Measurement: an evolutionary perspective

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The widespread diffusion of digital technologies and systems is making computational methods ubiquitous, and this implicitly challenges our understanding of measurement as an empirically-grounded process, as testified by the Introduction to the "Guide to the expression of uncertainty in measurement" which states that "measurement can be described as an experimental or computational process". Given that models in measurement are unavoidable, has the very distinction between measurement and computation to be reconsidered? where measurement ends and computation starts? This paper is devoted to propose some considerations on this issue, in the light of the key question of the role of measurement in the information-laden world we live in.

Introduction

In the previous papers of this series about fundamentals of measurement [1]-[4], we have proposed to understand measurement as a process producing information about empirical properties of objects in such a way that the trustworthiness of the reported value or values attributed to the property intended to be measured, i.e., the measurand, is somehow explicitly stated and therefore publicly knowable. This offers a justification for the *epistemic prestige* that measurement holds: the intended messages conveyed by, for example, "this is my opinion" and "this is a measurement result" are clearly different, and such a difference is rooted in the way the provided information is acquired, not necessarily in its quality, considering that both 'excellent opinion' and 'bad measurement' are acceptable concepts. Accordingly, it is in the structure of the process that we need to find what characterizes measurement and makes it different from other processes aimed at producing information like estimation, simulation, prediction, and, indeed, opinion making.

The extreme cases are simple to get rid of. On the one hand, the naive interpretation that conceives of measurement as an entirely empirical process, and thus superposes it with transduction, falls short in explaining how some information – measurement results are such – is produced by an empirical process. On the other hand, it is clear that purely informational processes, like arithmetic operations, are not measurements. The challenge is then to find a suitable, in-between place for measurement, and even before suitable criteria to justify such a placement, where a good example of the complexity of the issue is provided by the statement in the recently published official "Introduction" to the Guide to the expression of uncertainty in measurement (GUM) [5] that "measurement can be described as an experimental or computational process that, by comparison with a measurement standard, produces an estimate of the true value of a property, together with a statement of the uncertainty associated with that estimate, and intended for use in support of decision making". While we already discussed about the measurability of (non-quantitative) properties [1] and the role of true values in measurement [4], the idea of characterizing measurement as "an experimental or computational process" – where that disjunctive "or" implies that there can be measurements that are purely computational processes – calls us for deep reflections about the fundamentals of the field.

Moreover, with the widespread diffusion of digital technologies and systems – lately including also artificial intelligence entities whose cognitive sophistication and autonomy are quickly improving – that are making computational methods more and more pervasive, the challenge has only become more complex and critical: what is measurement then [6], and how is it different from computation? how is the empirical grounding of measurement connected to computation? given the now acknowledged unavoidability of models in measurement, where measurement ends and computation starts?

The concept of measurement as a moving target

A possible misconception has to be fixed first: the meaning of "measurement" was not given once and for all, and vice versa it changed in significant ways in the course of time.

The standard starting point of an even quick review can be Book V of the Euclid's Elements, which has been described as "the earliest contribution to the philosophy of measurement available in the historical record" [7], and in which one can read indeed that "a magnitude is a part of a[nother] magnitude, the lesser of the greater, when it *measures* the greater" [8] (emphasis added; here we take "quantity" and "magnitude" as synonyms). In the context of this Euclidean tradition we can interpret definitions like Charles Hutton's (1815) "measure: denotes any quantity, assumed as unity, or one, to which the ratio of other homogeneous or like quantities may be expressed" [9]. While "measure" could be meant accordingly as what today we call "measurement unit" (possibly by excluding units of less-than-ratio properties, like the Celsius degree), and therefore with an at least partial empirical connotation, this interpretation clashes with Thomas Reid's assertions that ‘‘mathematics contains properly the doctrine of measure; and the object of this science is commonly said to be quantity; therefore quantity ought to be defined as what may be measured" [10].

In this quick review another milestone, often mentioned as the seminal work of representational theories of measurement, is Otto Hölder's paper (1901) [11], whose original German title was translated as "The axioms of quantity and the theory of measurement", according to translators' understanding that "we now know precisely why some attributes are measurable and some not: what makes the difference is possession of quantitative structure, i.e., conformity to Hölder's axioms". The opening sentence of the paper is very clear on this matter: "by 'axioms of arithmetic' has been meant what I prefer to call 'axioms of quantity'", and the fact is that, aside from the title of the paper, the word "measurement" appears only once in the paper, in a sentence in which "the theory of the measurement" is meant to be "the modern theory of proportion".

Middle school teachers who ask pupils "to measure" the volume of a cube by computing the third power of the length of the side rightly speak then in conformity with this long and honorable tradition, actually stemming from the mentioned Euclid's Elements. But the point is that "in the geometrical constructions employed in the Elements [...] empirical proofs by means of measurement are strictly forbidden" [8], as stated by the English translator in his introductory notes, thus making it clear that in this sense it is possible *to measure without performing a measurement*. This should be sufficient to agree that we are facing here a mixture of meanings, that should be properly disentangled as already suggested by Mario Bunge, who highlighted the need to remove the confusion between 'measure' and 'measurement' [12], also given how the two concepts are so intertwined in daily activities and common discourse. And this also hints that the relationships between measurement and computation have been complex, and possibly complicated, from the very beginning of their historical development.

Since "measurement" is not a trademarked term, and the definitions that one can find in the relevant literature are many and diverse, with our aim to provide a justification of the characterization of measurement as a specific tool of information production we need "to open the box" and analyze measurement methods, as in particular related to the distinction between so-called "direct" and "indirect" methods, and compare them to purely computational methods.

Measurement and computation: an analysis of the structural differences based on two simple examples

Let us make our analysis as concrete as possible by referring to two simple examples about the process of evaluating (i.e., obtaining information in the form of values on) the quantitative property of an object, say the volume of a material body. For each of them we list the evaluation principle, the evaluation method (we use the term "evaluation method", instead of "measurement method", for the sake of generality), and the main assumptions it requires to be applied.

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Example 1: Liquid displacement method

Principle: The volume of the object is the same as the volume of the displaced liquid when the object is immersed.

Method: Completely immerse the object in a cylindrical container partially filled with liquid, and measure the displaced volume by measuring the difference of height of the liquid and multiplying it by the area of the cylinder.

Main assumptions: The object is fully immersed, is impermeable and non-soluble, with no absorption or chemical interaction with the liquid. No air bubbles or surface tension artifacts. The liquid is incompressible and insensitive to temperature. The container is cylindrical.

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Example 2: Mass-density method

Principle: The volume V of the object can be computed from its mass m and its assumed uniform density ρ by the relation $V = m / \rho$.

Method: Somehow obtain a value of the mass and a value of the density of the object, and divide the two values.

Main assumptions: The object is made of a material with uniform density, with no internal voids.

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These two examples have the same purpose – to evaluate the volume of an object – and therefore can be meaningfully compared. In view of our underlying aim of better understanding the relationships between measurement and computation, the key structural difference of the two examples is in the evaluation method: Method 1 specifies an empirical process to be performed, while in Method 2 is independent of the way the values of mass and density are obtained, so that for example the mass of the object could be actually obtained by some empirical process while its density could be read from a table under the assumption that the material of the object is known. Furthermore, Method 1 is based on a device that in principle is sensitive to the quantity intended to be measured (i.e., the measurand: the volume of the object) and operates by exploiting a physical effect of transduction, from the measurand to another quantity a value of which is assumed to be more easily obtained (i.e., the instrument indication: the difference of height of the liquid in the container). This implies that for such a device to be employed as a measuring instrument it must be calibrated, through its interaction with measurement standards of volume, i.e., of the same kind as the measurand. Vice versa, Method 2 does not require any measurand-related interaction, even in the case the values of both the mass and the density of the object are somehow obtained by empirical processes.

As described, both Methods include a computational step, but their function is completely different. In the case of Method 1, the operation of multiplying the value of the difference of height of the liquid, before and after having immersed the object, by the value of the area of the cylinder is only related to the device, and in fact it could be even avoided by suitably calibrating it and marking the displayed scale of heights with values of volume instead of length. In the case of Method 2, the operation of dividing the value of the mass by the value of the density of the object is what qualifies the method as related to the evaluation of volume, and is independent of the features of the measuring instruments that could be possibly exploited to produce such values.

On the basis of this simple analysis we are now ready to consider the crucial question: *are these differences sufficient to acknowledge a structural difference between Example 1 and Example 2 as cases of either measurement or computation?*

Measurement and computation: recovering or blurring the distinction?

Let us point out an interesting fact. The first edition of the *International Vocabulary of Metrology* (VIM) [13], published in 1984, included the definition of both ‘direct method of measurement’ and ‘indirect method of measurement’:

- ‘direct method of measurement’: “method of measurement in which the value of a measurand is obtained directly, rather than by measurement of other quantities functionally related to the measurand”
- ‘indirect method of measurement’: “method of measurement in which the value of a measurand is obtained by measurement of other quantities functionally related to the measurand”

Even though the first definition is questionable as written – ‘direct method’ almost circularly defined in terms of a value “obtained directly” – these two definitions clearly reflect the Examples we have been discussing: on the one hand, Example 1 is about a method requiring a measuring instrument designed to be sensitive to quantities of the same kind as the measurand, and as such *is “direct”*; on the other hand, Example 2 is about a method requiring a functional relation between the measurand and other, possibly measured, quantities, and as such *is “indirect”*, where the key point here is that such other quantities must be measured, and not generically evaluated, for the process to be acknowledged to be a measurement in turn. Explicitly, Method 2 applies also to the case the values of the mass and the density of the concerned object are, say, guessed, or chosen randomly, or even specified about a still non-existing object to be produced, and these would be cases of evaluation of volume, not of measurement. Vice versa, high school students asked to compute the volume of an object from values of mass and density read on a piece of paper would be performing a measurement task, thus treating measurement as a specific kind of computation: a position that would be consistent with the Euclidean tradition were "measurement" and "measure" considered as synonyms, but that would lose any justification for the *epistemic prestige* that measurement holds in the experimental sciences and in daily activities.

The interesting fact alluded to above is that such a fundamental distinction between direct and indirect methods of measurement completely disappeared in the second edition of the VIM [14], published in 1993, and is only mentioned as a possible way "to qualify" measurement methods in the current, third edition of the VIM [15], published in 2007 / 2008, but with no definition or further explanation. Why so?

While it is clearly not our purpose here to sketch an explanation that plausibly involved multiple actors in the search of a compromise, consensus position, a paragraph of the *Guide to the expression of uncertainty in measurement* (GUM, the "companion" document of the VIM in the context of the Joint Committee for Guides in Metrology, JCGM) [16] is very explicit in providing an answer to our question: "This Guide is also applicable to evaluating and expressing the uncertainty associated with the conceptual design and theoretical analysis of experiments, methods of measurement, and complex components and systems. Because a measurement result and its uncertainty may be conceptual and based entirely on hypothetical data, the term "result of a measurement" as used in this Guide should be interpreted in this broader context."

Let us analyze this message in the perspective of our interest to understand what measurement is and its differences with computation. The first sentence reports a matter of fact: the so-called "law of propagation of uncertainty" and the related methods and mathematical techniques apply not only to values obtained by means of empirical processes performed on empirical properties. For example, a college teacher could ask her students to compute both the volume of an object from values of mass and density and the combined standard uncertainty from the standard uncertainties of the measured values of the two input quantities, and the GUM would be an excellent reference to this purpose. The point is that, according to the second sentence of the quoted paragraph, the request of evaluating uncertainty *is sufficient to make it a measurement task*, even though the information may come from "entirely on hypothetical data", thus removing all constraints about an empirical component of the process. Notably, that "a measurement result and its uncertainty may be conceptual and based entirely on hypothetical data" is taken for granted here, and no justification of this position is given. And with no justification the doubt remains that this strategy is only based on the, however left implicit, principle that measurement is considered here to be any process whose products can be dealt with by means of GUM-related methods and mathematical techniques.

Hence, it is perhaps not surprising that the above quoted "Introduction" to the GUM confirms this understanding, and only adds the requirement that the process be performed "by comparison with a measurement standard", a condition that implies that also the comparison of the object under "measurement" and the measurement standards can be "entirely hypothetical", given that an empirical comparison with an "entirely hypothetical" entity is obviously not possible.

At stake here is the foundation of the *epistemic prestige* that measurement has gained in the last centuries: is it really independent of conditions about being at least partly empirical of the processes that we acknowledge to be measurements?

Some final thoughts

We already noted [1] that according to the International Office of Weights and Measures (BIPM) measurement is expected to be able to provide information worth the public trust because it is "reliable, traceable and comparable" [17]. Requiring that a measurement reports not only a measured value for the measurand but also some information on its trustworthiness – typically as a standard uncertainty but in simpler cases also, for example, as an appropriate number of significant digits of the value – is an implementation of this expectation. But this makes it a necessary condition for a process to be a measurement, not also a sufficient condition.

While pointing out once again that the issue we are discussing is not a matter of truth or falsehood, because there is nothing like the true meaning of "measurement", the idea that measurement can be a purely computational process seems so detached from the tradition rooted in Galilean science, i.e, what we have been calling "*experimental* method", that cannot be taken lightly.

Assuming that together with the value of the quantity also its uncertainty is evaluated, several scenarios can be considered. Moving roughly from the most conservative to the most extreme positions, are the following examples of measurements?

- The evaluation of the volume of an object at a temperature that differs from the one while the measurement is performed (by applying a physical law that takes thermal expansion into account).
- The evaluation of the volume of an object from values of its mass and density obtained by asking the opinion of an observer (who consciously stated also her uncertainty on this opinion).

– The evaluation of the volume of the Colossus of Rhodes from the available historical information (an object whose existence is considered certain but which was destroyed more than two thousand years ago).
– The evaluation of the volume of the body of Harry Potter from the descriptions found in the relevant literature.
– The evaluation of "rolume" of an object, where rolume is defined as the product of the fifth power of the electric resistance and the square root of the volume of the object.
  (This last example is inspired by Brian Ellis' famous discussion on "hage": "People can be arranged in a linear order, for example, by taking the product of their height and age. Let us say that they are then arranged in order of 'hage'." [18]; what Ellis commented then is interesting: "But we do not think that "hage" is the name of a genuine quantity like temperature or momentum.").

All of them are examples of evaluation of the property of an object by computation but, again, are they measurements? Is it really sufficient to complement the reported value with an uncertainty for making them measurements? The point is obvious then: can a non-empirically-existing property (possibly because not empirically existing anymore or yet) be measured? what kind of trustworthiness can we expect from the outcomes of a purely computational process applied to a purely hypothetical property? It is a fact that concepts may change with the evolution of knowledge and ethical values, and it is a sound principle that, while the available tools influence the way we understand, we should conceptualize first and then choose the tools accordingly (the risk of operating in reverse is well exemplified by the common saying that "if all you have is a hammer, everything looks like a nail": the fact that something is accompanied by a statement of uncertainty does not make it a measurement result). Precisely because our daily experience is more and more about "virtual" interactions with "virtual" objects, measurement can maintain its acknowledged role only if it remains different from computation, and solidly grounded on its empirical bases. Modeling and then computation is a key component of measurement, which is an empirical *and* computational process.

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Possible pull quotes:

- Measurement is a process producing information about empirical properties of objects that is worth the public trust.

● Measurement has been described as an experimental or computational process: is it an adequate position?

● The concept of measurement has changed over time.

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