ASTP720 - Computational Methods

Homework 1

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Problem 1

Write a library (i.e., in a separate file that you can call) for the three root-finding algorithms we discussed in class: Bisection, Newton, Secant. These functions should each take functions rather than data points. Make sure that each takes an optional argument the threshold and that it also takes a variable that allows the user to print out or return the number of iterations it took to hit that threshold.

Problem 2

For the pseudo-isothermal sphere, using your root-finding algorithms, numerically calculate the full width at half maximum, i.e., what is the width (in terms of r_c when $N_e(x) = N_0/2$, half the amplitude. Drawing pictures for yourself might be useful! Do so with each of your root-finding algorithms and show how many iterations each takes as a function of your threshold. Please plot the results.

Problem 3

You can probably see from the Gaussian Lens equations that if you have a light ray hitting x, and you know the other parameters of the lens (a, N_0, D, etc.), then you know what x' is. But that's boring and not what you actually observe. Let's instead say you are an observer in a "circular orbit" along the x' axis with radius 1 AU and a period of 1 year but centered at x' = 1 AU. Then, you know where your position x' is but not where the light rays from the source are intersecting the lens plane at x - as expected, analytically solving for x is not really an option.

Using one of your root-finding algorithms, solve the lens equation for each value of x' and make a raytracing plot as on the first page. Assume D = 1 kpc, a = 1 AU, $\lambda = 21$ cm, and $N_0 = 0.01$ pc $\text{cm}^{-}3$ (these are observer units, probably best to convert to something like $\text{cm}^{-}2$.

Problem 4

Repeat but for the pseudo-isothermal sphere with the same parameters but $r_c = 1$ AU.

Problem 5

Write a library for piecewise linear interpolation, given a set of x and y data points. This should return a function f that one can use to calculate a new point $f(x_{new}) \rightarrow y_{new}$.

Problem 6

In the file lens_density.txt are a series of values of x and $N_e(x)$ for some shape. Use your interpolator to plot the values of $N_e(x)$ halfway in between all of the given x values, i.e., when $x = 0.5, 1.5, \cdots$.