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HUDM 4125 Due September 16, 2021

HW 1

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1. Sketch the region in the xy-coordinate plane corresponding to $A \cup B$ and $A \cap B$ if

$$A = \{(x, y): 0 < x < 4, 0 < y < 3\}$$

$$B = \{(x, y): 2 < x < 4, 2 < y < 4\}$$

2. Consider the experiment consisting of tossing a coin three times, with H and T standing for heads and tails, respectively.

   a) The following set of outcomes is an incomplete list of the points of the sample space S of the experiment: {HHH, HTT, TTT, HHT, TTH, HTH, THH}. Find the missing outcome.

   b) List the outcomes of the following events:

   A_1 = "Head on the second toss"

   A_2 = "More than two heads"

   A_3 = "Head and tail alternate"

   c) List the outcomes of the event $B_1 \cap B_2$, where B_1 = "the first two tosses show the same face" and B_2 = "the last two tosses show the same face".

3. In a sample space S = {1, 2, 3, …, 10}, let $A = \{2, 3, 4, 5\}$, $B = \{1, 3, 7, 10\}$, $C = \{4, 5, 6, 7\}$, and $D = \{2, 3, 7\}$, find:

   a) $(A \cup B) \cap (C \cup D)$

   b) $(A \cup B) \cup (C \cap \bar{D})$

   c) $(\bar{A} \cap \bar{B}) \cap (\bar{C} \cap \bar{D})$

4. Define *symmetric difference* between two events A and B to be

$$A \div B = (A \cap \bar{B}) \cup (\bar{A} \cap B)$$

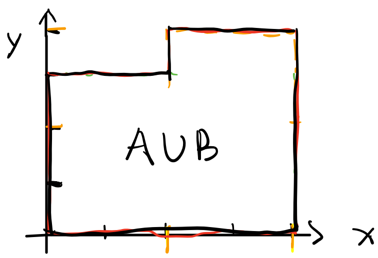
   Find the unknown event X if you know that for a given event A:

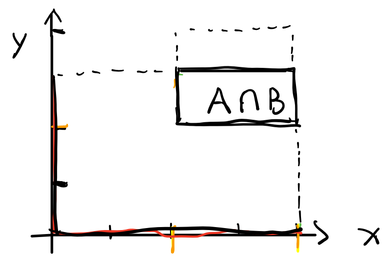
   a) $A \div X = \varnothing$

   b) $A \div X = A$ $(A \cap A) \cup (\bar{A} \cap \bar{A})$

   c) $A \div X = S$

1.





2. a) $\{HHH, HHT, HTT, TTT, THH, TTH, THT, HTH\}$
The missing one is THT.

b) $A_1 = \{HHH, HHT, THH, THT\}$
$A_2 = \{HHH, HHT, THH, HTH\}$
$A_3 = \{THT, HTH\}$

c) $B_1 = \{HHH, HHT, TTT, TTH\}$
$B_2 = \{HHH, HTT, TTT, THH\}$
$B_1 \cap B_2 = \{HHH, TTT\}$

3. a) $(A \cup B) \cap (C \cup D) = \{1,2,3,4,5,7,10\} \cap \{2,3,4,5,6,7\}$
$= \{2,3,4,5,7\}$

b) $(A \cup B) \cup (C \cap \bar{D}) = \{1,2,3,4,5,7,10\} \cup \{4,5,6\}$
$= \{1,2,3,4,5,6,7,10\}$

c) $(\bar{A} \cap \bar{B}) \cap (\bar{C} \cap \bar{D}) = \overline{A \cup B} \cap \overline{C \cup D} = \{8,9\}$

4. a) $A \dot{-} X = (A \cap \bar{X}) \cup (\bar{A} \cap X) = \phi \cup \phi \quad \therefore X = A.$

b) $A \dot{-} X = (A \cap \bar{X}) \cup (\bar{A} \cap X) = A \quad X = \phi$

c) $A \dot{-} X = (A \cap \bar{X}) \cup (\bar{A} \cap X) = S \quad X = \bar{A}$