Simon Chen

HUDM 4125 Due October 7, 2021

HW 4

1. Suppose the pdf of a random variable X is given by $f(x) = 3x^2$, $0 \leq x \leq 1$. Find $P\left(0 \leq X \leq \frac{1}{10}\right)$.

2. Suppose for a certain driver the time, in days, until he has an accident has Exp(λ) distribution. It is known that the probability that such a driver will be involved in an accident in the next two hundred and forty days is 0.59.

   a) Find the value of the parameter λ

   b) Find the probability that such a driver will be involved in an accident during the next three hundred and nineteen days.

3. From Devore's textbook:

   a) p. 170: Q10

   b) p. 171: Q16

1. $P(0 \le x \le \frac{1}{10}) = \int_0^{1/10} 3x^2\,dx = x^3 \Big]_0^{1/10} = (\frac{1}{10})^3 - 0^3 = 0.001$

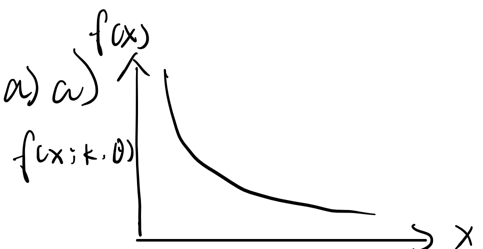
2. a) $F(240) = 1 - e^{-\lambda(240)} = 0.59$

$\therefore e^{-240\lambda} = 0.41$

$\lambda = 0.0037$

b) $F(319) = 1 - e^{-0.0037 \times 319} = 1 - 0.306 = 0.694$

3. a) a)



b) $f(x;k,\theta) = \frac{k\theta^k}{x^{k+1}}$

$\int_0^{\infty} \frac{k\theta^k}{x^{k+1}}\,dx \Rightarrow k\theta^k \int_0^{\infty} x^{-k-1}\,dx$

$k\theta^k \left(\frac{x^{-k}}{-k}\right)\Big]_0^{\infty} \Rightarrow -\theta^k x^{-k}\Big]_0^{\infty} \Rightarrow 0-(-1) = 1$

c) $P(x \le b) = \int_\theta^b f(x)\,dx = k\theta^k \cdot \left(-\frac{x^{k}}{k}\right)\Big]_\theta^b$

$= -\theta^k x^{-k}\Big]_\theta^b = (-\frac{\theta}{b})^k - (-1) = 1 - (\frac{\theta}{b})^k$

d) $P(a \le x \le b) = \int_a^b \frac{k\theta^k}{x^{k+1}}\,dx = -\theta^k x^{-k}\Big]_a^b$

$= \left(-\frac{\theta}{b}\right)^k - \left(-\frac{\theta}{a}\right)^k = \left(\frac{\theta}{a}\right)^k - \left(\frac{\theta}{b}\right)^k$

b. Q16. a) $P(X \le 1) = F(1) = \frac{1}{4}[1 + \ln(4)] = \frac{1}{4} + \frac{1}{4}\ln(4) = 0.597$

b) $P(1 \le X \le 3) = F(3) - F(1) = \frac{3}{4}[1 + \ln(\frac{4}{3})] - 0.597$

$= 0.966 - 0.597 = 0.369$

c) $F(x) = \frac{x}{4} + \frac{x}{4}\ln(\frac{4}{x})$

$f(x) = F'(x) = (\frac{x}{4})' \cdot [1 + \ln(\frac{4}{x})] + \frac{x}{4}[1 + \ln(\frac{4}{x})]'$

$= \frac{1}{4}[1 + \ln(\frac{4}{x})] + \frac{x}{4}[\frac{x}{4} \cdot \frac{x}{4}']$

$= \frac{1}{4}[1 + \ln(\frac{4}{x})] + \frac{x}{4}[\frac{x}{4} \cdot -\frac{4}{x}]$

$= \frac{1}{4} + \frac{\ln(\frac{4}{x})}{4} - \frac{1}{4} = \frac{\ln(\frac{4}{x})}{4}$