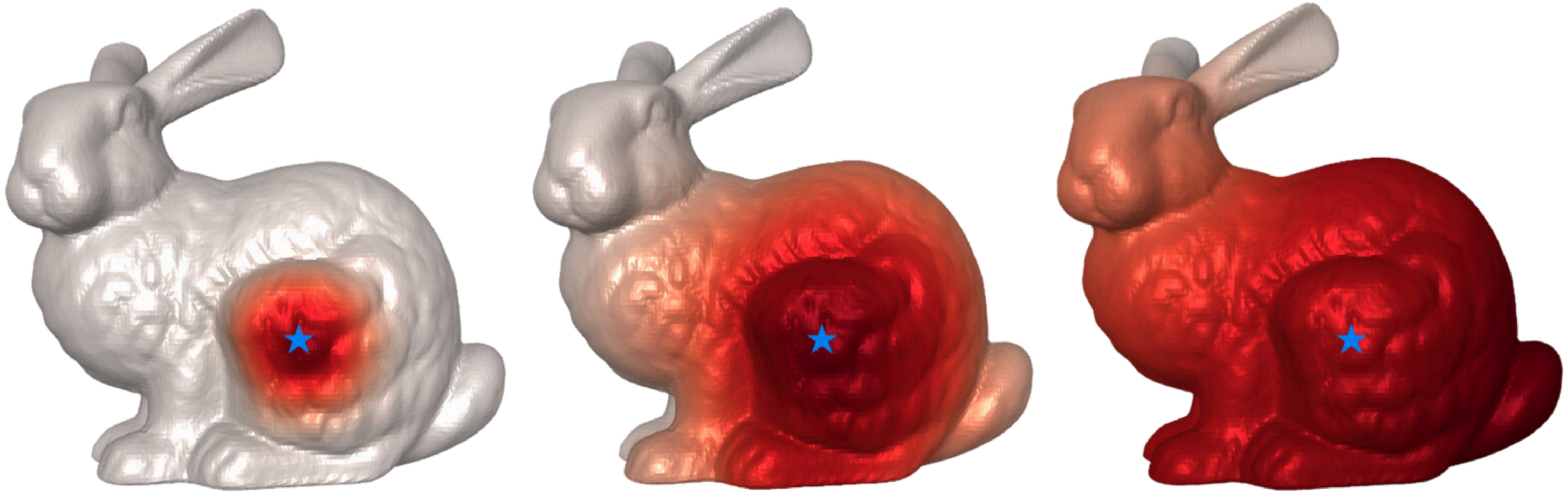
GEOMETRIC DATA ANALYSIS

Louis Martinez, Géraud Ilinca, Hadrien Levéchin

DiffusionNet: Discretization Agnostic Learning on Surfaces



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Context and Previous approach

Quite well resolved

- Rigid transformation invariance
- Permutation invariance

Not yet resolved

- Sampling Invariance
- Data structure Invariance

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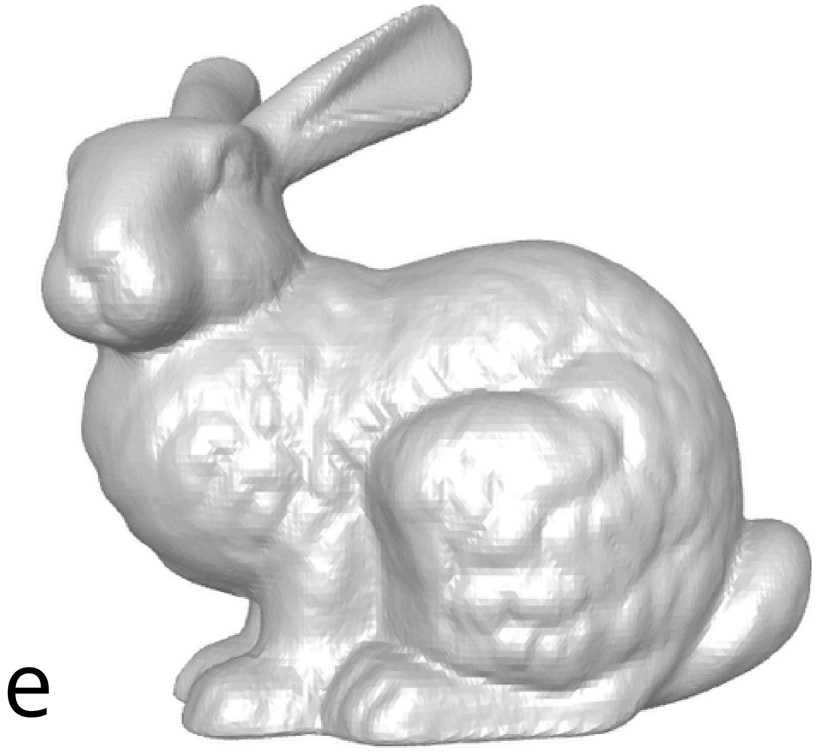

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- Sampling Invariance
- Data structure Invariance

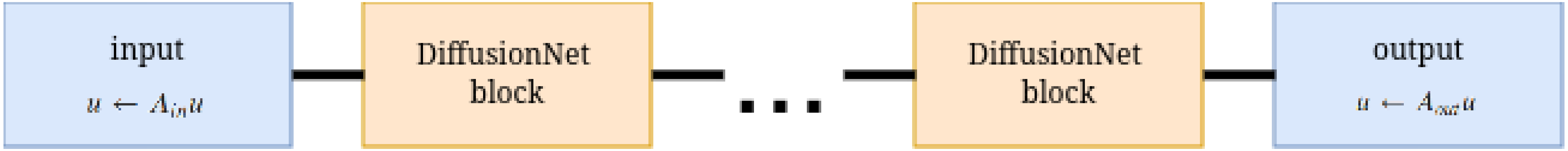
Agnosticity

Ground truth Prediction Representation Resolution Sampling

Architecture

DiffusionNet Block

Spatial Diffusion + Gradient Features + Point-wise MLP



Spatial Diffusion

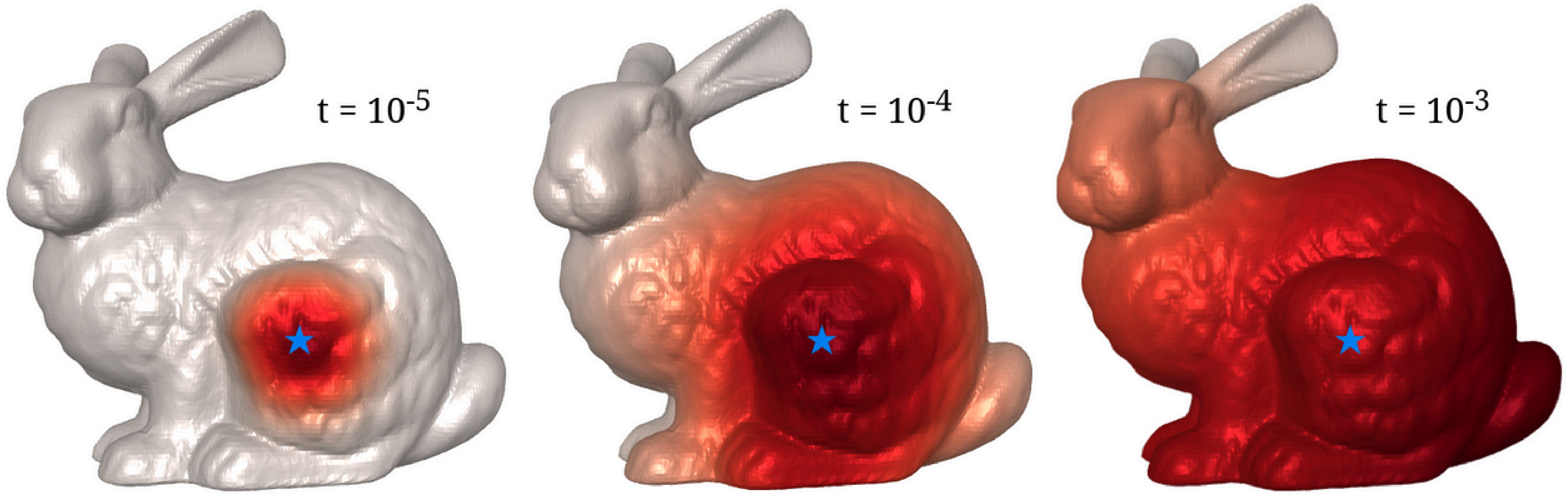
- **Why?** Allows to build reliable descriptors regardless of the representation
- Derived from the Heat Equation

$$\frac{\partial u}{\partial t} = \Delta u$$

scalar field defined on the surface

Laplace-Beltrami operator

↳ Well defined on surfaces



Learned Diffusion

- **Key idea:** One learnable time parameter per channel
    - Enables local and global information sharing on the surface
    - Optimized during training

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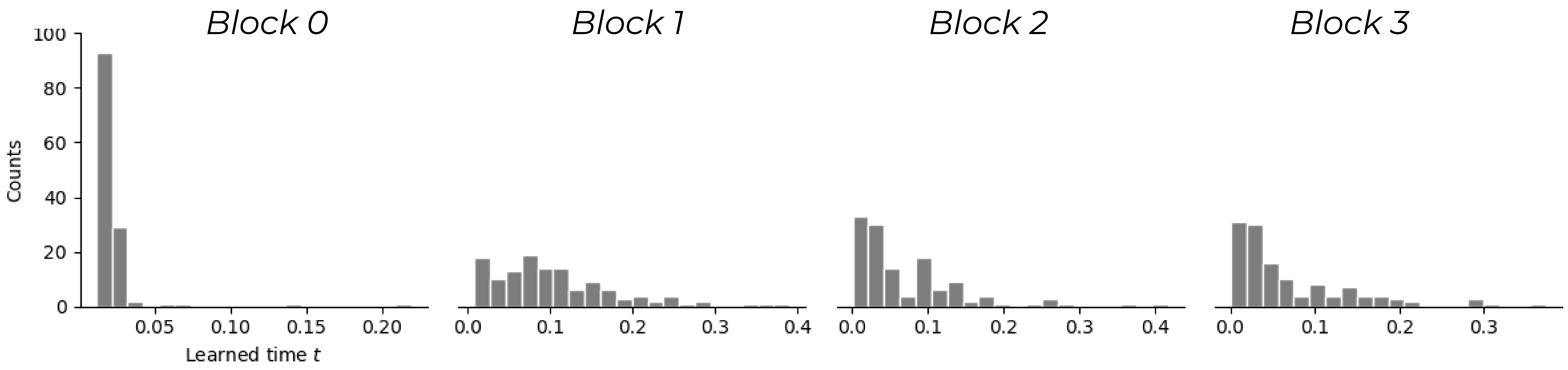
- Similar to receptive field in CNNs



Evaluating Diffusion

- Backward Euler
  ↳ Straightforward and simple
  ↳ Implies solving a large per channel system

$$h_t(u) := (M + tL)^{-1} M u$$

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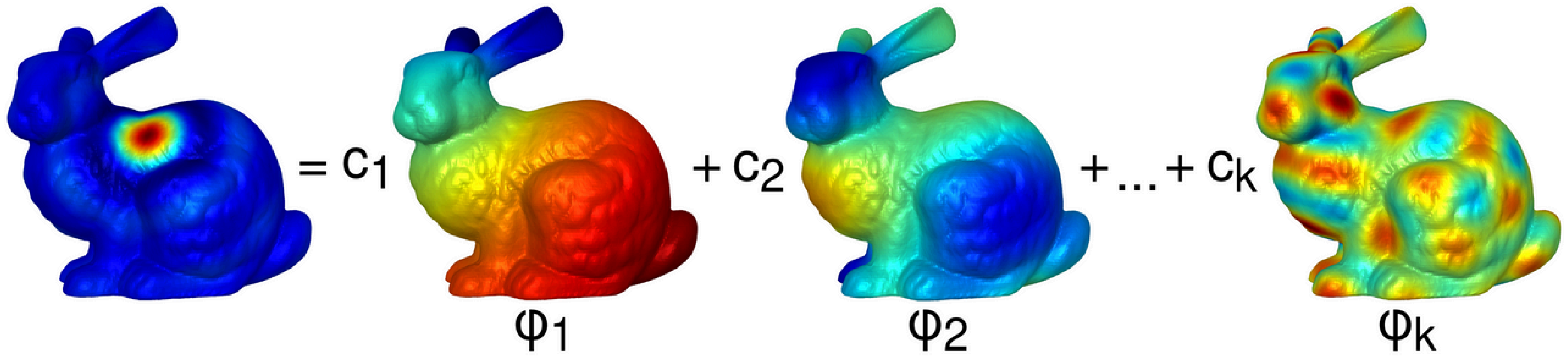
$$h_t(u) := (M + tL)^{-1} M u$$

- Laplacian Eigenbasis
  - ↳ Diffusion evaluated in closed form
  - ↳ Computationally cheaper
  - ↳ Small approximation error

$$h_t(u) := \Phi \begin{bmatrix} e^{-\lambda_0 t} \\ e^{-\lambda_1 t} \\ \vdots \end{bmatrix} \odot (\Phi^T M u)$$



⚠ Not a spectral method

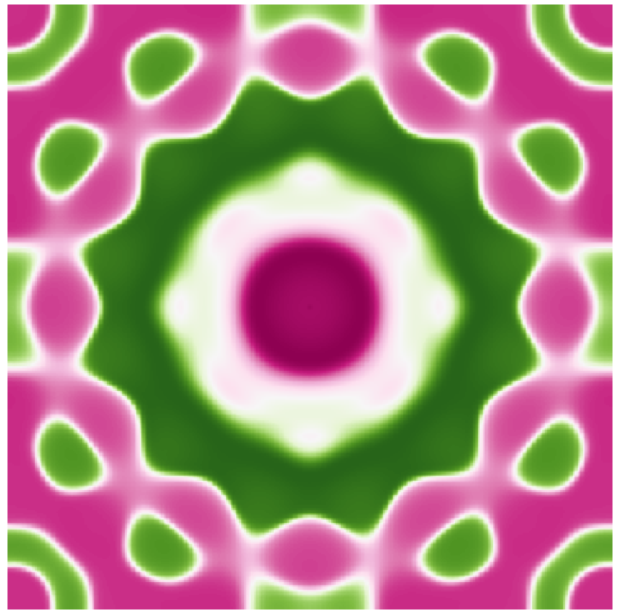
Spatial Gradient Features (SGF)

LEMMA

Radially Symmetric Filters

$\subset$

Diffusion + Pointwise MLP



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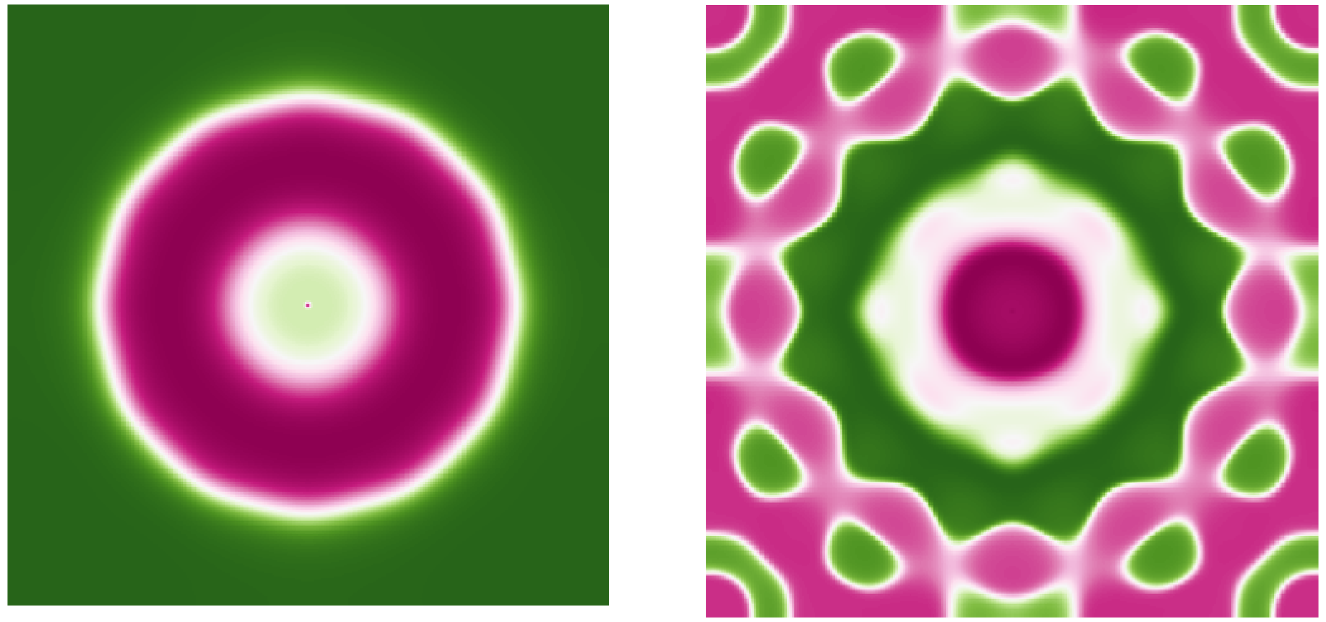
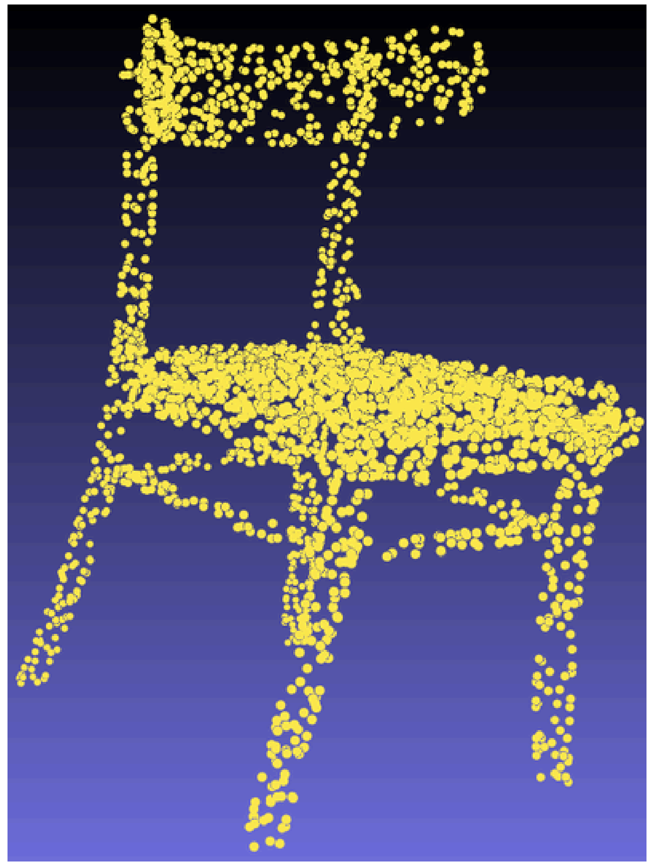


Restrictive !



Spatial Gradient Features (SGF)

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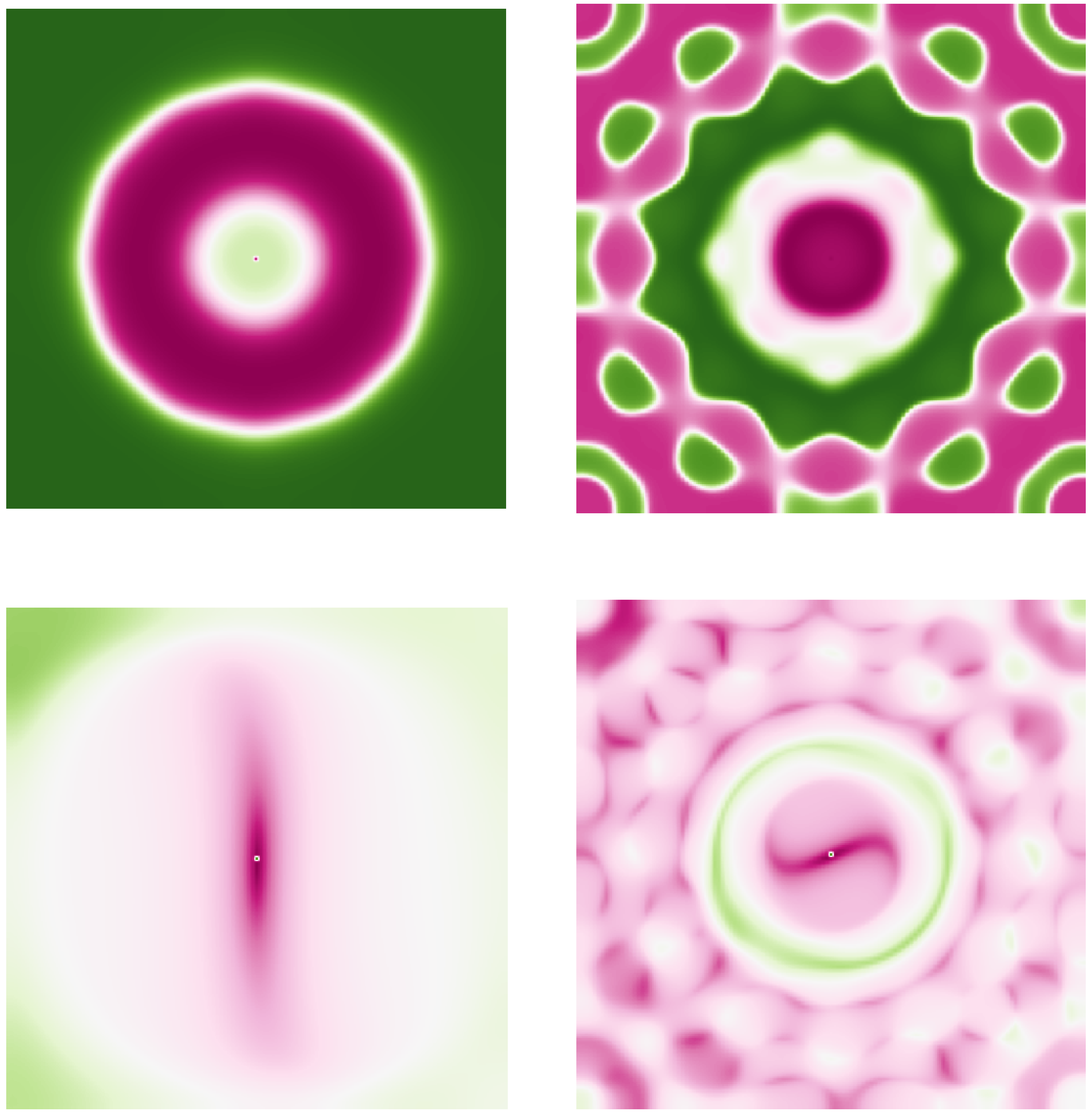
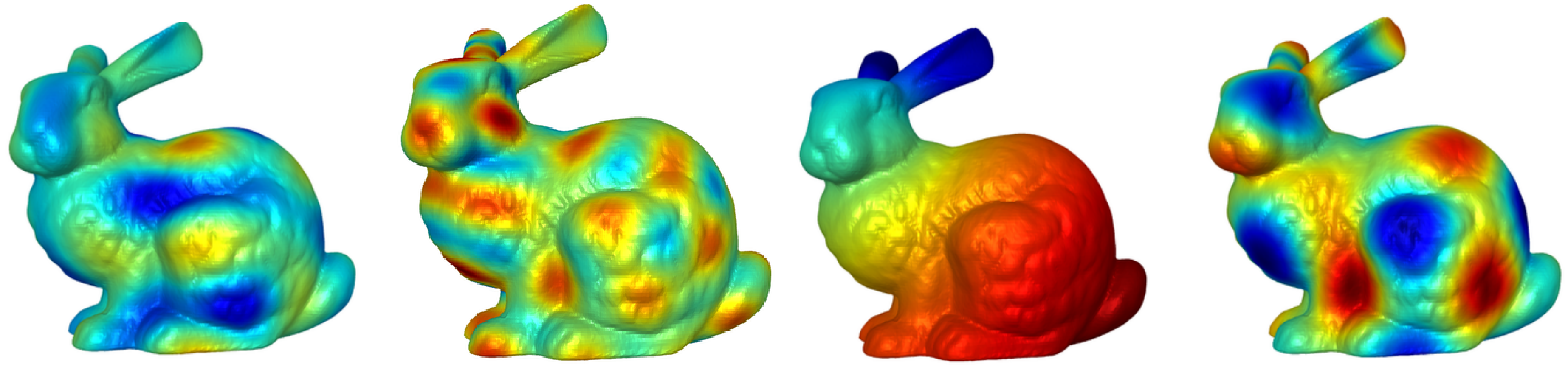
Without
SGF

With
SGF



Spatial Gradient Features

- Compare relative directional information contained in each feature



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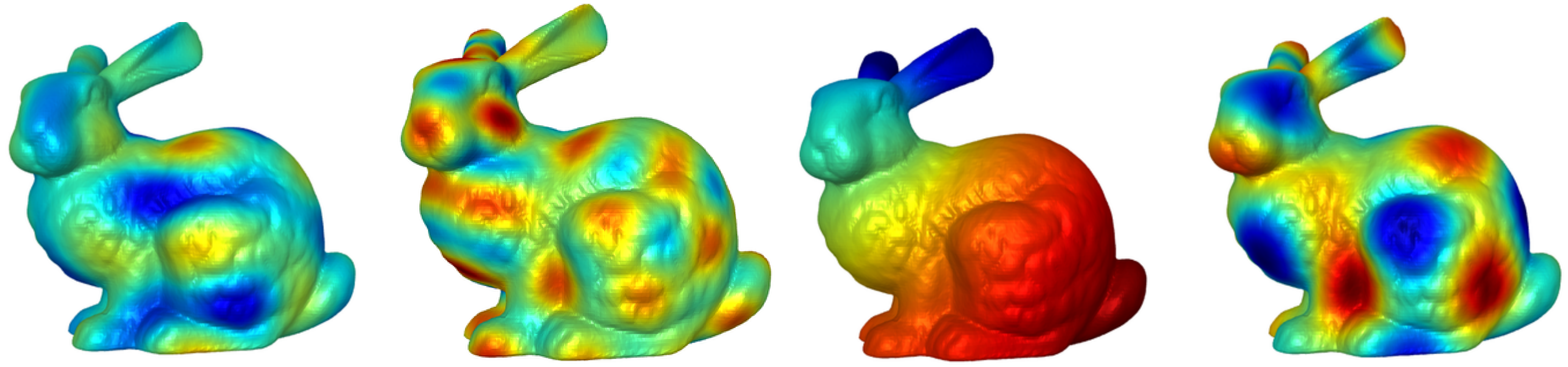
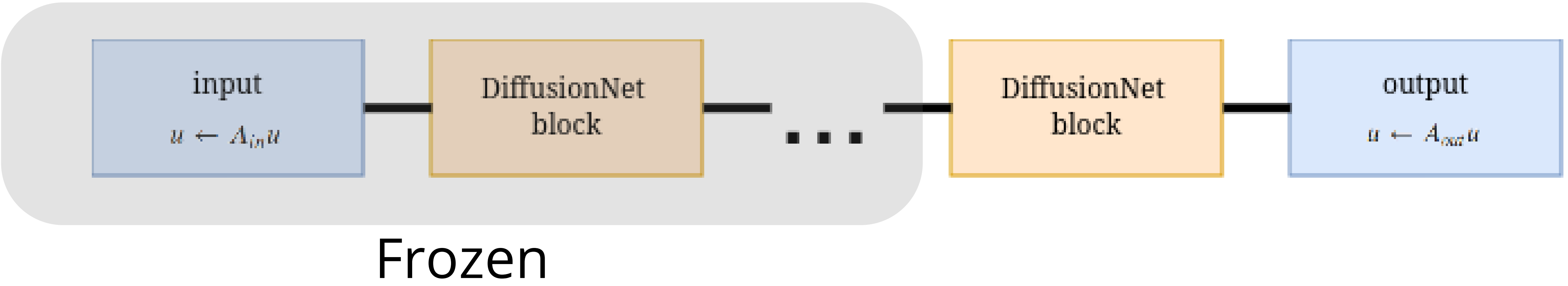


$$\boxed{g_j = \sum_{i=1}^{D} \langle \nabla f_j(p) \mid R_{i,j} \cdot \nabla f_i(p) \rangle}$$

- Invariant to global shape rotation
- Invariant to the choice of local bases
- Computationnaly efficient

Transferability and limitations

- Weak transferability of the learned representations
  - Pretraining on ShapeNet classification → 98% accuracy on validation set
  - Features extraction on SHREC'11 classification → 10% accuracy



- Struggles on more challenging datasets (ObjaVerse)
  - Several connected components per sample
  - Classification task : 60% Accuracy on validation set

DiffusionNet block