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HUDM 4125 Due September 30, 2021

HW 3

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1. A fair coin is tossed four times. What is the probability that the number of heads appearing on the first two tosses is equal to the number of heads appearing on the second two tosses?

2. A box contains 3 blue and 4 yellow marbles. Marbles are drawn out one at a time, the drawn marbles *not* being replaced. Let X = number of drawings made until all the marbles remaining in the box are of the same color. Construct the probability distribution of X. That is, a table with the possible values of X and their corresponding probabilities.

3. Urn 1 contains one white chip and three red chips; urn 2 has three white chips and two red chips. One chip is drawn at random from urn 1 and transferred to urn 2. Then one chip is drawn from urn 2. Suppose that a red chip was selected from urn 2. What is then the probability that the transferred chip was white?

4. From Devore's textbook:

    a) p. 109: Q13

    b) p. 110: Q14

1. {TTTT, THTH, THHT, HTTH, HTHT, HHHH}
$P = 6/16 = 0.375$

2. $P(X=3) = \frac{\binom{3}{3}}{\binom{7}{3}} = \frac{1}{35}$

$P(X=4) = \frac{\binom{3}{3}\binom{4}{1}}{\binom{7}{4}} \times \frac{1}{4} + \frac{\binom{4}{4}}{\binom{7}{4}} = \frac{4}{35}$

$P(X=5) = \frac{\binom{3}{2}\binom{4}{3}}{\binom{7}{5}} \times \frac{1}{3} + \frac{\binom{3}{1}\binom{4}{3}}{\binom{7}{4}} \times \frac{1}{3} = \frac{2}{7}$

$P(X=6) = \frac{\binom{3}{2}\binom{4}{3}}{\binom{7}{5}} \times \frac{1}{2} + \frac{\binom{3}{2}\binom{4}{3}}{\binom{7}{5}} \times \frac{1}{2} = \frac{4}{7}$

3. Let A = Drawing red from urn1
B = Drawing white from urn1
C = Selecting red chip from urn2

$P(A) = \frac{3}{4}$ $P(B) = \frac{1}{4}$ $P(C|A) = \frac{1}{2}$ $P(C|B) = \frac{2}{3}$

$P(B|C) = \frac{P(C|B) \times P(B)}{[P(C|A) \times P(A)] + [P(C|B) \times P(B)]} = \frac{\frac{2}{3} \times \frac{1}{4}}{\frac{3}{8} + \frac{1}{6}} = \frac{1}{6} \times \frac{24}{13} = \frac{4}{13}$

4. a) a) $P(x \leq 3) = 0.1 + 0.15 + 0.2 + 0.25 = 0.7$

b) $P(x < 3) = 0.15 + 0.1 + 0.2 = 0.45$

c) $P(x \geq 3) = 0.25 + 0.2 + 0.06 + 0.04 = 0.55$

d) $P(2 \leq x \leq 5) = 0.2 + 0.25 + 0.2 + 0.06 = 0.71$

e) $P = P(4) + P(3) + P(2) = 0.65$

f) $P(x \leq 2) = 0.2 + 0.15 + 0.1 = 0.45$

b) a) $k + 2k + 3k + 4k + 5k = 1$

$k = 1/15$

b) $P(y \le 3) = 1 - P(y > 3)$

$= 1 - 9k$

$= 1 - \frac{9}{15} = \frac{2}{5}$

c) $P(2 \le y \le 4) = 2k + 3k + 4k = 9k = \frac{3}{5}$

d) $P(y) = \frac{1}{30} + \frac{4}{30} + \frac{9}{30} + \frac{16}{30} + \frac{25}{30} = \frac{55}{30} > 1$

so $p(y) = y^2/30$ can't be y's pmf.