Introduction to Asset-Backed Securities

Abstract

12a	Calculate and interpret price, income and cross-price elasticities of demand and describe factors that affect each measure
12b	Compare substitution and income effects
12c	Distinguish between normal goods and inferior goods
12d	Describe the phenomenon of diminishing marginal returns
12e	Determine and interpret breakeven and shutdown points of production
12f	Describe how economies of scale and diseconomies of scale affect costs

Contents

1. Sources of Return

An investor of fixed-rate securities has three sources of return:

- receipt of the promised coupon and principal payments
- reinvestment of coupon payments
- potential capital gains (or losses) on the sale of the bond prior to maturity
- amortization of discount bonds - discount bonds are pulled to par across their tenors as the market discount rates and bond's carrying values adjust periodically to the issuing coupon rate and par values till they are in line.

This chapter fundamentally focus on *interest rate risk* rather than credit risk. One thing affecting the risks and returns is the holding period: investors planning to buy and hold till maturity are insensitive to the market value of the securities while investors with shorter investment windows depend on market prices to make profit.

The investor's **horizon yield** is the internal rate of return between the total sum of reinvested coupon payments and the sale price (or redemption value) and the purchase price of the bond. The horizon yield is an annualized holding-period rate of return.

The investor's horizon yield can match the yield-to-maturity if: (1) coupon payments are reinvested at the same interest rate (2) bond is sold at a price on the constant-yield price trajectory, which implies investors have no capital gains or losses when the bond is sold (bond is sold at fair value)

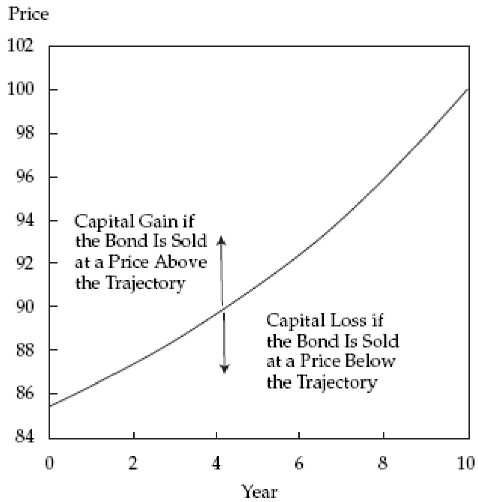

Figure 1. Constant-yield price trajectory

A point on the trajectory represents the **carrying value** of the bond at that time. That is, the purchase price plus the amortized amount of the discount if the bond is purchased below par. If bonds are purchased above par, the carrying value is less - it is the purchase price minus the amortized amount of the premium.
The amortized amount is the difference of price between two points. It is important to note that capital loss or gain is measured from the bond's carrying value, not from the original purchase price.
The investment horizon is therefore at the heart of understanding interest rate risk and return. There are two offsetting types of interest rate risk that affect bond investors: coupon reinvestment risk and market price risk. The future value of reinvested coupon payments increases as interest go up and decreases as interests go down.The sale price has a contrarian behavior, as it decreases if interest rates go up and increases if interest rates go up.
For instance, a buy-and-hold investor only has the coupon reinvestment risk.

2. Interest Rate Risk on Fixed-Rate Bonds

The two most common measures of interest rate risk are **duration** and **convexity**.

2.1 Macaulay, Modified and Approximate Duration

The *duration of a bond* measures the sensivity of the bond's full price to changes in the bond0s yield-to-maturity and changes in benchmark interest rates. Bond duration can be divided in **yield duration** and **curve duration**. Yield duration is the sensivity of the bond price with respect to the bond's own yield-to-maturity. Curve duration is the sensivity of the bond price with respect to a benchmark yield curve.
Yield duration statistics are *Macaulay duration*, *modified duration*, *money duration* and *the price value of a basis point* (PVBP).
Curve duration statistic is *effective duration*.

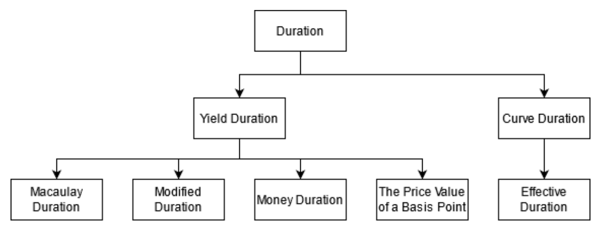

Figure 2

Macaulay Duration Macaulay duration is a weighted average of the time to receipt of the bond's promised payments where the weights are the shares of the full price (the full value of promised payments).

$$\text{MacDur} = \left\{ \begin{array}{l} (1 - t/T)\left[\dfrac{\dfrac{PMT}{(1+r)^{1-t/T}}}{PV^{Full}}\right] + (2 - t/T)\left[\dfrac{\dfrac{PMT}{(1+r)^{2-t/T}}}{PV^{Full}}\right] + \cdots + \\ (N - t/T)\left[\dfrac{\dfrac{PMT+FV}{(1+r)^{N-t/T}}}{PV^{Full}}\right] \end{array} \right\}$$

Figure 3

Consider a 10 year 8% annual coupon payment bond, YTM 10.40% and price of 85.50 per 100 of par value.

Period	Cash Flow	Present Value	Weight	Period × Weight
1	8	7.246377	0.08475	0.0847
2	8	6.563747	0.07677	0.1535
3	8	5.945423	0.06953	0.2086
4	8	5.385347	0.06298	0.2519
5	8	4.878032	0.05705	0.2853
6	8	4.418507	0.05168	0.3101
7	8	4.002271	0.04681	0.3277
8	8	3.625245	0.04240	0.3392
9	8	3.283737	0.03840	0.3456
10	108	40.154389	0.46963	4.6963
		85.503075	1.00000	7.0029

Figure 4

The first two columns represent the cash flow per period. The third column is the present value of each cash flow such that the sum of present values is the full price of the bond. The fourth value is the weight (present value of current payment divided by the sum of present values). Fifth column is the period times weight totaling the Macaulay duration (7.0029 years).
Now consider an example of Macaulay duration for calculations between coupon payment dates. A 6% semiannual payment corporate bond from 11 april 2019 to 14 february 2027 (16 periods).

Period	Time to Receipt	Cash Flow	Present Value	Weight	Time × Weight
1	0.6833	3	2.940012	0.02913	0.019903
2	1.6833	3	2.854381	0.02828	0.047601
3	2.6833	3	2.771244	0.02745	0.073669
4	3.6833	3	2.690528	0.02665	0.098178
5	4.6833	3	2.612163	0.02588	0.121197
6	5.6833	3	2.536080	0.02512	0.142791
7	6.6833	3	2.462214	0.02439	0.163025
8	7.6833	3	2.390499	0.02368	0.181959
9	8.6833	3	2.320873	0.02299	0.199652
10	9.6833	3	2.253275	0.02232	0.216159
11	10.6833	3	2.187645	0.02167	0.231536
12	11.6833	3	2.123927	0.02104	0.245834
13	12.6833	3	2.062065	0.02043	0.259102
14	13.6833	3	2.002005	0.01983	0.271389
15	14.6833	3	1.943694	0.01926	0.282740

Figure 5

Period	Time to Receipt	Cash Flow	Present Value	Weight	Time × Weight
16	15.6833	103	64.789817	0.64186	10.066535
			100.940423	1.00000	12.621268

Figure 6

The Macaulay duration is 12.621268 or 6.310634 years annualized (12.621268/2).
The Macaulay duration can also be calculated as a closed-form equation:

$$\text{MacDur} = \left\{ \frac{1+r}{r} - \frac{1+r+\left[N\times(c-r)\right]}{c\times\left[(1+r)^N-1\right]+r} \right\} - (t/T)$$

Figure 7

Modified Duration The modified duration (ModDur) statistic of a bond is:

$$ModDur = \frac{MacDur}{1+r} \tag{1}$$

For example, the modified duration of the 10 year, 8% annual payment bond with MacDur = 7.0029 is:

$$ModDur = \frac{7.0029}{1.1040} = 6.3432 \tag{2}$$

where r is $\frac{YTM}{n}$ and n is the number of compounding periods per year.
Modified duration provides an estimate of the percentage price change for a bond given a change in its yield-to-maturity.

$$\%\delta PV^{full} = -AnnualModDur \cdot \delta Yield \tag{3}$$

This equation establishes the relation between full bond price and annual Modified Duration. Prices and YTM move inversely.

Approximate Modified Duration An alternative approach to calculate modified duration is the approximate modified duration.

$$ApproxModDur = \frac{Pv_{-} - PV_{+}}{2\delta Yield \cdot PV_0} \tag{4}$$

The objective of the approximation is to estimate the slope of the line tangent to the price-yield curve.

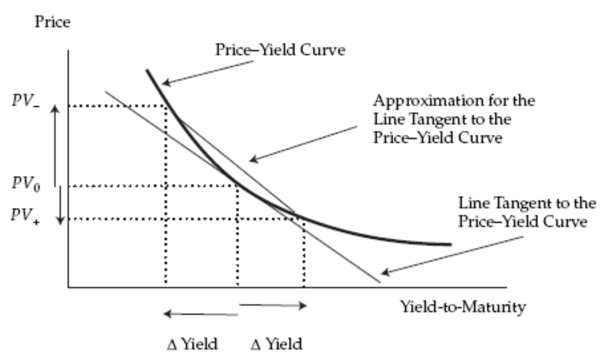

Figure 8. Approximate Modified Duration

Approximate Macaulay Duration The Macaulay duration can be approximated by:

$$ApproxMacDur = ApproxModDur(1+r) \tag{5}$$

2.2 Effective Duration

Another way of assessing the interest rate risk of a bond is to estimate the percentage change in price given a change in a benchmark yield curve (like the government par curve). The **effective duration** is thus the sensitivity of a bond's price to a change in a benchmark yield curve.
The difference between modified duration and effective duration is that modified duration is a **yield duration** measures the sensitivity in terms of a change in the bond's own yield-to-maturity while **effective duration** is a curve duration that measures the interest rate risk in terms of benchmark yield curve sensitivity.
Effective duration is used to measure the interest rate risk of a complex bond - like callable bonds - because future cash

flows are uncertain since they are contingent on future interest rates and the ability of the issuer to refinance at a lower costs. In brief, *callable bonds do not have a well-defined yield-to-maturity and therefore yield duration statistics like Macaulay duration or modified duration do not apply.*
Effective duration is the appropriate measure here. Thereby mortgage-backed bonds cannot be measured with yield duration statistics since homeowners have call options on their mortgage loans. Hence, effective duration applies to MBS.

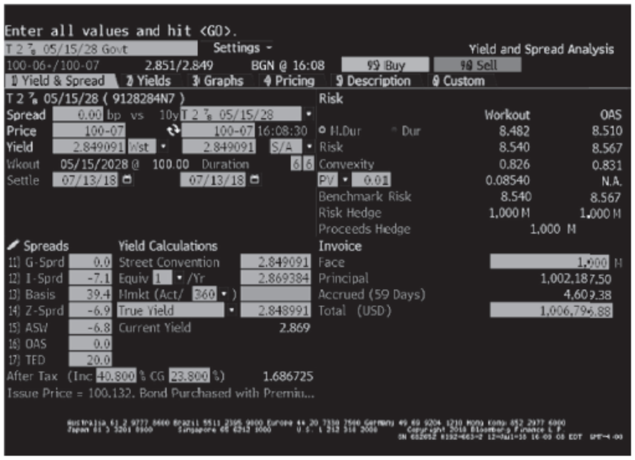

Figure 9. Bloomberg Yield and Spread (YAS) Analysis page for the 2.875% U.S. Treasury Note

Bloomberg Yield-and-Spread Analysis The quoted asked price for the bond is 100-07, which is equal to 100 and 7/32 which is 100.21875 in decimals (most US Treasuries are quoted in fractions, usually in 1/8 increments). The accrued interest uses actual/actual day-count. The settlement date is 59 days into a 184 day semiannual coupon payment period. The accrued interest is 0.4609375 per 100 of par value (59/184*0.02875/2)*100. The full price of the bond is 100.679688. The yield-to-maturity of the bond is 2.849091%, stated on a street-convention semiannual bond basis.
The modified duration for the bond is 8.482 which is the *conventional yield duration* statistic. The curve duration is 8.510. The effective duration is called OAS duration or Option-Adjusted Spread.

2.3 Key Rate Duration

The **key date duration** or **partial duration** is a measure of a bond's sensitivity to change in benchmark yield curve at a specific maturity segment. That is, effective duration assumes a parallel shift at all maturities, while key date duration allows the analyst to analyze the effect of benchmark changes for specific maturities such as short term maturities (¡2 years) or long term maturities (¿10 years).

2.4 Properties of Bond Duration

The effect of coupon payments As time passes during the coupon period, moving from the right to the left, that is, from further TTM dates to the bond's redemption (TTM=0), the Macaulay duration decreases. However, it declines smoothly and then jumps upward after the coupon is paid.

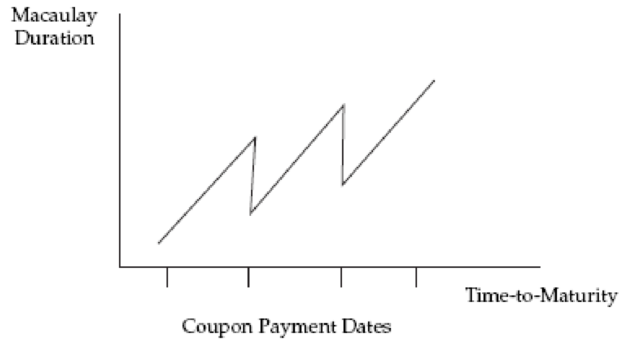

Figure 10. Macaulay Duration between Coupon Payments

Properties of the Macaulay Yield Duration This graph shows the Macaulay durations for coupon payments when t/T=0.

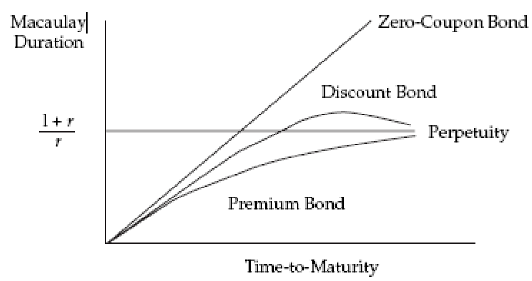

Figure 11. Macaulay Duration vs Time-to-Maturity

- The relationship between Macaulay duration and the time-to-maturity for a zero-coupon bond is a 45-degree line and Macaulay duration is its time-to-maturity.
- Perpetual bonds or consol is a bond that does not mature. Non-callable perpetuities have a Macaulay duration of $\frac{1+r}{r}$ as N approaches infinity.
- Macaulay duration is higher for zero-coupon bond than for low-coupon bond trading at discount. All else being equal, a lower coupon bond has a higher duration and more interest rate risk than a higher coupon bond. The same holds for yield-to-maturity.

The usual pattern is that longer times-to-maturity correspond to higher Macaulay duration statistics. However, for discount bonds the Macaulay duration increases for higher time-to-maturity until it exceeds $\frac{1+r}{r}$ and hits a maximum, approaching the threshold from above.

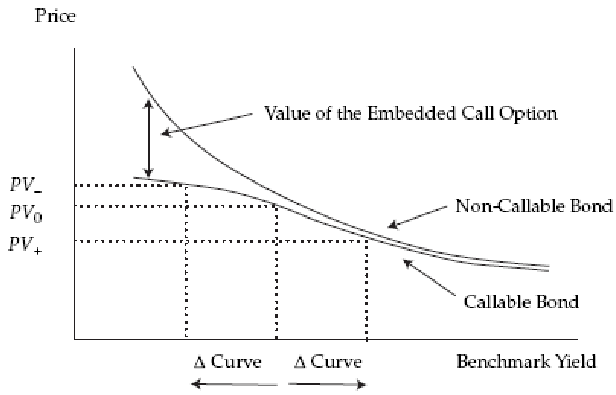

Figure 12. Callable Bond vs Non-Callable Bonds

Interest Rate Risk Characteristics of a Callable Bond The price of a non-callable bond is always greater than a callable bond of similar features. The difference is the *value of the embedded call option*. When interest rates are high, the value of the call option is low; when rates are low, the value of the call option is much greater because the issuer is much more likely to exercise the option to refinance debt at lower costs (*call risk*).
When interest rates are high, the effective durations of the callable and non-callable bonds are very similar. When interest rates are low, the effective duration of the callable bond is lower than the non-callable bond. Therefore, the embedded call option reduces the effective duration of the bond, specially when interest rates are lower.

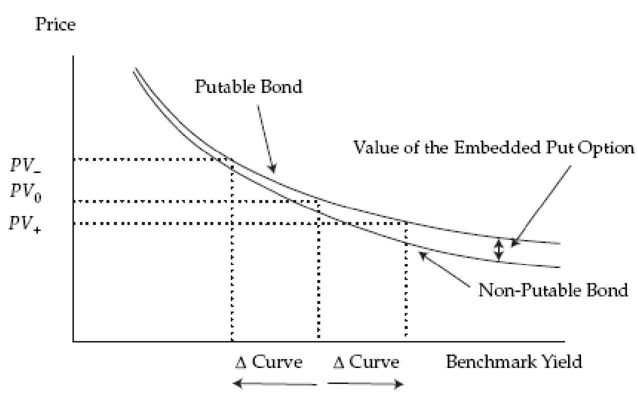

Figure 13. Putable Bond vs Non-Callable Bonds

Interest Rate Risk Characteristerics of a Putable Bond A putable bond allows the investor to sell the bond back to the issuer prior to maturity usually at par, protecting the investor from higher benchmark yields or higher credit spreads that would otherwise drive the bond to a discounted price.
An embedded put option reduces the effective duration of the bond, especially when rates are rising. If interest rates are low the value of the put option is low and therefore the putable bond is priced closely to non-putable bonds. When benchmark rates increase, the value of the put option increases and therefore the price of the putable bond since the investor can redeem those bonds at par.

2.5 Duration of a Bond Portfolio

There is two ways to calculate the duration of a bond portfolio:

- weighted average of time to receipt of the aggregate cash flows
- the weighted average of the individual bond durations

The first method is the theoretically correct approach but difficult to use in practice since bonds can be quite difference from one another, different options, uncertainty, capitalization periods, etc. The second method is commonly used by portfolio managers but has a few limitations.
The main advantage of the second approach is that it can be easily used as a measure of interest rate risk. For instance, if the YTM of the portfolio increases by 100 bps, the estimated drop in the portfolio value is 14,3725%. This assumes a **parallel shift** in the yield curve - all rates change by the same amount in the same direction. However, in reality, this isn't true as interest rates frequently change the yield curve.

2.6 Money Duration and the Price Value of a Basis Point

Money Duration The **money duration** of a bond is a measure of the price change in units of the currency in which the bond is denominated.

$$MoneyDur = AnnModDur \cdot PV^{full} \tag{6}$$

Price Value of a Basis Point The PVBP is an estimate of the change in the full price of a bond give a 1bp change in the yield-to-maturity.

$$PVBP = \frac{PV_{-} - PV_{+}}{2} \tag{7}$$

Where PV_{-} and PV_{+} are the full prices calculated by decreasing and increasing the yield-to-maturity 1 bp. The PVBP is also called *PV01* standing for the *price value of 1 bp*.

2.7 Bond Convexity

While duration measures the primary effect on a bond's percentage price change given a change in the YTM, **convexity** measures a secondary effect - the fact that the relation between the bond price and YTM is curved (convex) and not linear.

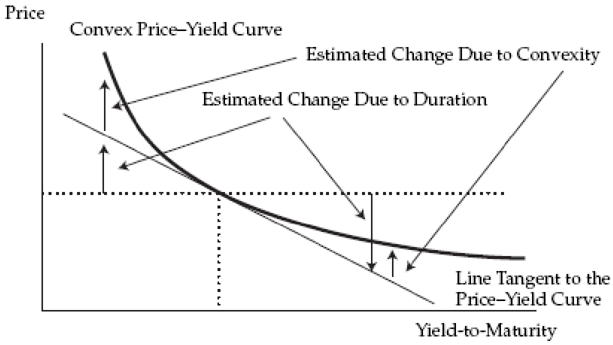

Figure 14. Convexity of a Traditional Option-Free Bond

Convexity is used to improve the estimate of the percentage price change provided by the modified duration alone. A convexity adjusted estimate of the percentage change in the bond's full price (this equation is similar to the Taylor Series expansions)

$$\%PV_{full} = (-AnnModDur \cdot \delta Yield) + [\frac{1}{2} \cdot AnnConvexity \cdot (\delta Yield)^2] \tag{8}$$

The first bracket is the *first-order effect* and the second bracket is the *second order effect* - the **convexity adjustment**.

Approximate Convexity The approximate convexity statistic can be calculated as:

$$ApproxCon = \frac{PV_- + PV_+ - 2 \cdot PV_0}{(\delta Yield)^2 \cdot PV_0} \tag{9}$$

Money Convexity The money convexity statistic (MoneyCon) is the second-order effect of the money duration

$$\%PV_{full} = (-MoneyDur \cdot \delta Yield) + [\frac{1}{2} \cdot MoneyConv \cdot (\delta Yield)^2] \tag{10}$$

A fixed rate bond with longer TTM, lower coupon rate and lower YTM has greater convexity than a bond with shorter time-to-maturity, higher coupon rate and higher yield to maturity.

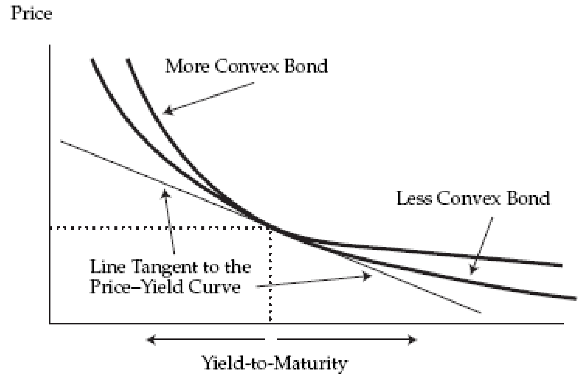

Figure 15. Convexity of a Traditional Option-Free Bond

For the same decrease in YTM, the more convex bond *appreciates more* in price; for the same increase in YTM the more convex bond *depreciates less*.

Effective Convexity The effective convexity is the secondary effect measure of a change in benchmark yield.

$$EffCon = \frac{[PV_- + PV_+] - [2 \cdot PV_0]}{(\delta Curve)^2 \cdot PV_0} \tag{11}$$

3. Interest Rate Risk and the Investment Horizon

3.1 Yield Volatility

The **term structure of yield volatility** is the relationship between the volatility of bond YTM and TTM.

3.2 Investment Horizon, Macaulay Duration and Interest Rate Risk

- When the investment horizon is greater than the Macaulay duration of a bond, the coupon reinvestment risk dominates market price risk aka the investor's risk is lower interest rates (negative duration gap).
- When the investment horizon is equal to the Macaulay duration of a bond, the coupon reinvestment risk offsets the market price risks (zero duration gap)
- When the investment horizon is less than the Macaulay duration of the bond, market price risk dominates coupon reinvestment risk. The investors risk are high interest rates (positive duration gap).

The difference between the Macaulay duration of a bond and the investment horizon is called **duration gap**. As the time passes, the investment horizon is reduced and the Macaulay duration of the bond also changes and therefore, the duration gap changes as well.

References

[cfa, 2019] 2019. *CFA program curriculum*. CFA Institute.