CSC_5RO01_TA
Hybrid Optimal Control
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Worksheet 4
Issued: Fri 12. 12. 25
Due: Thu 18. 12. 25

E 4.1 · *Hybrid Automata to Hybrid Program Translation for Lince*

For further training in the usage of Lince for simulation, recapitulate the derivation of Lince hybrid programs required based on the example of the two-tank system on worksheet 3.

E 4.2 · *Discrete Dynamic Programming of an Inverted Pendulum*

Implement the numerical approach to dynamic programming (DP) ('Dynamic Programming' slide 8) for the linear-quadratic (LQ) regulator (LQR) problem discussed in Exercise 2.3. Assume a maximum time span of 2 seconds for the inverted pendulum controller to achieve its goal, that is, $\mathbf{x}(2) = 0$, penalise all states at $t_\mathrm{f} = 2$ with $30000\,\phi^2(2)$, and define the admissible states and inputs to be $\mathcal{X} = [-1, 1] \times [-2, 1]$ and $\mathcal{U} = [-60, 10]$, respectively. Use $\Delta t = 0.1$, $\Delta u = 2$, and $\Delta\phi = \Delta\dot\phi = 0.05$ for the sampling resolutions of time, inputs, and state parameters. For approximation, you can discretise the system dynamics according to the pattern $x_{k+1} = x_k + \dot x_k \cdot \Delta t$.

Assessed Assignments

H 4.3 · *Dynamic Programming of Two Heated Tanks* (30 points)

Replace the LQR law(s) in the two-tank system (2.2/3.2) by two controllers, one for each tank, computed separately via discrete dynamic programming (DDP) according to the algorithm for value iteration ('Dynamic Programming' slide 9).

[**Note:** For the sake of simplicity, you can assume to have only one mode (i.e., no hybrid automaton (HA)) where both controllers are working simultaneously on their tanks.]

[**Note:** Optionally and for comparison, you can compute a single controller for heating both tanks.]

[**Note:** Optionally, you can model the two-tank system abstracting the heat transfer into a disturbance input applied by the other tank and the environment, and solve a zero-sum differential games (DGs) ('Differential Games' slide 6) for the two players to obtain your two controllers.]