

Geometric Tracking Control of an Unmanned Aerial Vehicle based on the Moving Mass Concept on SE(3)

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Abstract—This paper is focused on presenting the concept of geometric tracking control for a specific unmanned aerial vehicle (UAV) based on the moving mass concept. It has the ability to exploit its dynamic center of mass as a means of stabilization and control. A mathematical model of such system will be given as grounds for developing the nonlinear geometric tracking controller on the special Euclidean group SE(3). It will be shown that the chosen control terms have desirable properties. Finally, Gazebo simulation results for a selected trajectory tracking problem will be presented using a model of an aerial robot consisting of two moving masses distributed in a standard plus configuration.

I. INTRODUCTION

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II. MATHEMATICAL MODEL

First of all, it is necessary to introduce a fixed inertial reference frame $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ and a body-fixed frame $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$. As it was previously stated, μ MORUS UAV will exploit its shifting center of gravity (CoG) due to the moving masses in order to maneuver and stabilize itself. Therefore, a CoG vector from the origin of the body-fixed frame will be defined as follows:

$$\vec{r}_{cm} = \frac{m_b \vec{r}_{0,b} + \sum_{i=1}^n m_i \vec{r}_i}{m_b + \sum_{i=1}^4 m_i} = \frac{\sum_{i=1}^4 m_i \vec{r}_i}{m_t}, \quad (1)$$

The following terms are defined as:

- $\vec{r}_{cm} \in \mathbb{R}^3$ - Center of gravity with respect to the body-fixed frame
- $\vec{r}_i \in \mathbb{R}^3$ - Position of the i-th mass w.r.t. the body-fixed frame
- $\vec{r}_b \in \mathbb{R}^3$ - Position of UAV body w.r.t. the body-fixed frame. Note that because the body frame origin coincides with the rigid body CoG (without considering the moving masses) this term yields $\vec{r}_b = 0_{3 \times 1}$
- $m_b \in \mathbb{R}$ - Mass of the UAV body
- $m_i \in \mathbb{R}$ - Mass of the i-th moving mass attached to the UAV link
- $m_t \in \mathbb{R}$ - Mass of the whole UAV

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The equations of motion expressed in the inertial frame while taking in consideration that the CoG is located outside the origin of the body-fixed frame are as follows:

$$\dot{x} = v \quad (2)$$

$$m_t \dot{v} - m_t g e_3 - m_t R \vec{r}_{cm} \times \dot{\Omega} - m_t R \hat{\Omega} \vec{r}_{cm} \Omega = -f R e_3 \quad (3)$$

$$\dot{R} = R \hat{\Omega} \quad (4)$$

$$J \dot{\Omega} + \Omega \times J \Omega + m_t \vec{r}_{cm} \times R^T \dot{v} = M \quad (5)$$

The following terms are defined as:

- $J \in \mathbb{R}^{3 \times 3}$ - Moment of inertia matrix w.r.t. the body-fixed frame
- $R \in SO(3)$ - Rotation matrix from the body fixed frame to the inertial frame
- $\Omega \in \mathbb{R}^3$ - Angular velocity in the body-fixed frame
- $x \in \mathbb{R}^3$ - Location of the body-fixed frame in the inertial frame
- $v \in \mathbb{R}^3$ - Velocity of the body-fixed frame in the inertial frame
- $f \in \mathbb{R}$ - Total thrust produced by the UAV
- $M \in \mathbb{R}^3$ - Total moments acting in the body-fixed frame

The *hat map* is an operator equivalent to the expression $\hat{x}y = x \times y$. It maps elements of \mathbb{R}^3 to the $so(3)$ Lie algebra.

Moment of inertia matrix is defined as follows:

$$J = J_b + \sum_{i=1}^4 J_i \quad (6)$$

These equations describe the dynamical flow of a rotating and translating rigid body in terms of evolution of $(R, x, \Omega, \dot{x}) \in TSE(3)$ on the tangent bundle of SE(3).

Height and yaw of the UAV is controlled by variations in rotor velocity, whereas roll and pitch by moving the masses along UAV links placed in plus configuration. It is assumed that first and third propeller rotate clockwise, while second and fourth rotate counter-clockwise. The relation between moments, thrust and rotor velocity is the following:

$$f_i = b_f \omega_i^2 \quad (7)$$

$$\tau_i = (-1)^i b_m f_i \quad (8)$$

Where the following terms are defined as:

- $f_i \in \mathbb{R}$ - Thrust of the i-th motor
- $\tau_i \in \mathbb{R}$ - Moment i-th motor produces

- $b_f \in \mathbb{R}$ - Motor thrust constant
- $b_m \in \mathbb{R}$ - Motor moment constant
- $\omega_i \in \mathbb{R}$ - Rotation velocity of the i-th rotor

Napisat cemu je jedanaka ukupna sila i momenti,
napisati onu matricu

III. MID-RANGING CONTROL CONCEPT

TODO: Control...

IV. SIMULATION

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V. EXPERIMENTS

Experiments...

VI. CONCLUSION

Conclusions

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