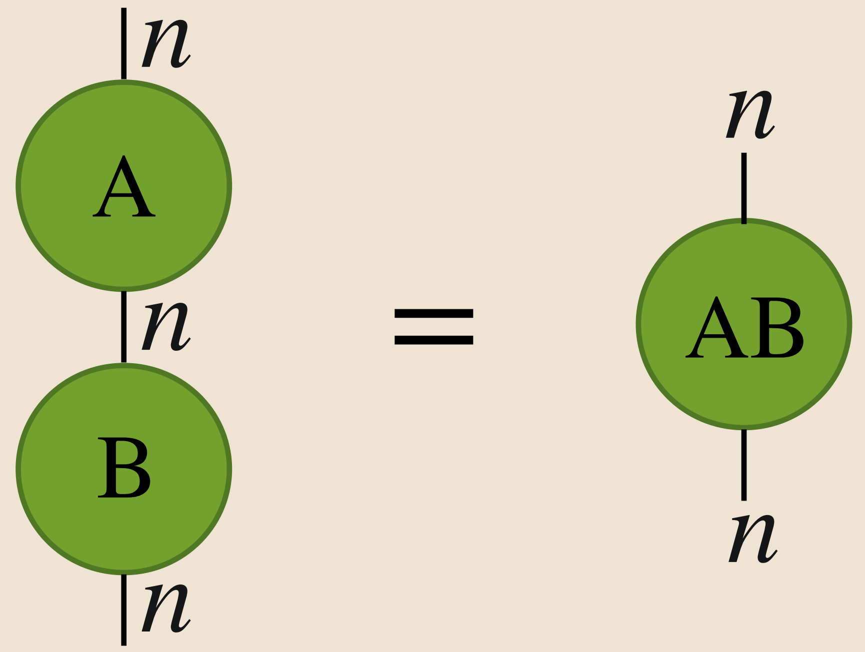
$$\langle\psi|\phi\rangle = \sum_i^n \psi_i^* \phi_i =$$



$$O(n)$$

- Then, the contraction between two order-2 tensor is simply the matrix-matrix multiplication:

$$(\mathrm{AB})_{ik} = \sum_j a_{ij} b_{jk} =$$

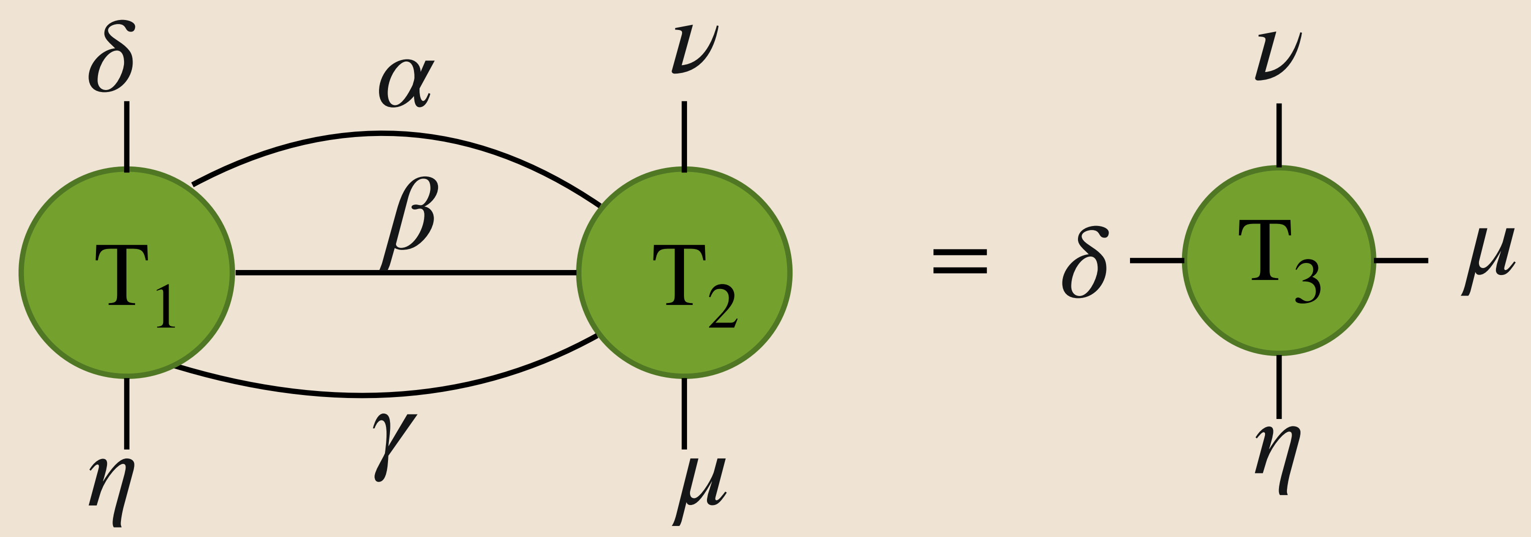


$$O(n^3)$$

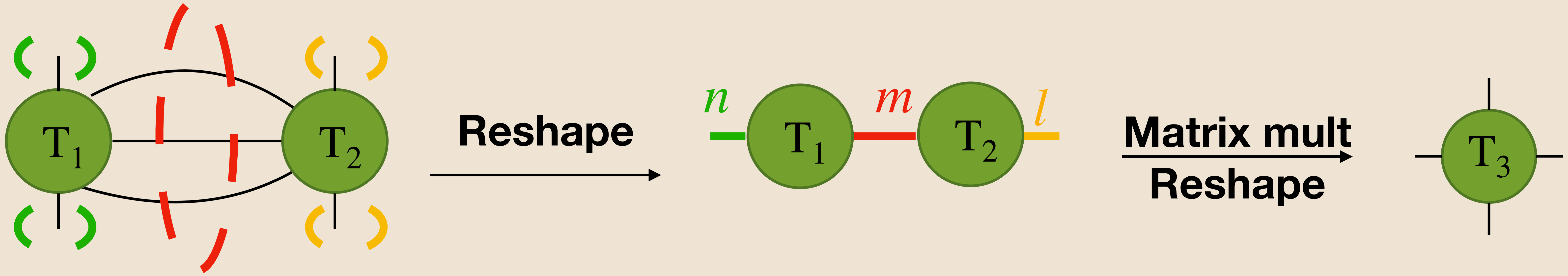
Tensor contraction

- In general, we can contract any leg of an order-n tensor:

$$T_3 = \sum_{\alpha\beta\gamma} \mathrm{T}_{1,\alpha\beta\gamma\delta\eta}\mathrm{T}_{2,\alpha\beta\gamma\mu\nu} =$$

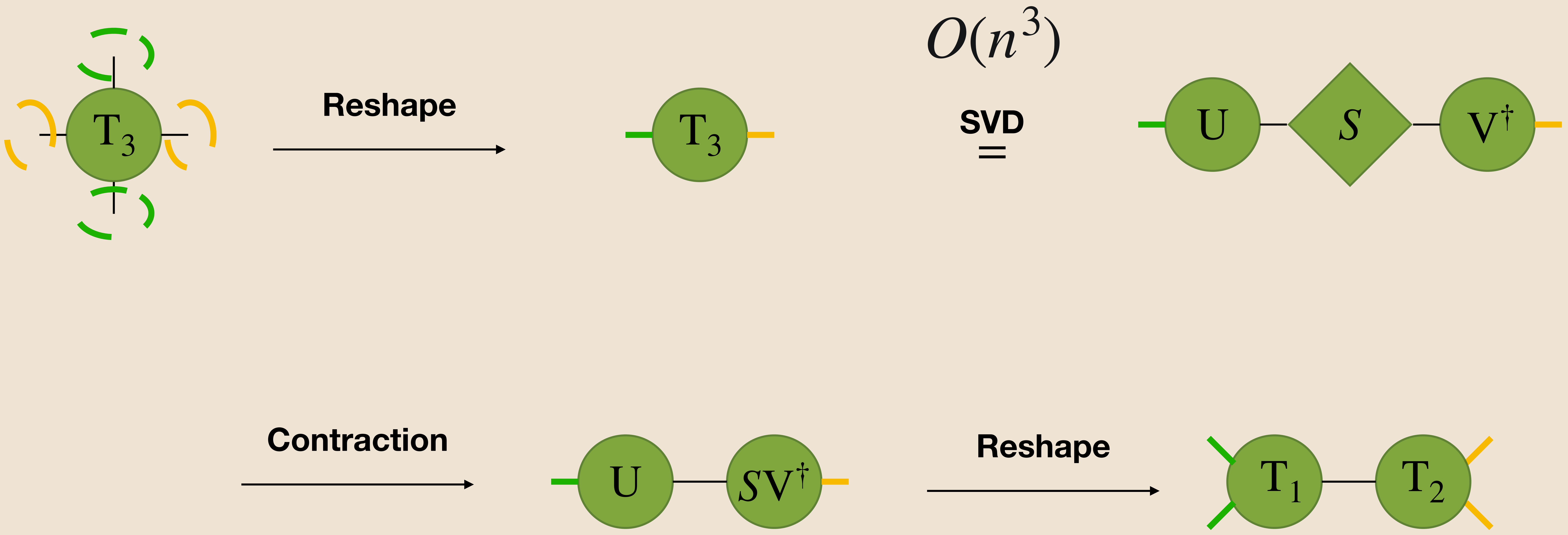


- What will be done in practice by the simulator, however, will be a little different:



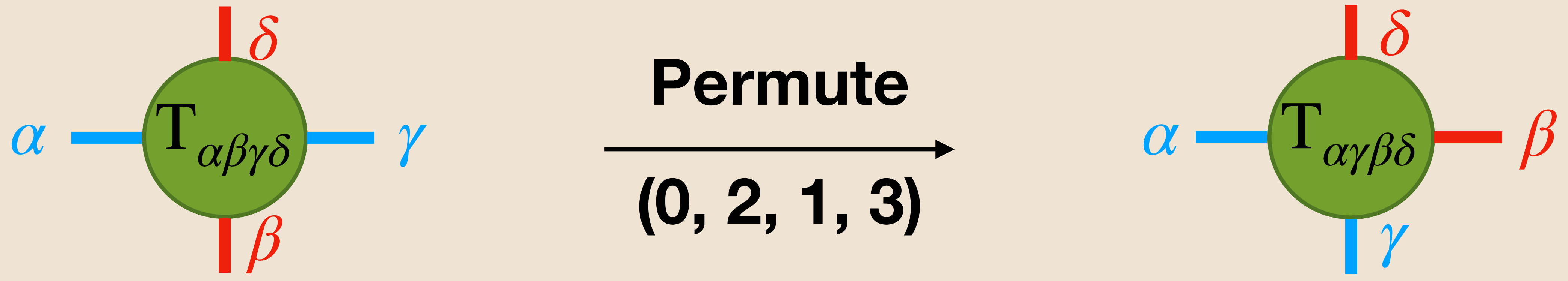
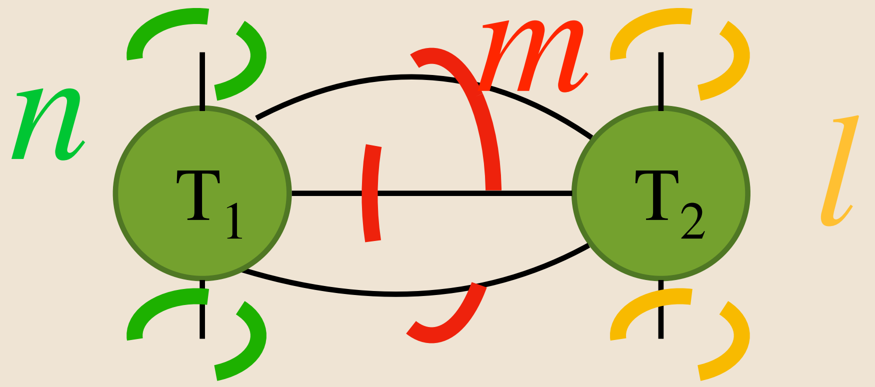
$O(nml)$

SVD: RECAP



T_3
Reshape
T_3
$O(n^3)$
SVD
=
U
S
$V^\dagger$
Contraction
U
$SV^\dagger$
Reshape
T_1
T_2

Tensor operations

Reshaping		-
Permutation		$O(n)$
Contraction		$O\left(2nml\right)$
QR		$O\left(\frac{2}{3}n^3\right)$
SVD		$O\left(4m^2n+22n^3\right)$