10-Minute In-Class Quiz 2: Survival Analysis (Chapters 4–7)

Instructions

- You have **10 minutes** to complete this quiz.
- Answer all questions *concisely*.
- Show all relevant calculations where applicable.

Question 1 (5 points) – Left-Truncated Data

The table below presents left-truncated survival data for **6 individuals** in a study.

ID	Entry Time T_L	Event/Censoring Time X	Event Indicator δ (1: event; 0: censoring)
1	0	2.2	1
2	1.5	3.8	0
3	1.8	4.1	1
4	3.0	6.3	0
5	3.5	7.0	1
6	5.0	8.0	1

- Compute the Kaplan-Meier estimator $\hat{S}(t)$ at times $t = 2, 4, 6.5$, and 8. (Expressions are sufficient; no need to compute numerical values.)

Solution: The unique event times are $t_1 = 2.2$, $t_2 = 4.1$, $t_3 = 7.0$, and $t_4 = 8.0$.

- At $t_1 = 2.2$, there are $n_j = 3$ (ID = 1, 2, 3) subjects at risk and $d_j = 1$ failure;
- At $t_2 = 4.1$, there are $n_j = 3$ (ID = 3, 4, 5) subjects at risk and $d_j = 1$ failure;
- At $t_3 = 7.0$, there are $n_j = 2$ (ID = 5, 6) subjects at risk and $d_j = 1$ failure;
- At $t_4 = 8.0$, there is $n_j = 1$ (ID = 6) subject at risk and $d_j = 1$ failure.

From Table 2, $\hat{S}(2) = 1$, $\hat{S}(4) = 0.667$, $\hat{S}(6.5) = 0.444$, and $\hat{S}(8) = 0$.

Table 2: Survival probabilities at observed time points.

t_j	d_j	n_j	$S(t_j) = \prod_{l=1}^{j}(1 - d_l/n_l)$
2.2	1	3	0.667
4.1	1	3	0.444
7.0	1	2	0.222
8.0	1	1	0.000

Question 2 (5 points) – Time-Varying Treatment Effect in Cox Model

Consider a Cox proportional hazards model:

$$\lambda(t \mid Z) = \lambda_0(t) \exp(\beta Z), \tag{1}$$

where Z is a binary treatment indicator ($Z = 1$ for treatment, $Z = 0$ for control).

Now suppose the log-hazard ratio is not constant over time, but follows a **quadratic function**:

$$\text{HR}(t) = \frac{\lambda(t \mid Z = 1)}{\lambda(t \mid Z = 0)} = \gamma_0 + \gamma_1 t + \gamma_2 t^2.$$

- To specify the above form of $\text{HR}(t)$, add time-varying covariates in model (1) that are interactions between Z and certain functions of time.

Solution:

Set

$$\lambda(t \mid Z) = \lambda_0(t) \exp\{Z \log(\gamma_0 + \gamma_1 t + \gamma_2 t^2)\},$$

then

$$\text{HR}(t) = \frac{\lambda_0(t) \exp\{\log(\gamma_0 + \gamma_1 t + \gamma_2 t^2)\}}{\lambda_0(t)} = \gamma_0 + \gamma_1 t + \gamma_2 t^2.$$

However, this will be a non-standard Cox model.