AND Cheat Sheet

1 Vocabulary

HYPERBOLIC EQ.POINTS $\mathcal{R}\{x\} \neq 0$
NONHYPERBOLIC PROBLEMS Equilibria with EV on the imaginary axis.

2 Basics on cont.-time dynamical systems

$$\dot{x} = f(t, x, p), \quad x(t_0) = x_0 \Leftrightarrow \dot{x} = f(x, p), \quad x(0) = x_0$$

2.1 Flow of a system

Solution is the flow $\phi_t(x_0)$

$$x(t; x_0, p) = \phi_t(x_0, p)$$

FLOW AXIOMS

$$\phi_0(x_0, p) = x_0$$
$$\phi_{t_1}[\phi_{t_2}(x_0, p), p] = \phi_{t_1+t_2}(x_0, p)$$

2.2 Existence and uniqueness

EXISTENCE AND UNIQUENESS OF LOCAL (IN TIME) SOLUTIONS
If f is Lipschitz in x and piecewise continuous in t then the IVP has a unique solution $x(t) = \phi(t; t_0)x_0$ over a finite time interval $t \in [t_0 - \tau, t_0 + \tau]$.
Allows for finite-escape times.

2.3 Stability

Def.: An equilibrium point is a state x^* s.t. $f(x^*) = 0$
Def.: An eq.point x^* is stable if for any $\epsilon > 0$ there exists a constant $\delta > 0$ such that

$$\forall x_0 : \|x_0 - x^*\| \leq \delta \Rightarrow \|x(t) - x^*\| \leq \epsilon \quad \forall t \geq 0$$

ATTRACTOR
x^* is attractive if $\lim_{t\to\infty} \|x(t) - x^*\| = 0 \quad \forall x_0 \in \mathcal{S}$ with $\mathcal{S}$ being the domain of attraction.
Eq points may be attractive without being stable (ex. Vinograd's system).
ASYMPTOTIC STABILITY
An eq.point x^* is asymptotically stable if it is stable and attractive.
EXPONENTIAL STABILITY
An eq.point x^* is exponentially stable in $\mathcal{S}$ if it is stable and there are constants $a, \lambda > 0$ such that
$\forall x_0 \in \mathcal{S} : \|x(t) - x^*\| \leq a\|x_0 - x^*\|e^{-\lambda t}$

3 Linear systems

3.1 LTI systems

3.2 LTV systems and Floquet theory

4 Nonlinear flows

4.1 Local Theory

HARTMAN-GROBMAN
Gives idea of the behavior of nonlinear systems in a vicinity of hyperbolic eq. points by looking at the (Taylor) linearization.
CENTER MANIFOLD THEOREM

$$\frac{dh}{dx_z}[A_z x_z + f_z(x_z, h(x_z))] = A_s h(x_z) + f_s(h(x_z), x_z)$$
$$h(0) = 0, \quad \frac{dh(0)}{dx_z} = 0$$

4.2 Non-local phenomena

5 Bifurcations of vector fields

5.1 Bifurcations of SSs

5.2 Bifurcations of trajectories