Lecture 7

Network Models of Cities

7.2 Fractals and Fractal Dimension, Mobility and Interactions

IUS 3.2.2: Part 2, pp 77-82

Four Principles of Urban Organization

1) Cities are mixing populations (networks) over built space and time **Jacobs, Wirth, Burgess**

2) Personal effort is limited **Park, Milgram, Zahavi, Simon**

3) City infrastructure as decentralized but hierarchical networks **Alexander**

4) Socioeconomic products of cities are the result of interactions, **Jacobs**

subject to spatial costs **Alonso**

already in the amorphous model, but more to come…

To get closer to the right answer need:

To understand fundamental constraints on human interactions

To understand the general characteristics of urban spaces

A better model of social interactions over built space

To better compute costs of transportation and land rents

Socioeconomic interactions are proportional to local social interactions

Net benefits (interactions):

$$y_i^m = g_m k_i^m$$

agent i's "income" from interactions of type m

value per interaction

number of interactions of type m (degree)

sum over all agents in population

$$Y^m = g_m \sum_i k_i^m$$

Total income of type m in population

$$Y = \sum_{i,m} g_m k_i^m$$

Total income of all type (like GDP)

We want to write these quantities in terms of lifepaths

lifepaths —> interactions networks —> degree

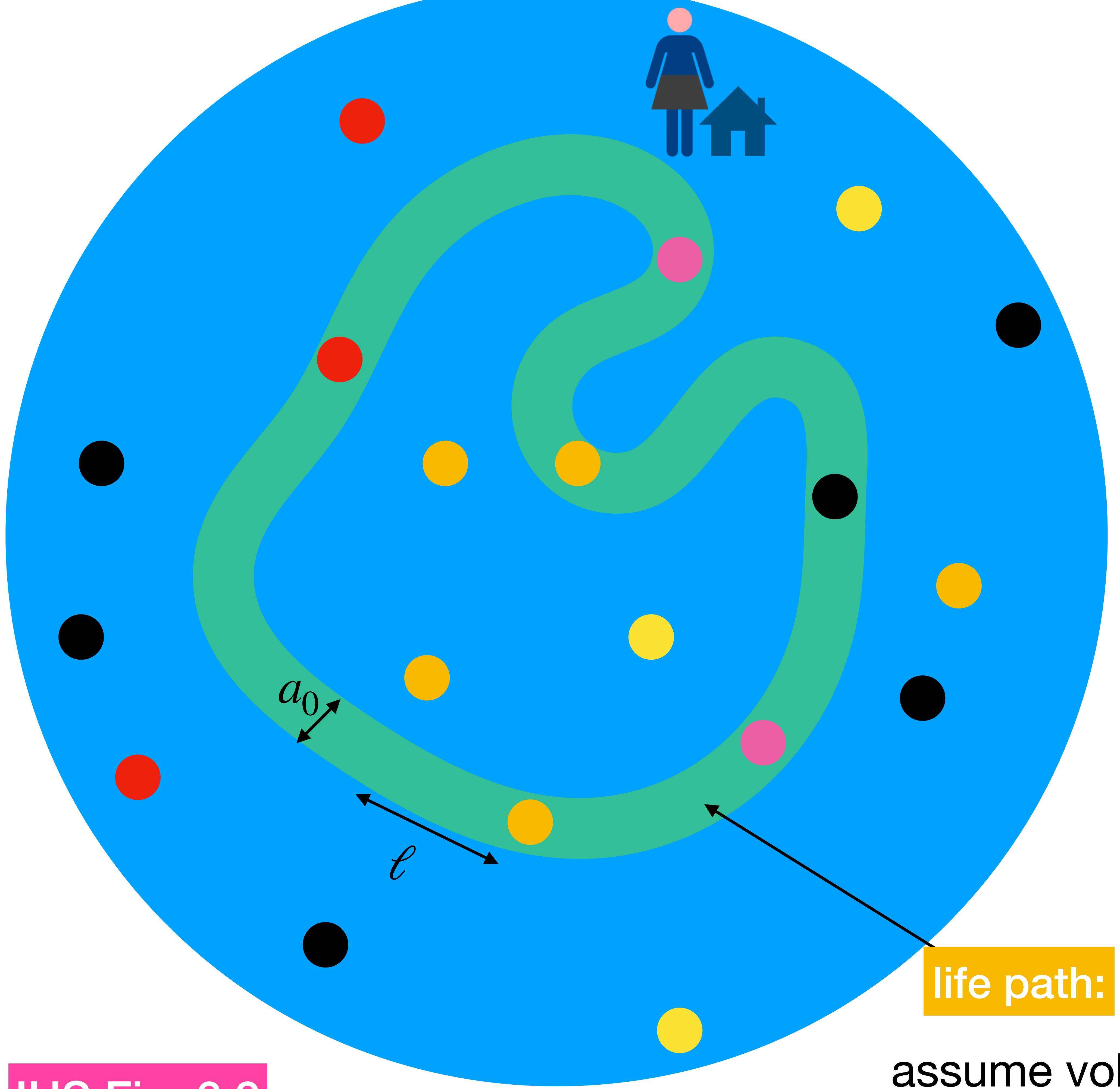
1. Interactions between agents i, j of type m

$$Y = \sum_{i,j;m} g_m \, F_{ij}^m$$

network of interactions of type m (over some time)

recall that $k_i = \sum_j F_{ij}$

IUS pp 80-81



life path: $x_i[t]$

IUS Fig. 3.9

Interactions of type *m* for individual *i*:

happen when lifepaths overlap in space and time

$$F_{ij}^m = P(m\,|\,i,j)F_{ij} \simeq P(m)F_{ij}$$

probability of type m interaction, given agents are i,j

mean field approximation (probability of interaction type is independent of specific pair)

$$k_i^m = \int dt \sum_j P(m\,|\,i,j)\ \Gamma^m(x_j[t] - x_i[t]) \simeq \frac{P(m)}{A_n}\int dt\ d^Dx\ \Gamma(x - x_i[t])$$

$$= P(m)a_0\ell\frac{N-1}{A_n}$$

counts coincidences in spacetime

assume volume of world line is the same length+ width regardless of city: conserved human effort

on average: $$y \simeq \bar{g}\frac{a_0\ell}{A_n}N = G\frac{N}{A_n}$$

$$\bar{g} = \sum_m P(m)g_m, \quad G = a_0\ell\bar{g}$$

keeps track of time

This is an *average* result (over time and city population):
different people have different lifepaths varying over time, expressing their preferences, constraints, growth, etc

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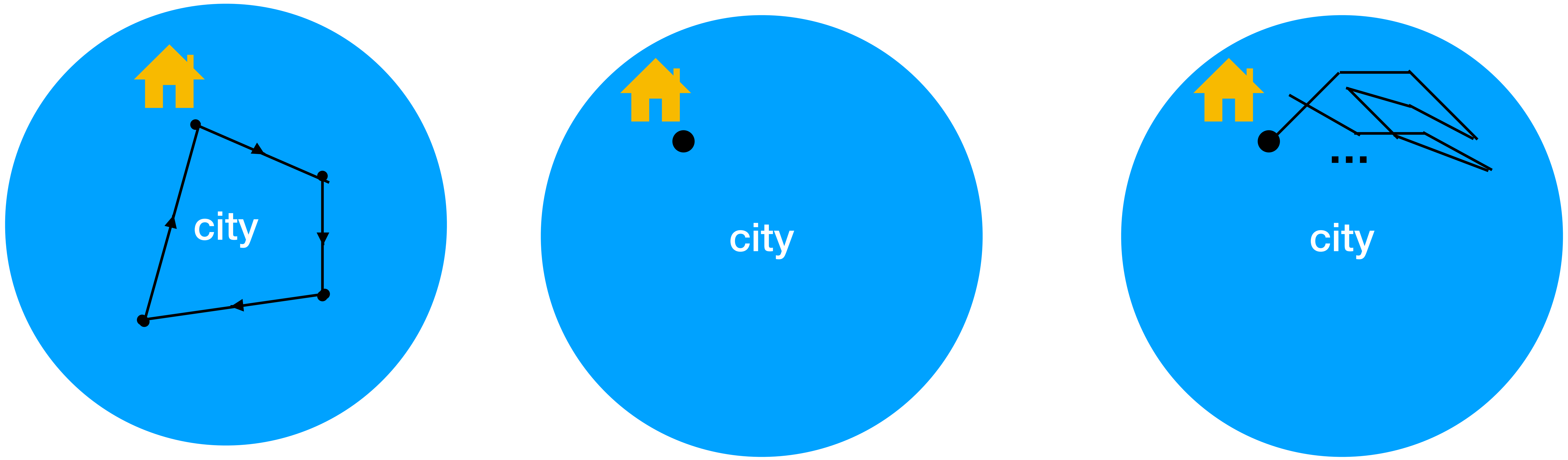
How do people move and interact in a city?



lines

(intentional behavior along a path)

point

(stay at home)

area

(roam anywhere)

These behaviors entail different (fractal) dimension of movement in the city…

IUS Figure 3.8

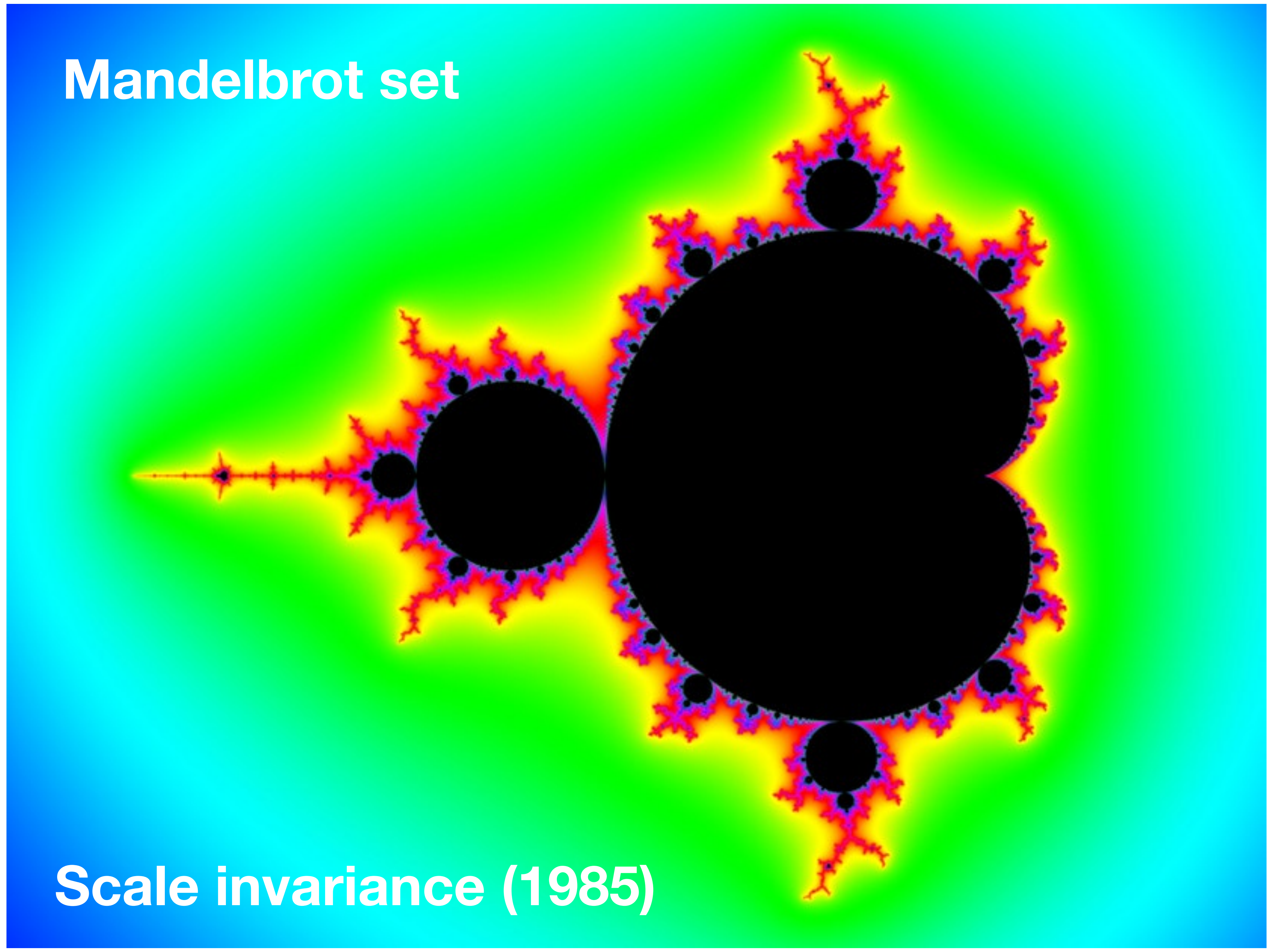
The life path has a different spatial dimension

Concept (a detour):

Fractal Dimensions and Fractal Geometries

Benoit Mandelbrot

fractals and “weird” (spatial) dimensions



solutions (in complex plane) of $c : f_c(z) = z^2 + c$ remains bounded

11.5 x 200 = 2300 km

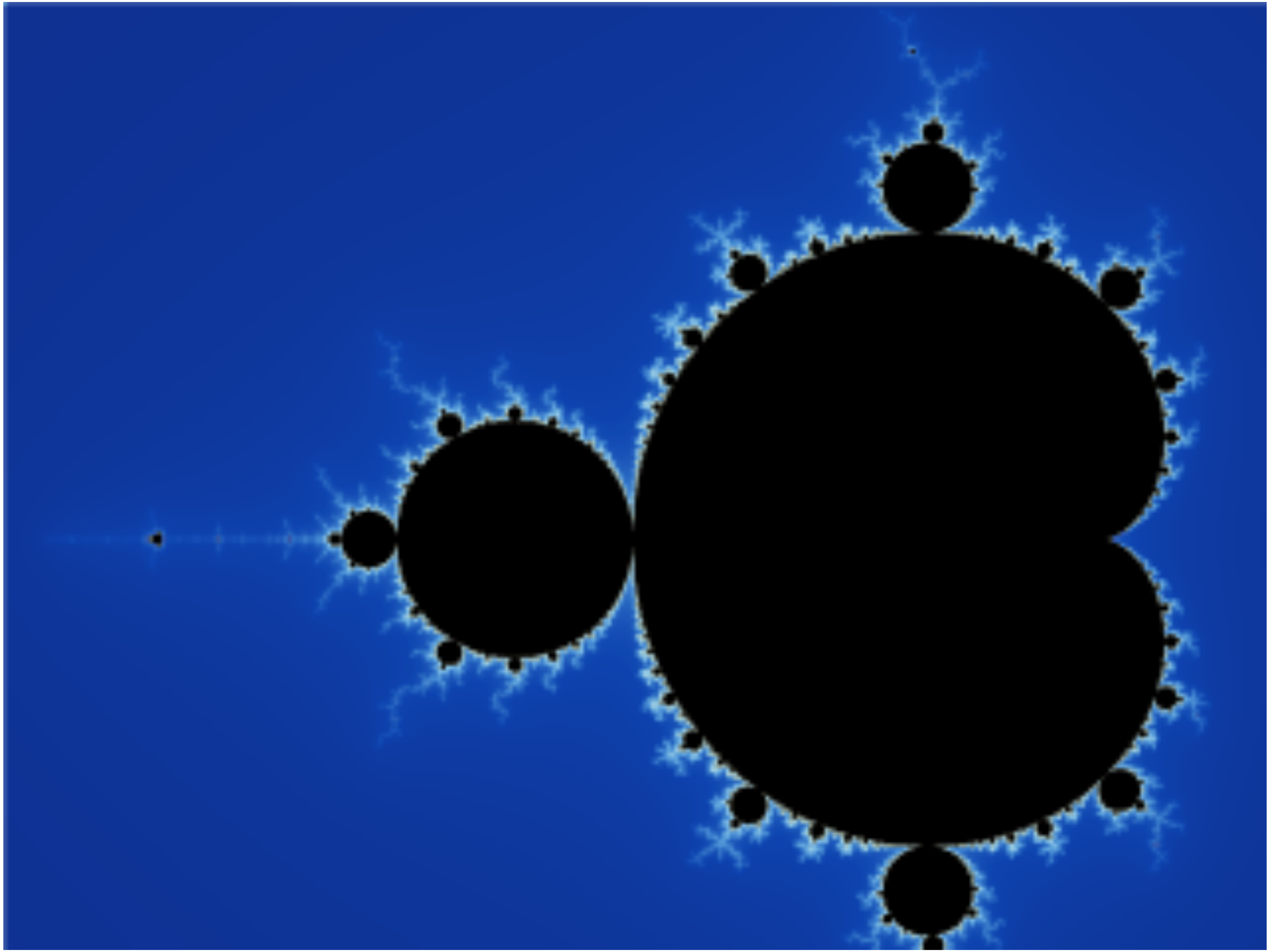
28 x 100 = 2800 km

70 x 50 = 3500 km

Figure 1. As the length of the measuring stick is scaled smaller and smaller, the total length of the coastline measured increases.

credit: wikipedia

https://en.wikipedia.org/wiki/Mandelbrot_set

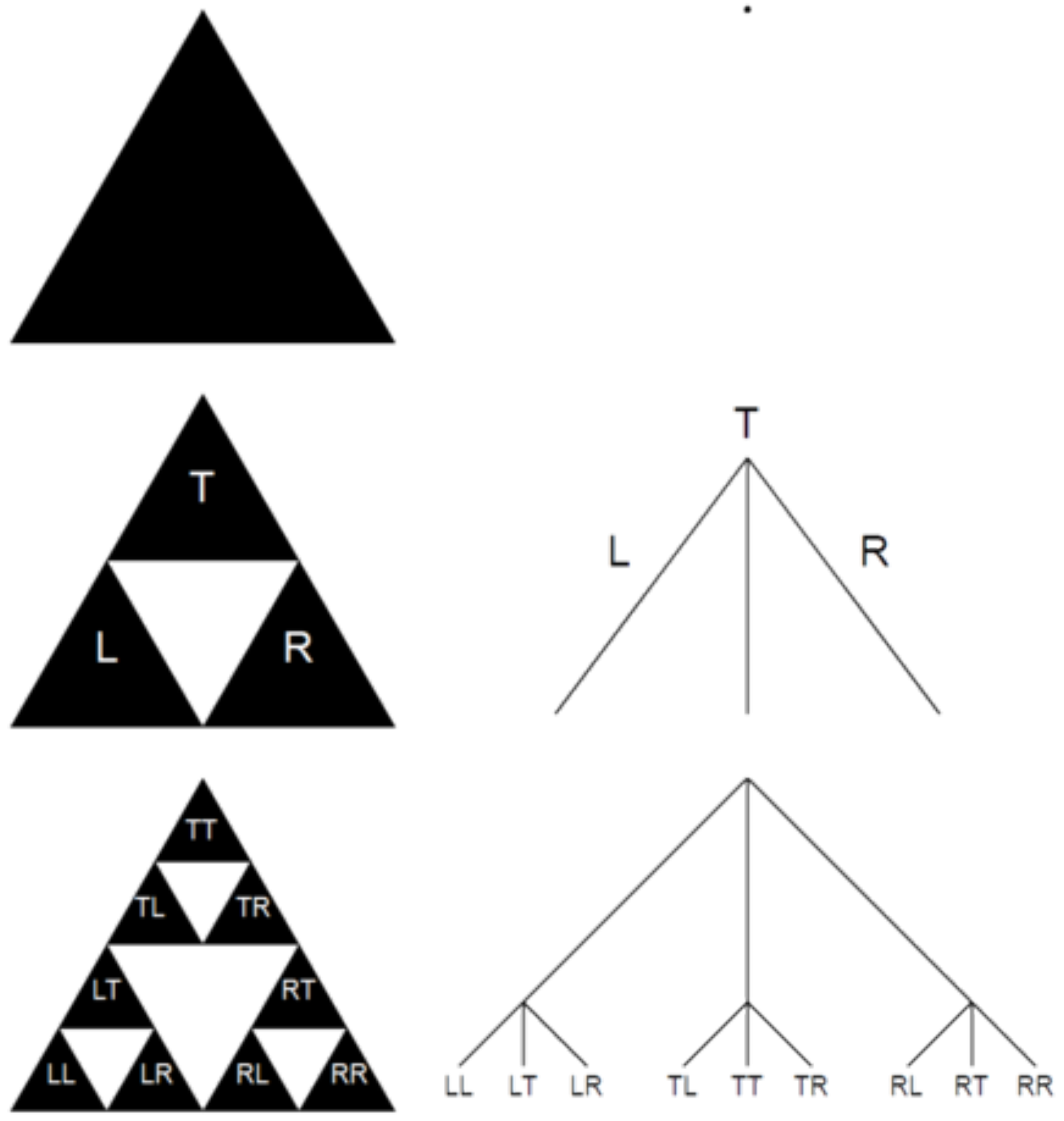


Sierpinski Triangle

Hausdorff dimension of log(3)/log(2)≈1.58

https://en.wikipedia.org/wiki/Sierpiński_triangle

Floor : Santa Maria Trastevere, Rome

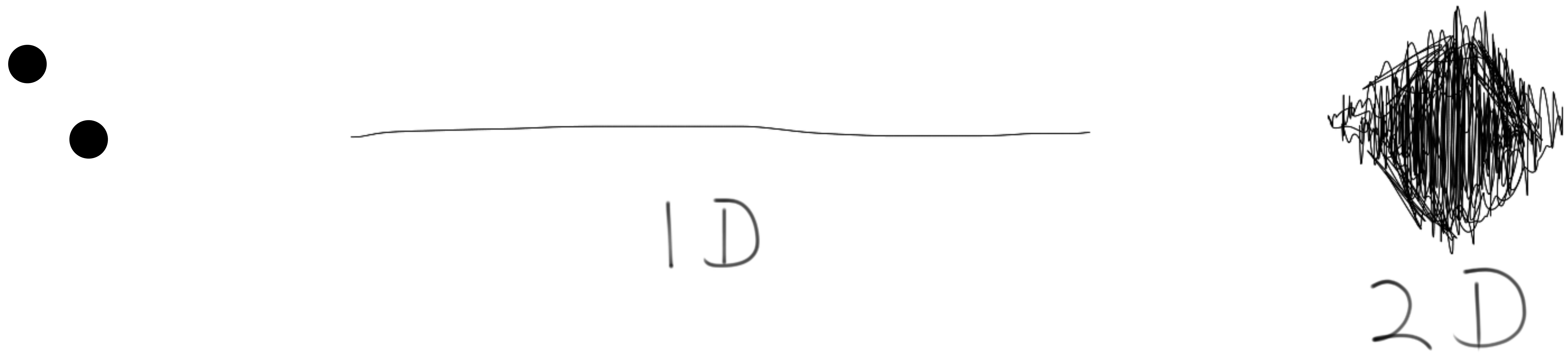


We will use this idea to fill urban land
with buildings
and
a hierarchy of infrastructure networks

Let’s use these ideas first in the amorphous settlement model:

lifepaths (urban mobility) can have different (fractal) dimensions

depending on the space they take up



H_m=0 H_m=1 H_m=2

We will take our world line to possibly have different fractal dimensions

$$\ell = A^{H_m/2}$$

IUS pp 77

The Amorphous Settlement Model

maximal spatial limit to the city

per individual:

net benefits~expected number of interactions = costs of movement

$$y = G\frac{N}{A}$$

lower bound on income

$$C = c_{T_0} R^{H_m}$$

fractal dimension of movement

Area:

$$A(N) = \left(\frac{\sqrt{\pi}G}{c_{T_0}}\right)^{\frac{2}{2+H_m}} N^{\frac{2}{2+H_m}}$$

sublinear

D is the dimension of space, usually *D*=2

H_m is the fractal dimension of mobility, usually $H_m = 1$

exponent: $\alpha = \dfrac{D}{D + H_m}$

Expresses how these two spaces fit together