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HUDM 4125 Due December 20, 2021

HW 10

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1. A politician is running for reelection. A newly released poll claims to have contacted a random sample of one hundred and twenty of the politician's current supporters and found that seventy-two of them were men. In last election, exit polls indicated that 65% of those who voted for him were men. Using $\alpha = 0.05$ level of significance, test the null hypothesis that the proportion of his male supporters has remained the same vs. the alternative that the proportion has dropped.

    a) State the two hypotheses in terms of the parameter being tested.
    b) Calculate the value of the test statistic.
    c) State the rejection rule.
    d) Draw a conclusion.
    e) Compute the p-value.

2. Derive the formula for the power function $\pi(\mu)$ for testing H_0: $\mu = 80$ vs. H_a: $\mu \neq 80$ at $\alpha = 0.07$ if the sample $X_1, \ldots, X_{16}$ is drawn from $N(\mu, 7)$ population. Sketch the graph of the power function or use software to draw it.

3. From Devore's textbook:

- Q9 on p. 435
- Q31 on p. 449

1. a) p = proportion of men who support politician.

null hypothesis: $H_0 = p = 0.65$

alternate hypothesis: $H_1 = p < 0.65$

b) $TS = \frac{\hat{p} - P}{\sqrt{\frac{P(1-P)}{n}}} \sim N(0,1)$ $\quad \hat{p} = 72/120 = 72.5/120 = 0.6042$

$= \frac{0.6042 - 0.65}{\sqrt{\frac{0.65(1-0.65)}{120}}} = \frac{-0.0458}{0.04354} = -1.0519$

c) Left tailed test. Z table $\alpha = 0.05$

$Z_{1-0.05} = Z_{0.95} = -1.64485$

Reject the null hypothesis if the test statistics less than critical value.

d) Because $TS > Z_{0.95}$

so we don't reject the null hypothesis.

at 5% level of significance the proportion has remained same.

e) $p(Z < \text{Test Statistics}) = P(Z < -1.0519)$

from z table, $P = 0.1464$

2. $P(Z < -1.812) = 0.035$ $\quad P(-1.812 < Z < 1.812) = 0.93$

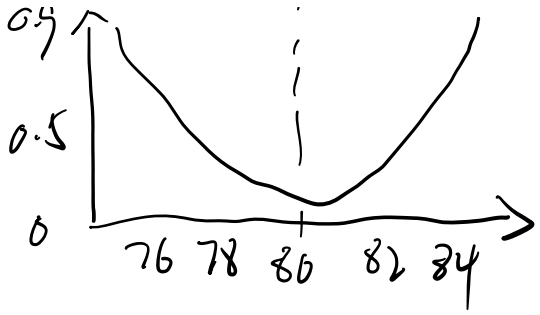
$\pi(\mu) = 1 - P(80 - 1.812 \times \frac{7}{\sqrt{16}} < \bar{x} < 80 + 1.812 \times \frac{7}{\sqrt{16}})$

$= 1 - P(76.829 < \bar{x} < 83.171)$

Convert to a standard normal variable

$\pi(\mu) = 1 - P(\frac{76.829 - \mu}{\frac{7}{\sqrt{16}}} < Z < \frac{83.171 - \mu}{\frac{7}{\sqrt{16}}})$

$= 1 - \Phi(< \frac{83.171 - \mu}{1.75}) + \Phi(\frac{76.829 - \mu}{1.75})$



3. a) Since the alternate hypothesis is $H_a: p \neq 0.5$.
So rejection region should be two tailed
so $R_1 = \{x : x \leq 7 \text{ or } x \geq 18\}$ is most appropriate.

b) Type I error: Reject the null hypothesis when it is true.
It can be judge that a majority favour one of the two companies when that is not the case.

Type II error: Fail to reject the null hypothesis when it false potential subscribes are evenly split between the two companies when they are not.

c) $X \sim Bin(25, 0.5)$

$\alpha = P(\text{type I error})$

$= P(X \leq 7 \text{ or } X \geq 18, \text{ when } p = 0.5)$

$= P(X \leq 7 \text{ or when } p = 0.5) + P(X \geq 18, \text{ when } p = 0.5)$

$= B(7; 25, 0.5) + 1 - P(\leq 17 \text{ when } p = 0.5)$

$= 0.022 + (1 - 0.978)$

$= 0.044$

d) $\beta(0.3) = P\{\text{Accepting } H_0 \mid H_0 \text{ is false}\}$

$= P\{7 \leq X \leq 17 \mid p = 0.3\}$

$= B(17, 25, 0.3) - B(7, 25, 0.3)$

$= 1 - 0.512 = 0.488$

$$\beta(0.4) = P\{7 \le x \le 17 \mid p = 0.4\}$$
$$= B(17, 25, 0.4) - B(7, 25, 0.4)$$
$$= 0.999 - 0.154 = 0.845$$

$$\beta(0.6) = P\{7 \le x \le 17 \mid p = 0.6\}$$
$$= B(17, 25, 0.6) - B(7, 25, 0.6)$$
$$= 0.846 - 0.001 = 0.845$$

$$\beta(0.7) = P\{7 \le x \le 17 \mid p = 0.7\}$$
$$= B(17, 25, 0.7) - B(7, 25, 0.7)$$
$$= 0.488 - 0 = 0.488$$

a) $x = 6$ lies in rejection region
so null hypothesis is rejected.

4. μ denote true average adoption time $\mu_0 = 7$ sec

Hypothesis $H_0: \mu = 7$ vs. $H_a: \mu < 7$

$$\therefore t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{6.32 - 7}{\frac{1.65}{\sqrt{9}}} = \frac{-0.68 \times 3}{1.65} = -1.24$$

at $\alpha = 0.1$, $df = n - 1 = 8$

$t_{0.1,8} = 1.397$

Since $-1.24 > -1.397$

So, the data does not contradict.