Two Highest Penalties: A Modified Vogels Approximation Method to Find Initial Basic Feasible Solution of Transportation Problem

Bilqis Amaliah
Department of Informatics
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
bamaliah@gmail.com

Chastine Fatichah
Department of Informatics
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
chastine@if.its.ac.id

Erma Suryani
Department of Information System
Institut Teknologi Sepuluh Nopember
Surabaya, Indonesia
erma.suryani@gmail.com

2021 13th International Conference on Information & Communication Technology and System (ICTS) | 978-1-6654-4059-2/21/$31.00 ©2021 IEEE | DOI: 10.1109/ICTS52701.2021.9608005

***Abstract*— Vogel's Approximation Method (VAM) is one of the methods to find Initial Basic Feasible Solution (IBFS) of Transportation Problem (TP), which is mostly used to find the solution with minimum cost. Unfortunately, VAM has a limitation, i.e., if there are more than one highest penalty, VAM allows to select one arbitrarily. It causes ambiguity on penalty selection, which leads to the production of several alternative final solutions. In order to answer the challenge, Logical Development Of Vogel's Approximation Method (LD-VAM) turned up by selecting penalty in conflict using cell with lowest cost value. This technique triggered another ambiguity when there are several cells with the same minimum cost value. To avoid the ambiguity, Two Highest Penalties Method (THP) is proposed. The proposed method can reduce the cost of transportation problem as it uses Max-Min penalty, select two highest penalties, and use minimum (cost x allocation) to allocate values to the cell. THP still inherits some of VAM and LD-VAM concepts and computation procedures, yet it also introduces a new algorithm to select the suitable cell when ambiguity arises. Numerical examples have been used at this research to prove that THP can solve ambiguity, providing only one final solution and showing better final solution compared to those of VAM and LD-VAM. The result of THP is 98% accurate with optimal solution from TORA Program, which is used as reference.**

Keywords*— *Initial Basic Feasible Solution, Transportation Problem, Vogel's Approximation Method

I. INTRODUCTION

Vogel's Approximation Method (VAM) is a method mostly used to find Initial Basic Feasible Solution (IBFS) at minimum cost transportation problem [1]. Many researchers have tried to improve the technique, which meant that many derivative methods have been VAM proposed. These VAM derivatives have similar purpose to that of VAM, finding final solution by value closest to optimal value. IBFS is the foundation and important step to get the total minimum cost of transportation problem [2]–[4].

Some of the VAM derivatives are as follows. Joseph [5] used to optimize energy flow in welding. The problem in Transportation Problem [TP] was usually correlated to budget plan optimization to distribute logistics into one supply chain [6]. Serkan Ball et al. [7] suggested IVAM (Improved VAM), which used total opportunity cost to solve TP. Although IVAM final solution worked better than VAM's, IVAM mechanism did not solve ambiguities during penalty cost selection on a row or column as well as cost selection on a cell. In the meantime, Meenakshi [8] analyzed some VAM variants.

Abdul Sattar Soomro et al. [9] recommended another technique to obtain penalty cost on row by calculating the difference between two maximum costs on every column. Penalty cost on a column was still calculated as the way it was in VAM. Other calculation procedures were unchanged, which made ambiguity during selection process may still arose. Muwafaq Alkubaisi [10] used median range on every row and column as penalty cost. This method was appropriate for data with distinct cell value. Ambiguity on row or column selection, as well as on cell selection, still occurred when more than one row or column has identical median. Sharif Uddin et al. [11] proposed AVAM (Advanced VAM) to compute penalty cost by a method better than VAM, a method that ignored two or more identical minimum costs and used the next minimum cost (min2 – min1). Unfortunately, since AVAM had similar calculation process to that of VAM, ambiguity problem in selecting row or column as the result of similar penalty costs had not been settled yet. Das et al. [12] suggested LD-VAM (Logical Development VAM) to answer the ambiguity problem. The technique used was effortless by simply using the lowest cost on a row or column that has similar penalty cost. Ambiguity problem occurred again when there were some minimum costs on the selected row or column.

Two Highest Penalties Method (THP) is proposed to solve those ambiguities. The proposed method can decrease the cost of transportation problem as it uses Max-Min penalty, chooses two highest penalty costs, and uses minimum (cost x allocation) to allocate values to the cell. THP still inherits some concepts and computation procedures from VAM and LD-VAM, yet it also introduces a new algorithm to select the suitable cell when ambiguous problem arises.

The numerical examples have been used at this research to show that THP could solve the ambiguities, provide only one final solution, and show better final solution compared to VAM and LD-VAM. Matrix 3 x 3, 4 x 4, and 5 x 5 are used to evaluate the performance of the proposed method. The result of THP will be compared with optimal solution from TORA program [13].

II. TRANSPORTATION PROBLEM

Generally, Transportation Problem can be described by the following diagram as shown in Fig. 1. Transportation Problem modeling on Fig. 1 can be tabulated into the transportation table as shown in Table 1. C_{ij} is transportation cost to transport unit quantity from supply i to demand j. When the total amounts of supply and demand are imbalance, it is called unbalanced transportation problem.

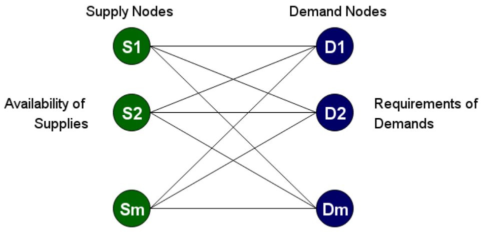

TABLE 1. TRANSPORTATION PROBLEM

		Destinations				
		D_1	D_2	…	D_n	**Supply**
Sources	S_1	c_{11}	c_{12}	…	c_{1n}	s_1
	S_2	c_{21}	c_{22}	…	c_{2n}	s_2
	…	…	…	…	…	…
	S_m	c_{m1}	c_{m2}	…	c_{mn}	s_m
	Demand	d_1	d_2	…	d_n	

III. VAM ALGORITHM

Vogel's Approximation Method is a technique to find an initial solution of a transportation problem. The result of this VAM is usually better than North West and lowest cost methods. Steps in applying VAM technique is presented as follows:

Step 1. Preliminaries
Determine whether total supply = total demand. Otherwise, add dummy rows or columns to balance the total supply and total demand.

Step 2. Determine the Penalty for each row and column:
a. Identify the lowest cell cost and the next lowest cell cost.
b. Penalty is the difference between the lowest cell cost and the next lowest cell cost.
c. If there are more than one lowest cell costs, the penalty is zero.

Step 3. Select the Penalty and Cell
a. Select the row or column with the highest penalty.
b. If there are more than one penalty with similar value, choose the penalty arbitrarily.
c. Select the lowest cost cell on the chosen penalty.
d. If there are more than one lowest cost cells with similar value, choose arbitrarily.

Step 4. Allocate the Cell
a. Allocate as much as possible to the chosen cell.
b. Subtract the supply and demand with maximum allocation from point a.

Step 5. Eliminate Row or Column
a. Find out each row and column with zero supply or demand.
b. Eliminate rows or columns with zero supply or demand.
c. Repeat Step 2 until all rows and columns are eliminated.

VAM algorithm has some limitations, especially on step_3: if there are more than one highest penalty, VAM allows to select one arbitrarily. By choosing arbitrarily, VAM final solution might not be optimal. We need to try all highest penalty costs to get the optimal solution. Thus, we will end up with more than one alternative final solution.

IV. LD-VAM ALGORITHM

Sharif Uddin [10] observed VAM limitation and proposed an improved approach called LD-VAM. In this method, when largest penalty cost appears more than once, LD-VAM can select penalty in conflict using cell with lowest cost. The steps of LD-VAM algorithm proposed by Sharif Uddin et al. [10] are as follows:

Step 1. Preliminaries
Determine whether total supply = total demand. Otherwise, add dummy rows or columns to balance the total supply and total demand.

Step 2. Determine the Penalty for each row and column:
a. Identify the lowest cell cost and the next lowest cell cost.
b. Penalty is the difference between the lowest cell cost and the next lowest cell cost.
c. If there are more than one lowest cell cost, the penalty is zero.

Step 3. Select the Penalty Cost and Cell
a. Select the row or column with the highest penalty.
b. If there are more than one penalty with similar value, select a row or column with the lowest cost in cell.
c. If the lowest cost appears more than once, choose arbitrarily.

Step 4. Allocate the Cell
a. Allocate as much as possible to the chosen cell.
b. Subtract the supply and demand with maximum allocation from point a.

Step 5. Eliminate Row or Column
a. Find out each row and column with zero supply or demand.
b. Eliminate rows or columns with zero supply or demand.
c. Repeat Step 2 until all rows and columns are eliminated.

From the above illustrations, it is concluded that the difference between VAM and LD-VAM lies on Step 3, on which we need to choose row or column with similar penalty. Ambiguity in selecting a cell might still occur when some lowest costs appear on the chosen row or column (Step 3, point c). Meanwhile, other phases remain the same.

V. NEW PROPOSED METHOD: THP

VAM and LD-VAM schemes still contain ambiguities on choosing highest penalty and cell, which leads to several alternative final solutions. Those methods, either VAM or LD-VAM, allow arbitrary selection of highest penalty or cell, and simply suggest applying North West Method (NWM). In some cases, ignoring NWM during the selection stage will give different final solution from the process with applying NWM.

The more ambiguity in selecting penalty cost and cell during calculation iteration, the more various final solutions will develop; meaning that more energy is needed to achieve optimal result. Therefore, a new technique is essential in order to reduce ambiguity during calculation iteration and to obtain better final solution.

This section will explain a new method namely Two Highest Penalties Method (THP) that aims at reducing, or eliminating, selection ambiguity which may happen. Detail of THP is shown as follows:

Step 1. Preliminaries
Determine whether total supply = total demand. Otherwise, add dummy rows or columns to balance the total supply and total demand.

Step 2. Determine the Penalty for each row and column:

a. Identify the minimum cost and maximum cost.
b. Penalty is the difference between minimum cost and maximum cost.

Step 3. Select the Penalty and Cell

a. Select the highest penalty and the next highest penalty.
b. When highest penalty appears more than once, choose all of them. When the next highest penalty appears more than once, select all of them.
c. For each penalty, choose cell with the lowest cost.
d. For each cell, calculate multiplication between cost and allocation, CA = cost x allocation.
e. From point d, select the lowest CA.
f. If lowest CA occurs more than once, choose cell with the lowest cost.

Step 4. Allocate the Cell

a. Allocate as much as possible to the chosen cell.
b. Subtract the supply and demand with maximum allocation from point a.

Step 5. Eliminate Row or Column

a. Find out each row and column with zero supply or demand.
b. Eliminate rows or columns with zero supply or demand.
c. Repeat Step 2 until all rows and columns are eliminated.

From the detail of THP, it can be concluded that the differences among THP, VAM, and LD-VAM lie on Step 2 and Step 3. On Step 2, Max-Min Penalty is applied to calculate the penalty. In other words, penalty cost is the difference between the lowest and highest costs.

Meanwhile, Step 3 in VAM, LD-VAM, and THP is different. While Step 3 in VAM and LD-VAM only includes selecting one highest penalty cost, Step 3 in THP includes choosing two highest penalty costs. Cell selection in THP is carried out by multiplying lowest cost with allocation in each penalty cost, then choosing the lowest cost from the calculation.

In some cases, calculation in THP may minimize or even eliminate ambiguity during selection that might occur in the middle of calculation process. Arbitrary cell selection may still happen but with minimum probability as the selection goes through several examinations; unlike VAM and LD-VAM that allow arbitrary cell selection since the beginning of calculation iteration.

VI. NUMERICAL ILLUSTRATION

This section portrays some case studies to describe Transportation Problem solution by THP. The case study 1 is shown in Table 2.

A. Case Study 1

Calculation process works on the following flow:

1. Compute penalty cost for each row and column. Penalty cost in each row and column is calculated by subtracting maximum cost with minimum cost. For column *D1*, maximum cost =10 and minimum cost =6, so penalty cost *D1*=4 as shown in Table 3.
2. Select two highest penalty costs, they are penalty cost with value 8 and 6. Penalty cost with value 8 is on *D4*. Penalty cost with value 6 is on *D3, S2*, and *S3*.
3. Select cell based on the result of multiplication process of cost and allocation, CA = cost * allocation.

   Based on the multiplication, three selections are available,
   a) cell 1,3 with cost=1, allocation=30, cost * allocation=1*30=30 (*D3*)
   b) cell 2,4 with cost=1, allocation=55, cost * allocation=1*55=55 (*D4, S2*)
   c) cell 3,2 with cost=4, allocation=65, cost * allocation=4*65=260 (*S3*)

   From the above variety, cell 1,3 is selected as it contains the lowest cost * allocation, namely 30.
4. Allocate cell.
   Allocate 30 on cell 1,3.
5. Eliminate row and column.
   Column Demand *D3* and row Supply S1 is subtracted by 30. Since column Demand *D3* is used up, it is marked with grey and eliminated from the next iteration. Meanwhile, Row S1 still has 20 (50 – 30 = 20).
6. Continue to Iteration 2.

Calculation process works on the following flow:

1. Recalculate penalty cost for each row and column.
2. Select two highest penalty costs, namely penalty costs with value 8 and 6, on *D4, S2* and *S3* as shown in Table 4.
3. Select a cell. Since there are two cells are available, namely:
   a) cell 2,4 with cost=1, allocation=55, cost*allocation=1*55=55 (*D4, S2*)
   b) cell 3,2 with cost=4, allocation=65, cost*allocation=4*65=260 (*S3*)

   cell 2,4 is chosen as it contains the lowest cost*allocation, namely 55.

TABLE 3. ITERATION 1 OF CASE STUDY 1

	D1	D2	D3[5]	D4	Supply	Penalty[1]
S1	6	3	1[3a]	4	50,20	5
			30[4]			
S2	7	6	2	1[3b]	55	6[2]
S3	10	4[3c]	5	9	75	6[2]
S4	7	7	7	3	60	4
Demand	90	65	30,0	55		
Penalty[1]	4	4	6[2]	8[2]		

TABLE 4. ITERATION 2 OF CASE STUDY 1

	D1	D2	D3	D4[4]	Supply	Penalty[1]
S1	6	3	1	4	50,20	3
			30			
S2	7	6	2	1[3a]	55,0	6[2]
				55		
S3	10	4[3b]	5	9	75	6[2]
S4	7	7	7	3	60	4
Demand	90	65	30,0	55,0		
Penalty[1]	4	4	0	8[2]		

4. Allocate a cell
   Allocate 55 on cell 2,4.
5. Subtract row *S2* and column *D4* with 55.
6. Eliminate row *S2* and column *D4* as they have been used up.
7. Continue to Iteration 3.

Calculation process works on the following flow:

1. Recalculate penalty cost for each row and column.
2. Select two highest penalty costs; those are penalty costs with value 6 and 4 on *D1, D2*, and *S3* as shown in Table 5.
3. Select a cell. Since there are two cells are available, namely:
   a) cell 1,1: cost=6, allocation=20, cost*allocation = 6*20 =120 (*D1*)
   b) cell 1,2: cost=3, allocation=20, cost*allocation = 3*20 =60 (*D2*)
   c) cell 3,2: cost=4, allocation=65, cost*allocation = 4*65=260 (*S3*)
   cell 1,2 is chosen as it contains the lowest cost*allocation, namely 60.
3. Allocate a cell
   Allocate 20 on cell 1,2.
4. Subtract row *S1* and column *D2* with 20.
5. Eliminate row *S1* as it has been used up.
6. Continue to Iteration 4.

Calculation process works on the following flow:

1. Recalculate penalty cost for each row and column.
2. Select two highest penalty costs; those are penalty costs with value 6 and 3 on *D1, D2*, and *S3* as shown in Table 6.
3 Select a cell. Since there are two cells are available, namely:
   a) cell 3,2: cost=4, allocation=45, cost*allocation = 4*45=180 (*D1,S3*)
   b) cell 4,1: cost=7, allocation=60, cost*allocation = 7*60=420 (*S4*)
   cell 3,2 is chosen as it contains the lowest cost*allocation, namely 180.
4. Allocate a cell
   Allocate 45 on cell 3,2.
5. Subtract row S3 and column *D2* with 45.
6. Eliminate column *D2* as it has been used up. Meanwhile, row S3 still has 30 (75-45=30).
7. Continue to Iteration 5.

Calculation process works on the following flow:

1. Recalculate penalty cost for each row and column. There is only penalty *D1* that can be calculated
2. There is only penalty *D1* that can be chosen as shown in Table 7.
3. Select a cell
   Only cell 4,1 available: cost=7, allocation=60, cost*allocation=7*60=420 (*D1*)
4. Allocate a cell
   Allocate 60 on cell 4,1.
5. Subtract row *S4* and column *D1* with 60.
6. Eliminate row *S4* as it has been used up.
7. Continue to Iteration 6.

Calculation process works on the following flow:
Since there is one cell left, final allocation is directly done on cell 3,1, i.e., 30 as shown in Table 8.
Final solution of the calculation is:
(1x30) + (1x55) + (3x20) + (4x45) + (10x30) + (7x60) = 1045

B. *Case Study 2*

Calculation process for case study 2 is shown in Table 9, and Table 10 shows the final result:

1. Iteration-1:
   a. Two highest penalty costs: 8 and 6 (*D2, S3*)
   b. Cell options: cell 3,1 (cost*allocation=700); cell 3,2 (cost*allocation=5*100=500).
   c. Cell selection by the lowest cost*allocation, cell 3,2 is chosen.
2. Iteration-2:
   a. Two highest penalty costs: 8 and 4 (*S1, S2, S3*)
   b. Cell options: cell 3,1 (cost*allocation=4*175=700); cell 2,1 (cost*allocation=1225); cell 1,1 (cost*allocation=900).
   c. Cell selection by the lowest cost*allocation, cell 3,1 is chosen.
3. Iteration-3:
   a. Two highest penalty costs: 4 and 1 (*S1, S2, D1, D3*).
   b. Cell options: cell 1,1 (cost*allocation=6*25=150); cell 2,1 (cost*allocation=175); cell 1,3 (cost*allocation=1500)
   c. Cell selection by the lowest cost*allocation, cell 1,1 is chosen.

TABLE 5. ITERATION 3 OF CASE STUDY 1

	D1	D2	D3	D4	Supply	Penalty$^{(1)}$
S1	$6^{(3a)}$	$3^{(3b)}$	1	4	50,20,0	3
		$20^{(4)}$	30			
S2	7	6	2	1	55,0	0
				55		
S3	10	$4^{(3c)}$	5	9	75	$6^{(2)}$
S4	7	7	7	3	60	0
Demand	90	65,45	30,0	55,0		
Penalty$^{(1)}$	$4^{(2)}$	$4^{(2)}$	0	0		

TABLE 6. ITERATION 4 OF CASE STUDY 1

	D1	D2$^{(5)}$	D3	D4	Supply	Penalty$^{(1)}$
S1	6	3	1	4	50,20,0	0
		20	30			
S2	7	6	2	1	55,0	0
				55		
S3	10	$4^{(3a)}$	5	9	75,30	$6^{(2)}$
		$45^{(4)}$				
S4	$7^{(3b)}$	7	7	3	60	0
Demand	90	65,45,0	30,0	55,0		
Penalty$^{(1)}$	$3^{(2)}$	$3^{(2)}$	0	0		

4. b. Cell selection by minimum cost is cell 1,3 with cost*allocation = 1.250.
5. Iteration-5:
   Only one cell left, namely cell 2,1, with cost*allocation = 11*175 = 1925.

Based on the iteration process, final solution of the calculation:

(5x100) + (4x175) + (6x25) + (10x125) + (11x175) = 4525

C. Case Study 3

Calculation process for case study 3 is shown in Table 11, and Table 12 shows the final result:

1. Iteration-1:
   a. Two highest penalty costs: 88 and 87 (*D3, S2*)
   b. Cell options: cell 1,3 (cost*allocation=108), cell 2,4(cost*allocation=60).
   c. Cell selection by the lowest cost*allocation, cell 2,4 is chosen.
2. Iteration-2:
   a. Two highest penalty costs: 87 and 83 (*D3, S2*)
   b. Cell options: cell 1,3 (cost*allocation=108), cell 2,5 (cost*allocation=30).
   c. Cell selection by the lowest cost*allocation, cell 2,5 is chosen.
3. Iteration-3:
   a. Two highest penalty costs: 78 and 75 (*D3, D5, S5*)
   b. Cell options: cell 1,3 (cost*allocation=108), cell 5,5 (cost*allocation=99).
   c. Cell selection by the lowest cost*allocation, cell 5,5 is chosen.
4. Iteration-4:
   a. Two highest penalty costs: 78 and 64 (*D3, S1, S5*)
   b. Cell options: cell 1,3 (cost*allocation = 108), cell 5,2 (cost*allocation=80).
   c. Cell selection by the lowest cost*allocation, cell 5,2 is chosen.
5. Iteration-5:
   a. Two highest penalty costs: 71 and 64 (*D3, S1*)
   b. Cell options: cell 1,3 (cost*allocation=108)
   c. If there is only one option, choose cell 1,3
6. Iteration-6:
   a. Two highest penalty costs: 51 and 39 (*D1, D3*)
   b. Cell options: cell 4,1 (cost*allocation=594), cell 4,3 (cost*allocation=52).
   c. Cell selection by the lowest cost*allocation, cell 4,3 is chosen.
7. Iteration-7:
   a. Two highest penalty costs: 39 and 31 (*D1, S3*)
   b. Cell options: cell 4,1 (cost*allocation=594), cell 3,2 (cost*allocation=868).
   c. Cell selection by the lowest cost*allocation, cell 4,1 is chosen.
8. Iteration-8:
   a. Only one penalty cost left: 31 (*S3*)
   b. Cell selection by minimum cost is cell 3,2.
9. Iteration-9:
   Only one cell left, namely cell 3,1.

Based on the iteration process, final solution of the calculation =

(5x14) + (10x3) + (9x11) + (20x4) + (6x18) + (26x2) + (54x11) + (62x14) + (93x5) = 2366

VII. CONCLUSION

By examining the calculation processes of the above case studies, especially for the first case study, cell selection goes through a long procedure, yet comes up with final solution nearest to optimal solution. The final solution for the second and third case studies is exactly the same with optimal final solution. VAM and optimal solutions are obtained through TORA [13].

This long journey to choose a cell is a proposed method that aims at reducing ambiguity during cell selection process. This new scheme is called Two Highest Penalties Method (THP). Illustration of the above case studies evidently proves that there is no ambiguity during cell selection.

This is different from VAM and LD-VAM which have ambiguity since the beginning of the calculation process, which leads to some alternative final solutions. THP successfully eliminates cell selection ambiguity. Thus, THP creates single final solution.

THP final solution is also smaller than VAM and LD-VAM with 98% accuracy, and it is better than VAM and LD-VAM as shown in Table 13. For the three examples, VAM and LD-VAM cannot reach the optimal solution.

TABLE 7. ITERATION 5 OF CASE STUDY 1

	D1	D2	D3	D4	Supply	Penalty(1)
S1	6	3	1	4	50,20,0	0
		20	30			
S2	7	6	2	1	55,0	0
				55		
S3	10	4	5	9	75,30,0	--
		45				
S4	7(3)	7	7	3	60,0	0
	60(4)					
Demand	90,60,30	65,45,0	30,0	55,0		
Penalty(1)	3(2)	0	0	0		

TABLE 8. ITERATION 6 OF CASE STUDY 1

	D1	D2	D3	D4	Supply	Penalty(1)
S1	6	3	1	4	50,20,0	0
		20	30			
S2	7	6	2	1	55,0	0
				55		
S3	10	4	5	9	75,30,0	--
	30	45				
S4	7	7	7	3	60,0	0
	60					
Demand	90,60,30, 0	65,45,0	30,0	55, 0		
Penalty(1)	--	0	0	0		

TABLE 9. CASE STUDY 2

	D1	D2	D3	Supply
S1	6	8	10	150
S2	7	11	11	175
S3	4	5	12	275
Demand	200	100	300	

TABLE 10. DETAIL OF CASE STUDY 2

	D1	D2	D3	Supply
S1	6[3]	8	10[4]	150
	25		125	
S2	7	11	11[5]	175
			175	
S3	4[2]	5[1]	12	275
	175	100		
Demand	200	100	300	

TABLE 11. CASE STUDY 3

	D1	D2	D3	D4	D5	Supply
S1	70	37	6	76	17	**18**
S2	59	90	93	5	10	**17**
S3	93	62	77	47	62	**19**
S4	54	55	26	9	84	**13**
S5	53	20	84	15	9	**15**
Demand	**16**	**18**	**20**	**14**	**14**	

TABLE 12. DETAIL OF CASE STUDY 3

	D1	D2	D3	D4	D5	Supply
S1	70	37	6[5]	76	17	18
			18			
S2	59	90	93	5[1]	10[2]	17
				14	**3**	
S3	93[9]	62[8]	77	47	62	19
	5	**14**				
S4	54[7]	55	26[6]	9	84	13
	11		**2**			
S5	53	20[4]	84	15	9[3]	15
		4			**11**	
Demand	16	18	20	14	14	

TABLE 13. IBFS RESULT OF VAM, LD-VAM AND WITH ACCURATION

No	**IBFS**				**Accuration (%)**		
	VAM	**LD-VAM**	**THP**	**Optimal**	**VAM**	**LD-VAM**	**THP**
1	1095	1095	1045	985	89	89	94
2	5125	5125	4525	4525	87	87	100
3	2388	2388	2366	2366	99	99	100
Average					92	92	**98**

REFERENCES

[1] O. Kirca and A. Satir, “A Heuristic for Obtaining an Initial Solution for the Transportation Problem,” *Journal Operational Research Society*, vol. 41, no. 9, pp. 865–871, 1990, doi: https://doi.org/10.1057/jors.1990.124.

[2] B. Amaliah, C. Fatichah, and E. Suryani, “Total opportunity cost matrix – Minimal total: A new approach to determine initial basic feasible solution of a transportation problem,” *Egyptian Informatics Journal*, vol. 20, no. 2, pp. 131–141, 2019, doi: 10.1016/j.eij.2019.01.002.

[3] B. Amaliah, C. Fatichah, and E. Suryani, “A new heuristic method of finding the initial basic feasible solution to solve the transportation problem,” *Journal of King Saud University - Computer and Information Sciences*, no. xxxx, 2020, doi: 10.1016/j.jksuci.2020.07.007.

[4] B. Amaliah, C. Fatichah, E. Suryani, and A. Muswar, “Total Opportunity Cost Matrix – Supreme Cell : A New Method to Obtain Initial Basic Feasible Solution of Transportation Problems,” pp. 151–156, 2020, doi: https://doi.org/10.1145/3411174.3411198.

[5] J. I. Achebo, “Optimization of Fluence Energy in Relation to Weld Properties Based on Vogel Approximation Method,” no. Juli, 2012, [Online]. Available: https://www.researchgate.net/publication/265749198_Optimization_of_Fluence_Energy_in_Relation_to_Weld_Properties_Based_on_Vogel_Approximation_Method.

[6] S. Singh, G. C. Dubey, and R. Shrivastava, “A Various Method to Solve the Optimality for the Transportation Problem Methods of obtaining Intial Basic Feasible Solutions,” vol. 1, no. 4, pp. 21–28, 2012.

[7] S. Korukoğlu and S. Ballı, “A Improved Vogel’s Approximatio Method for the Transportation Problem,” *Mathematical and Computational Applications*, vol. 16, no. 4, pp. 370–381, 2011, doi: 10.3390/mca16020370.

[8] M. Mathirajan and B. Meenakshi, “Experimental Analysis of Some Variants of Vogel’s Approximation Method,” *Asia-Pacific Journal of Operational Research (APJOR)*, vol. 21, no. 04, pp. 447–462, 2004, doi: https://doi.org/10.1142/S0217595904000333.

[9] A. S. Soomro *et al.*, “Modified Vogel ’ s Approximation Method For Solving Transportation Problem,” *Mathematical Theory and Modeling*, vol. 5, no. 4, pp. 32–43, 2015.

[10] M. Alkubaisi, “Modified VOGEL Method to Find Initial Basic Feasible Solution (IBFS) Introducing a New Methodology to Find Best IBFS,” *Business and Management Research*, vol. 4, no. 2, pp. 22–36, 2015, doi: 10.5430/bmr.v4n2p22.

[11] U. K. Das, A. Babu, A. R. Khan, and S. Uddin, “Advanced Vogel’s Approximation Method (AVAM): A New Approach to Determine Penalty Cost for Better Feasible Solution of Transportation Problem,” *International Journal of Engineering Research & Technology*, vol. 3, no. 1, pp. 182–187, 2014.

[12] U. K. Das, R. Ashraful Babu, A. Khan, and U. Helal, “Logical Development Of Vogel’s Approximation Method (LD-VAM): An Approach To Find Basic Feasible Solution Of Transportation Problem,” *International Journal of Scientific & Technology Research*, vol. 3, no. 2, 2014.

[13] H. A. Taha, *Operations Research: An Introduction*, 8th ed. New Jersey: Pearson Prentice Hall, 2007.