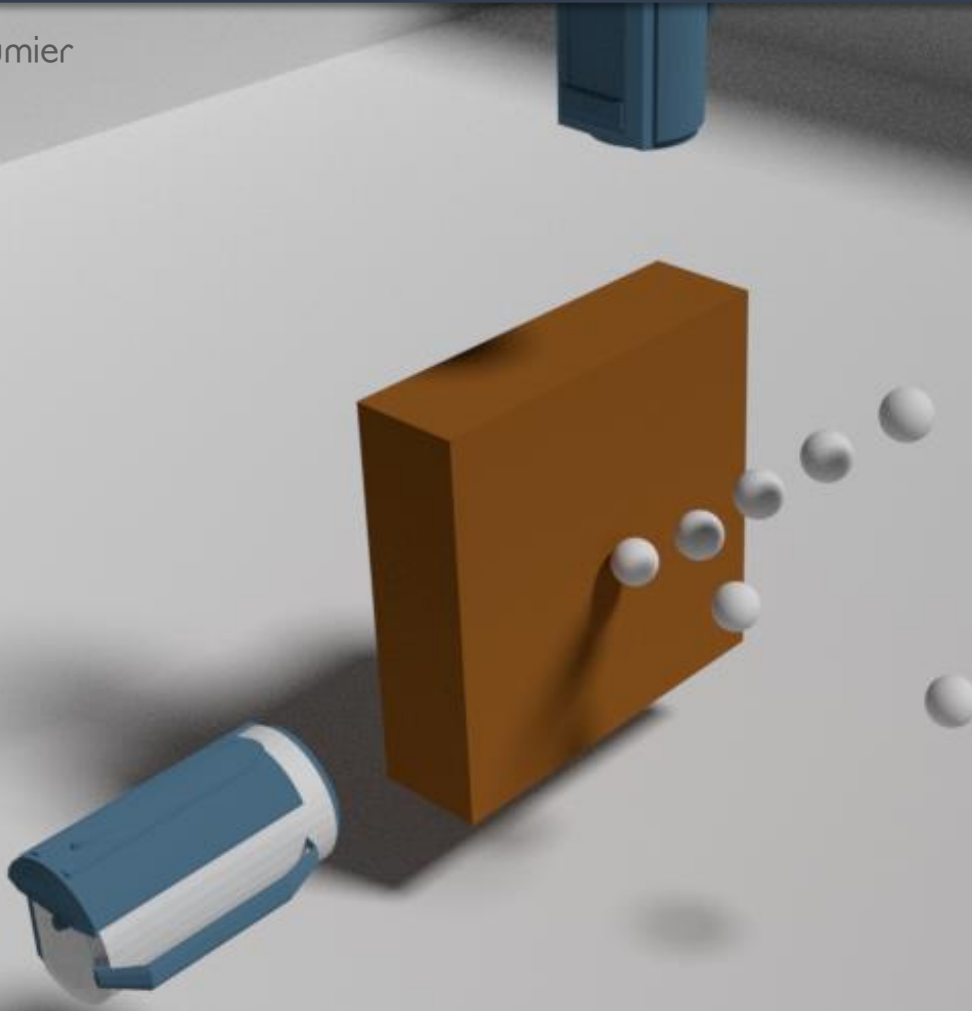


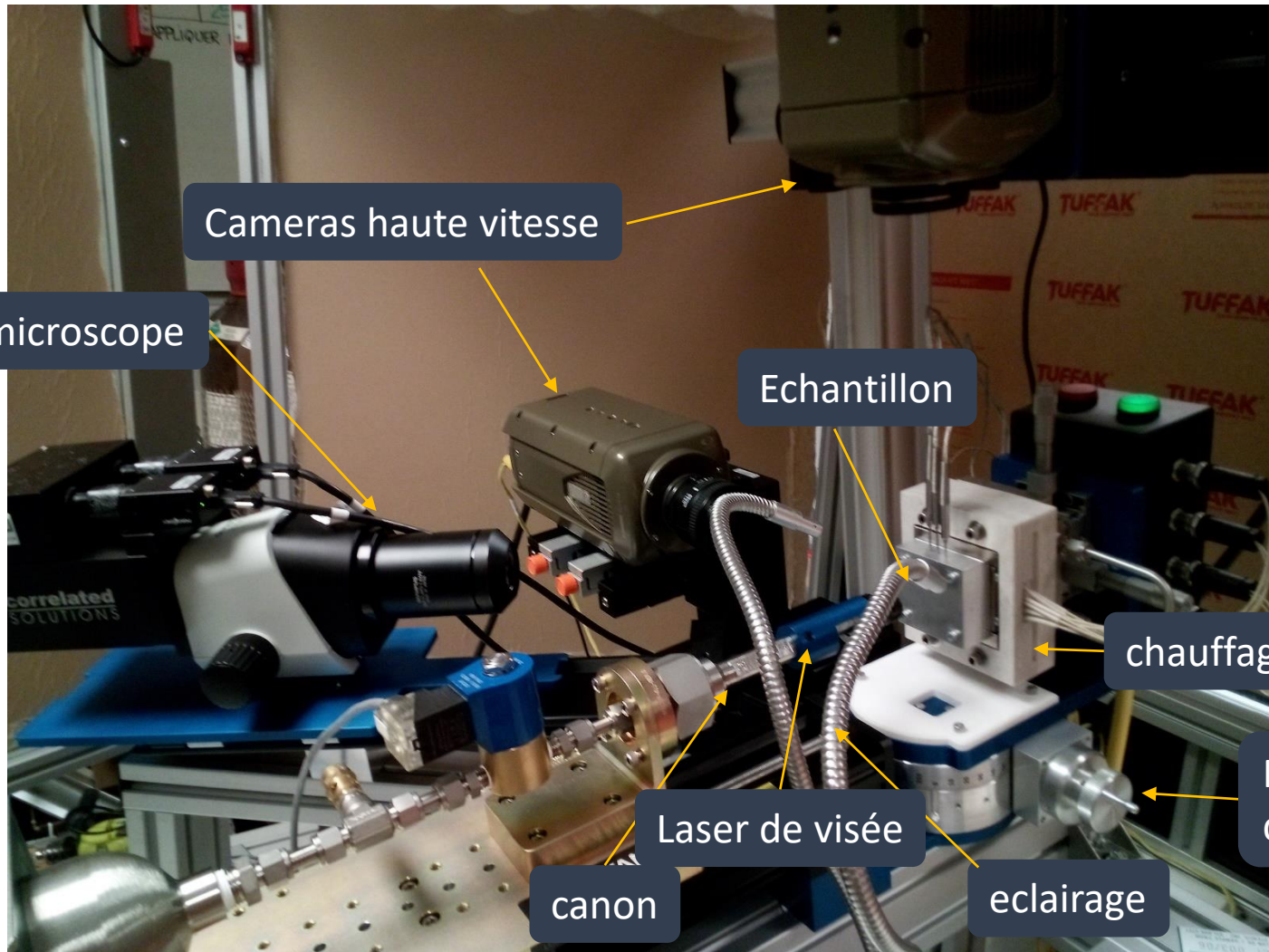
# Avancement

Simon Breumier

10/12/2020



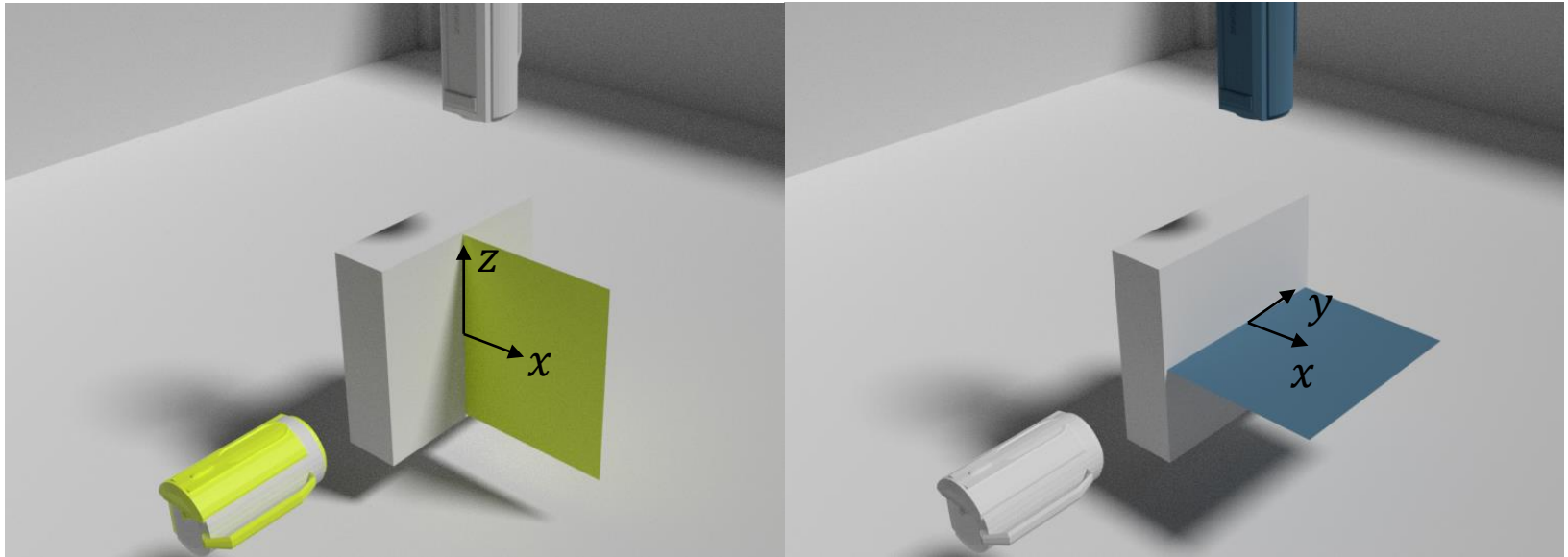
# Canon à bille



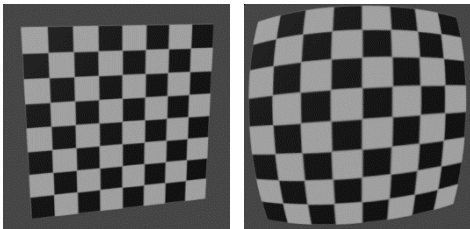
# Mesure de trajectoire en 3D

## Principe

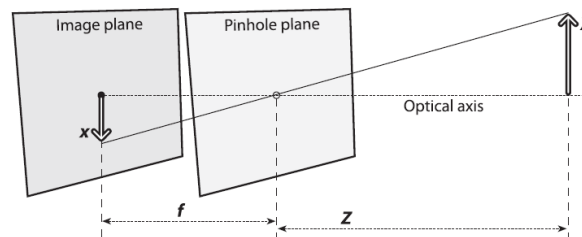
Chaque caméra filme à peu près dans un des plans principaux plan de l'échantillon



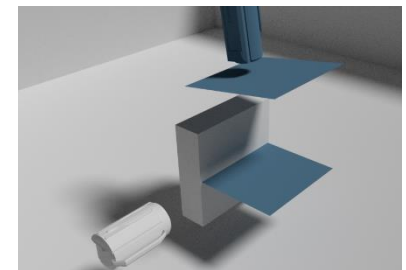
Corrections à prendre en compte:



Effets de lentille



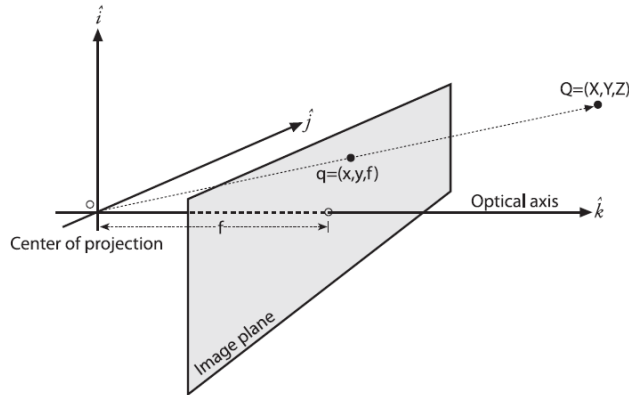
Effets de perspective



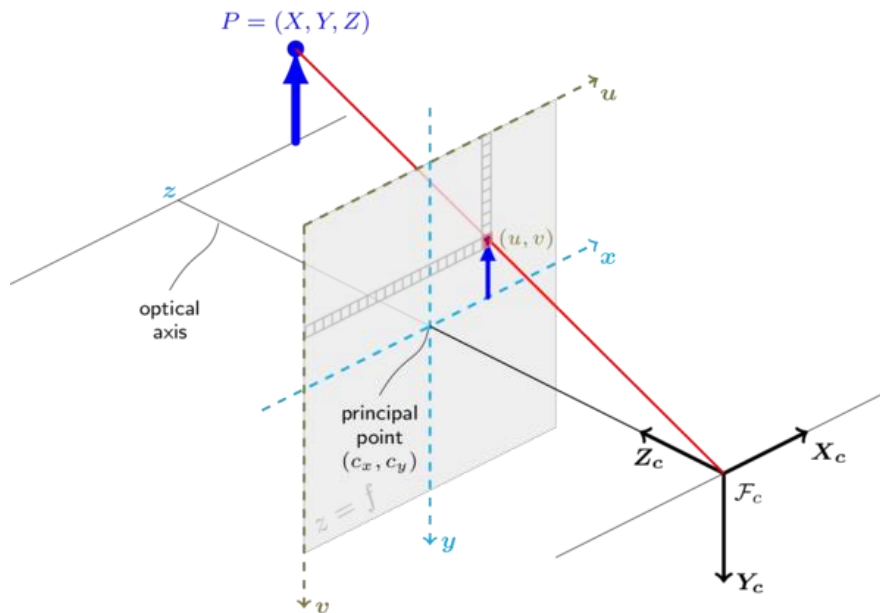
Alignement des caméras

# Mesure de trajectoire en 3D

## Pinhole camera model



$$x_{\text{screen}} = f_x \left( \frac{X}{Z} \right) + c_x, \quad y_{\text{screen}} = f_y \left( \frac{Y}{Z} \right) + c_y$$



$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t$$

$$x' = x/z$$

$$y' = y/z$$

$$x'' = x' \frac{1+k_1 r^2+k_2 r^4+k_3 r^6}{1+k_4 r^2+k_5 r^4+k_6 r^6} + 2p_1 x' y' + p_2 (r^2 + 2x'^2)$$

$$y'' = y' \frac{1+k_1 r^2+k_2 r^4+k_3 r^6}{1+k_4 r^2+k_5 r^4+k_6 r^6} + p_1 (r^2 + 2y'^2) + 2p_2 x' y'$$

$$\text{where } r^2 = x'^2 + y'^2$$

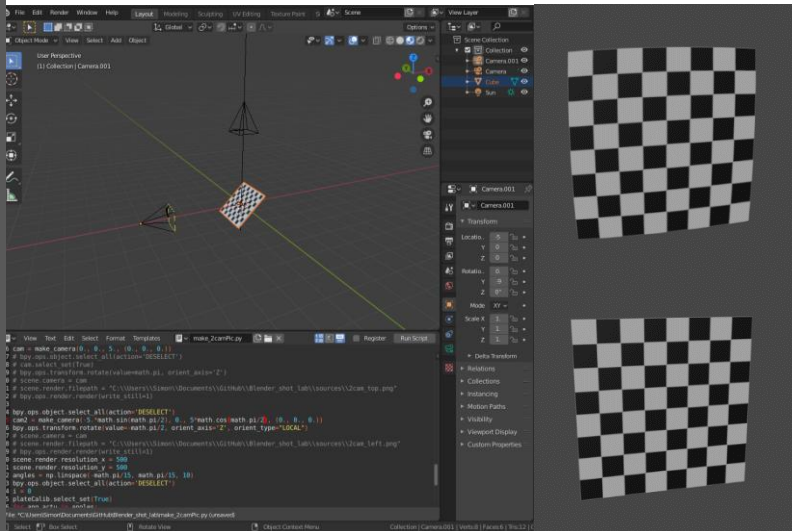
$$u = f_x * x'' + c_x$$

$$v = f_y * y'' + c_y$$

## Validation: Blender comme laboratoire virtuel



**POLYTECHNIQUE  
MONTREAL**



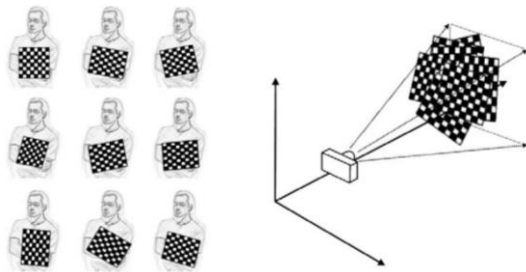
- ▶ Logiciel modélisation 3D open source
- ▶ Scriptable en python
- ▶ Chaque paramètre est isolable:
  - Caméra ( focale, position, distorsion de lentilles...)
  - Lumière
  - Effet de perspective (désactivable)

## Création d'un « laboratoire virtuel » pour tester la démarche de calibration

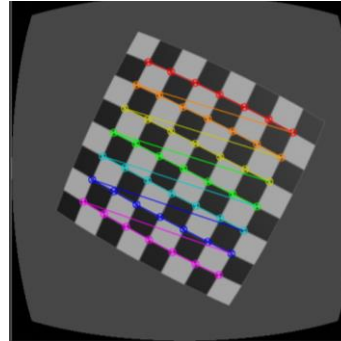


# Mesure de trajectoire en 3D

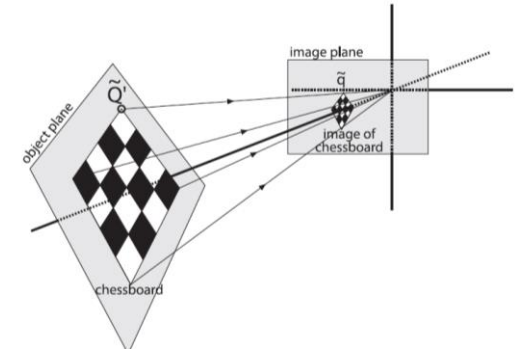
## 1/ Calibration des paramètres caméras



Images du damier dans différentes position



Détection des points du damier



Détermination de la position du pattern (homographie)

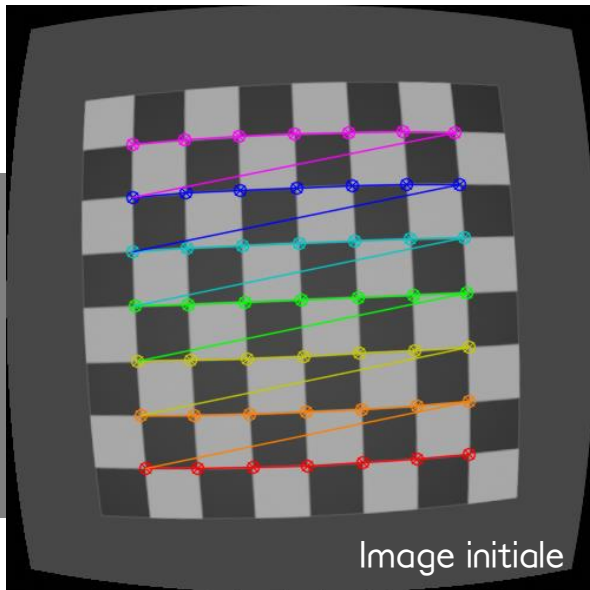


Image initiale

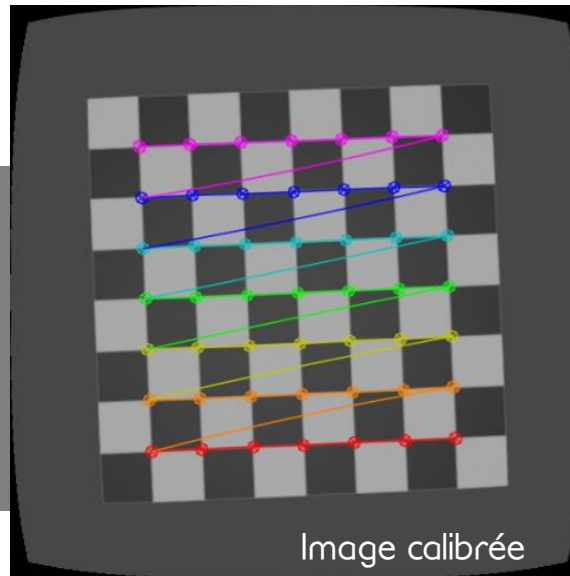


Image calibrée

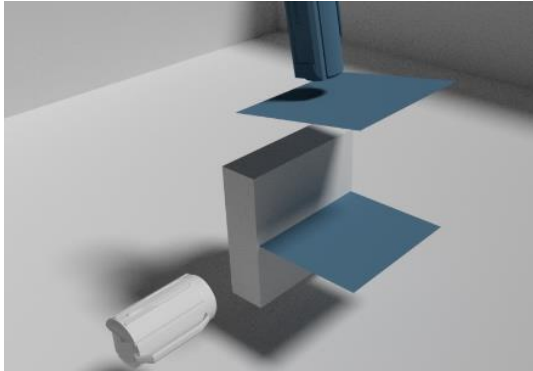
$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Connaissant les dimensions et la structure du damier:

détermination des  
paramètres de la caméra

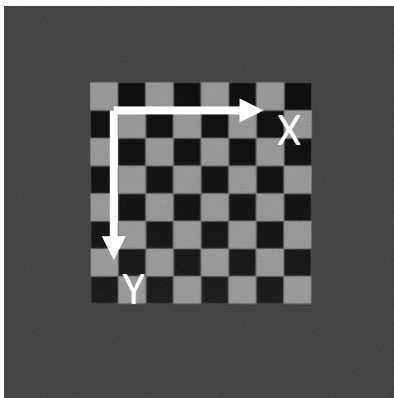
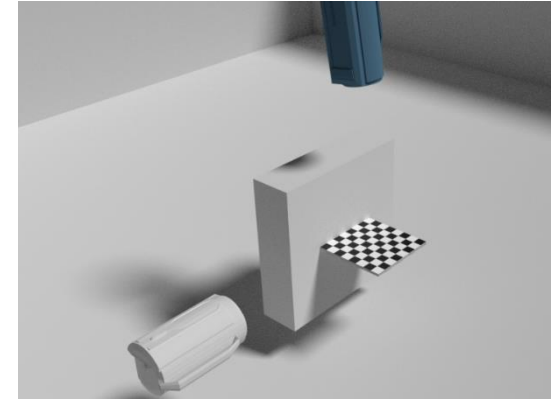
# Mesure de trajectoire en 3D

## Position de l'échantillon



$$s \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Détermination de la matrice de passage:  
Caméra => échantillon



OpenCV: solvePnP

Translation caméra échantillon: (0,0,5) bu

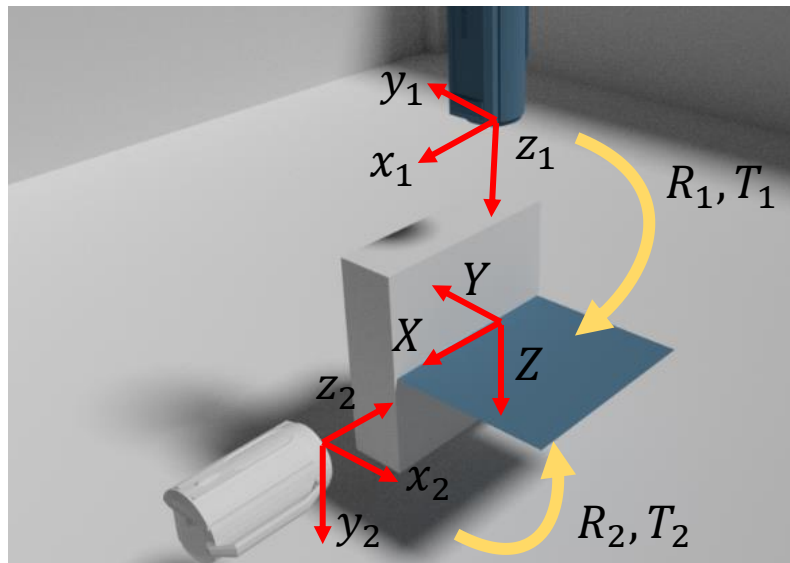
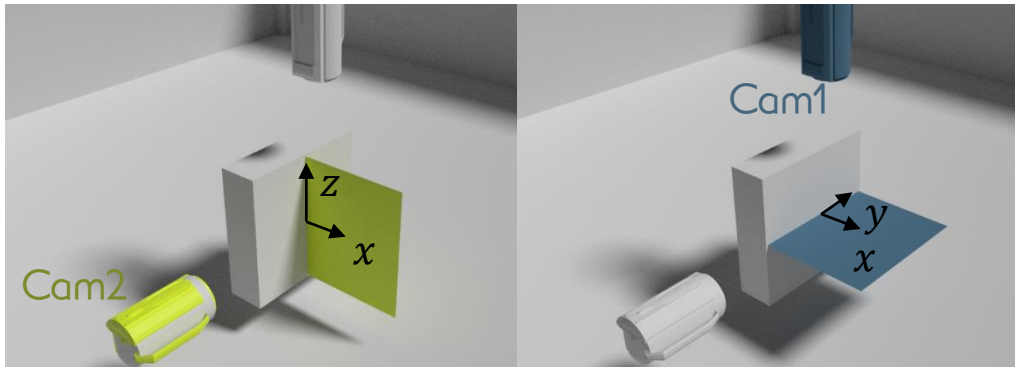
Rotation: (0,0,0.1°)



```
Rotation angle (°)
0.09520404348568402
Rotation axis
[[ 0.99565159]
 [-0.09224152]
 [ 0.01301592]]
Translation vector
[[-0.75068381]
 [-0.75213112]
 [ 4.9606228 ]]
```

# Mesure de trajectoire en 3D

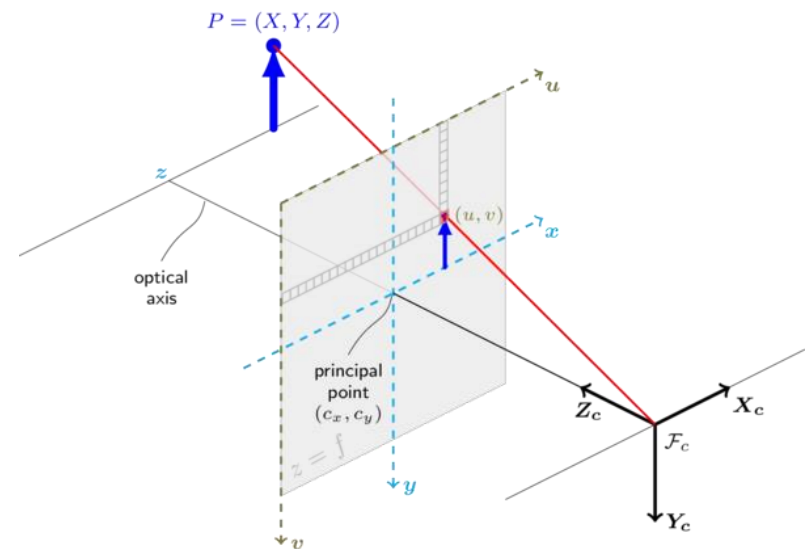
## Calcul de la trajectoire



Repère échantillon -> Repère caméra

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix}_{Cam1} = F_{Cam1} \left( R_1 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + T_1 \right)$$

$$\begin{pmatrix} u_2 \\ v_2 \end{pmatrix}_{Cam2} = F_{Cam2} \left( R_2 \begin{pmatrix} -Y \\ Z \\ -X \end{pmatrix} + T_2 \right)$$



Repère caméra -> Coordonnée écran 8



# Mesure de trajectoire en 3D

## Calcul de la trajectoire

$$\begin{pmatrix} u_1 \\ v_1 \\ c_1 \end{pmatrix}_{Cam1} = \begin{pmatrix} f_{x,1} & 0 & c_{x,1} \\ 0 & f_{y,1} & c_{y,1} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{xx,1} & r_{xy,1} & r_{xz,1} & t_{x,1} \\ r_{yx,1} & r_{yy,1} & r_{yz,1} & t_{y,1} \\ r_{zx,1} & r_{yz,1} & r_{zz,1} & t_{z,1} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} u_2 \\ v_2 \\ c_2 \end{pmatrix}_{Cam2} = \begin{pmatrix} f_{x,2} & 0 & c_{x,2} \\ 0 & f_{y,2} & c_{y,2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{xx,2} & r_{xy,2} & r_{xz,2} & t_{x,2} \\ r_{yx,2} & r_{yy,2} & r_{yz,2} & t_{y,2} \\ r_{zx,2} & r_{yz,2} & r_{zz,2} & t_{z,2} \end{pmatrix} \begin{pmatrix} -Y \\ X \\ Z \\ 1 \end{pmatrix}$$

---


$$\begin{pmatrix} u_1 \\ v_1 \\ c_1 \end{pmatrix}_{Cam1} = \begin{pmatrix} f_{x,1} & 0 & c_{x,1} \\ 0 & f_{y,1} & c_{y,1} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{xx,1}X + r_{xy,1}Y + r_{xz,1}Z + t_{x,1} \\ r_{yx,1}X + r_{yy,1}Y + r_{yz,1}Z + t_{y,1} \\ r_{zx,1}X + r_{zy,1}Y + r_{zz,1}Z + t_{z,1} \end{pmatrix}$$


---

$$\begin{pmatrix} u_1 \\ v_1 \\ c_1 \end{pmatrix}_{Cam1} = \begin{pmatrix} (f_{x,1}r_{xx,1} + c_{x,1}r_{zx,1})X + (f_{x,1}r_{xy,1} + c_{x,1}r_{zy,1})Y + (f_{x,1}r_{xz,1} + c_{x,1}r_{zz,1})Z + f_{x,1}t_{x,1} + c_{x,1}t_{z,1} \\ (f_{y,1}r_{yx,1} + c_{y,1}r_{zx,1})X + (f_{y,1}r_{yy,1} + c_{y,1}r_{zy,1})Y + (f_{y,1}r_{yz,1} + c_{y,1}r_{zz,1})Z + f_{y,1}t_{y,1} + c_{y,1}t_{z,1} \\ r_{zx,1}X + r_{zy,1}Y + r_{zz,1}Z + t_{z,1} \end{pmatrix}$$

$$\begin{pmatrix} u_2 \\ v_2 \\ c_2 \end{pmatrix}_{Cam1} = \begin{pmatrix} -(f_{x,2}r_{xx,2} + c_{x,2}r_{zx,2})Y + (f_{x,2}r_{xy,2} + c_{x,2}r_{zy,2})X - (f_{x,2}r_{xz,2} + c_{x,2}r_{zz,2})Z + f_{x,2}t_{x,2} + c_{x,2}t_{z,2} \\ -(f_{y,2}r_{yx,2} + c_{y,2}r_{zx,2})Y + (f_{y,2}r_{yy,2} + c_{y,2}r_{zy,2})X + (f_{y,2}r_{yz,2} + c_{y,2}r_{zz,2})Z + f_{y,2}t_{y,2} + c_{y,2}t_{z,2} \\ -r_{zx,2}Y + r_{zy,2}X + r_{zz,2}Z + t_{z,2} \end{pmatrix}$$

# Mesure de trajectoire en 3D

## Calcul de la trajectoire



POLYTECHNIQUE  
MONTRÉAL

$$\begin{pmatrix} u_1 \\ v_1 \\ c_1 \end{pmatrix}_{Cam1} = \begin{pmatrix} (f_{x,1}r_{xx,1} + c_{x,1}r_{zx,1})X + (f_{x,1}r_{xy,1} + c_{x,1}r_{zy,1})Y + (f_{x,1}r_{xz,1} + c_{x,1}r_{zz,1})Z + f_{x,1}t_{x,1} + c_{x,1}t_{z,1} \\ (f_{y,1}r_{yx,1} + c_{y,1}r_{zx,1})X + (f_{y,1}r_{yy,1} + c_{y,1}r_{zy,1})Y + (f_{y,1}r_{yz,1} + c_{y,1}r_{zz,1})Z + f_{y,1}t_{y,1} + c_{y,1}t_{z,1} \\ r_{zx,1}X + r_{zy,1}Y + r_{zz,1}Z + t_{z,1} \end{pmatrix}$$

$$\begin{pmatrix} u_2 \\ v_2 \\ c_2 \end{pmatrix}_{Cam1} = \begin{pmatrix} -(f_{x,2}r_{xx,2} + c_{x,2}r_{zx,2})Y + (f_{x,2}r_{xy,2} + c_{x,2}r_{zy,2})X - (f_{x,2}r_{xz,2} + c_{x,2}r_{zz,2})Z + f_{x,2}t_{x,2} + c_{x,2}t_{z,2} \\ -(f_{y,2}r_{yx,2} + c_{y,2}r_{zx,2})Y + (f_{y,2}r_{yy,2} + c_{y,2}r_{zy,2})X + (f_{y,2}r_{yz,2} + c_{y,2}r_{zz,2})Z + f_{y,2}t_{y,2} + c_{y,2}t_{z,2} \\ -r_{zx,2}Y + r_{zy,2}X + r_{zz,2}Z + t_{z,2} \end{pmatrix}$$

$$\begin{pmatrix} U_{ecran} \\ V_{ecran} \end{pmatrix} = \frac{1}{c} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$U_1(r_{zx,1}X + r_{zy,1}Y + r_{zz,1}Z + t_{z,1}) = (f_{x,1}r_{xx,1} + c_{x,1}r_{zx,1})X + (f_{x,1}r_{xy,1} + c_{x,1}r_{zy,1})Y + (f_{x,1}r_{xz,1} + c_{x,1}r_{zz,1})Z + f_{x,1}t_{x,1} + c_{x,1}t_{z,1}$$

$$V_1(r_{zx,1}X + r_{zy,1}Y + r_{zz,1}Z + t_{z,1}) = (f_{y,1}r_{yx,1} + c_{y,1}r_{zx,1})X + (f_{y,1}r_{yy,1} + c_{y,1}r_{zy,1})Y + (f_{y,1}r_{yz,1} + c_{y,1}r_{zz,1})Z + f_{y,1}t_{y,1} + c_{y,1}t_{z,1}$$

$$U_2(-r_{zx,2}Y + r_{zy,2}X + r_{zz,2}Z + t_{z,2}) = -(f_{x,2}r_{xx,2} + c_{x,2}r_{zx,2})Y + (f_{x,2}r_{xy,2} + c_{x,2}r_{zy,2})X - (f_{x,2}r_{xz,2} + c_{x,2}r_{zz,2})Z + f_{x,2}t_{x,2} + c_{x,2}t_{z,2}$$

$$V_2(-r_{zx,2}Y + r_{zy,2}X + r_{zz,2}Z + t_{z,2}) = -(f_{y,2}r_{yx,2} + c_{y,2}r_{zx,2})Y + (f_{y,2}r_{yy,2} + c_{y,2}r_{zy,2})X + (f_{y,2}r_{yz,2} + c_{y,2}r_{zz,2})Z + f_{y,2}t_{y,2} + c_{y,2}t_{z,2}$$

$$A \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = b$$

# Mesure de trajectoire en 3D

## Résultats

