ENSTa
IP PARIS

CSC_5RO01_TA
Hybrid Optimal Control
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Worksheet 3
Issued: Fri 5. 12. 25
Due: Thu 11. 12. 25

E 3.1 · *Setting Up and Testing* Lince

Lince[1] is a web-based hybrid system simulation tool. A brief overview of its inputs language, a variation of hybrid programs (HPs), is given in Mendes et al. (2024).

(a) Load the bouncing ball (cf. Lecture 1) from "Examples" into the "Hybrid Program" editor.

(b) How does that hybrid program in Lince correspond to the hybrid automaton from the slides?

(c) Independent of the initialisation (i.e., concrete values for state variables), what are the differences between a trajectory of the hybrid automaton and that hybrid program?

Assessed Assignments

H 3.2 · *From Switching to Optimal Modal Control* *(25 points)*

Reconsider the switched system of two tanks heated alternatingly by a gas burner in Exercise 1.4. In Exercise 2.2, a naïve controller for mode ℓ_1 performing a step-stimulus (h_1) was replaced with P- and LQR controllers. Now, modify the hybrid automaton to one with optimal state-feedback control in modes ℓ_i, $i = 1, 2$.

(a) Design an LQR for mode i with set-point x_i^r, for example, $100\,°C$. Replace h_i with the resulting control law. Derive the closed-loop system in state-space formulation.

(b) Design parametric guards with a steady-state-error tolerance t_i for each mode, for example, $x_1 \geq x_1^r - t_1 \wedge x_2 \leq 0.8x_1^r$. Resolve the non-deterministic choice between *toggle* and *turnoff* as well as when to stop an evolution.

(c) Translate the modified hybrid automaton into a hybrid program for Lince. The *evolution period* of *continuous statements* can be determined by measuring the transient response length.

   [**Note:** For the sake of simplicity, you can ignore the *mode invariant I* in all modes. However, in place of I, you will need to use the guard regions prioritised in (b).]

(d) Simulate your result with Lince for a single initial state or a range of initial states.

References

Mendes, Pedro et al. (2024). "Formal Simulation and Visualisation of Hybrid Programs". In: *FMAS, Workshop*. Vol. 411. Open Publishing Association, pp. 20–37. DOI: 10.4204/eptcs.411.2.

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[1] Version 2.0 is available under https://lmf.di.uminho.pt/lince-2.0/.