Measurement: designing the acquisition of knowledge

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This paper is devoted to introduce measurement as a process aimed at producing knowledge according to an explicitly designed procedure, supported by explicitly designed and suitably operated devices. Some subjects of the paper are: the functional structure of a measurement process, the role that measuring systems have in such a process, as either comparators or transducers, and the related direct synchronous and direct asynchronous methods of measurement.

Introduction

This is the third paper of a series, mainly aimed at practitioners who, though not already specifically experts in the foundational aspects of measurement science, are interested in some better understanding on this matter. The hypothesis underlying the entire series is that *the basic purpose of measurement is to improve our knowledge about empirical properties in a socially reliable way*. Hence "measurement" and "knowledge" are the two keywords characterizing all papers of the series. The first paper, "Measurement: knowledge from information about empirical properties" [1], introduces the meaning of the key relation

property of an object = value of a property (1)

an example of which, when the property is quantitative, is

length of a given steel rod = 1.2345 m (2)

as conveying the information that one and the same property is identified as both the property of the given object (the length of the rod) and the given multiple of the chosen unit (1.2345 m). This shows that these are actual equations (and indeed we propose to call them "Basic Evaluation Equations" [2]), and not only generic representations. Hence, the definition of measurement scales, and therefore in particular of measurement units, is preliminary to the very possibility to design and perform a measurement. On this ground, the second paper, "Measurement: the social spread of knowledge" [3], discusses the conditions to guarantee that such Basic Evaluation Equations can be in principle interpreted in the same way by different subjects, in different context and times. These conditions are implemented in a metrological system through the definition of measurement units, the development of measurement standards realizing these definitions, and the social dissemination of such standards by means of calibrations. The aim of such an organization is to guarantee an appropriate metrological traceability of the results produced by measuring systems.

Our next step now focuses on measuring systems themselves, in their role of property-related information producers. Indeed, complementary to the condition of intersubjectivity inherited via their calibration, measuring systems are expected to provide information that is referred to the measurands (i.e., the properties intended to be measured, according to the International Vocabulary of Metrology (VIM) [4]) and not to other properties, including the opinions of the measurer. In short, the basic purpose of a measuring system is to produce information that is sufficiently both

– *intersubjective*, in the sense that it can be interpreted in the same way by different subjects, and this is what metrological traceability aims at, and

– *objective*, in the sense that it is about the intended object of the measurement, i.e., the measurand.

The subject of the present paper is this aboutness, and therefore the conditions for a measurement process to be able to produce sufficiently objective results.

The purpose and function of measuring systems

We design measuring systems with the purpose to support us in the effective production of information about empirical properties in the form of a Basic Evaluation Equation, i.e., equation (1) (and in fact an extended version of it, including information about the empirical trustworthiness of the equation itself: this will be discussed in a further paper of this series). This condition is understood by interpreting such an equation as resulting from a process, where the property of the concerned object is an input and the value of a property is the sought output. In other words, given [1]

– a measurand x, i.e., an individual property that is an instance of a general property X ("is of the kind X") and is known as the empirical property of an object, and

– a measured value y, i.e., an individual property that is an instance of the same general property X and is known as an element of a classification of X,

we conclude that $x = y$ because we have been able to design and perform a process whose input is the measurand x and whose output is the measured value y, and thus realizing a function f such that $y = f(x)$ (the VIM does not give f a name: it could be called a Measurement Mapping, to maintain it distinct from 'measurement function' that the VIM defines as a function from values to values [4]). *Measuring systems are devices that make this possible*.

Let us discuss this claim, in the simple and usual case of a ratio quantity, like length or mass.

The first point to consider is that for equation (1) to be true the entities x and y must be the same individual property, though known in different ways:

– the input, x, is a quantity to which we have an *empirical* access: hence, the measuring system must be able of empirical interaction, and then must have an empirical component in it, usually one or more physical sensors;

– the output, y, is a quantity to which we have an *informational* access, as the multiple of a given quantity we conventionally chose as the unit: hence, the measuring system must be able of information generation, and then must have an informational component in it.

That in equation (1) one term is empirical and the other informational is a key feature of measurement. Indeed, consider again our previous statement. At least in simple or ideal cases, in which measurement uncertainty can be neglected, the process of measuring is modeled as a Measurement Mapping $y = f(x)$ and its result reported as the equation $x = y$. Shouldn't we conclude then that f is the identity function, and therefore that, at least in simple or ideal cases, measurement is a useless process?

In fact, this is sometimes claimed, possibly by also describing measurement as a process whose "input [...] is the true value of the variable [and whose] output is the measured value of the variable [, so that] in an ideal measurement system, the measured value would be equal to the true value" [5]. However, this conclusion is obviously wrong: yes, by means of measurement we assess an equality of properties; but, no, such an equality is not trivial. Before measurement, we know, say, both that a given object has a mass, x, and that masses can be identified by taking multiples and submultiples of a given mass chosen as the unit u, like the kilogram, i.e, we know that $y = z$ u for a non-negative number z. Thanks to measurement, we discover the number z such that x and z u are the same mass.

The category mistake that trivializes measurement as being ideally modeled as an identity function is then clear: the input to a measurement process, and to a measuring system, is not a value (true or whatever), but, of course, an empirical property. Hence, this mistake is avoided by maintaining the distinction between *how things are*, i.e., ontology, and *what we know of things*, i.e., epistemology. In the language of philosophy, Basic Evaluation Equations are indeed [2]

– *ontologically* irrelevant, given that, if they are true, they simply instantiate the tautology that an individual quantity is equal to itself, but

– *epistemologically* significant, given that, if they are true, they inform us that two individual quantities that were known according to different criteria are in fact one and the same.

Moreover, the fact that its input is empirical and its output is informational requires the measuring system to be able to bridge such two realms [6], [7]. Today we take this ability for granted, but having learned, millennia ago, how to fulfill these challenges hints the ingenuity of human beings.

The fundamental operations of measuring systems

The implementation of the conditions that lead to a Basic Evaluation Equation – and more generally to a result including measurement uncertainty – is based on the assumption that somehow we know how to reliably compare (by ratio, in the case considered above) empirical quantities of the kind X, as made possible for example by an equal-arm balance for X = mass (of course under the assumption of operating in a uniform gravitational field). Indeed, measuring systems are such reliable comparators.

This justifies choosing an empirical quantity as the unit u of the kind X – for example, u could be the mass that is the kilogram – and from it identifying some other empirical quantities x_2, x_3, ... such that, e.g., $x_2/\mathrm{u} = 2$, $x_3/\mathrm{u} = 3$, and so on. A measurement scale $s_\mathrm{u}(x_k) = k$ is thus generated, where for each k the actual meaning of $s_\mathrm{u}(x_k) = k$ is $x_k = k\ \mathrm{u}$, in analogy with what we have considered above about Measurement Mappings and Basic Evaluation Equations: x_k and u are empirical properties and k, being a number, is an informational property. Indeed, *measurement scales are the core components of the empirical-informational mapping* realized by measurement [2]:

– their arguments, i.e., the reference quantities x_k, are empirical, but typically accessible in controlled "laboratory" conditions;

– their values, i.e., the multipliers k, are numbers, and as such informational.

Once a measurement scale is available, the process that leads to a measured value y for the measurand x, and therefore to a Basic Evaluation Equation $x = y$, is straightforward (see Figure 1):

1) the reference quantity x_k is found that is the same as the measurand, $x_k = x$ (this is an empirical operation);

2) the scale is applied and x_k is mapped to k, $s_{\mathrm{u}}(x_k) = k$, and therefore $x_k = k$ u (this is partly an empirical and partly an informational operation);

3) by transitivity from 1) and 2) the Basic Evaluation Equation $x = k$ u is obtained (this is an informational operation).

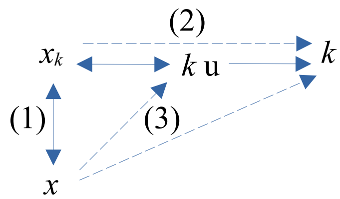

Fig. 1 - Schematic description of the process that maps ratio measurand x to a numerical value k, under the assumption that the measurement scale $s_{\mathrm{u}}(x_k) = k$ has been already constructed.

It is remarkable that this fundamental structure of measurement is independent of the algebraic structure of the concerned property, and therefore its type: ratio in the example above, but possibly interval, or ordinal, or even nominal [8]. Indeed, while, as mentioned above, in the construction of a measurement scale for a ratio quantity we exploit the algebraic structure of the quantity through the multipliers $x_k/\mathrm{u} = k$, the three-step process represented in Figure 1 is performed by means of 1) an empirical comparison, 2) the application of a pre-defined function, and 3) the application of a mathematical property, all of them which do not rely on any specific scale type. Hence, it is only due to ties with the tradition that the condition that measurement can only be performed on quantitative properties can be justified [9], [10]. Rather, at its core, *the structure of measurement is the same for all empirical properties*, independently of their type.

Furthermore, this process is also independent of the nature, either physical or psychosocial, of the properties in question. Indeed, it is structurally not so different from what, for example, fair teachers could in principle do to grade student competences:

1) somehow compare the competence x of the student under evaluation with their reference set of competences, and find the reference competence x_k that is the same as x;

2) retrieve the mark k they consider corresponding to the reference competence x_k;

3) attribute the mark k to the competence x of the given student.

From a structural point of view, the main difference with respect to the previous case is that student competence is plausibly only an ordinal, not a ratio, property. As a consequence, there is not a "unit of student competence" u here, and the reference competence x_k is not mapped to k times u by a measurement scale *s*, given that a scale mark *k* corresponds here to a value that is not (*k* u), but something like (*k* in s), for example (1 in the Mohs scale) for the mineral hardness of talc: the difference is critical in terms of how the scale can be constructed and values can be processed accordingly, but largely immaterial for our purposes.

The methods of operations of measuring systems

The first of the three steps in the process represented in Figure 1 has been described above as a direct, synchronous comparison of the measurand x and the reference quantities in a measurement scale, aimed at finding the reference quantity x_k such that $x_k = x$, which is in fact the operation performed for example by an equal-arm balance. However, this measurement method – that can be then called *direct synchronous* [11] – is seldom performed as such, due to inconvenience of requiring the availability of some measurement standards that realize the definition of the reference quantities to be available while making the measurement. More usually, the object under measurement is put in interaction with a measuring system, which is designed as a transducer, not a comparator, that is sensitive to the measurand and produces a new property, i.e., an instrument reading or indication, as output. Again in the case of masses in a gravitational field, a spring balance is a simple example of such a measuring system, which transduces the applied force to the spring elongation. Since the operation of the system produces a length, not a mass, the key issue here is how to produce the required information about the measurand from the instrument output, i.e., a value of mass from a length.

This is obtained by calibrating the system, by putting it in interaction with some measurement standards that embody quantities of the same kind as the measurand and of known value, so that from the indication a value for the measurand can be obtained. The method based on a transducer can be then called *direct asynchronous*, to emphasize that the measuring system directly interacts with both the object under measurement and some measurement standards, but the two interactions usually happen in different times, thanks to the mediation of the system itself and under the assumption of the stability over time of the measuring system input/output relationship.

A schematic representation of this process requires now two main stages.

First, the *measuring system calibration* (sketched in Figure 2), in which the system transduces a set of reference quantities x_k to indications y_j (1), for each of them a corresponding marker j is obtained (2) via a system-related indication scale $y_j \rightarrow j$ (markers are information entities, possibly numerical values). Finally, by assuming the commutativity of the diagram, the previously known numerical values k associated to the reference quantities x_k are mapped to a set of system-related markers j (3); the result is the mapping $k \to j$ (or its equivalent about values of quantities, if measurement units are taken into account), sometimes called a calibration function:

$$
\begin{tikzcd}
y_j \arrow[r, "(2)"] & j \\
x_k \arrow[u, "(1)"] \arrow[r] & k \arrow[u, dashed, "(3)"']
\end{tikzcd}
$$

Fig. 2 - Schematic description of a measuring system calibration.

In the example considered above about a spring balance, reference masses with associated known numerical values are transduced to spring lengths (1), each corresponding to a marker in the balance scale (2), so that numerical values of masses can be mapped to markers (3).

Second, the *measuring system operation* (sketched in Figure 3), in which the system transduces the measurand x (1), thus obtaining an indication y_j that via the system-related indication scale $y_j \to j$ is mapped to a marker / numerical value j (2), that via the inverse of the calibration function is mapped to the sought numerical value k for the measurand (3):

$$
\begin{tikzcd}
y_j \arrow[r, "(2)"] & j \arrow[d, "(3)"] \\
x \arrow[u, "(1)"] \arrow[r, dashed] & k
\end{tikzcd}
$$

Fig. 3 - Schematic description of a calibrated measuring system operation.

In the considered example, the measurand is transduced to a spring length (1), that corresponds to a marker in the balance scale (2), that is mapped to a numerical value of mass (3).

For a given measurement scale $s_u(x_k) = k$, in reference to which the measuring system was calibrated, this outcome is effectively summarized by the Basic Evaluation Equation $x = k$ u (once again, also measurement uncertainty should be taken into account, as we will do in a next paper of this series).

As for direct synchronous methods, also direct asynchronous methods are not restricted to the measurement of physical properties. Indeed, class assignments administered by teachers can be considered transduction devices, from the tested competence of given students to raw scores, and as such their calibration and operation can be characterized, as shown in Figures 2 and 3 (some structural differences with the physical case should not be underestimated though, in particular that nothing like the International System of Units exists for psychosocial properties).

The objectivity of the output of measuring systems

On this basis, we can now discuss our initial claim, that the basic purpose of measuring systems is to produce information that is about the intended object of the measurement, i.e., the measurand, and that in this sense they aim at producing sufficiently *objective* information. As either comparators (in direct synchronous methods) or transducers (in direct asynchronous methods), a critical stage of the operation of measuring systems is their empirical interaction with the object carrying the measurand. In this interaction, the structural condition that characterizes the quality of a measuring system is its ability to produce an output that depends on the measurand and nothing else, i.e., to find the reference quantity that is the same as the measurand in direct synchronous methods, and to correctly perform the transduction of the measurand in direct asynchronous methods. In this regards, then, a perfectly objective measuring system behaves like an ideal filter, which is fully sensitive to the measurand and fully insensitive to all other properties, that the VIM calls “influence properties” [4]. In this sense, key metrological skills are the abilities to design, manufacture, set up, and operate measuring systems so that their output are as objective as the available resources allow.

References

[1] L. Mari, D. Petri, “Fundamentals of Measurement: knowledge from information about empirical properties”, *IEEE Instr. Meas. Magazine*, vol. 26, no. 1, 2023.

[2] L. Mari, M. Wilson, A. Maul, “Measurement across the sciences – Developing a shared concept system for measurement”, Springer Nature, second edition, 2023 (open access: https://link.springer.com/book/10.1007/978-3-031-22448-5).

[3] L. Mari, D. Petri, “Fundamentals of Measurement: the spread of social knowledge”, *IEEE Instr. Meas. Magazine*, vol. 26, no. 4, 2023.

[4] Joint Committee for Guides in Metrology, JCGM 200, International Vocabulary of Metrology – Basic and General Concepts and Associated Terms (VIM) (3rd ed.), 2012. [Online]. Available: https://www.bipm.org/documents/20126/2071204/JCGM_200_2012.pdf.

[5] J. P. Bentley, “Principles of measurement systems”, New York: Pearson, 2005.

[6] L. Mari, D. Petri, “Measurement science: constructing bridges between reality and knowledge”, *IEEE Instr. Meas. Magazine*, vol. 17, no. 6, 2014.

[7] D. Petri, L. Mari, P. Carbone, “A structured methodology for measurement development”, *IEEE Trans. Instr. Meas.*, vol. 64, no. 9, 2015.

[8] S. S. Stevens, “On the theory of scales of measurement”, *Science*, 103(2684), 677-680, 1946.

[9] M. Bunge, “On confusing ‘measure’ with ‘measurement’ in the methodology of behavioral science”, in M. Bunge (Ed.), The Methodological unity of science (pp. 105–122). Dordrecht: Reidel, 1973.

[10] L. Mari, A. Maul, D. Torres Irribarra, M. Wilson, “Quantities, quantification, and the necessary and sufficient conditions for measurement”, *Measurement*, 100, 115-121, 2017.

[11] A. Maul, L. Mari, D. Torres Irribarra, M. Wilson, “The quality of measurement results in terms of the structural features of the measurement process”, *Measurement*, 116, 611-620, 2018.

POSSIBLE HIGHLIGHTS

- The process of measuring can be effectively described in **functional** way.
- Measurement bridges the empirical realm and the information realm thanks to **measurement scales**.
- Measuring systems, operating as either comparators or transducers, are the key devices to guarantee a sufficient **objectivity** of the information produced by a measurement.

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