How can we know if a $G(CFG)$ is $LL(1)$?
First, we should separate the

$$FIRST(X),\ FOLLOW(X),\ SELECT(X \to \alpha)$$

sets of this $G(CFG)$.

Tips: all the $FIRST(X)$ is the most left terminal state of the productions of X.
We use the symbol α as the production of X.

To separate the $FIRST(X)$ sets, there are 3 rules:

1. If X the production of X is a terminal state, then $FIRST(X) = \alpha$.
2. If α is a nonterminal state, and $FIRST(X) \to \alpha_1\alpha_2\alpha_{3\ldots}\alpha_k\ (k \geq 1)$
   2.1 If α_k not include ε, then $FIRST(X) = FIRST(\alpha)$, until the next nonterminal state α has a production of terminal state.
   2.2 If α_k include ε, then $FIRST(X) = FIRST(\alpha_k) - \{\varepsilon\} + FIRST(\alpha_{k+1})$, until the next nonterminal state has a production of terminal state or α_{k+1} is a terminal state.
3. If $FIRST(X) \to \varepsilon$, add the $\{\varepsilon\}$ into the $FIRST(X)$.

For example, in the $G(A)$:

$A \to BCc \mid gDB$
$B \to bCDE \mid \varepsilon$
$C \to DaB \mid ca$
$D \to dD \mid \varepsilon$
$E \to gAf \mid c$

$FIRST(A) = FIRST(B) - \{\varepsilon\} + FIRST(C) + g = \{a, b, c, d, g\}$
$FIRST(B) = \{b, \varepsilon\}$
$FIRST(C) = FIRST(D) - \{\varepsilon\} + a + c = \{a, c, d\}$
$FIRST(D) = \{d, \varepsilon\}$
$FIRST(E) = \{c, g\}$

To separate the $FOLLOW(X)$ sets, there are also 5 rules:

1. If X is the initial state, add the terminal symbol Z_0 into $FOLLOW(X)$.
2. Find all the Xs in the productions (right side)
   2.1 If X is in the end of the production, add $FOLLOW(A)$ into $FOLLOW(X)$
   2.2 If the nonterminal state α next to X not include ε, add $FIRST(\alpha)$ into $FOLLOW(X)$.
   2.3 If the nonterminal state α next to X include ε, add $FIRST(\alpha_k) - \{\varepsilon\}$ into $FOLLOW(X)$ until there is no more nonterminal states or reach a terminal state. If it reaches the end of all the productions, you have to also add the $FOLLOW(A)$ into $FOLLOW(X)$ as the rule in 2.1.

For example, in the $G(A)$:

$A \to BCc \mid gDB$
$B \to bCDE \mid \varepsilon$
$C \to DaB \mid ca$
$D \to dD \mid \varepsilon$
$E \to gAf \mid c$

$FOLLOW(A) = f + Z_0 = \{f, Z_0\}$
$FOLLOW(B) = FIRST(C) + FOLLOW(A) + FOLLOW(C) = \{a, c, d, g, f, Z_0\}$
$FOLLOW(C) = c + FIRST(D) - \{\varepsilon\} + FIRST(E) = \{c, d, g\}$
$FOLLOW(D) = FIRST(B) - \{\varepsilon\} + FOLLOW(A) + a + FIRST(E) = \{a, b, c, g, f, Z_0\}$
$FOLLOW(E) = FOLLOW(B) \ = \{a, c, d, g, f, Z_0\}$

To get the $SELECT(A \to X)$, we should get all the $FIRST(X)$s and $FOLLOW(X)$s from the last step, there are 2 rules to get the $SELECT(A \to X)$ sets:

1. Suppose we have two productions of a nonterminal state:
$$A \to \alpha$$
$$A \to \beta$$
One of the necessary and sufficient conditions to prove a $LL(1)$ grammar is, to two productions of a nonterminal state, $A \to \alpha$, $A \to \beta$, must be met
$$SELECT(A \to \alpha) \cap SELECT(A \to \beta) = \emptyset$$
Tips: the productions α and β can't be both ε.
2. Calculate each $FIRST(\alpha)$ of each production:
in the $G(A)$
$A \to BCc \mid gDB$
$B \to bCDE \mid \varepsilon$
$C \to DaB \mid ca$
$D \to dD \mid \varepsilon$
$E \to gAf \mid c$

$FIRST(BCc) = FIRST(B) - \{\varepsilon\} + FIRST(C) = \{a, b, c, d\}$
$FIRST(gDB) = \{g\}$
$FIRST(bCDE) = \{b\}$
$FIRST(\varepsilon) = \{\varepsilon\}$
$FIRST(DaB) = FIRST(D) - \{\varepsilon\} + a = \{a, d\}$
$FIRST(ca) = \{c\}$
$FIRST(dD) = \{d\}$
$FIRST(gAf) = \{g\}$
$FIRST(c) = \{c\}$
3. Get the $SELECT(A \to \alpha)$ sets
3.1 If $\alpha \nrightarrow \varepsilon$ then

$$SELECT(A \rightarrow \alpha) = FIRST(\alpha)$$

3.2 If $\alpha \rightarrow \varepsilon$ then

$$SELECT(A \rightarrow \alpha) = (FIRST(\alpha) - \{\varepsilon\}) \cup FOLLOW(A)$$

For example, in the $G(A)$:

$A \rightarrow BCc \mid gDB$
$B \rightarrow bCDE \mid \varepsilon$
$C \rightarrow DaB \mid ca$
$D \rightarrow dD \mid \varepsilon$
$E \rightarrow gAf \mid c$

$SELECT(A \rightarrow BCc) = \{a, b, c, d\}$
$SELECT(A \rightarrow gDB) = \{g\}$
$SELECT(B \rightarrow bCDE) = \{b\}$
$SELECT(B \rightarrow \varepsilon) = (FIRST(\varepsilon) - \{\varepsilon\}) \cup FOLLOW(A) = \{a, c, d, g, f, Z_0\}$
$SELECT(C \rightarrow DaB) = \{a, d\}$
$SELECT(C \rightarrow ca) = \{c\}$
$SELECT(D \rightarrow dD) = \{d\}$
$SELECT(D \rightarrow \varepsilon) = (FIRST(\varepsilon) - \{\varepsilon\}) \cup FOLLOW(D) = \{a, b, c, g, f, Z_0\}$
$SELECT(E \rightarrow gAf) = \{g\}$
$SELECT(E \rightarrow c) \rightarrow \{c\}$

And we calculate the intersections of all the $SELECT(A \rightarrow \alpha)$s from the same nonterminal:

$SELECT(A \rightarrow BCc) \cap SELECT(A \rightarrow gDB) = \{a, b, c, d\} \cap \{g\} = \emptyset$
$SELECT(B \rightarrow bCDE) \cap SELECT(B \rightarrow \varepsilon) = \{b\} \cap \{a, c, d, g, f, Z_0\} = \emptyset$
$SELECT(C \rightarrow DaB) \cap SELECT(C \rightarrow ca) = \{a, d\} \cap \{c\} = \emptyset$
$SELECT(D \rightarrow dD) \cap SELECT(D \rightarrow \varepsilon) = \{d\} \cap \{a, b, c, g, f, Z_0\} = \emptyset$
$SELECT(E \rightarrow gAf) \cap SELECT(E \rightarrow c) = \{g\} \cap \{c\} = \emptyset$

As can be seen from above, all the intersections are empty, so the grammar is a LL(1) grammar.