Worksheet 2

Issued: Fri 28. 11. 25
Due: Thu 4. 12. 25

E 2.1 · *Asymptotic Regulation of a Heater*

Consider the room heating (worksheet 1, example 3) and use Python CONTROL facilities:

(a) Gradually enhance your P controller towards a SISO PID controller.

[**Note:** The PID transfer function from the slides can be composed using `parallel` and the P-, I-, and D-transfer functions `tf(kp, 1)`, `tf(1, [Ti, 0])`, `tf([Td, 0], [T, 1])`.]

[**Note:** This simple example allows a manual guessing of the corresponding parameters, e.g., k_P.]

(b) Compare the PID law with the PI, PD, and P laws. What can you see?

(c) Change system parameters (e.g., k_s). What happens?

Assessed Assignments

H 2.2 · *Asymptotic Regulation of Two Heated Tanks* *(10 points)*

Use Python CONTROL for the following steps:

(a) Implement a multiple-input/multiple-output (MIMO) P-controller for the heated tanks model (worksheet 1) *focusing on mode* $\mathbf{l}_1$ *only.*

[**Note:** You can compose the transfer functions of the P laws (see previous exercise) using `append`.]

(b) Synthesise an LQR law as an alternative to the P law.

H 2.3 · *Optimal Control of a Simple Inverted Pendulum* *(20 points)*

We explore infinite-horizon linear-quadratic (LQ) control of a simple inverted pendulum (Figure 1). We want to find an optimal state-feedback controller $u^* = \mathbf{k}^* \cdot \mathbf{x}$ that stabilises the pendulum rod of length $l = 0.68\,\text{m}$ and mass $m = 10.67\,\text{kg}$ at $\phi = 0$ using a *minimal* momentum u applied somehow[1] to the rod. The second-order state-space formulation *linearised* around the goal location $\bar{\phi} = 0$ is given by

$$\underbrace{\begin{bmatrix}\dot{\phi}\\ \ddot{\phi}\end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix}0 & 1\\ \frac{3g}{2l} & 0\end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix}0\\ \frac{3}{ml^2}\end{bmatrix}}_{\mathbf{B}} u, \qquad \underbrace{\begin{bmatrix}\phi(0)\\ \dot{\phi}(0)\end{bmatrix}}_{\mathbf{x}(0)} = \underbrace{\begin{bmatrix}\phi_0\\ \dot{\phi}_0\end{bmatrix}}_{\mathbf{x}_0} = \begin{bmatrix}0.52\\ 0\end{bmatrix}, \qquad y = \underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix}\phi\\ \dot{\phi}\end{bmatrix}}_{\mathbf{x}} = \phi\,.$$

Construct an LQ cost functional and identify the optimal controller $\mathbf{k}^* \in \mathbb{R}^{1\times 2}$ using the LQ solution method.

[1] Classical variants of the inverted pendulum involve a carriage under the rod to be moved to generate the momentum.

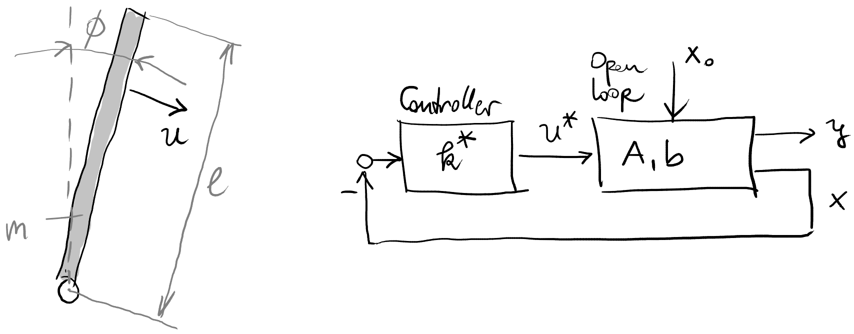

Figure 1: A simple inverted pendulum (left), where ϕ and $\dot{\phi}$ are the angular position and velocity, and a basic control block diagram (right)