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Forecasting exchange rates with a large Bayesian VAR

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Abstract

Models based on economic theory have serious problems forecasting exchange rates better than simple univariate driftless random walk models, especially at short horizons. Multivariate time series models suffer from the same problem. In this paper, we propose to forecast exchange rates with a large Bayesian VAR (BVAR), using a panel of 33 exchange rates vis-a-vis the US Dollar. Since exchange rates tend to co-move, a large set of them can contain useful information for forecasting. In addition, we adopt a driftless random walk prior, so that cross-dynamics matter for forecasting only if there is strong evidence of them in the data. We produce forecasts for all 33 exchange rates in the panel, and show that our model produces systematically better forecasts than a random walk for most of the countries, and at all forecast horizons, including 1-step-ahead.
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1. Introduction

Is it possible to devise a model which is able to forecast exchange rates better than a simple no-change random walk forecast? Generations of economists have struggled with this question, since the seminal work of Meese and Rogoff (1983), who provided evidence that exchange rate models based on economic theory produce forecasts which are strikingly outperformed by a simple driftless random walk.

Several papers have tried to build and estimate models able to outperform the random walk in out-of-sample forecasting performance, and some papers have documented progress in this respect (Chinn & Meese, 1995; MacDonald & Marsh, 1997; MacDonald & Taylor, 1994; Mark, 1995).

However, such evidence of predictability is typically limited to only very long forecast horizons, and reasonable gains in forecasting performance generally begin at around 3 years ahead. Chinn and Meese (1995) estimate several structural exchange rate models, and their findings confirm that fundamental exchange rate models forecast no better than a random walk model for short-term prediction horizons, but

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that, for longer horizons, error correction terms can explain exchange rate movements significantly better than a no change forecast. Mark (1995) shows that the deviation of the log exchange rate from its fundamental value contains relevant information for forecasting long-horizon changes in log nominal exchange rates, and that the out-of-sample point predictions generally outperform the driftless random walk at the longer horizons.

Moreover, several papers have claimed that even the rather limited existing evidence supporting fundamental-based forecasts of exchange rates is very weak, not robust to the inference procedures used, or very sensitive to the choice of the sample and the data vintage (see Berben & Van Dijk, 1998; Berkowitz & Giorgianni, 2001; Faust, Rogers, & Wright, 2003; Groen, 1999; and Kilian, 1999).

All of the papers cited above share one key feature, namely, they try to forecast the exchange rates using economic fundamentals and models based on economic theory. Having a model which is both theory consistent and forecasts well is very appealing, as the economic foundations allow economists to explain the forecasts, and a good forecasting performance itself provides evidence in favour of the theory. However, the simple task of forecasting is important in its own right.

In this paper we take a completely different perspective, and consider the task of forecasting the exchange rates per se, using a purely time series approach that exploits information in a rather large panel of exchange rates. Given that the best forecasts of exchange rates seem to be produced by a driftless random walk, it is natural to believe a priori that exchange rates do follow such a process, and to incorporate such information in the forecasting model. This can be done by using a Bayesian Vector Autoregression (BVAR), in which the VAR coefficients are shrunk towards a random walk representation.

Besides giving the opportunity of including a priori information in the picture, the BVAR approach efficiently summarizes the information contained in large datasets, whereas a simple multivariate linear model would encounter the curse of dimensionality problem. Although the good forecasting performance of BVAR was documented years ago by Doan, Litterman, and Sims (1984) and Litterman (1986), only recently Banbura, Giannone, and Reichlin (2007) have shown that the Bayesian VAR is a natural and effective tool for forecasting and performing structural analysis with a large information set.

We propose to forecast exchange rates using a large information set (a panel of 33 exchange rates) and a BVAR with a driftless random walk prior. The proposed prior takes a normal-inverted Wishart form, and closed form solutions for the posteriors are available. Moreover, the prior features a Kronecker structure, which dramatically reduces the computational costs involved in using a large information set. The overall tightness of the prior is chosen by using a data-driven procedure.

We produce forecasts for the whole panel of 33 exchange rates vis-a-vis the US Dollar, and we provide evidence that this strategy may systematically outperform the random walk for most of the variables. On average the forecast gains are in the range of 2%–3%, but in some relevant cases such as the Euro–Dollar and the GBP–Dollar, the gains can go up to 6%–9%. Importantly, the forecast gains can arise at any forecast horizon, including 1-step-ahead. Given this fact, BVAR forecasts might also become the new benchmark for the evaluation of more economic theory based models of exchange rates.

The paper is structured as follows. Section 2 describes the BVAR, with a focus on the specification of the prior distribution for the VAR parameters. Section 3 describes the forecasting exercise and discusses the results. Section 4 summarizes and concludes.

2. The BVAR with the driftless random walk prior

A random walk without drift is overall a very competitive model in forecasting exchange rates. Therefore, it seems reasonable to build a forecasting model in which exchange rates are a priori following such a process. However, the model should not completely discard potentially useful information from dynamic comovements in exchange rates. Hence, in this paper we adopt a Bayesian approach, imposing a univariate driftless random walk prior on the parameters of a vector autoregression for a large set of exchange rates. Such a prior can be considered as a normal-inverted Wishart version of the traditional Minnesota prior, proposed originally by

Doan et al. (1984) and Litterman (1986), which has the advantage of avoiding the inconvenient assumption of a fixed and diagonal residual variance matrix. The use of this prior for forecasting was originally proposed by Kadiyala and Karlsson (1993, 1997), and recently Banbura et al. (2007) have shown that it performs well in forecasting with large datasets. Sims and Zha (1998) have provided additional results for such priors in the context of structural vector autoregressions.

In what follows, we denote the exchange rate of currency i vis-a-vis the US Dollar at time t as $y_{i,t}$, and we collect all of the exchange rates in the N-dimensional vector $Y_t = (y_{1,t}, y_{2,t}, \ldots, y_{N,t})'$. Consider the following vector autoregression:

$$Y_t = \Phi_{0,h} + \Phi_{1,h} Y_{t-h} + e_t; \quad e_t \sim \text{IIDN}(0, \Psi). \quad (1)$$

Note that, unlike Kadiyala and Karlsson (1993, 1997), in the above model Y_t is regressed directly onto Y_{t-h}, which means that for each forecast horizon, h, a different model is employed. Such an approach, which is known as "direct" forecasting, focuses on minimizing the relevant loss function for each forecast horizon, i.e., the h-step-ahead forecast error, while the traditional powering up strategy implies that the only loss function considered is based on the 1-step-ahead forecast error. For a discussion and comparison of these alternative methods, see, e.g., Marcellino, Stock, and Watson (2006).

The h-step-ahead forecast produced by a driftless random walk forecast is $\hat{y}_{i,t+h} = y_{i,t}$. In order to devise a model which produces such a forecast a priori, we just need to impose a priori that $\Phi_{0,h} = 0$ and $\Phi_{1,h} = I$. The restriction $\Phi_{0,h} = 0$ imposes the absence of drift, while the restriction $\Phi_{1,h} = 1$ sets all of the coefficients in each equation to zero except the own lag, which is set to 1, and clearly enforces a univariate random walk representation for each of the variables at hand.

We assume a very tight prior for $\Phi_{0,h}$. In particular, we set its prior mean to zero, with a variance of 10^{-20}. As for Φ_1, we assume that it is conditionally distributed as:

$$\Phi_{1,h} | \Psi \sim N(I, V[\Phi_{1,h}]); \quad V[\Phi_{1,h}^{(ij)}] = \theta \sigma_i^2 / \sigma_j^2, \quad (2)$$

where V is the variance operator and $\Phi_1^{(ij)}$ denotes the element in position (i, j) in the matrix Φ_1.

The prior variance matrix $V[\Phi_{1,h}^{(ij)}]$ depends on the shrinkage parameter θ and on the scaling factors σ_i^2/σ_j^2. The scaling factors account for the difference in scale and variability of the data, and to set the scale parameters σ_i^2 we follow common practice (see e.g. Litterman, 1986, and Sims & Zha, 1998) in setting them equal to the variance of the residuals from a univariate autoregressive model for the variables.

The parameter θ measures the tightness of the prior. When $\theta = 0$, the prior is imposed exactly, the data do not influence the estimates, and the model produces the random walk forecasts, while as $\theta \to \infty$ the prior becomes loose and the posterior estimates approach the OLS estimates. To set the parameter θ, we adopt a data driven procedure, based on past forecasting performance, which we will describe in detail in the next section. For the time being, we stress the fact that the parameter θ will be set to small values, i.e., we will use a tight prior. This allows us to put a lot of weight on the a priori belief that exchange rates follow a driftless random walk. Besides this, as the number of variables in the VAR increases, smaller values of the tightness parameter θ are needed in order to avoid overfitting (see Banbura et al., 2007).

Finally, the prior specification is completed by assuming an inverted Wishart prior distribution for the variance covariance matrix of the errors in Eq. (1), $\Psi \sim iW(\Psi_0, \alpha_0)$, where α_0 and Ψ_0 are such that the prior expectation of Ψ is equal to a fixed diagonal residual variance matrix, as in the traditional Minnesota prior, namely, $\mathrm{E}[\Psi] = \mathrm{diag}(\sigma_1^2, \ldots, \sigma_N^2)$.

To derive the posterior distributions, it is useful to rewrite the VAR in Eq. (1) in the form of a multivariate regression model. Defining $B = (\Phi_{0,h}, \ \Phi_{1,h})'$ and $X_t = (1, Y_{t-h})'$, Eq. (1) can be compactly written as:

$$Y_t = B' X_t + e_t. \quad (3)$$

Rewriting Eq. (3) in data-matrix notation yields:

$$Y = XB + E. \quad (4)$$

In Eq. (4), the observations are by row, and equations by column, so $Y = (Y_1, \ldots, Y_T)'$ is a $T \times N$ matrix of dependent variables and $X = (X_1, \ldots, X_T)'$ is a $T \times M$ matrix of explanatory variables. The matrix $E = (e_1, e_2, \ldots, e_T)'$ is the matrix of disturbances, where the generic column is $\varepsilon_i \sim \text{IIDN}(0, \Psi \otimes I)$.

The normal-inverted Wishart prior takes the form:

$$B | \Psi \sim N(B_0, \Psi \otimes \Omega_0), \quad \Psi \sim IW(\Psi_0, \alpha_0), \quad (5)$$

where the subscript 0 denotes that the parameters are those of the prior distribution. Integrating out Ψ, the marginal distribution of B can be obtained, and it is a matricvariate t-distribution with α_0 degrees of freedom and prior mean B_0: $B \sim MT(\Omega_0^{-1}, \Psi_0, B_0, \alpha_0)$. For a derivation and description of the properties of the matricvariate t-distribution, see Zellner (1973).

The prior at hand can be implemented in the form of dummy variable observations. In particular, the addition of T_d dummy observations Y_d and X_d to the system is equivalent to imposing this prior with $\Omega_0 = (X_d' X_d)^{-1}$, $\Psi_0 = (Y_d - X_d B_0)'(Y_d - X_d B_0)$, $B_0 = (X_d' X_d)^{-1} X_d' Y_d$, and $\alpha_0 = T_d - M - N - 1$. Banbura et al. (2007) give details of how to construct the dummy variables needed to set B_0 equal to the desired values in Eq. (5).

The conditional posterior distributions are also of the normal-inverted Wishart form:

$$B|\Psi, Y \sim N(\bar{B}, \Psi \otimes \bar{\Omega}), \qquad \Psi|Y \sim IW(\bar{\Psi}, \bar{\alpha}), \tag{6}$$

where the bar denotes that the parameters are those of the posterior distribution. Also, in this case, by integrating out Ψ it is possible to obtain the marginal posterior distribution of B, which is again matricvariate t: $B|Y \sim MT(\bar{\Omega}^{-1}, \bar{\Psi}, \bar{B}, \bar{\alpha})$. Given the prior parameters $\Omega_0, \Psi_0, B_0, \alpha_0$ and defining $\hat{B}$ and $\hat{E}$ as the traditional OLS estimates, the posterior parameters are given by $\bar{\Omega} = (\Omega_0^{-1} + X'X)^{-1}$, $\bar{\Psi} = \hat{B}'X'X\hat{B} + B_0'\Omega_0^{-1}B_0 + \Psi_0 + \hat{E}'\hat{E} - \hat{B}'\bar{\Omega}^{-1}\hat{B}$, $\bar{B} = \bar{\Omega}(\Omega_0^{-1}B_0 + X'X\hat{B})$, $\bar{\alpha} = T + \alpha_0$. For a compete derivation see Zellner (1973).

If the prior is specified in the form of dummy observations, the posterior parameters can be computed with a simple OLS regression, after augmenting the model in Eq. (4) with the dummy variables. Defining the augmented model as

$$Y_* = X_* B_* + E_*, \tag{7}$$

the posterior parameters are given by $\bar{\Omega} = (X_*' X_*)^{-1}$, and $\bar{B} = (X_*' X_*)^{-1} X_*' Y_*$ (details are given by Kadiyala & Karlsson, 1997).

We will now use the BVAR model defined so far to forecast a large set of exchange rates.

3. Forecasting exchange rates

In this section we first describe the data and the forecasting exercise, then discuss the results of the forecast evaluation, and finally try to explain the good performance of our BVAR model.

3.1. Forecasting exercise

The data used in the paper are the monthly averages of the exchange rates vis-a-vis the dollar for 33 currencies, and are described in Table 1. All data are taken from Datastream, from three different international sources. The source for each series is identified by an acronym in Table 1. The sources are WMR/Reuters (WMR), Global Trade Information Services (GTIS), and the New York FED (FED). The second column in Table 1 contains a short code which is used to label the currencies. A plot of the data is displayed in Fig. 1.

The forecasting exercise is performed in pseudo real time, using a rolling estimation window of 7 years (84 months), and projecting the models forward up to 12 steps ahead. Using a short rolling estimation window is a natural way to reduce problems of instability (see e.g. Pesaran & Timmermann, 2005, for a discussion), and direct forecasting can also be helpful, since it is in general more robust than the standard iterated method in the presence of model misspecification. The initial estimation window is 1995:1–2000:12 (with data from 1994 used for initialization), and the initial forecast window is 2001:1–2001:12. The last estimation window is 2001:4–2007:4, while the last forecast window is 2007:5–2008:4.

We will evaluate our results in terms of the Mean Squared Forecast Error (MSFE) generated by model M when forecasting the exchange rate (vis-a-vis the US Dollar) of currency i at horizon h. Defining $\hat{y}^M_{i,t+h|t}$ as the h-step-ahead forecast of $y_{i,t+h}$, given the information available at time t, the h-step-ahead forecast error at time t is:

$$\mathrm{FE}^M_{h,t} = \hat{y}^M_{i,t+h|t} - y_{i,t+h}, \tag{8}$$

and the h-step-ahead MSFE is defined as:

$$\mathrm{MSFE}^M_{i,h} = \frac{1}{T_0}\sum_{t=1}^{T_0}\left(\mathrm{FE}^M_{h,t}\right)^2, \tag{9}$$

where T_0 is the total number of computed forecasts.

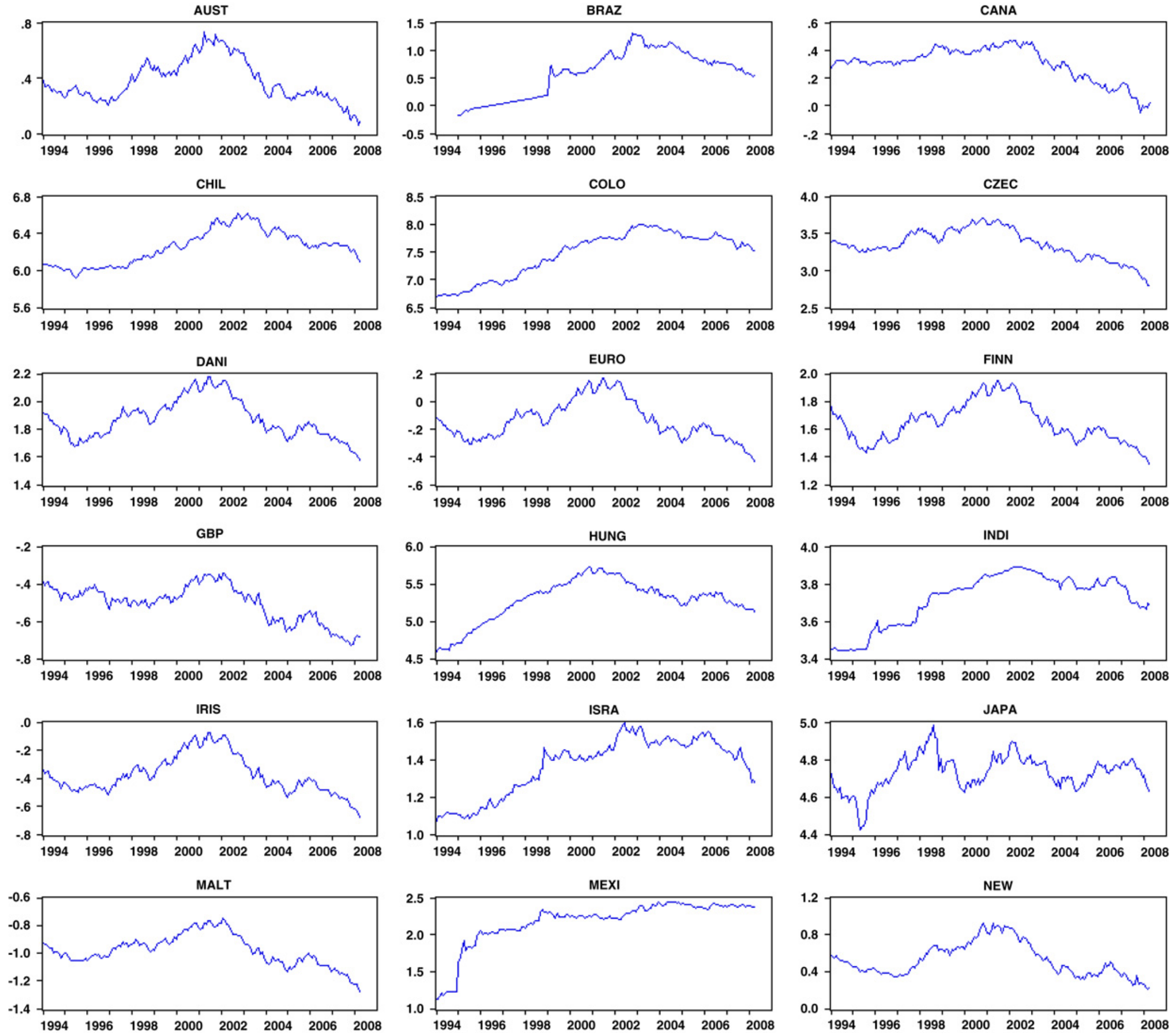

Fig. 1. Monthly exchange rate data (in natural logarithms).

Similarly, the Mean Absolute Forecast Error (MAFE) can be defined as:

$$\mathrm{MAFE}_{i,h}^{M} = \frac{1}{T_0}\sum_{t=1}^{T_0} |FE_{h,t}^{M}|, \tag{10}$$

and it is useful to include it in the evaluation since it assigns a smaller weight to large forecast errors than the MSFE.

The benchmark model is a driftless random walk, which produces the following h-step-ahead forecast of the exchange rate:

$$\hat{y}_{i,t+h} = y_{i,t}. \tag{11}$$

In the case at hand, the BVAR forecast of the vector of exchange rates $Y_t = (y_{1,t}, y_{2,t}, \ldots, y_{N,t})'$ at time $t+h$ is:

$$\hat{Y}_{t+h} = \hat{\Phi}_{0,h} + \hat{\Phi}_{1,h} Y_t \tag{12}$$

where $\hat{\Phi}_{0,h}$ and $\hat{\Phi}_{1,h}$ are the posterior means of the matrices of coefficients in Eq. (1).

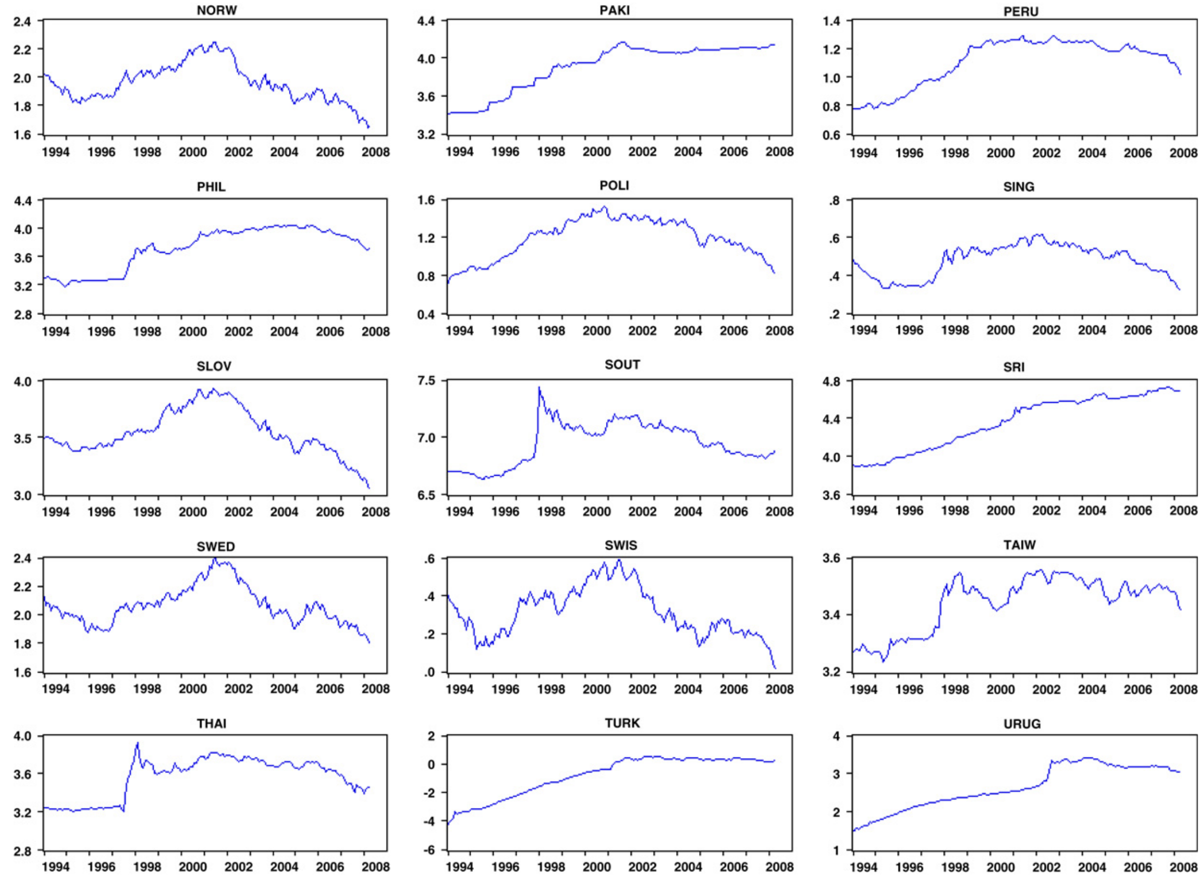

Fig. 1. (*continued*)

The shrinkage parameter θ is chosen period by period using a real time data driven procedure as follows. At each point in time, the BVAR is estimated for a grid of values for θ, then the h-step-ahead forecast is produced with the model based on the value θ^* that provided the smallest total squared forecast error (computed over all the variables) in the previous period:

$$\theta^*_{t,h} = \arg\min_{\theta} \left\{ \sum_{i=1}^{N} (FE_{h,t}^{BVAR})^2 \right\}. \tag{13}$$

The grid used is $\theta \in 10^{-4} * \{0.01, 0.1, 0.5, 1, 1.5, 2, 2.5, 5\}$. Such a grid enforces a tight prior, putting a lot of weight on the a priori belief that exchange rates follow a driftless random walk. Moreover, small values of the tightness parameter allow us to avoid overfitting when the cross sectional dimension of the dataset is large (see Banbura et al., 2007). Finally, note that this data-driven procedure can not be implemented at the beginning of the experiment, not until the h-step-ahead forecast error of the previous period is observed. Hence, until the forecast error for the desired forecast horizon becomes available, we set θ equal to $10^{-4} * 0.01$.

We also include in the comparison a simple autoregressive model, where the lag length L^* is chosen according to the Bayesian Information Criterion (BIC). The reported results are based on AR(L^*) forecasts of the exchange rate of currency i at time $t + h$, obtained as:

$$\hat{y}_{i,t+h} = \hat{\alpha}_{ih} + \hat{\beta}_{ih}(L^*)\hat{y}_{i,t}, \tag{14}$$

Table 1
Data.

NAME	CODE
1 AUSTRALIAN DOLLAR TO US Dollar (FED)– EXCHANGE RATE	AUST
2 BRAZILIAN REAL TO US Dollar (FED) NOON NY – EXCHANGE RATE	BRAZ
3 CANADIAN DOLLAR TO US Dollar (FED) NOON NY – EXCHANGE RATE	CANA
4 CHILEAN PESO TO US Dollar (WMR) – EXCHANGE RATE	CHIL
5 COLOMBIAN PESO TO US Dollar (WMR) – EXCHANGE RATE	COLO
6 CZECH KORUNA TO US Dollar (GTIS) – EXCHANGE RATE	CZEC
7 DANISH KRONE TO US Dollar (FED) NOON NY – EXCHANGE RATE	DANI
8 EURO TO US Dollar (FED)	EURO
9 FINNISH MARKKA TO US Dollar (GTIS) – EXCHANGE RATE	FINN
10 UK £ to US Dollar (WMR) – EXCHANGE RATE	GBPP
11 HUNGARIAN FORINT TO US Dollar (GTIS) – EXCHANGE RATE	HUNG
12 INDIAN RUPEE TO US Dollar (FED) NOON NY – EXCHANGE RATE	INDI
13 IRISH PUNT TO US Dollar (FED) – EXCHANGE RATE	IRIS
14 ISRAELI SHEKEL TO US Dollar (GTIS) – EXCHANGE RATE	ISRA
15 JAPANESE YEN TO US Dollar (FED) NOON NY – EXCHANGE RATE	JAPA
16 MALTESE LIRA TO US Dollar (GTIS) – EXCHANGE RATE	MALT
17 MEXICAN PESO TO US Dollar NOON NY – EXCHANGE RATE	MEXI
18 NEW ZEALAND DOLLAR TO US Dollar – EXCHANGE RATE	NEWZ
19 NORWEGIAN KRONE TO US Dollar NOON NY – EXCHANGE RATE	NORW
20 PAKISTAN RUPEE TO US Dollar (WMR) – EXCHANGE RATE	PAKI
21 PERUVIAN NUEVO SOL TO US Dollar (WMR) – EXCHANGE RATE	PERU
22 PHILIPPINE PESO TO US Dollar (GTIS) – EXCHANGE RATE	PHIL
23 POLISH ZLOTY TO US Dollar (GTIS) – EXCHANGE RATE	POLI
24 SINGAPORE DOLLAR TO US Dollar NOON NY – EXCHANGE RATE	SING
25 SLOVAK KORUNA TO US Dollar (GTIS) – EXCHANGE RATE	SLOV
26 SOUTH KOREAN WON TO US Dollar (FED) NOON NY – EXCHANGE RATE	SOUT
27 SRI LANKAN RUPEE TO US Dollar (FED) NOON NY – EXCHANGE RATE	SRIL
28 SWEDISH KRONA TO US Dollar (FED) NOON NY – EXCHANGE RATE	SWED
29 SWISS FRANC TO US Dollar (FED) NOON NY – EXCHANGE RATE	SWIS
30 TAIWAN new DOLLAR TO US Dollar (FED) NOON NY – EXCHANGE RATE	TAIW
31 THAI BAHT TO US Dollar (FED) NOON NY – EXCHANGE RATE	THAI
32 TURKISH LIRA TO US Dollar (GTIS) – EXCHANGE RATE	TURK
33 URUGUAYAN PESO FIN TO US Dollar (GTIS) – EXCHANGE RATE	URUG

where $\hat{\alpha}_{ih}$ and $\hat{\beta}_{ih}(L^*)$ are the coefficients of a regression of $y_{i,t}$ on $y_{i,t-h}, y_{i,t-h-1}, \ldots, y_{i,t-h-L^*}$. We also considered a more parsimonious AR specification, with a fixed lag length of 1, but the resulting models are outperformed by those based on the BIC lag length selection. These additional results are available upon request.

We also report results for VAR based forecasts. As we will see in the next subsection, such forecasts are definitely worse than those of the other models, but they are of interest as they provide a good illustration of how shrinkage can solve the curse of dimensionality problem, even in large datasets. The VAR forecast at time $t + h$ is given by:

$$\hat{Y}_{t+h} = \hat{A}_h + \hat{B}_h Y_t, \tag{15}$$

where, again, $\hat{A}_h$ and $\hat{B}_h$ are respectively a vector and a matrix of coefficients of a regression of $\hat{Y}_t$ on Y_{t-h}.

Finally, it is interesting to consider an alternative strategy for dealing with the "curse of dimensionality" problem, i.e. using a factor model. In particular, we consider the following specification:

$$\hat{Y}_{t+h} = \hat{A}_h^f + \hat{B}_h^f F_t, \tag{16}$$

where F_t are the first r principal components of the exchange rates at time t. When r is smaller than the cross sectional dimension, this reduces the number of parameters in the model, which might lead to gains in out-of-sample forecast accuracy.

3.2. Results

To facilitate the comparison, we provide results in terms of the Relative Mean Squared Forecast Error (RMSFE) of a given model against the driftless random walk:

$$\text{RMSFE}_{i,h}^{M} = \frac{\text{MSFE}_{i,h}^{M}}{\text{MSFE}_{i,h}^{RW}}. \tag{17}$$

An RMSFE below 1 denotes that the model at hand outperforms the RW in out-of-sample forecast accuracy. The results of the forecasting experiment are summarized in Tables 2 and 4–6 for, respectively, the BVAR, the AR(L^*), the VAR, and the factor model (each against the random walk). Each row in the tables refers to a different variable (exchange rate of currency i vis-a-vis the US Dollar), and each column to a different forecast horizon, ranging from 1 to 12. The last rows in the tables report the average RMSFE computed over all of the currencies for each forecast horizon.

The main message from Table 2 is that, overall, the BVAR with the random walk prior produces fairly good forecasts. In particular, the average RMSFE across currencies is below 1 for all forecast horizons, with gains ranging from 2% to 4%. The pattern of the gains has a U-shape; namely, there are gains around 2% at both very short and very long forecast horizons, and larger gains at intermediate forecast horizons.

These results are confirmed by a more disaggregate investigation, which reveals that the BVAR with the random walk prior outperforms the random walk for most currencies and forecast horizons. In particular, for $h = 1$ the BVAR outperforms the random walk in 30 cases out of 33 (the three exceptions being Mexico, Uruguay, and Taiwan). For $h = 3$ and 6 the BVAR is better in 24 cases out of 33, and in 23 cases for $h = 12$.

It is also interesting to focus on the forecasting performance for some prominent currencies, such as the euro, the GB pound, and the yen. For the Euro–Dollar and the GBP–Dollar exchange rates, the BVAR outperforms the random walk at all horizons, with gains of respectively 2% and 1.4% for $h = 1$, 2.7% and 4.8% for $h = 3$, and up to 6.9% and 9.6% for $h = 6$. For $h = 12$, the gain in forecasting the Euro–Dollar rate is 2.4% and 1% for the GBP–Dollar rate. For the Yen–Dollar rate the evidence is more mixed, with the BVAR providing better forecasts only at longer horizons, with smaller gains.

Regarding the two major trading partners of the US, Canada and Mexico, the BVAR performs very well for the former country, with gains ranging from 1% for $h = 1$ to 17% for $h = 12$, and only slightly worse for the latter country at short horizons, with losses smaller than 4% and gains of about 1.5% for $h = 12$.

Finally, the stars in the table denote rejection of the null of equal forecast accuracy of the models at 1%, 5%, and 10%, according to the Giacomini and White (2006) statistic. This is a test of equal forecasting accuracy, and as such can handle forecasts based on both nested and non-nested models, regardless of the estimation procedures used for the derivation of the forecasts, including Bayesian methods. As is clear, although the RMSFE across currencies is below 1 in several instances, in only a few cases are the differences in the forecasts statistically significant.

Fig. 2 displays the time path of forecast errors for these five currencies (respectively the Canadian Dollar, Euro, GB Pound, Yen, and Mexican Peso). Each figure plots the forecast errors for the BVAR and the RW, for 3- 6- 9- and 12-step-ahead forecast horizons. Interestingly, the main differences in the errors from BVAR and RW arise at the end of the evaluation sample, i.e. around 2007–2008, which is a period characterized by large swings in the US Dollar exchange rate. As we shall see in the next subsection, in this period the cross-sectional information is important.

Next, we evaluate whether the good performance of the BVAR is robust to a change in the loss function. In Table 3 we report results for the RMAFE, where large forecast errors receive a smaller weight than in the RMSFE. The overall picture is unaffected. From the average across currencies results, the BVAR still outperforms the random walk at each forecast horizon, with gains in the range of 1%–3%. From the detailed country by country results, the BVAR remains better for the vast majority of countries and forecast horizons. As for the statistical difference in the forecasts, it is again limited to only a few cases.

Moving now to the forecasting performance of the AR models, Table 4 shows that the average RMSFE is above 1 for all of the forecast horizons, signaling that overall the AR model is not able to outperform the random walk. This finding is confirmed by the country

Table 2
RMSFE of BVAR with the driftless random walk prior vs the random walk.

Cou:	Hor:											
	1	2	3	4	5	6	7	8	9	10	11	12
AUST	0.983	0.987	0.978	0.949	0.939	0.915	0.911	0.927	0.910	0.911	0.907	0.928
BRAZ	0.995	0.983	0.973	0.962	0.985	1.002	1.013	1.001	0.960	0.916	0.897	0.900
CANA	0.989	0.990	0.979	0.967	0.957	0.941	0.917	0.899	0.875	0.857	0.833	0.831
CHIL	0.998	0.999	0.981	0.964	0.978	0.992	1.004	1.011	0.993	0.959	0.943	0.917
COLO	0.994	1.002	1.023	1.021	1.023	1.032	1.040	1.032*	0.982	0.932	0.928	0.914
CZEC	0.978	0.970	0.968	0.967	0.959	0.943	0.936	0.943	0.934	0.927	0.899	0.896
DANI	0.980	0.965	0.971	0.985	0.965	0.931	0.915	0.927	0.941	0.955	0.959	0.975
EURO	0.980	0.966	0.973	0.985	0.966	0.931	0.917	0.928	0.941	0.955	0.959	0.976
FINN	0.980	0.966	0.974	0.983	0.965	0.930	0.916	0.931	0.944	0.957	0.961	0.979
GBPP	0.986	0.973	0.952	0.934	0.915	0.904	0.904	0.901	0.913	0.944	0.969	0.990
HUNG	0.985	0.999	1.019	1.005	0.970	0.935	0.926	0.945	0.950	0.974	0.991	1.012
INDI	0.981	0.969	0.962	0.941	0.932	0.920	0.919	0.922	0.931	0.945	0.951	0.966
IRIS	0.981	0.968	0.975	0.982	0.963	0.929	0.916	0.929	0.941	0.953	0.958	0.976
ISRA	0.996	1.005	1.015	1.017	1.030	1.005	0.989	0.983	0.997	0.989	1.012	1.016
JAPA	0.998	1.005	1.008	1.014**	1.010	0.995	0.989	0.993	0.999	0.993	0.990	0.998
MALT	0.978	0.969	0.960	0.953	0.933	0.902*	0.898	0.899	0.904	0.920	0.924	0.936
MEXI	1.012	1.034	1.035	1.015	1.005	1.015	1.037	1.020	1.009	0.998	1.001	0.985
NEWZ	0.991	0.992	0.989	0.984	0.978	0.949	0.932	0.939	0.926	0.939	0.950	0.971
NORW	0.994	0.988	0.997	0.999	0.969	0.932	0.926	0.948	0.947	0.960	0.957	0.958
PAKI	0.933	0.838	0.884	1.056	1.254	1.367	1.357	1.362	1.372	1.428	1.461	1.481
PERU	0.987	0.973	1.066	1.097	1.090	1.035	1.008	0.982	0.890	0.837	0.878	0.919
PHIL	0.980	0.949	0.958	1.048	1.085	1.109	1.125	1.128	1.102	1.070	1.036***	1.017
POLI	0.988	0.980	0.945**	0.912***	0.924	0.912	0.914	0.905	0.872	0.869	0.861	0.866
SING	0.966*	0.961*	0.983	0.992	0.943	0.896*	0.867*	0.896	0.933	0.943	0.937	0.941
SLOV	0.952*	0.922	0.900	0.888	0.871	0.846	0.840	0.839	0.846	0.879	0.891	0.910
SOUT	0.974	1.001	1.012	1.008	1.010	0.995	1.000	1.064	1.113	1.119	1.144	1.192
SRIL	0.975	0.933	0.925*	0.941	0.888*	0.828**	0.788**	0.753**	0.793*	0.850	0.898	0.934
SWED	0.978	0.968	0.966	0.970	0.965	0.957	0.962	0.981	0.990	1.008	1.010	1.018
SWIS	0.994	0.990	1.006	1.033	1.024	0.997	0.993	1.003	1.012	1.011	1.001	1.000
TAIW	1.002	0.983	0.983	0.999	1.011	1.004	1.001	1.031	1.047	1.017	1.008	1.007
THAI	0.930**	0.919*	0.924	0.924	0.929	0.920	0.907	0.921	0.924	0.915	0.916	0.934
TURK	0.913	0.936	0.942	0.896	0.911	0.993	1.059	1.083	1.061	1.071***	1.087***	1.079
URUG	1.012*	1.013	1.010	1.011	1.014	1.019	1.021	1.019	0.998	0.989	1.001	1.006
Average	0.981	0.972	0.977	0.982	0.981	0.969	0.965	0.971	0.968	0.969	0.973	0.983

* Denotes rejection of the null of equal forecast accuracy at the 10% level, according to the Giacomini and White (2006) test.
** Denotes rejection of the null of equal forecast accuracy at the 5% level, according to the Giacomini and White (2006) test.
*** Denotes rejection of the null of equal forecast accuracy at the 1% level, according to the Giacomini and White (2006) test.

by country results, in which the AR outperforms the random walk in only a few cases. However, interestingly, the AR performs well for Taiwan and Japan, with large gains for the Japanese Yen, two exchange rates where the BVAR was not so good. Looking at Fig. 1, the explanation for this result seems to be the large variability in these two exchange rates that most likely requires more dynamics in the model, which is allowed in the AR case, but not in the BVAR specification, where only one lag is included.

Table 5 shows that the VAR produces very poor forecasts for all of the currencies in the panel. This finding, combined with the good performance of the BVAR, indicates that there is some relevant information in the joint dynamics of the exchange rates under analysis, which is lost in the random walk models, but also in the large unconstrained parameterization of the VAR.

As discussed in the previous section, an alternative way to efficiently summarize the information in a large

Table 3
RMAFE of BVAR with the driftless random walk prior vs the random walk.

Cou:	Hor: 1	2	3	4	5	6	7	8	9	10	11	12
AUST	1.001	0.977	0.985	0.957	0.950	0.926	0.929	0.923	0.923	0.925	0.925	0.933
BRAZ	0.994	0.989	0.978	0.970	0.982	1.001	1.009	1.004	0.968	0.962	0.966	0.966
CANA	0.997	1.005	1.019	0.999	0.992	0.980	0.968	0.967	0.956	0.943	0.933	0.910
CHIL	0.997	0.992	0.993	0.985	0.988	1.008	1.020	1.019	1.001	0.962	0.933	0.927
COLO	0.987	1.005	1.032	1.034	1.041	1.047	1.042	1.035	0.987	0.966	0.967	0.965
CZEC	0.975	0.985	0.986	0.955	0.952	0.927	0.929	0.931	0.913	0.909	0.910	0.924
DANI	0.987	0.994	0.967	0.959	0.953	0.916	0.901	0.925	0.926	0.925	0.930	0.938
EURO	0.989	0.998	0.969	0.962	0.955	0.918	0.904	0.929	0.928	0.926	0.930	0.938
FINN	0.991	0.989	0.970	0.962	0.954	0.918	0.906	0.926	0.926	0.924	0.930	0.939
GBPP	0.993	0.996	0.941*	0.946	0.928	0.911	0.909	0.907	0.932	0.948	0.972	0.995
HUNG	0.978*	1.011	1.010	1.003	1.002	0.967	0.971	0.958	0.950	0.952	0.956	0.962
INDI	0.993	0.977	0.986	0.962	0.962	0.953	0.952	0.966	0.975	0.991	1.004	1.011
IRIS	0.991	0.991	0.971	0.961	0.953	0.917	0.904	0.925	0.925	0.923	0.929	0.938
ISRA	0.996	1.002	1.006	1.005	1.012	1.003	1.003	1.003	1.000	0.998	1.000	0.996
JAPA	1.002	1.003	1.005	1.018*	1.014	1.001	0.998	0.995	1.004	1.001	0.994	0.997
MALT	0.987	1.001	0.962	0.954	0.936	0.905	0.900	0.919	0.924	0.925	0.930	0.938
MEXI	1.007	1.024	1.027	1.025	1.009	1.029	1.042	1.031	1.033	1.030	1.035	1.038
NEWZ	0.981	0.991	1.000	0.970	0.973	0.955	0.960	0.956	0.931	0.930	0.930	0.937
NORW	0.987	0.991	0.987	0.993	0.970	0.978	0.959	0.946	0.936	0.954	0.948	0.939
PAKI	0.992	0.946	0.969	1.030	1.068	1.101	1.098	1.106	1.110	1.126	1.136	1.139
PERU	1.002	0.973	0.995	0.996	0.994	0.999	0.996	0.981	0.936	0.891*	0.941	0.982
PHIL	0.992	0.969	0.980	1.028	1.040	1.055	1.075	1.082	1.082	1.066	1.028**	0.999
POLI	0.995	0.990	0.961*	0.922***	0.911*	0.919	0.921	0.933	0.925	0.928	0.938	0.947
SING	0.983	0.980	0.985	0.995	0.966	0.927*	0.905*	0.932	0.949	0.958	0.962	0.968
SLOV	0.961***	0.963	0.923*	0.920	0.919	0.891	0.892	0.902	0.905	0.914	0.921	0.929
SOUT	1.008	1.003	1.011	1.016	1.028	1.010	0.999	1.023	1.052	1.078	1.097	1.086
SRIL	0.995	0.943**	0.943	0.946	0.909*	0.855***	0.858*	0.832**	0.866*	0.903	0.945	0.970
SWED	0.999	0.996	0.978	0.973*	0.974	0.961	0.975	0.970	0.953	0.949	0.949	0.957
SWIS	1.002	1.010	1.023	1.020	1.016	0.995	1.003	1.006	0.998	0.990	0.979	0.971
TAIW	1.013**	0.996	0.995	0.993	1.011	1.007	1.006	1.018	1.028	1.015	1.011	1.022
THAI	0.969**	0.968	0.967	0.975	0.994	0.980	0.976	0.980	0.982	0.990	0.995	0.998
TURK	0.979	0.971	0.965	0.956	0.952	0.994	1.035	1.074	1.056	1.060***	1.064***	1.064*
URUG	1.048***	1.054	1.051	1.045	1.066	1.079	1.084	1.083	1.081	1.077	1.076	1.069
Average	0.993	0.990	0.986	0.983	0.981	0.971	0.971	0.975	0.972	0.971	0.975	0.978

* Denotes rejection of the null of equal forecast accuracy at the 10% level, according to the Giacomini and White (2006) test.
** Denotes rejection of the null of equal forecast accuracy at the 5% level, according to the Giacomini and White (2006) test.
*** Denotes rejection of the null of equal forecast accuracy at the 1% level, according to the Giacomini and White (2006) test.

dataset is to use a factor model. Table 6 reports the results of the factor model with four factors.[1] As is clear from the table, the factor model is outperformed by the RW, but systematically (with some noticeable exceptions for some currencies at the 1-step-ahead horizon) gives better forecasts than the unrestricted VAR. This confirms that using a factor structure does help in efficiently summarizing the information contained in the dataset, but also that shrinking towards a RW provides better results for the dataset at hand.

3.3. Understanding the BVAR results

To provide a better understanding of the BVAR results, we focus on the θ parameter, which, as described in Section 2, controls the overall shrinkage

[1] We considered all specifications from $r = 1$ to $r = 5$, but to economize on space we only report the results for the case $r = 4$, as this specification provided the best overall forecasting performance. Results for the remaining specifications are available upon request.

Table 4
RMSFE of AR(L*) vs the random walk.

Cou:	Hor: 1	2	3	4	5	6	7	8	9	10	11	12
AUST	1.063	1.107	1.155	1.196	1.337	2.215	2.236	2.256	2.259	2.324	2.285	2.254
BRAZ	1.054	1.119	1.169	1.233	1.272	1.286	1.296	1.297	1.316	1.339	1.392	1.457
CANA	1.072	1.111	1.145	1.190	1.210	1.195	1.177	1.227	1.316	1.398	1.716	1.873
CHIL	1.082	1.124	1.194	1.244	1.302	1.346	1.629	1.755	1.833	1.850	1.936	2.116
COLO	1.097	1.076	1.125	1.169	1.275	1.395	1.367	1.309	1.347	1.366	1.399	1.412
CZEC	1.056	1.122	1.189	1.261	1.305	1.365	1.431	1.489	1.536	1.585	1.802	1.932
DANI	1.077	1.152	1.194	1.210	1.294	1.412	1.496	1.591	1.619	1.644	1.775	1.963
EURO	1.081	1.156	1.196	1.208	1.336	1.423	1.452	1.578	1.605	1.648	1.775	1.949
FINN	1.073	1.149	1.193	1.213	1.260	1.386	1.497	1.581	1.607	1.634	1.798	1.919
GBPP	1.072	1.143	1.196	1.239	1.255	1.286	1.341	1.410	1.453	1.475	1.488	1.370
HUNG	1.085	1.161	1.257	1.338	1.519	1.689	1.699	1.671	1.736	1.845	1.827	1.775
INDI	1.032	1.117	1.154	1.164	1.237	1.232	1.190	1.148	1.145	1.136	1.132	1.174
IRIS	1.088	1.167	1.213	1.235	1.298	1.429	1.535	1.663	1.712	1.742	1.976	2.069
ISRA	1.044	1.093	1.135	1.155	1.187	1.213	1.246	1.278	1.286	1.303	1.317	1.302
JAPA	0.952	0.910	0.873	0.829	0.786	0.777	0.735	0.801	0.811	0.767	0.707	0.581
MALT	1.081	1.150	1.194	1.219	1.240	1.358	1.426	1.486	1.469	1.537	1.928	2.461
MEXI	1.131	1.209	1.243	1.344	1.560	1.830	2.056	2.184	2.292	2.295	2.165	2.051
NEWZ	1.086	1.169	1.240	1.289	1.360	1.873	1.989	2.109	2.395	2.311	2.344	2.392
NORW	1.059	1.094	1.118	1.146	1.210	1.251	1.342	1.428	1.513	1.631	1.754	1.898
PAKI	1.005	1.046	1.046	1.135	1.201	1.351	1.593	1.916	2.325	2.802	3.232	3.520
PERU	1.111	1.310	1.515	1.650	1.749	1.797	1.788	1.757	1.792	1.822	1.833	1.762
PHIL	1.055	1.123	1.174	1.217	1.275	1.338	1.424	1.522	1.605	1.643	1.665	1.694
POLI	1.100	1.246	1.417	1.511	1.515	1.477	1.423	1.380	1.365	1.327	1.314	1.280
SING	1.026	1.037	1.070	1.069	1.090	1.069	1.095	1.119	1.140	1.154	1.177	1.232
SLOV	1.103	1.201	1.283	1.332	1.380	1.462	1.565	1.683	1.787	1.903	1.899	1.872
SOUT	0.965	1.070	1.130	1.204	1.335	1.448	1.578	1.764	2.004	2.238	2.519	2.748
SRIL	0.996	0.979	1.024	1.100	1.162	1.271	1.397	1.468	1.565	1.654	1.762	2.696
SWED	1.055	1.094	1.135	1.381	1.221	1.253	1.281	1.495	1.758	2.031	2.310	2.488
SWIS	1.047	1.107	1.162	1.202	1.265	1.328	1.394	1.645	1.921	2.012	2.050	2.045
TAIW	0.952	1.008	0.946	0.917	0.901	0.916	0.945	1.016	1.122	1.215	1.230	1.292
THAI	0.998	1.019	1.054	1.113	1.186	1.235	1.285	1.300	1.281	1.287	1.299	1.313
TURK	0.961	0.915	0.886	0.899	0.940	0.993	1.040	1.070	1.157	1.986	2.099	2.415
URUG	2.128	7.692	6.224	3.353	1.958	2.419	10.514	15.832	21.689	23.396	21.218	20.427
Average	1.052	1.109	1.157	1.207	1.264	1.372	1.436	1.512	1.596	1.685	1.778	1.884

* The average does not include the Uruguay Peso.

of the BVAR parameters toward the random walk prior. As mentioned, the value of θ is chosen empirically, period by period, using a real time data driven procedure. At each point in time, the BVAR is estimated for a grid of values for θ, then the h-step-ahead forecast is produced with the model based on the value which provided the smallest total forecast error in the previous period, see Eq. (13). Therefore, it is worth looking at the time path of the selected $\theta^*_{t,h}$, which is depicted in Fig. 3. The figure displays 12 panels, each corresponding to a different forecast horizon, and each panel reports the time path of the selected θ. The figure indicates that, for shorter forecast horizons, the tightness parameter changes substantially, while at longer horizons it tends to stay fixed at a given value, and, noticeably, an extremely tight prior is chosen for long horizons. At the end of the sample, when large swings in the US Dollar exchange rate are observed, the selected value of θ tends to be higher, signaling a stronger effect of the information contained in the cross-section.

One might then wonder whether the BVAR forecasting gains come simply from the fact that choosing the optimal value of θ in real time allows more flexibility than the $\theta = 0$ choice underlying

Table 5
RMSFE of VAR vs the random walk.

Cou:	Hor: 1	2	3	4	5	6	7	8	9	10	11	12
AUST	2.571	4.980	4.929	3.759	5.936	3.447	4.941	3.809	3.721	5.601	3.860	1.947
BRAZ	4.537	6.172	5.188	2.725	9.782	3.345	4.546	10.506	18.543	3.610	3.472	5.764
CANA	2.664	3.480	3.623	2.956	2.745	1.839	1.891	3.344	1.567	1.785	1.730	1.794
CHIL	2.523	3.652	4.756	4.421	3.327	1.725	2.367	3.302	3.683	3.824	2.834	8.282
COLO	2.764	4.124	4.924	3.940	4.721	3.965	4.844	4.132	5.408	2.276	7.484	11.258
CZEC	3.042	3.569	4.571	5.565	2.984	3.691	4.889	4.605	3.054	3.227	3.539	7.879
DANI	2.416	2.473	3.339	4.054	3.272	3.080	3.619	3.559	2.301	5.344	5.449	5.818
EURO	2.423	2.402	3.273	4.050	3.074	3.086	3.782	3.871	2.226	5.190	5.401	5.757
FINN	2.489	2.585	3.719	4.498	3.232	2.978	3.932	3.959	2.257	5.318	5.404	5.396
GBPP	1.752	2.466	2.751	4.531	4.923	3.717	3.086	1.634	3.003	2.723	4.266	4.032
HUNG	2.189	2.145	3.347	5.896	3.041	2.137	2.353	3.919	4.278	5.158	6.072	4.139
INDI	1.908	1.656	2.135	2.061	2.075	2.754	2.095	2.588	1.925	3.036	3.287	2.924
IRIS	2.419	2.696	3.390	4.513	3.396	3.160	3.060	4.028	2.471	5.708	5.819	5.072
ISRA	3.035	2.801	3.012	2.812	3.011	3.750	2.946	2.826	6.577	1.923	3.077	3.469
JAPA	4.189	7.632	4.682	8.202	2.654	6.467	9.377	11.774	4.513	8.036	16.711	4.225
MALT	2.233	2.173	2.474	3.777	3.158	2.778	2.929	2.691	1.717	2.760	4.720	4.323
MEXI	2.805	3.529	4.029	1.862	2.574	6.420	4.601	3.493	2.669	6.060	5.881	11.151
NEWZ	2.198	3.952	6.263	5.641	3.940	2.162	2.444	2.968	3.477	4.523	4.870	3.666
NORW	2.117	2.554	2.967	4.122	2.754	3.309	2.094	3.050	2.326	4.067	5.272	4.683
PAKI	5.913	7.836	7.823	5.567	3.397	5.310	11.283	5.872	26.388	19.549	12.491	16.269
PERU	2.943	4.612	3.360	4.469	5.054	3.965	2.935	7.328	12.903	14.545	13.861	6.304
PHIL	3.915	5.630	6.932	13.449	11.839	8.175	6.643	14.826	16.792	16.201	19.065	10.096
POLI	2.653	4.265	4.419	4.593	2.308	3.053	7.163	6.192	3.295	3.892	4.498	3.625
SING	3.304	6.471	8.035	7.562	3.541	7.997	3.971	5.943	10.751	4.760	6.402	9.582
SLOV	2.258	2.453	3.308	4.222	3.299	2.356	3.015	2.559	3.494	3.914	4.826	4.968
SOUT	5.178	10.939	13.725	12.667	15.298	18.051	19.892	15.492	16.327	4.745	21.364	14.198
SRIL	3.147	2.510	4.795	6.502	9.857	7.356	5.477	3.738	4.602	6.805	1.920	1.011
SWED	2.368	2.582	3.072	3.443	2.408	2.383	2.073	2.537	1.393	3.095	3.662	4.835
SWIS	2.441	2.304	3.205	4.360	2.995	2.837	4.393	4.463	2.284	5.128	6.803	5.719
TAIW	4.405	5.763	4.457	5.984	7.926	7.684	3.812	3.698	5.689	7.649	15.161	11.945
THAI	5.643	11.687	12.560	13.299	7.116	6.201	7.435	9.697	15.300	8.172	10.010	5.611
TURK	1.418	2.291	2.286	2.234	3.744	5.017	9.490	7.304	5.724	2.912	5.075	9.498
URUG	1.805	2.243	4.627	6.260	4.125	4.836	4.646	4.692	4.013	7.248	4.811	2.221
Average	2.959	4.140	4.727	5.273	4.652	4.516	4.910	5.285	6.202	5.721	6.942	6.287

the random walk model. In other words, changing the value of θ allows us to use the random walk when the cross-sectional information is not relevant or persistence is substantial (or when the random walk specification is convenient to reduce the negative forecasting effects of structural changes), and switch to alternative models in the remaining periods. This consideration is correct, but it is important to stress that the alternative specifications (based on values of $\theta \neq 0$) must in any case be strongly shrunk toward the random walk prior in order for this strategy to produce gains (i.e., θ has to be very small in any case).

To provide further evidence on this point, we have repeated the forecasting exercise for the random walk and AR(L^*) models only, selecting between the two specifications at each point in time based on their past squared forecast error averaged across all countries, as for the θ selection in the case of the BVAR. In practice, cross-sectional information is excluded a priori, but the persistence is allowed to vary, even more than in the BVAR case. As a result, the AR(L^*) model was *never* selected, and this strategy simply produces a random walk forecast. Therefore, the general forecasting advantages from the BVAR are related to the possibility of using cross-sectional information when needed, rather than changing persistence in the exchange rate processes.

Table 6
RMSFE of PC vs the random walk.

Cou:	Hor: 1	2	3	4	5	6	7	8	9	10	11	12
AUST	2.093	1.441	1.195	0.927	0.762	0.671	0.585	0.618	0.711	0.900	1.136	1.352
BRAZ	3.012	2.198	1.880	1.825	1.596	1.285	1.114	1.044	1.122	0.901	0.758	0.631
CANA	5.591	3.118	2.422	1.893	1.482	1.090	0.877	0.694	0.545	0.431	0.385	0.395
CHIL	3.827	2.261	1.735	1.346	1.144	1.012	0.887	0.787	0.708	0.591	0.441	0.358
COLO	6.973	3.733	2.820	2.455	2.208	1.929	1.634	1.461	1.482	1.544	1.722	1.814
CZEC	2.578	2.278	2.265	2.357	2.442	2.523	2.674	2.705	2.834	2.843	2.836	2.850
DANI	1.087	0.867	0.886	1.069	1.217	1.393	1.665	1.989	2.383	2.616	2.819	2.968
EURO	1.055	0.852	0.879	1.067	1.217	1.385	1.653	1.970	2.367	2.601	2.809	2.973
FINN	1.020	0.868	0.896	1.077	1.219	1.378	1.649	1.970	2.364	2.590	2.768	2.919
GBPP	1.958	1.405	1.345	1.239	1.163	1.131	1.117	1.153	1.219	1.226	1.265	1.309
HUNG	2.994	2.393	2.383	2.524	2.642	2.743	2.875	3.050	3.311	3.465	3.597	3.669
INDI	2.508	1.316	0.974	0.822	0.788	0.789	0.837	0.893	0.928	0.922	0.998	1.119
IRIS	0.978	0.905	0.988	1.210	1.337	1.473	1.718	2.034	2.453	2.678	2.844	2.999
ISRA	7.531	3.899	3.022	2.595	2.517	2.516	2.415	2.203	1.888	1.572	1.355	1.290
JAPA	7.903	5.109	4.302	3.638	3.456	3.726	3.734	4.594	4.687	4.814	4.713	4.252
MALT	1.252	0.902	0.877	0.994	1.053	1.108	1.224	1.423	1.698	1.886	2.072	2.175
MEXI	5.104	2.284	1.545	1.354	1.418	1.623	1.808	1.933	1.877	1.646	1.547	1.485
NEWZ	1.956	1.322	1.035	0.765	0.662	0.665	0.813	1.002	1.294	1.557	1.783	2.053
NORW	2.037	1.649	1.659	1.845	1.935	2.029	2.248	2.415	2.489	2.522	2.474	2.423
PAKI	18.067	10.558	7.070	5.890	5.218	5.050	5.579	6.589	7.661	9.718	10.873	11.291
PERU	21.945	12.036	10.160	8.790	7.303	5.943	5.288	5.278	5.622	5.725	5.776	4.913
PHIL	13.181	7.456	6.660	5.984	7.101	8.400	8.127	7.698	6.568	4.637	4.010	3.453
POLI	5.329	3.716	3.292	3.423	3.431	3.362	3.273	3.062	3.085	3.033	3.145	2.990
SING	2.587	1.950	1.854	1.567	1.390	1.390	1.409	1.490	1.554	1.562	1.544	1.645
SLOV	2.174	2.040	2.195	2.381	2.418	2.460	2.513	2.527	2.663	2.652	2.662	2.642
SOUT	5.190	2.752	2.312	2.344	3.266	3.625	3.949	4.201	4.435	4.506	4.030	3.135
SRIL	7.547	4.480	5.015	4.470	3.823	3.471	3.320	3.316	3.326	3.273	3.415	3.857
SWED	1.777	1.359	1.331	1.362	1.345	1.344	1.415	1.530	1.629	1.758	1.834	1.887
SWIS	1.281	1.126	1.178	1.359	1.546	1.753	2.025	2.347	2.667	2.725	2.770	2.864
TAIW	5.027	2.450	1.947	1.696	1.796	2.151	2.461	3.167	3.776	4.027	4.059	3.885
THAI	8.472	4.849	4.261	4.043	4.382	4.596	3.829	3.322	2.952	2.747	2.666	2.681
TURK	1.466	1.332	1.313	1.362	1.464	1.556	1.579	1.598	1.743	1.934	2.302	2.674
URUG	5.078	2.991	2.538	2.245	2.017	1.815	1.651	1.523	1.431	1.357	1.298	1.219
Average	4.866	2.966	2.552	2.361	2.326	2.345	2.362	2.472	2.590	2.635	2.688	2.672

3.4. The role of emerging countries and trading strategies

This section considers two issues. First, we explore in more detail the nature of the cross-sectional information picked up by the BVAR, and in particular we try to assess whether there is cross-sectional dependence among groups of currencies. Second, we also evaluate the economic value of the BVAR forecasts by implementing a very simple trading strategy based on the BVAR forecasts.

For the first issue, we split the sample into ‘developed’ and ‘emerging’ currencies, and run the forecasting exercise again using two BVARs (one for each group) of smaller dimensions. Such an experiment allows us to explore whether the original BVAR model is picking up cross-sectional information among developed currencies, or cross-sectional dependence among emerging currencies, or links between developed and emerging currencies.

The results of the experiment are displayed in Table 7. An interesting pattern emerges. While the developed countries do systematically better when the large BVAR is used, the results are more mixed for emerging countries (though the results are still

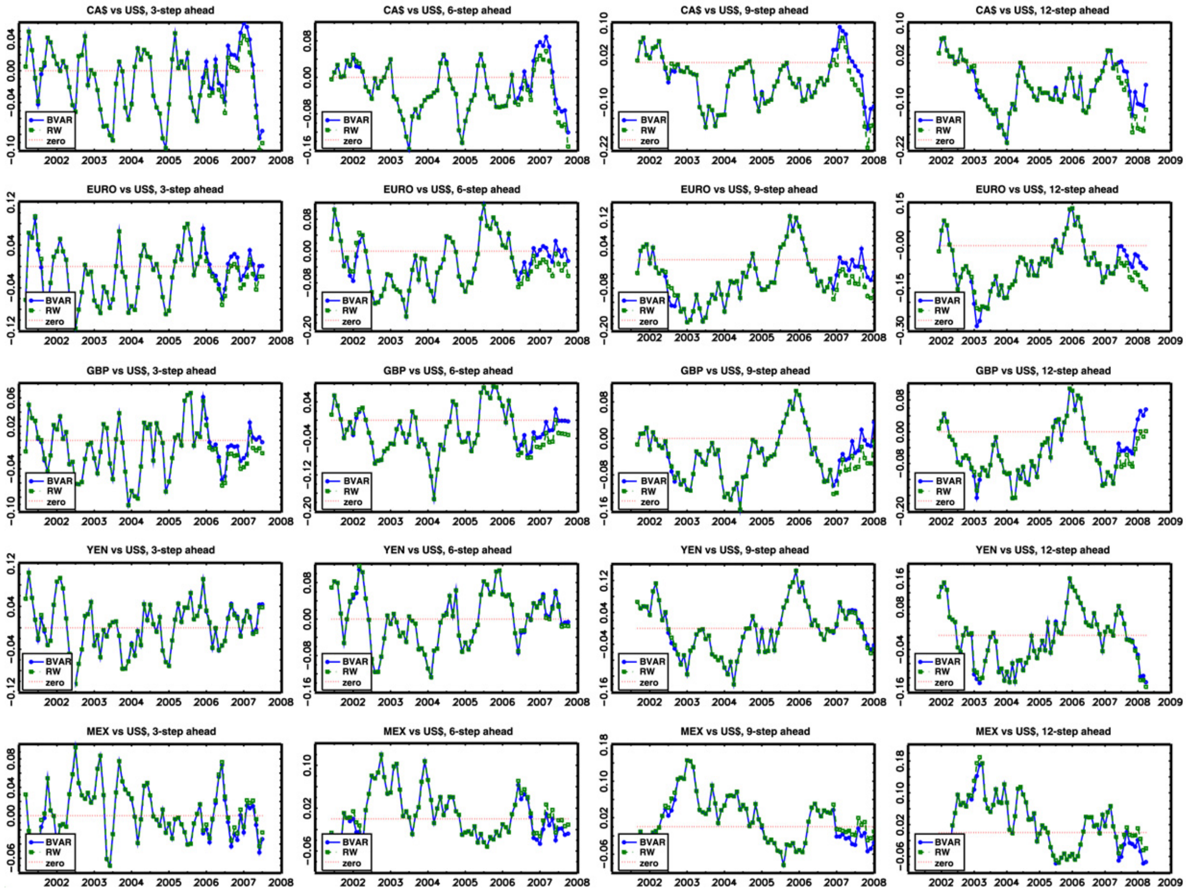

Fig. 2. Forecast errors for different forecast horizons.

better than the RW for the majority of cases). In particular, the results for the emerging countries are often similar, and in some cases better, when only the smaller BVAR of emerging countries is used. This suggests that the information on the emerging countries is critical in improving the forecasting accuracy of the BVAR. Such a result is likely to be linked to the faster pace of globalization and the larger and larger role that the emerging countries are playing in the world economy, considering that our forecast sample covers only the period after 2001.

We now turn to the second issue, namely considering the gains obtained by using a trading strategy based on the BVAR forecasts. Such exercise is close in spirit to that proposed by De Zwart, Markwat, Swinkels, and van Dijk (2007), although we use a simpler strategy, working as follows.[2] The investor owns capital in US dollars, and at each point in time takes the decision on whether to invest it in a foreign currency. The investment decision is based on the prediction made by a model (we consider the BVAR and the AR): if the model predicts that the foreign currency will appreciate, then the investor will go short in dollars and long in the foreign currency, while if the model predicts a depreciation the investor will hold his position and stay long in dollars. We assume that at each point in time the investor realizes the gain/loss and reinvests the initial capital.

Table 8 displays the results of such a trading strategy for the BVAR and the AR. For each of the two

[2] See De Zwart et al. (2007) for a discussion of more complex trading strategies based on both fundamentals and chartists analysis.

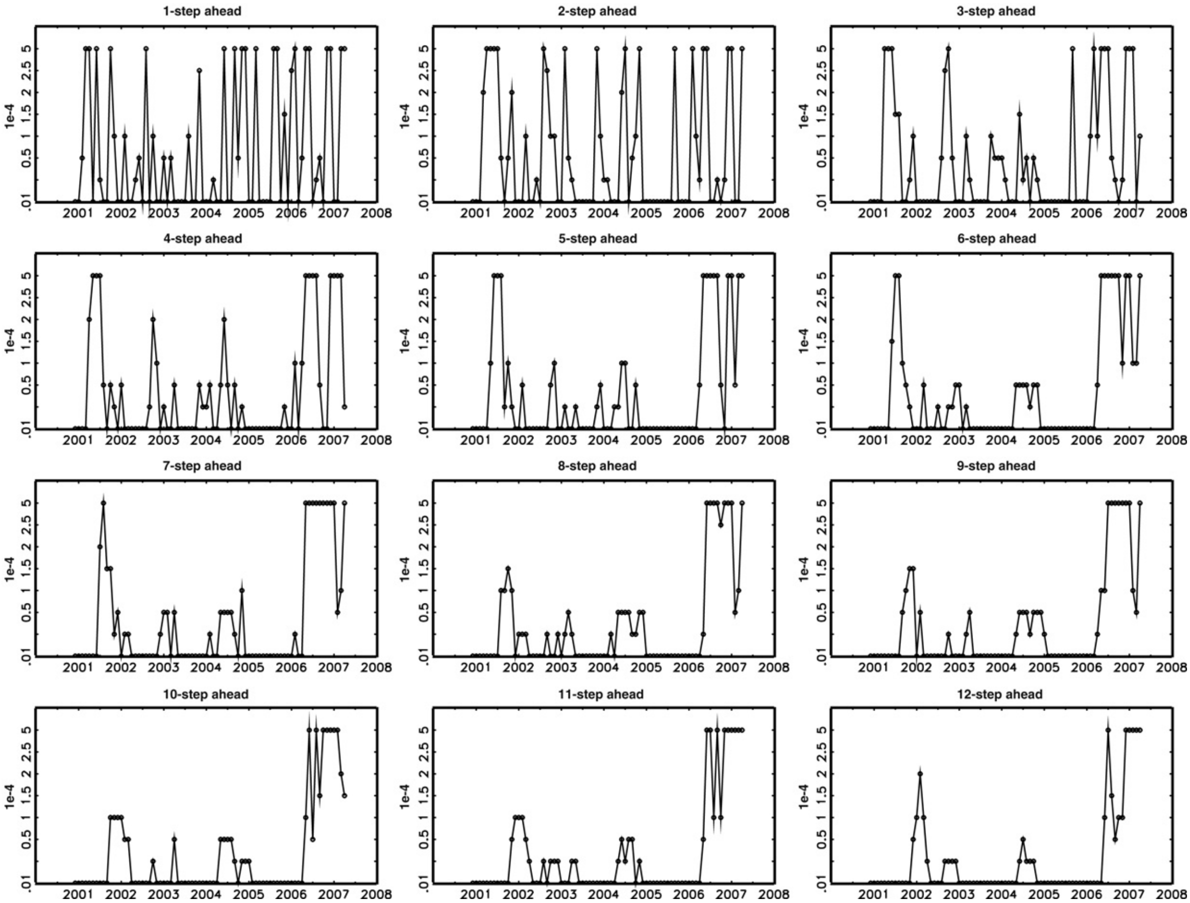

Fig. 3. Selected values of the tightness parameter θ, over time.

panels in the table the first column displays the average return, the second the standard deviation, and the third the Sharpe ratio, which is a quick way of assessing the mean–variance trade-off. The last column in the table contains the difference between the Sharpe ratio obtained by using the BVAR and that obtained by using the AR.

As is clear from the table, overall the strategy based on the BVAR provides positive returns. Moreover, the BVAR strategy performs better than the one based on the AR in terms of both returns and standard deviation, as is shown by the Sharpe ratios, which are higher in 26 out of 33 cases. Finally, it is worth noting that the BVAR strategy involved systematically fewer transactions than the AR, i.e., the BVAR model induces the investor to change his position less often, which means that the transaction costs associated with such strategy would be smaller.

4. Summary and conclusions

Having a forecasting model which is consistent with economic theory and also forecasts well is very appealing, but the simple task of forecasting is important in its own right. In this paper we focused on the task of forecasting a large panel of exchange rates, using a purely time series approach.

As the random walk without drift has proven to be a very competitive model in forecasting exchange rates,

Table 7
RMSFE of BVAR vs the random walk, emerging and developed countries.

Developed countries	Hor:											
	1	2	3	4	5	6	7	8	9	10	11	12
AUST	0.991	0.993	0.991	0.968	0.961	0.945	0.947	0.956	0.950	0.951	0.960	0.967
CANA	0.990	0.984	0.980	0.965	0.955	0.946	0.946	0.942	0.926	0.911	0.900	0.897
DANI	0.987	0.975*	0.978	0.971	0.968	0.954	0.960	0.976	0.988	0.990	0.995	0.994
EURO	0.987	0.976	0.978	0.971	0.968	0.953	0.960	0.976	0.987	0.989	0.995	0.994
FINN	0.987	0.977	0.980	0.971	0.968	0.953	0.960	0.977	0.988	0.990	0.995	0.995
GBPP	0.990	0.983	0.972	0.954*	0.950	0.946	0.955	0.961	0.973	0.988	0.999	1.001
IRIS	0.988	0.978	0.980	0.971	0.967	0.953	0.960	0.976	0.987	0.989	0.994	0.994
JAPA	1.002	1.006*	1.005	1.004	0.998	0.988	0.988	0.995	1.003	1.007	1.009	1.009
NEWZ	0.991	0.989	0.994	0.980	0.977	0.959	0.955	0.960	0.960	0.971	0.992	1.004
NORW	0.994	0.989	0.988	0.975	0.962	0.947*	0.958	0.976	0.980	0.980	0.985	0.979
SWED	0.987	0.978	0.977*	0.971*	0.974	0.972	0.983	0.998	1.008	1.012	1.016	1.012
SWIS	0.994	0.987	0.993	0.993	0.992	0.981	0.997	1.019	1.035	1.033	1.029	1.018
Emerging countries												
BRAZ	0.987	0.968	0.952	0.936	0.946	0.960	0.982	0.978	0.958	0.939	0.955	0.973
CHIL	1.002	0.992	0.977	0.961	0.973	0.973	0.989	1.000	0.987	0.992	1.019	1.026
COLO	0.990	0.985	0.979	0.975	0.964	0.946	0.958	0.955	0.915	0.888	0.913	0.935
CZEC	0.984	0.979	0.988	0.982	0.987	0.963	0.966	0.971	0.955	0.935	0.907	0.901
HUNG	0.998	1.019	1.039	1.024	1.024	0.998	0.993	1.011	1.004	1.019	1.039	1.052
INDI	0.976	0.967	0.962	0.945	0.943	0.947	0.955	0.970	0.987	1.005	1.018	1.055
ISRA	0.983	1.000	0.992	0.986	0.984	0.992	1.001	1.024	1.035	1.047	1.075	1.077
MALT	0.984	0.972	0.969	0.965	0.961	0.937	0.942	0.947	0.945	0.956	0.960	0.967
MEXI	1.005	1.023	1.030	1.010	0.971	0.959	0.985	0.963	0.947	0.950	0.968	0.951
PAKI	0.986	0.954	1.009	1.156	1.317	1.406	1.427	1.434	1.484	1.585	1.668	1.729
PERU	0.998	0.998	1.029	1.059	1.082	1.044	1.013	0.994	0.925	0.891	0.923	0.969
PHIL	1.012	0.953	0.973	1.033	1.072	1.055	1.082	1.070	1.023	1.002	0.981	0.955
POLI	0.996	0.984	0.950**	0.922***	0.949	0.929	0.938	0.935	0.902	0.888	0.885	0.884
SING	0.976	0.966	1.001	1.001	0.987	0.947	0.933	0.977	0.991	0.980	0.973	0.979
SLOV	0.963	0.936	0.933	0.919	0.919	0.895	0.898	0.900	0.899	0.931	0.947	0.960
SOUT	0.994	1.022	1.020	1.011	1.041	1.046	1.046	1.111	1.142	1.136	1.160	1.206
SRIL	0.935	0.937	0.903**	0.887***	0.840***	0.793***	0.765***	0.749***	0.806	0.882	0.929	0.965
TAIW	1.012	0.993	0.982	0.990	1.009	1.016	1.008	1.040	1.050*	1.031***	1.019	1.046
THAI	0.967	0.947	0.952	0.942	0.964	0.956	0.942	0.950	0.949	0.943	0.952	0.981
TURK	0.905*	0.905	0.936	0.914	0.925	0.960*	1.031	1.071***	1.072***	1.101***	1.196*	1.299*
URUG	0.997	0.983	0.966	0.970	0.971	0.968	0.976	0.982	0.971	0.969	0.986	0.996

* Denotes rejection of the null of equal forecast accuracy at the 10% level, according to the Giacomini and White (2006) test.
** Denotes rejection of the null of equal forecast accuracy at the 5% level, according to the Giacomini and White (2006) test.
*** Denotes rejection of the null of equal forecast accuracy at the 1% level, according to the Giacomini and White (2006) test.

it is reasonable to build a model in which exchange rates are a priori following such a process. However, the model should also take into account information from the large panel of exchange rates, when needed. Therefore, we have developed a Bayesian vector autoregression with a normal-inverted Wishart prior, imposing a univariate driftless random-walk representation a priori, but allowing the data to speak about the relevance of other available information.

In addition to giving the opportunity of including a priori information in the picture, the Bayesian VAR approach allows the efficient handling of large datasets, whereas using a simple multivariate linear model would encounter the curse of dimensionality problem.

We used the proposed BVAR model to forecast a panel of 33 exchange rates vis-a-vis the US Dollar, finding that it can lead to gains in forecast accuracy for

Table 8
Performance of trading strategies.

	BVAR			AR			$\Delta(SR)$
	Avg Return	StDev	SR	Avg Return	StDev	SR	
AUST	0.3191	2.0605	0.1548	0.0548	2.0481	0.0267	0.1281
BRAZ	0.0773	1.3269	0.0583	0.1728	1.9035	0.0908	−0.0325
CANA	0.3364	1.77	0.19	0.1501	1.9286	0.0778	0.1122
CHIL	0.0391	1.1107	0.0352	−0.053	0.7385	−0.0717	0.1069
COLO	0.0453	0.7752	0.0584	0	0	0	0.0584
CZEC	0.6202	2.8869	0.2148	0.3861	2.3815	0.1621	0.0527
DANI	0.2118	1.8638	0.1137	0.1463	1.7394	0.0841	0.0296
EURO	0.2119	1.8557	0.1142	0.1246	1.7004	0.0733	0.0409
FINN	0.1738	1.9152	0.0907	0.0504	1.8545	0.0272	0.0635
GBPP	0.1753	1.8787	0.0933	0.0012	1.8086	0.0007	0.0926
HUNG	0.072	2.5355	0.0284	−0.0463	2.1011	−0.022	0.0504
INDI	0.0771	0.9815	0.0786	0.0846	0.4264	0.1985	−0.1199
IRIS	0.19	1.8935	0.1003	0.0569	1.6334	0.0349	0.0654
ISRA	0.0819	0.3982	0.2056	−0.0129	1.4087	−0.0092	0.2148
JAPA	−0.0376	1.8315	−0.0206	0.2736	1.8726	0.1461	−0.1667
MALT	0.2968	1.8128	0.1637	0.14	1.6463	0.0851	0.0786
MEXI	0	0	0	−0.1205	1.4216	−0.0848	0.0848
NEWZ	0.406	2.4361	0.1667	0.2136	2.1907	0.0975	0.0692
NORW	0.3345	2.5854	0.1294	0.1211	1.6783	0.0722	0.0572
PAKI	−0.0177	0.0702	−0.2521	0.0121	0.2966	0.041	−0.2931
PERU	0.0801	0.6715	0.1193	0.0178	0.173	0.1031	0.0162
PHIL	0.1051	0.3457	0.3041	0.0012	0.6937	0.0017	0.3024
POLI	0.409	2.2524	0.1816	0.1612	1.4127	0.1141	0.0675
SING	0.0961	0.7811	0.1231	0.148	0.8724	0.1697	−0.0466
SLOV	0.4095	2.1098	0.1941	0.2909	1.9323	0.1506	0.0435
SOUT	0.3293	1.6708	0.1971	0.2359	1.636	0.1442	0.0529
SRIL	0	0	0	−0.0634	0.7247	−0.0874	0.0874
SWED	0.0725	2.2674	0.032	−0.0854	1.7463	−0.0489	0.0809
SWIS	0.186	2.2782	0.0816	0.1481	2.2038	0.0672	0.0144
TAIW	−0.0305	0.6105	−0.05	0.0486	0.9872	0.0493	−0.0993
THAI	0.3072	1.1081	0.2772	0.1566	1.2627	0.124	0.1532
TURK	0.0531	0.3392	0.1565	0.0348	0.8766	0.0397	0.1168
URUG	0	0	0	0.2012	1.4241	0.1413	−0.1413

the large majority of the exchange rates under analysis. The gains arise at *all* forecast horizons, including the very short ones where the random walk forecast is typically extremely hard to outperform. The forecast gains are typically in the range of 2%–3%, but in some relevant cases, such as the Euro–Dollar and the GBP–Dollar, they can reach 6%–9%. Moreover, a simple trading strategy based on the BVAR forecasts provides positive returns which are higher than those from RW forecasts.

Finally, the good performance of the BVAR appears to be related more to the intermittent use of information in the large panel than to changes in the persistence of the exchange rates. In addition, in the post-2000 period the information in the exchange rates of emerging countries seems to matter for forecasting those of the developed countries more than the reverse, a finding that deserves additional research.

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