Supplementary Material of "Towards a Pareto Front Shape Invariant Multi-Objective Evolutionary Algorithm Using Pair-Potential Functions"

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1 Qualitative Results

Figures 1 to 14 show approximation sets with the median HV value among 30 independent runs obtained by NSGA-III and each version of NSGA-III-$\mathcal{K}$ on DTLZ1-DTLZ7 and DTLZ1^{-1}-DTLZ7^{-1} with 3 objective functions, respectively. Figures 1 to 4 show that all the versions of NSGA-III-$\mathcal{K}$ maintain the good performance of NSGA-III on MOPs with regular *Pareto front* shapes. On the other hand, Figures 5 to 14 show approximation sets on MOPs with irregular *Pareto front* shapes. Figure 5 and Figure 6 show that all the versions of NSGA-III-$\mathcal{K}$ improve the distribution of solutions on MOPs with degenerate *Pareto front* shapes. Figure 7 and Figure 14 show that all versions of NSGA-III-$\mathcal{K}$ obtain better distribution of solutions than NSGA-III on MOPs with disconnected *Pareto front* shapes. From Figures 8 to 13, it can be seen that all versions of NSGA-III-$\mathcal{K}$ outperform NSGA-III on MOPs with inverted linear and inverted convex *Pareto front* shapes.

Figures 15 to 18 show the approximation sets with the median HV value among 30 independent runs obtained by NSGA-III and each version of NSGA-III-$\mathcal{K}$ on DTLZ1, DTLZ4, DTLZ1^{-1}, and DTLZ4^{-1} with 10 objective functions, respectively. It can be seen from Figure 15 and Figure 16 that all versions of NSGA-III-$\mathcal{K}$ preserve the good performance of NSGA-III on MOPs with regular *Pareto front* shapes. On the other hand, Figure 17 and Figure 18 show that NSGA-III is not capable to fully cover the *Pareto front* while all versions of NSGA-III-$\mathcal{K}$ outperform NSGA-III on MOPs with irregular *Pareto front* shapes.

These results show the capability of the selection mechanism based on niching and pair-potential functions to preserve the good performance of NSGA-III on MOPs with regular *Pareto front* shapes. At the same time, it increases the performance of NSGA-III on MOPs with irregular *Pareto front* shapes.

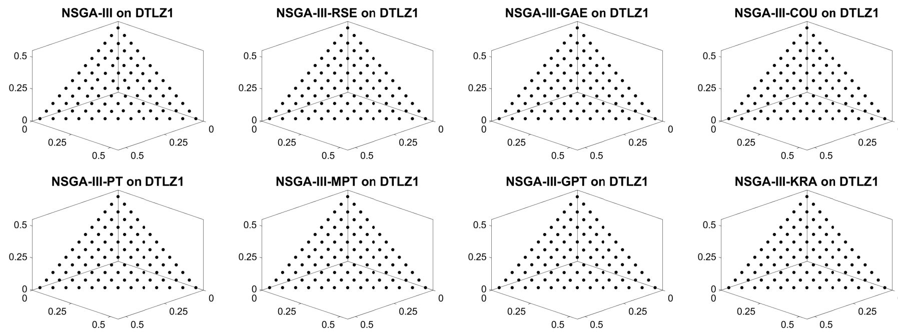

Fig. 1. Approximation sets with the median HV value among 30 independent runs on DTLZ1 with 3 objectives.

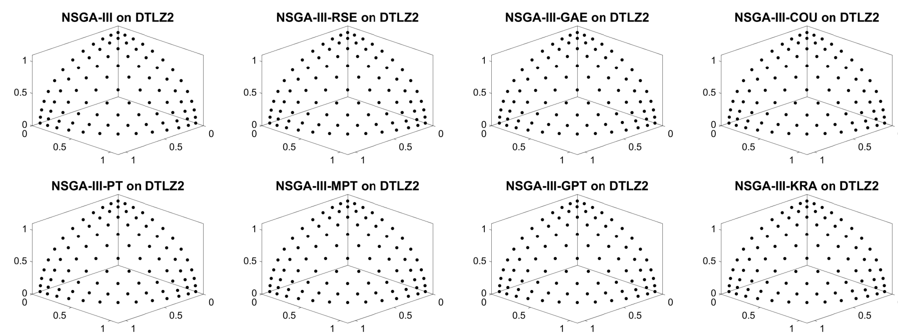

Fig. 2. Approximation sets with the median HV value among 30 independent runs on DTLZ2 with 3 objectives.

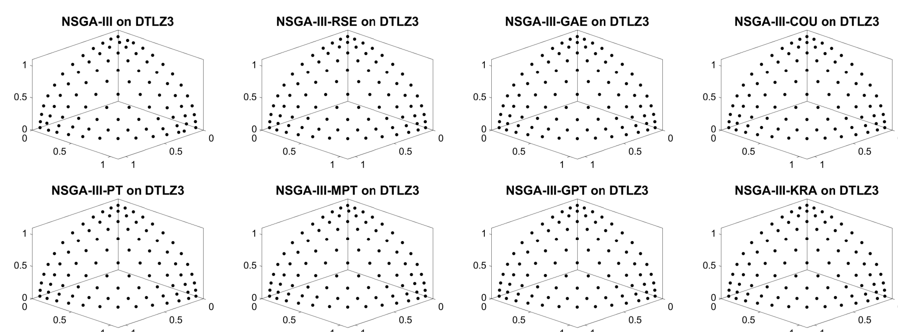

Fig. 3. Approximation sets with the median HV value among 30 independent runs on DTLZ3 with 3 objectives.

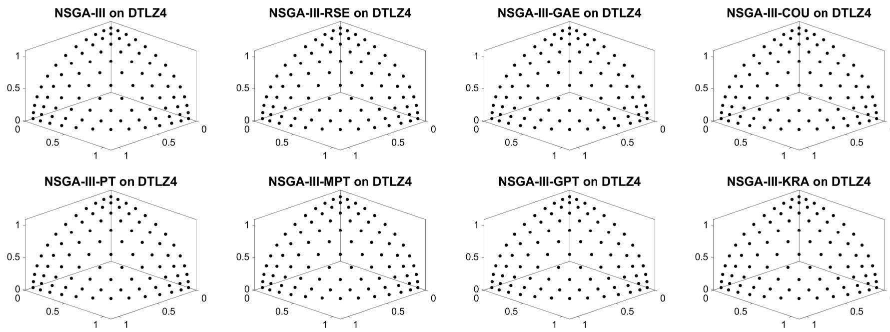

Fig. 4. Approximation sets with the median HV value among 30 independent runs on DTLZ4 with 3 objectives.

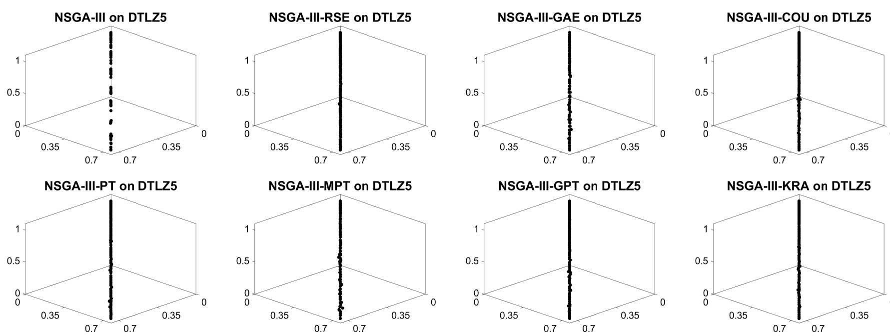

Fig. 5. Approximation sets with the median HV value among 30 independent runs on DTLZ5 with 3 objectives.

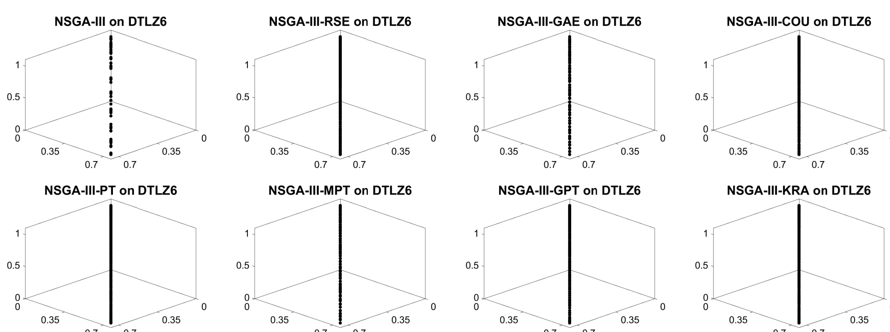

Fig. 6. Approximation sets with the median HV value among 30 independent runs on DTLZ6 with 3 objectives.

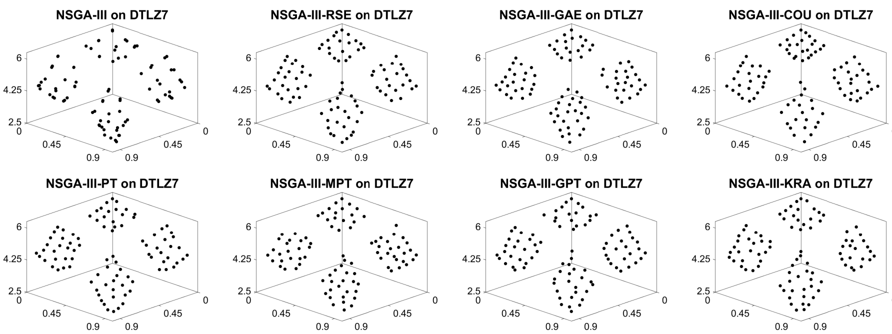

Fig. 7. Approximation sets with the median HV value among 30 independent runs on DTLZ7 with 3 objectives.

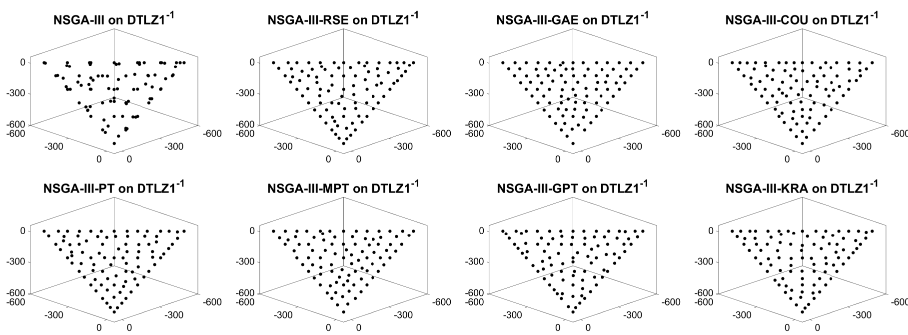

Fig. 8. Approximation sets with the median HV value among 30 independent runs on DTLZ1^{-1} with 3 objectives.

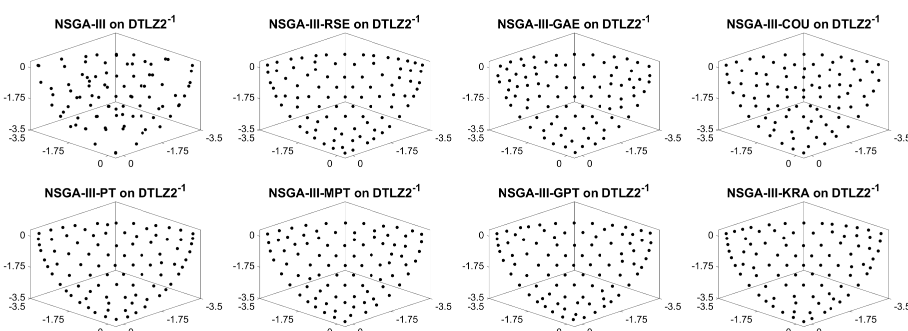

Fig. 9. Approximation sets with the median HV value among 30 independent runs on DTLZ2^{-1} with 3 objectives.

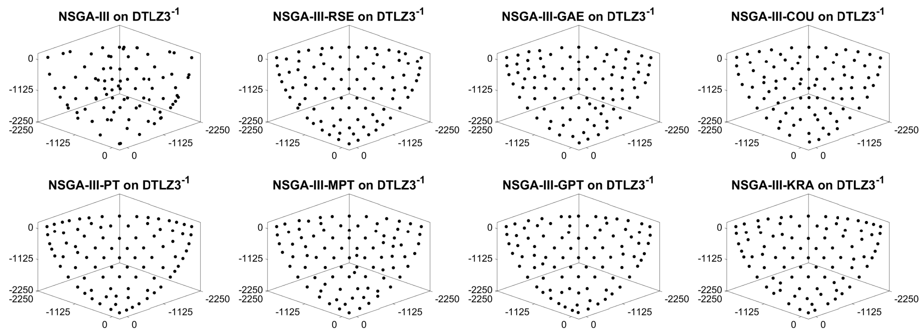

Fig. 10. Approximation sets with the median HV value among 30 independent runs on DTLZ3^{-1} with 3 objectives.

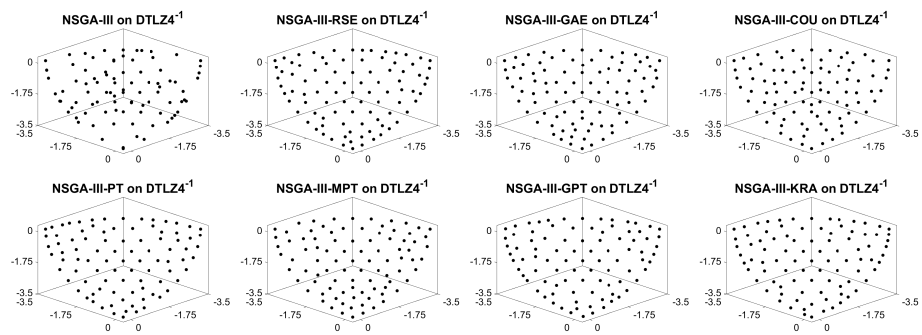

Fig. 11. Approximation sets with the median HV value among 30 independent runs on DTLZ4^{-1} with 3 objectives.

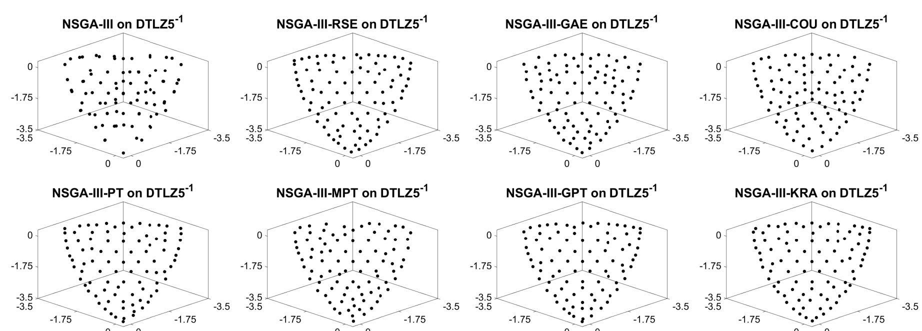

Fig. 12. Approximation sets with the median HV value among 30 independent runs on DTLZ5^{-1} with 3 objectives.

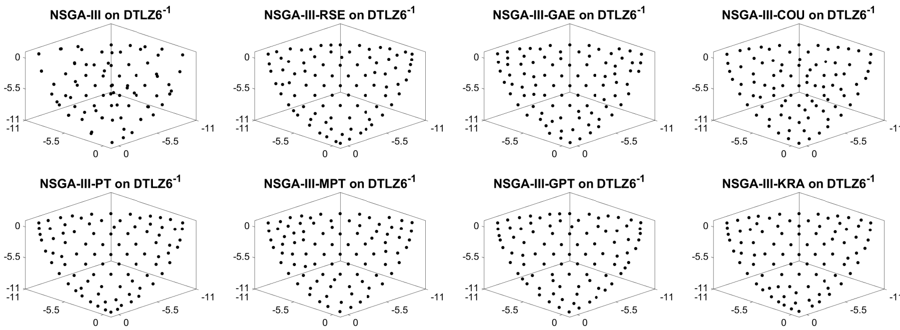

Fig. 13. Approximation sets with the median HV value among 30 independent runs on DTLZ6^{-1} with 3 objectives.

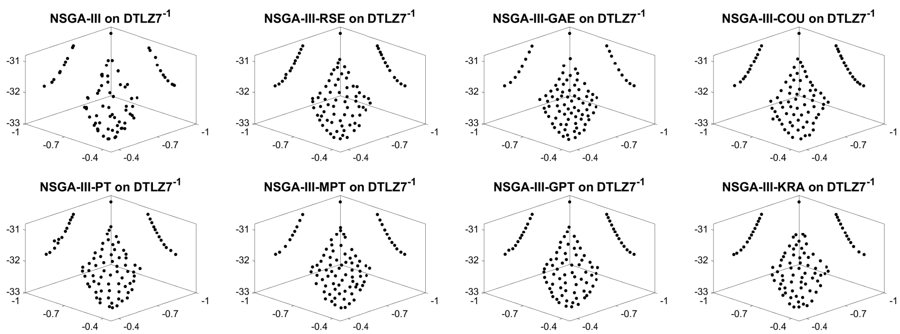

Fig. 14. Approximation sets with the median HV value among 30 independent runs on DTLZ7^{-1} with 3 objectives.

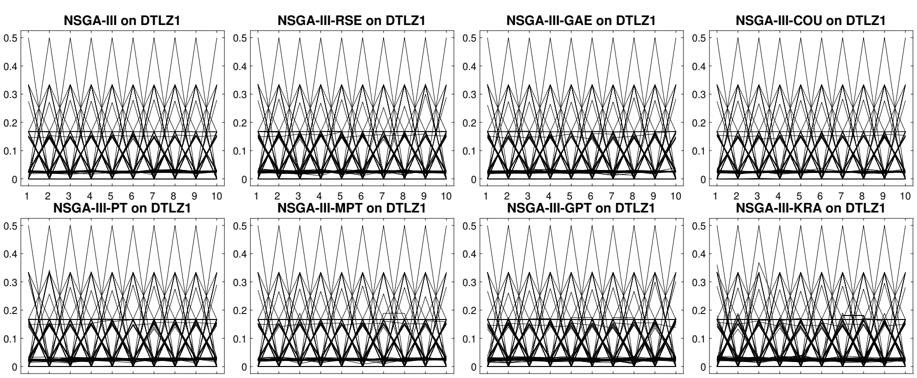

Fig. 15. Approximation sets with the median HV value among 30 independent runs on DTLZ1 with 10 objectives.

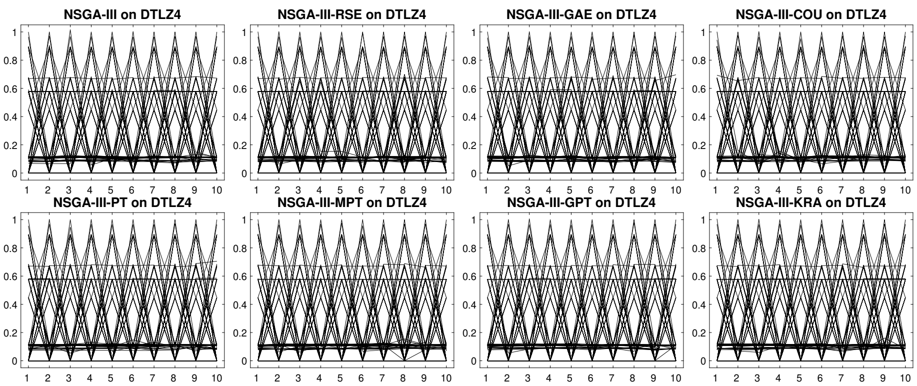

Fig. 16. Approximation sets with the median HV value among 30 independent runs on DTLZ4 with 10 objectives.

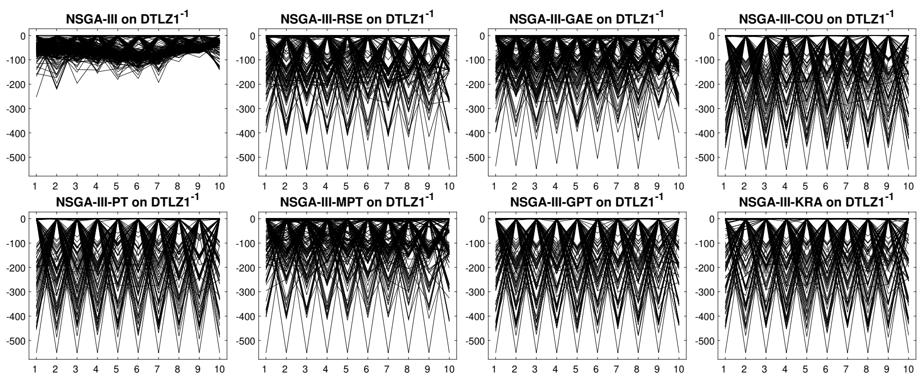

Fig. 17. Approximation sets with the median HV value among 30 independent runs on DTLZ1^{-1} with 10 objectives.

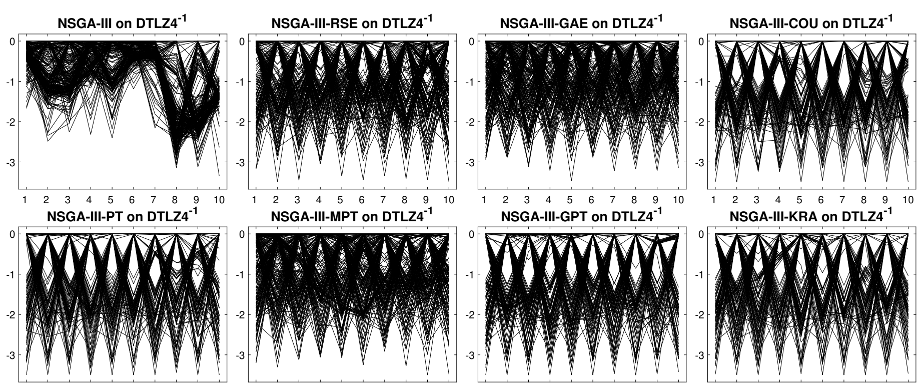

Fig. 18. Approximation sets with the median HV value among 30 independent runs on DTLZ4^{-1} with 10 objectives.