10-Minute In-Class Quiz 3: Survival Analysis (Chapters 8-9)

Instructions

- You have **10 minutes** to complete this quiz.
- Answer all questions *concisely*.
- Show all relevant calculations where applicable.

Question 1 (5 points) – Recurrent Events

Figure 1 below shows the follow-up of six patients for repeated infections.

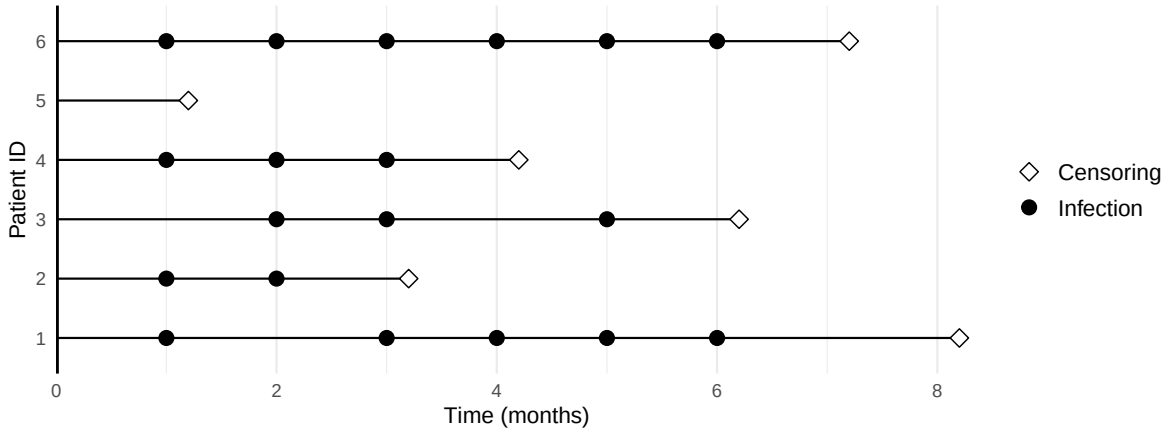

Figure 1: Follow-up of six patients for repeated infections.

- Calculate the (Nelsen–Aalen-type) estimates of the **average number of infections per patient by month 2, 4, and 6**.
  - `d_j`: observed number of events d_j
  - `n_j`: observed number of patients at risk n_j

– `cum_avg`: $\sum_{l=1}^{j} d_l/n_l$

Table 1: Nelsen–Aalen-type estimates of the cumulative average number of infections.

month	d_j	n_j	d_j/n_j	cum_avg
1	4	6	0.67	0.67
2	4	5	0.80	1.47
3	4	5	0.80	2.27
4	2	4	0.50	2.77
5	3	3	1.00	3.77
6	2	3	0.67	4.44

- Why is the following arithmetic biased:

$$\frac{\text{Total number of black dots by month } 2, 4, 6}{6 \text{ subjects}}$$

This is equivalent to setting $n_j = 6$ for all months. However, not all six patients are at risk when infections occur.

Question 2 (5 points) – Mean Model

Let $N^*(t)$ denote the average number of recurrent events by time t (such as estimated in Question 1). Let $Z = 1, 0$ denote treatment and control groups, respectively. Under the multiplicative mean model:

$$E\{N^*(t) \mid Z\} = \exp(\beta Z)\mu_0(t),$$

- What is the mean function in each group, i.e., $E\{N^*(t) \mid Z = z\}$ for $z = 1, 0$?
  - $E\{N^*(t) \mid Z = 0\} = \mu_0(t)$;
  - $E\{N^*(t) \mid Z = 1\} = \exp(\beta)\mu_0(t)$.
- Suppose $\hat{\beta} = \log(0.8)$, interpret the treatment effect.

The treatment group has a **20% reduction** $(1 - \exp(\hat{\beta}))$ in the average number of recurrent events compared to the control group.