Geometric Tracking Control of an Unmanned Aerial Vehicle Based On Variations in Center of Gravity

Antun Ivanović, Marko Car, Tomislav Haus, Matko Orsag, Stjepan Bogdan, Lovro Marković

***Abstract*— This paper is focused on presenting the concept of geometric tracking control for an unmanned aerial vehicle (UAV) based on variations in center of gravity. It has the ability to exploit its dynamic center of mass as a means of stabilization and control. A mathematical model of such system will be given as grounds for developing the nonlinear geometric tracking controller on the special Euclidean group SE(3). It will be shown that the chosen control terms have desirable properties. Finally, two sets of Gazebo simulation results for a selected trajectory tracking problem with a different UAV concept will be presented.**

I. Introduction

Spomenuti za mmc / manipulator na letjelici clanke, tko je prije radio tip upravljanja itd.

Spomenuti mikro Morus letjelicu

The goal of this paper is to attempt to apply the already developed concept of geometric control on a specific type of Unmanned Aerial System (UAS). The vehicles studied in this paper are built as a classic quadrotor UAV set up in plus configuration. However, unlike a standard UAV it utilizes variations in its center of gravity in order to achieve attitude tracking. Essentially, this means that such variations, which would usually be considered a disturbance in the system, could be exploited as a means of controlling the UAS.

One of the ways these variations will be achieved is by implementing the moving mass concept on the standard quadrotor UAV. This includes mounting moving blocks on the UAV axes, whose offset will act as the control input of the system along with rotor speed variation.

Another way such variations could be achieved is by mounting two 2-DOF manipulators to the UAV and having them carry a relatively heavy payload - compared to the total UAV mass. In this case position of the payload will directly determine any offset in center of gravity.

The proposed nonlinear geometric control concept for the described UAS will primarily be used for trajectory tracking, given smooth control inputs for position $x_d(t)$ and heading $\vec{b}_{1,d}$. Before presenting the control terms, the UAS dynamic model needs to be stated.

Since the geometric controller is used in this paper, the dynamics need to be expressed on SE(3) configuration manifold. Similar quadrotor dynamics have already been considered in other research papers e.g. [1], [2], [3]. Within the classic quadrotor dynamic model, the center of gravity lies in the origin of the body-fixed frame. However, in this paper this is not the case. Due to the variations in the center of gravity a different dynamic model will be taken in consideration.

Having chosen the dynamic model on SE(3) configuration manifold, control terms for total thrust and moments in the body fixed-frame can be selected. These control terms are assumed to have instantaneous effect on the system and will be treated as such when considering the stability of error dynamics.

The controller behavior will be tested on a prescribed trajectory tracking problem. Simulation is going to be conducted in the Gazebo simulator within ROS environment. It is important to note that in such realistic environment effects actuator dynamics, rotor velocity change etc. will be included. The paper is organized as follows...

Dovršiti introduction

Contributions...

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II. Mathematical model

Promijeniti jednadzbe na nacin - vektori podebljano, Matrice text

First of all, it is necessary to introduce a fixed inertial reference frame $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ and a body-fixed frame $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$. As it was previously stated, one of the ways μMORUS UAV will exploit its shifting center of gravity (CoG) due to the moving masses in order to maneuver and stabilize itself. Therefore, a CoG vector from the origin of the body-fixed frame will be defined as follows:

$$\vec{r}_{CoG} = \frac{m_b \vec{r}_{0,b} + \sum_{i=1}^{n} m_i \vec{r}_i}{m_b + \sum_{i=1}^{4} m_i} = \frac{\sum_{i=1}^{4} m_i \vec{r}_i}{m_t}, \quad (1)$$

Dodati r cog jednadzbu za manipulator / payload, reći da se linkovi manipulatora poništavaju

.

The following terms are defined as:

- $\vec{r}_{CoG} \in \mathbb{R}^3$ - Center of gravity with respect to the body-fixed frame
- $\vec{r}_i \in \mathbb{R}^3$ - Position of the i-th mass w.r.t. the body-fixed frame
- $\vec{r}_b \in \mathbb{R}^3$ - Position of UAV body w.r.t. the body-fixed frame. Note that because the body frame

origin coincides with the rigid body CoG (without considering the moving masses) this term yields $\vec{r}_b = 0_{3x1}$
- $m_b \in \mathbb{R}$ - Mass of the UAV body
- $m_i \in \mathbb{R}$ - Mass of the i-th moving mass attached to the UAV link
- $m_t \in \mathbb{R}$ - Mass of the whole UAV

Spomenuti jakobijan za racunanje pomaka u manipulatoru(?)

The equations of motion expressed in the inertial frame while taking in consideration that the CoG is located outside the origin of the body-fixed frame are as follows[4]:

Referenecirati se na prošireni model iz dodatka, ali ostati pri ovome.

$$\dot{x} = v \tag{2}$$

$$m_t\dot{v} + m_t g e_3 - m_T R\vec{r}_{CoG} \times \dot{\Omega} - m_t R\hat{\Omega}\hat{\vec{r}}_{CoG}\Omega = fRe_3 \tag{3}$$

$$\dot{R} = R\hat{\Omega} \tag{4}$$

$$J\dot{\Omega} + \Omega \times J\Omega + m_t\vec{r}_{CoG} \times R^T\dot{v} = M \tag{5}$$

The following terms are defined as:
The *hat map* is an operator equivalent to the expression $\hat{x}y = x \times y$. It maps elements of $\mathbb{R}^3$ to the so(3) Lie algebra.

- $J \in \mathbb{R}^{3\times3}$ - Moment of inertia matrix w.r.t. the body-fixed frame
- $R \in SO(3)$ - Rotation matrix from the body fixed frame to the inertial frame
- $\Omega \in \mathbb{R}^3$ - Angular velocity in the body-fixed frame
- $x \in \mathbb{R}^3$ - Location of the body-fixed frame in the inertial frame
- $v \in \mathbb{R}^3$ - Velocity of the body-fixed frame in the inertial frame
- $f \in \mathbb{R}$ - Total thrust produced by the UAV
- $M \in \mathbb{R}^3$ - Total moments acting in the body-fixed frame

Moment of inertia matrix expressed in the body-fixed frame is defined as follows:

$$J = J_b + \sum_{i=1}^{4} J_i \tag{6}$$

where J_b is body and J_i is i-th mass moment of inertia. Using the parallel axis theorem, one is able to calculate J_i while knowing moment of inertia around its CoG:

$$J_i = J_{i,CoG} + m_i(\vec{r}_i^T \cdot \vec{r}_i I_{3x3} - \vec{r}_i \cdot \vec{r}_i^T) \tag{7}$$

Equations 2, 3, 4 and 5 describe the dynamical flow of a rotating and translating rigid body in terms of evolution of $(R, x, \Omega, \dot{x}) \in$ TSE(3) on the tangent bundle of SE(3).

Height and yaw of the UAV is controlled by variations in rotor velocity, whereas roll and pitch by moving the masses along UAV links placed in plus configuration. It is assumed that first and third propeller rotate clockwise, while second and fourth rotate counter-clockwise. The relation between moments, thrust and rotor velocity is the following:

$$f_i = b_f\omega_i^2 \tag{8}$$

$$\tau_i = (-1)^i b_m f_i \tag{9}$$

Where the following terms are defined as:
- $f_i \in \mathbb{R}$ - Thrust of the i-th motor
- $\tau_i \in \mathbb{R}$ - Moment i-th motor produces
- $b_f \in \mathbb{R}$ - Motor thrust constant
- $b_m \in \mathbb{R}$ - Motor moment constant
- $\omega_i \in \mathbb{R}$ - Rotation velocity of the i-th rotor

Total thrust can be expressed as:

$$f = \sum_{1}^{4} f_i \tag{10}$$

and total moment acting in the body-fixed frame as:

$$M = [m_1 g d_x \cdot b_{3,dx}, m_2 g d_y \cdot b_{3,dy}, b_m(-f_1 + f_2 - f_3 + f_4)] \tag{11}$$

Using f and M as control inputs of the system one is able to obtain the desired force of each rotor and the desired offset of each moving mass d_x and d_y. Note that the offset of masses on the same axis is equal.
Actuator dynamics of moving masses and the change in desired rotor force will be regarded as instantaneous while presenting the controller synthesis and stability conditions. They will, however, be included within the Gazebo simulation environment.

III. Geometric control on SE(3)

In this section a nonlinear tracking controller will be developed. The main focus will be put on position tracking, therefore the trajectory will consist of a desired position $x_d(t)$ and desired heading $\vec{b}_{1,d}$ of the body-fixed frame. Since the given position is known ahead of time, one is able to calculate both desired linear velocity $v_d(t)$ and acceleration $a_d(t)$ which will also inherently be included as inputs.
The controller will be developed on the nonlinear Lie group SE(3) whose subgroups are the rotation group SO(3) and translation group T(3). The main advantage of using the SO(3) rotation group is to avoid any singularities or ambiguities that may arise when representing rotations with Euler angles or quaternions.
Firstly, chosen position and orientation errors will be presented which will also lie on the SE(3) manifold and its tangent space. Using previously defined errors, nonlinear control terms can be chosen. Finally, the stability conditions of the tracking errors will be presented.

A. Tracking errors

Compatible attitude error function and transport map between tangent bundles of SO(3) are chosen as suggested in [5] and confirmed in research regarding geometric control with aerial vehicles [3], [2], [6], [1]. Attitude error function on SO(3) along with its compatible transport map are chosen as:

$$\Psi(R, R_d) = \frac{1}{2}tr[I - R_d^T R] \tag{12}$$

$$\mathcal{T}(R, R_d) = R^T R_d \tag{13}$$

Linear position and velocity tracking errors are defined as follows:

$$e_x = x - x_d \tag{14}$$

$$e_v = v - v_d \tag{15}$$

Defining attitude and angular velocity tracking errors is not as straight-forward. It is shown in [5] that the attitude tracking error should be chosen as a left-differential of the attitude error function $\Psi(R, R_d)$. It is chosen as follows:

$$e_R = \frac{1}{2}(R_d^T R - R^T R_d)^{\vee} \tag{16}$$

Due to the fact that angular velocities $\Omega \in T_R SO(3)$ and $\Omega_d \in T_{R_d} SO(3)$ lie in different tangential bundles, the proposed left transport map 13 needs to be applied when calculating the tracking error:

$$e_\Omega = \Omega - R^T R_d \Omega_d \tag{17}$$

B. Control terms

Taking in consideration the proposed system dynamics 3 and 5, the force and moment control terms are chosen as follows:

$$\begin{aligned} A = (&-k_x e_x - k_v e_v \\ &+ mge_3 + m\ddot{x}_d \\ &- mR\vec{r}_{cm} \times \dot{\Omega} - mR\hat{\Omega}\hat{\vec{r}}_{cm}\Omega) \\ f =& A \cdot Re_3 \end{aligned} \tag{18}$$

$$\begin{aligned} M = &-k_R e_R - k_\Omega e_\Omega \\ &+ \Omega \times J\Omega - J(\hat{\Omega}R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d) \\ &+ m\vec{r}_{cm} \times R^T \ddot{x} \end{aligned} \tag{19}$$

When error dynamics will be presented, it can be seen that the control terms are chosen in order to negate the undesirable system dynamics.

Desired rotation matrix is constructed in the traditional way when considering geometric control of aerial vehicles [1], [6], [2]. The proposed desired rotation matrix is constructed as $R_d = [\vec{b}_{1,c}, \vec{b}_{3,d} \times \vec{b}_{1,c}, \vec{b}_{3,d}]$ where component vectors of R_d are calculated in the following way:

$$\vec{b}_{3,d} = \frac{A}{||A||} \tag{20}$$

$$\vec{b}_{1,c} = -\frac{(\vec{b}_{3,d} \times (\vec{b}_{3,d} \times \vec{b}_{1,d}))}{||\vec{b}_{3,d} \times \vec{b}_{1,d}||} \tag{21}$$

It is also assumed that:

$$||A|| \neq 0 \tag{22}$$

The chosen constraint for the trajectory tracking problem differs slightly from the one proposed in [1] due to the fact that different model dynamics are considered in this paper. New trajectory constraints are presented as follows:

$$||mge_3 + m\ddot{x}_d - mR\vec{r}_{cm} \times \dot{\Omega} - mR\hat{\Omega}\hat{\vec{r}}_{cm}\Omega|| < B \tag{23}$$

where B is some positive constant.

Desired angular velocity and acceleration also need to be considered in this trajectory tracking problem. One is able to calculate the desired angular velocity and acceleration using R_d and its derivatives in the following way:

$$\hat{\Omega}_d = R_d^T \dot{R}_d \tag{24}$$

$$\dot{\hat{\Omega}}_d = -\hat{\Omega}_d \hat{\Omega}_d + R_d^T \ddot{R}_d \tag{25}$$

Derivatives of R_d are easily calculated using the backwards differentiation method. It has to be noted that due to stability issues, computation rate of desired angular velocity and acceleration has to be lesser than the overall simulation rate. For further implementation details, please refer to [7].

C. Error dynamics and stability discussion

In this section linear and angular error dynamics will be presented. First of all, derivatives over time need to be calculated for linear 15 and angular 17 tracking errors:

$$\dot{e}_v = \ddot{x} - \ddot{x}_d \tag{26}$$

$$\dot{e}_\Omega = \dot{\Omega} + \hat{\Omega}R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d \tag{27}$$

After including 3 and 5 in 26 and 27 respectively, the following equations are obtained:

$$\begin{aligned} m\dot{e}_v = &- mge_3 - m\ddot{x}_d \\ &+ mR\vec{r}_{cm} \times \dot{\Omega} + mR\hat{\Omega}\hat{\vec{r}}_{cm}\Omega \\ &+ A + X \end{aligned} \tag{28}$$

$$\begin{aligned} J\dot{e}_\Omega = &M - \Omega \times J\Omega \\ &+ J(\hat{\Omega}R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d) \\ &+ m\vec{r}_{cm} \times R^T \ddot{x} \end{aligned} \tag{29}$$

Note that in 28 $A \in \mathbb{R}^3$ is regarded as a control force for the translational dynamics, mentioned in 18, while $X \in \mathbb{R}^3$ is a bounded term that arises when deriving this equation which equals:

$$X = \frac{f}{(R_d e_d)^T Re_3}(R_d e_3 - ((R_d e_d)^T Re_3)Re_3) \tag{30}$$

After substituting control force from 18 and 19 in 28 and 29 respectively the final form of error dynamics is obtained:

$$m\dot{e}_v = -k_x e_x - k_v e_v + X \tag{31}$$

$$J\dot{e}_\Omega = -k_R e_r - k_\Omega e_\Omega \tag{32}$$

Having started with a different mathematical model of the UAV than the previous research done on this subject, applying the newly formed control terms 18 and 19 and

taking in consideration initial assumptions 22 and 23 one is able to derive identical translational and rotational error dynamics as found in [3], [1].
Therefore, to avoid redundancy, the full stability proof will not be presented in this paper. However, the final conclusions for exponential asymptotic stability of the attitude error function and attraction to the zero-equilibrium state of tracking errors will be outlined.

If the initial UAV configuration satisfies the following conditions:

$$\Psi(R(0), R_d(0)) < 2 \tag{33}$$

$$||e_\Omega(0)||^2 < \frac{2}{\lambda_{min}(J)} k_R(2 - \Psi(R(0), R_d(0)) \tag{34}$$

it can be shown that tracking errors of the whole system will reach zero-equilibrium state and the attitude function will be exponentially bounded as:

$$\Psi(R(t), R_d(t)) \leq \min\{2, \alpha e^{-\beta t}\} \tag{35}$$

for some positive constants α and β.

IV. Simulation

Reći da se provode 2 seta experimenate.

Dodati još i parametre za slucaj sa manipulatorima

In this section simulation results will be presented and analysed. Simulations are conducted in the Gazebo simulator within the ROS environment. UAV used in experiments is the μMorus with moving-masses which can be found in the *mmuav_gazebo* repository [7], along with its parameters. Control parameters are chosen as follows:

$$k_x = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 50 \end{bmatrix}, \; k_v = \begin{bmatrix} 3.75 & 0 & 0 \\ 0 & 3.75 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$k_R = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \; k_\Omega = \begin{bmatrix} 0.65 & 0 & 0 \\ 0 & 0.65 & 0 \\ 0 & 0 & 1.54 \end{bmatrix}$$

It is important to note that the actuator dynamics of the moving masses is taken in consideration within the Gazebo simulation environment. Furthermore there is a slight transient delay when increasing or decreasing rotor velocity which results in a non-instantaneous control force change. These phenomena were not taken in consideration while modeling the system and choosing control terms.

The chosen trajectory tracking problem is formulated as a 20 second rotating spiral:

$$\vec{x}_d(t) = [0.4\mathrm{t};\, 0.5\mathrm{sin}(\pi \mathrm{t});\, 0.6\mathrm{cos}(\pi \mathrm{t}) + 2]$$

$$\vec{b}_{1,d}(t) = [\cos\left(\frac{\pi}{5}\mathrm{t}\right); \sin\left(\frac{\pi}{5}\mathrm{t}\right); 0]$$

Initial position and orientation is chosen at the start of the trajectory.

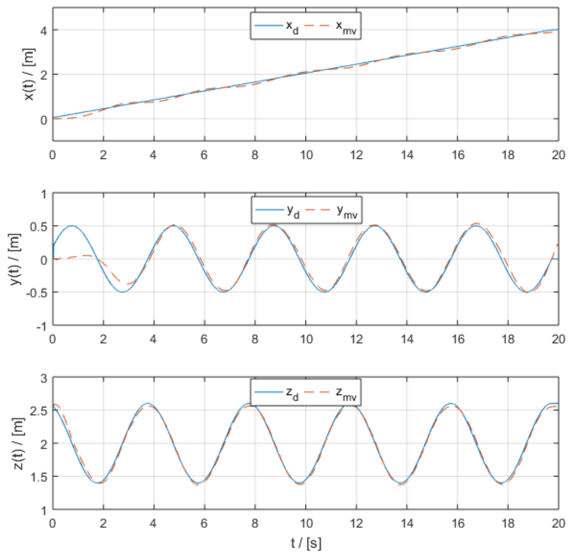


Fig. 1: Comparison of the desired $\vec{x}_d$ and measured position values $\vec{x}_{mv}$. (Using the moving mass control concept.)

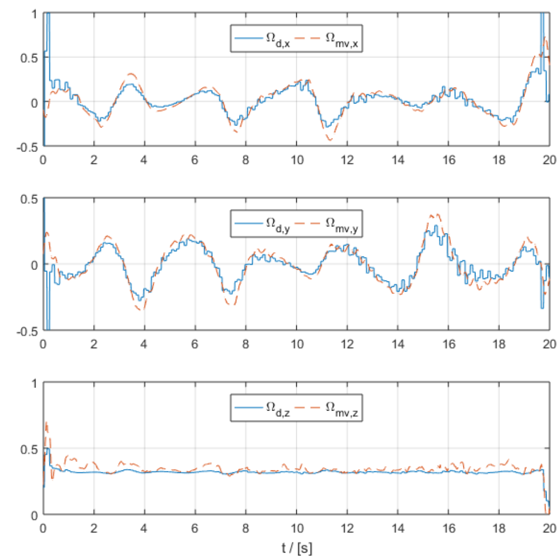


Fig. 2: Comparison of desired Ω_d and measured Ω_{mv} angular velocity values. (Using the moving mass control concept.)

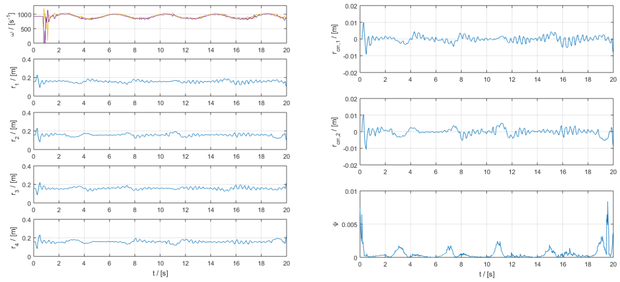
V. Conclusion

Conclusions

APPENDIX

A. Extended UAV dynamics

General form of Euler-Lagrange dynamics for a rotating rigid body in SE(3) configuration manifold in the body-



a) b)

Fig. 3: Control inputs are shown in figure a) in the following order: rotor velocities ω_i and mass offsets r_i. Figure b) shows first two components of CoG vector $\vec{r}_{CoG}$ (third component is always zero since masses only move in x-y plane of the body-fixed frame) and attitude error function Ψ. (Using the moving mass control concept.)

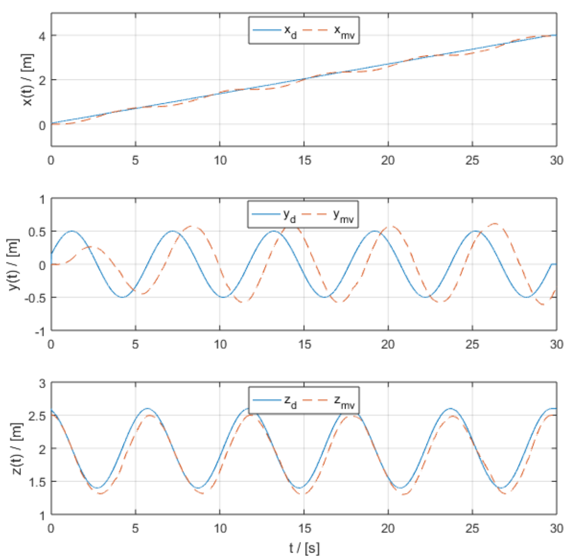


Fig. 4: Comparison of the desired $\vec{x}_d$ and measured position values $\vec{x}_{mv}$. (Using the manipulator control control.)

fixed frame:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \omega}\right) + \omega \times \frac{\partial \mathcal{L}}{\partial \omega} + v \times \frac{\partial \mathcal{L}}{\partial v} + \sum_{i=1}^{3} r_i \times \frac{\partial \mathcal{L}}{\partial r_i} = 0 \tag{36}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial v}\right) + \omega \times \frac{\partial \mathcal{L}}{\partial v} - R^T \frac{\partial \mathcal{L}}{\partial x} = 0 \tag{37}$$

For the the proposed UAS with variations in center of mass the Lagrangian is:

$$\mathcal{L}(R, x, \Omega, v) = \frac{1}{2}\omega^T J \omega + m\omega^T S(r_{cm}) v + \frac{1}{2} m v^T v - U(R, x) \tag{38}$$

where $U(R, x)$ is the potential energy of the system. It

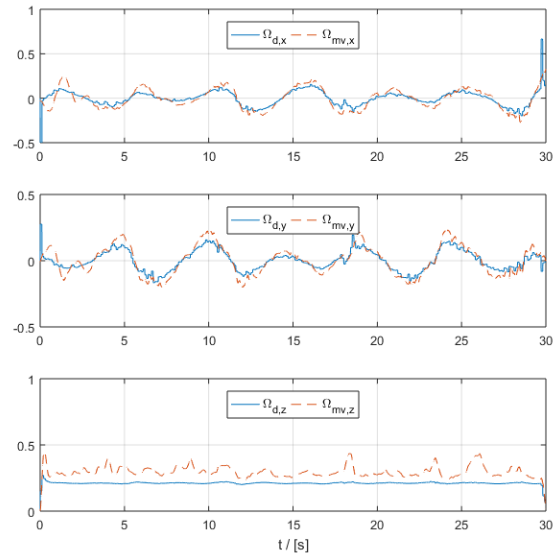
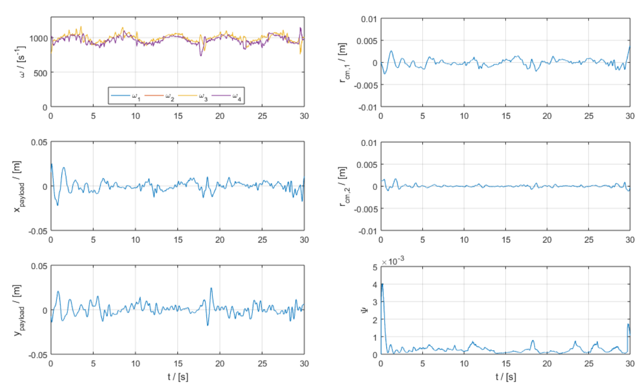


Fig. 5: Comparison of desired Ω_d and measured Ω_{mv} angular velocity values. (Using the manipulator control concept)



a) b)

Fig. 6: Control inputs: rotor velocities ω_i and payload position; are shown in in figure a). Figure b) shows first two components of CoG vector $\vec{r}_{CoG}$ and attitude error function Ψ.

is important to note that J and r_{cm} are changing over time.

Lagrangian derivatives needed for the general form equation are:

$$\frac{\partial \mathcal{L}}{\partial \omega} = J\omega + mS(r_{cm})v \tag{39}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \omega}\right) = \dot{J}\omega + J\dot{\omega} + m\dot{r}_{cm} \times v + mr_{cm} \times \dot{v} \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial v} = mv - mr_{cm} \times \omega \tag{41}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial v}\right) = m\dot{v} - m\dot{r}_{cm} \times \omega - mr_{cm} \times \dot{\omega} \tag{42}$$

It is of interest to transfer rotation and translation dynamics in the inertial frame. This can be done using the following relations:

$$v = R^T\dot{x} \tag{43}$$

$$\dot{v} = R^T\ddot{x} - \omega \times (R^T\dot{x}) \tag{44}$$

$$r_{cm} = R^T(x_{cm} - x) \tag{45}$$

$$\dot{r}_{cm} = R^T(\dot{x}_{cm} - \dot{x}) + R^T S(\omega)x_{cm} - S(\omega)R^T(x_{cm} - x) \tag{46}$$

After plugging in 39, 40, 41, 42 in 36, 37 and using 43, 44 as transformations of velocity and acceleration to inertial frame the following equations are obtained:

$$\begin{aligned} &J\dot{\omega} + mr_{cm} \times R^T\ddot{x} \\ &+ \dot{J}\omega + m\dot{r}_{cm} \times R^T\dot{x} \\ &+ \omega \times J\omega + \sum_{i=1}^{3} r_i \times \frac{\partial \mathcal{L}}{\partial r_i} = 0 \end{aligned} \tag{47}$$

$$\begin{aligned} &m\ddot{x} - mRS(r_{cm})\dot{\omega} - mRS(\dot{r}_{cm})\omega \\ &- mRS(\omega)S(r_{cm})\omega + \frac{\partial U}{\partial x} = 0 \end{aligned} \tag{48}$$

After plugging in the center of mass transform 45, 46 the final form of dynamics is obtained:

$$\begin{aligned} &J\dot{\omega} + mR^T(x_{cm} - x) \times R^T\ddot{x} + \dot{J}\omega \\ &+ mS(R^T\dot{x}_{cm})R^T\dot{x} - R^T\dot{x} \times (R^T S(\omega)x_{cm}) \\ &- mS(\omega)R^T(x_{cm} - x) \times R^T\dot{x} \\ &+ \omega \times J\omega + \sum_{i=1}^{3} r_i \times \frac{\partial \mathcal{L}}{\partial r_i} = 0 \end{aligned} \tag{49}$$

$$\begin{aligned} m\ddot{x} &- mRS(R^T(x_{cm} - x))\dot{\omega} \\ &- mRS(R^T(\dot{x}_{cm} - \dot{x}))\omega \\ &- mR[(R^T S(\omega)x_{cm}) \times \omega] \\ &+ \frac{\partial U}{\partial x} = 0 \end{aligned} \tag{50}$$

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