Geometric Tracking Control of an Unmanned Aerial Vehicle based on the Moving Mass Concept on SE(3)

Antun Ivanović, Marko Car, Tomislav Haus, Matko Orsag, Stjepan Bogdan, Lovro Marković

***Abstract*— This paper is focused on presenting the concept of geometric tracking control for a specific unmanned aerial vehicle (UAV) based on the moving mass concept. It has the ability to exploit its dynamic center of mass as a means of stabilization and control. A mathematical model of such system will be given as grounds for developing the nonlinear geometric tracking controller on the special Euclidean group SE(3). It will be shown that the chosen control terms have desirable properties. Finally, Gazebo simulation results for a selected trajectory tracking problem will be presented using a model of an aerial robot consisting of two moving masses distributed in a standard plus configuration.**

I. INTRODUCTION

TODO: Introduction

TODO: Contribution

II. MATHEMATICAL MODEL

First of all, it is necessary to introduce a fixed inertial reference frame $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ and a body-fixed frame $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$. As it was previously stated, μMORUS UAV will exploit its shifting center of gravity (CoG) due to the moving masses in order to maneuver and stabilize itself. Therefore, a CoG vector from the origin of the body-fixed frame will be defined as follows:

$$\vec{r}_{cm} = \frac{m_b \vec{r}_{0,b} + \sum_{i=1}^{n} m_i \vec{r}_i}{m_b + \sum_{i=1}^{4} m_i} = \frac{\sum_{i=1}^{4} m_i \vec{r}_i}{m_t}, \qquad (1)$$

The following terms are defined as:

- $\vec{r}_{cm} \in \mathbb{R}^3$ - Center of gravity with respect to the body-fixed frame
- $\vec{r}_i \in \mathbb{R}^3$ - Position of the i-th mass w.r.t. the body-fixed frame
- $\vec{r}_b \in \mathbb{R}^3$ - Position of UAV body w.r.t. the body-fixed frame. Note that because the body frame origin coincides with the rigid body CoG (without considering the moving masses) this term yields $\vec{r}_b = 0_{3x1}$
- $m_b \in \mathbb{R}$ - Mass of the UAV body
- $m_i \in \mathbb{R}$ - Mass of the i-th moving mass attached to the UAV link
- $m_t \in \mathbb{R}$ - Mass of the whole UAV

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The equations of motion expressed in the inertial frame while taking in consideration that the CoG is located outside the origin of the body-fixed frame[1] are as follows:

$$\dot{x} = v \qquad (2)$$

$$m_t \dot{v} + m_t g e_3 - m_T R \vec{r}_{cm} \times \dot{\Omega} - m_t R \hat{\Omega} \hat{\vec{r}}_{cm} \Omega = f R e_3 \qquad (3)$$

$$\dot{R} = R\hat{\Omega} \qquad (4)$$

$$J\dot{\Omega} + \Omega \times J\Omega + m_t \vec{r}_{cm} \times R^T \dot{v} = M \qquad (5)$$

The following terms are defined as:

- $J \in \mathbb{R}^{3x3}$ - Moment of inertia matrix w.r.t. the body-fixed frame
- $R \in SO(3)$ - Rotation matrix from the body fixed frame to the inertial frame
- $\Omega \in \mathbb{R}^3$ - Angular velocity in the body-fixed frame
- $x \in \mathbb{R}^3$ - Location of the body-fixed frame in the inertial frame
- $v \in \mathbb{R}^3$ - Velocity of the body-fixed frame in the inertial frame
- $f \in \mathbb{R}$ - Total thrust produced by the UAV
- $M \in \mathbb{R}^3$ - Total moments acting in the body-fixed frame

The *hat map* is an operator equivalent to the expression $\hat{x}y = x \times y$. It maps elements of $\mathbb{R}^3$ to the so(3) Lie algebra.
Moment of inertia matrix expressed in the body-fixed frame is defined as follows:

$$J = J_b + \sum_{i=1}^{4} J_i \qquad (6)$$

where J_b is body and J_i is i-th mass moment of inertia. Using the parallel axis theorem, one is able to calculate J_i while knowing moment of inertia around its CoG:

$$J_i = J_{i,CoG} + m_i(\vec{r}_i^T \cdot \vec{r}_i I_{3x3} - \vec{r}_i \cdot \vec{r}_i^T) \qquad (7)$$

Equations 2, 3, 4 and 5 describe the dynamical flow of a rotating and translating rigid body in terms of evolution of $(R, x, \Omega, \dot{x}) \in$ TSE(3) on the tangent bundle of SE(3).

Height and yaw of the UAV is controlled by variations in rotor velocity, whereas roll and pitch by moving the masses along UAV links placed in plus configuration. It is assumed that first and third propeller rotate clockwise, while second and fourth rotate counter-clockwise. The relation between moments, thrust and rotor velocity is the following:

$$f_i = b_f \omega_i^2 \tag{8}$$
$$\tau_i = (-1)^i b_m f_i \tag{9}$$

Where the following terms are defined as:

- $f_i \in \mathbb{R}$ - Thrust of the i-th motor
- $\tau_i \in \mathbb{R}$ - Moment i-th motor produces
- $b_f \in \mathbb{R}$ - Motor thrust constant
- $b_m \in \mathbb{R}$ - Motor moment constant
- $\omega_i \in \mathbb{R}$ - Rotation velocity of the i-th rotor

Total thrust can be expressed as:

$$f = \sum_{1}^{4} f_i \tag{10}$$

and total moment acting in the body-fixed frame as:

$$M = [m_1 g d_x, m_2 g d_y, b_m(-f_1 + f_2 - f_3 + f_4)] \tag{11}$$

Using f and M as control inputs of the system one is able to obtain the desired force of each rotor and the desired offset of each moving mass d_x and d_y. Note that the offset of masses on the same axis is equal.
Actuator dynamics for the moving masses are not considered in this paper. It is also assumed that the change in desired rotor force is instantaneous.

III. Geometric control on SE(3)

In this section a nonlinear tracking controller will be developed. The main focus will be put on position tracking, therefore the trajectory will consist of a desired position $x_d(t)$ and desired heading $\vec{b}_{1,d}$ of the body-fixed frame. Since the given position is known ahead of time, one is able to calculate both desired linear velocity $v_d(t)$ and acceleration $a_d(t)$ which will also inherently be included as inputs.
The controller will be developed on the nonlinear Lie group SE(3) whose subgroups are the rotation group SO(3) and translation group T(3). The main advantage of using the SO(3) rotation group is to avoid any singularities or ambiguities that may arise when representing rotations with Euler angles or quaternions.
Firstly, chosen position and orientation errors will be presented which will also lie on the SE(3) manifold and its tangent space. Using previously defined errors, nonlinear control terms can be chosen. Finally, the stability conditions of the tracking errors will be presented.

A. Tracking errors

Attitude error function on SO(3) is chosen as:

$$\Psi(R, R_d) = \frac{1}{2} tr[I - R_d^T R] \tag{12}$$

Linear position and velocity tracking errors are defined as follows:

$$e_x = x - x_d \tag{13}$$
$$e_v = v - v_d \tag{14}$$

Attitude and velocity tracking errors are defined as follows:

$$e_R = \frac{1}{2}(R_d^T R - R^T R_d)^{\mathrm{V}} \tag{15}$$
$$e_\Omega = \Omega - R^T R_d \Omega_d \tag{16}$$

Napisati mozda jos nesto o pogreskama...

B. Control terms

Force and moment control terms are chosen as follows:

$$\begin{aligned} A = (&-k_x e_x - k_v e_v \\ &+ m g e_3 + m\ddot{x}_d \\ &+ mR\vec{r}_{cm} \times \dot{\Omega} + mR\hat{\Omega}\hat{\vec{r}}_{cm}\Omega) \\ f =& A \cdot R e_3 \end{aligned} \tag{17}$$

$$\begin{aligned} M = &-k_R e_R - k_\Omega e_\Omega \\ &+ \Omega \times J\Omega - J(\hat{\Omega} R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d) \\ &+ m\vec{r}_{cm} \times R^T \ddot{x} \end{aligned} \tag{18}$$

Desired rotation matrix is constructed as $R_d = [\vec{b}_{1,c}, \vec{b}_{3,d} \times \vec{b}_{1,c}, \vec{b}_{3,d}]$ where component vectors of R_d are calculated in the following way:

$$\vec{b}_{3,d} = \frac{A}{||A||} \tag{19}$$

It is assumed that:

$$||A|| \neq 0 \tag{20}$$

and for a given trajectory tracking problem the following holds:

$$||mge_3 + m\ddot{x}_d + mR\vec{r}_{cm} \times \dot{\Omega} + mR\hat{\Omega}\hat{\vec{r}}_{cm}\Omega|| < B \tag{21}$$

for a positive constant B.

Due to the fact that desired heading $\vec{b}_{1,d}$ will not always lie in the plane which has the desired thrust vector $\vec{b}_{3,d}$ as its normal, it needs to be adjusted accordingly:

$$\vec{b}_{1,c} = -\frac{1}{||\vec{b}_{3,d} \times \vec{b}_{1,d}||}(\vec{b}_{3,d} \times (\vec{b}_{3,d} \times \vec{b}_{1,d})) \tag{22}$$

Desired angular velocity and acceleration also need to be considered in this trajectory tracking problem. One is able to calculate the desired angular velocity and acceleration using R_d and its derivatives in the following way:

$$\hat{\Omega}_d = R_d^T \dot{R}_d \tag{23}$$
$$\dot{\hat{\Omega}}_d = -\hat{\Omega}_d \hat{\Omega}_d + R_d^T \ddot{R}_d \tag{24}$$

Derivatives of R_d are simply calculated by using the backwards differentiation method. It has to be noted that due to stability issues, computation rate of desired angular velocity and acceleration has to be lesser than the overall simulation rate. For further implementation details, please refer to [2].

C. Error dynamics and stability discussion

In this section linear and angular error dynamics will be presented. First of all, derivatives over time need to be calculated for linear 14 and angular 16 tracking errors:

$$\dot{e}_v = \ddot{x} - \ddot{x}_d \tag{25}$$

$$\dot{e}_\Omega = \dot{\Omega} + \hat{\Omega} R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d \tag{26}$$

After including 3 and 5 in 25 and 26 respectively, the following equations are obtained:

$$\begin{aligned} m\dot{e}_v =& - mge_3 - m\ddot{x}_d \\ & + mR\vec{r}_{cm} \times \dot{\Omega} + mR\hat{\Omega}\hat{\vec{r}}_{cm}\Omega \\ & + A + X \end{aligned} \tag{27}$$

$$\begin{aligned} J\dot{e}_\Omega =& M - \Omega \times J\Omega \\ & + J(\hat{\Omega} R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d) \\ & + m\vec{r}_{cm} \times R^T \ddot{x} \end{aligned} \tag{28}$$

Note that in 27 $A \in \mathbb{R}^3$ is regarded as a control force for the translational dynamics, mentioned in 17, while $X \in \mathbb{R}^3$ is a bounded term that arises when deriving this equation which equals:

$$X = \frac{f}{(R_d e_d)^T R e_3}(R_d e_3 - ((R_d e_d)^T R e_3) R e_3) \tag{29}$$

After substituting control force from 17 and 18 in 27 and 28 respectively the final form of error dynamics is obtained:

$$m\dot{e}_v = -k_x e_x - k_v e_v + X \tag{30}$$

$$J\dot{e}_\Omega = -k_R e_r - k_\Omega e_\Omega \tag{31}$$

Napisi konacni rezultat za eksponencijalnu stabilnost

IV. Simulation

TODO: Simulation...

V. Experiments

Experiments...

VI. Conclusion

Conclusions

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