

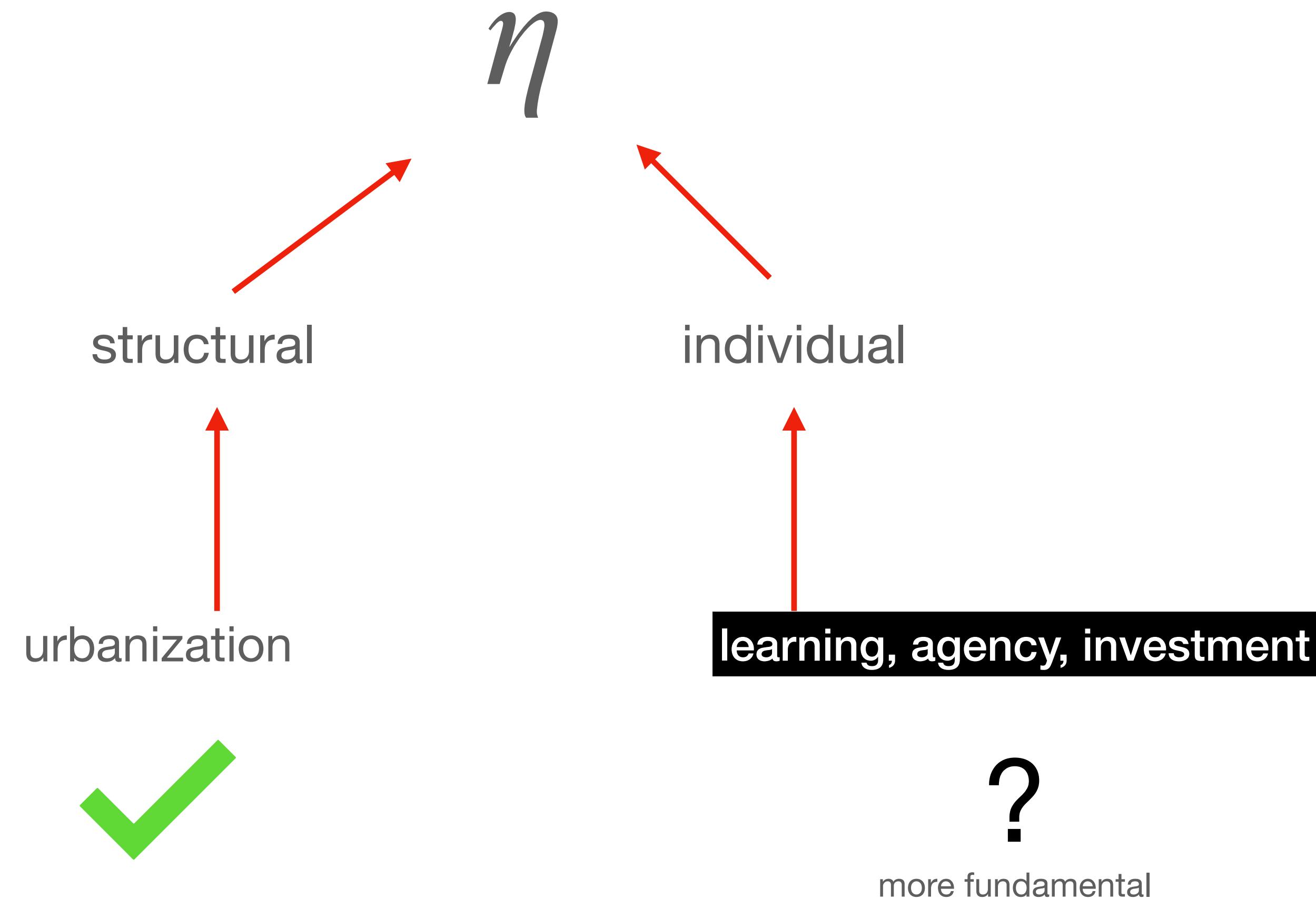
## **Lecture 17**

# **Economic Growth, Information and Cities**

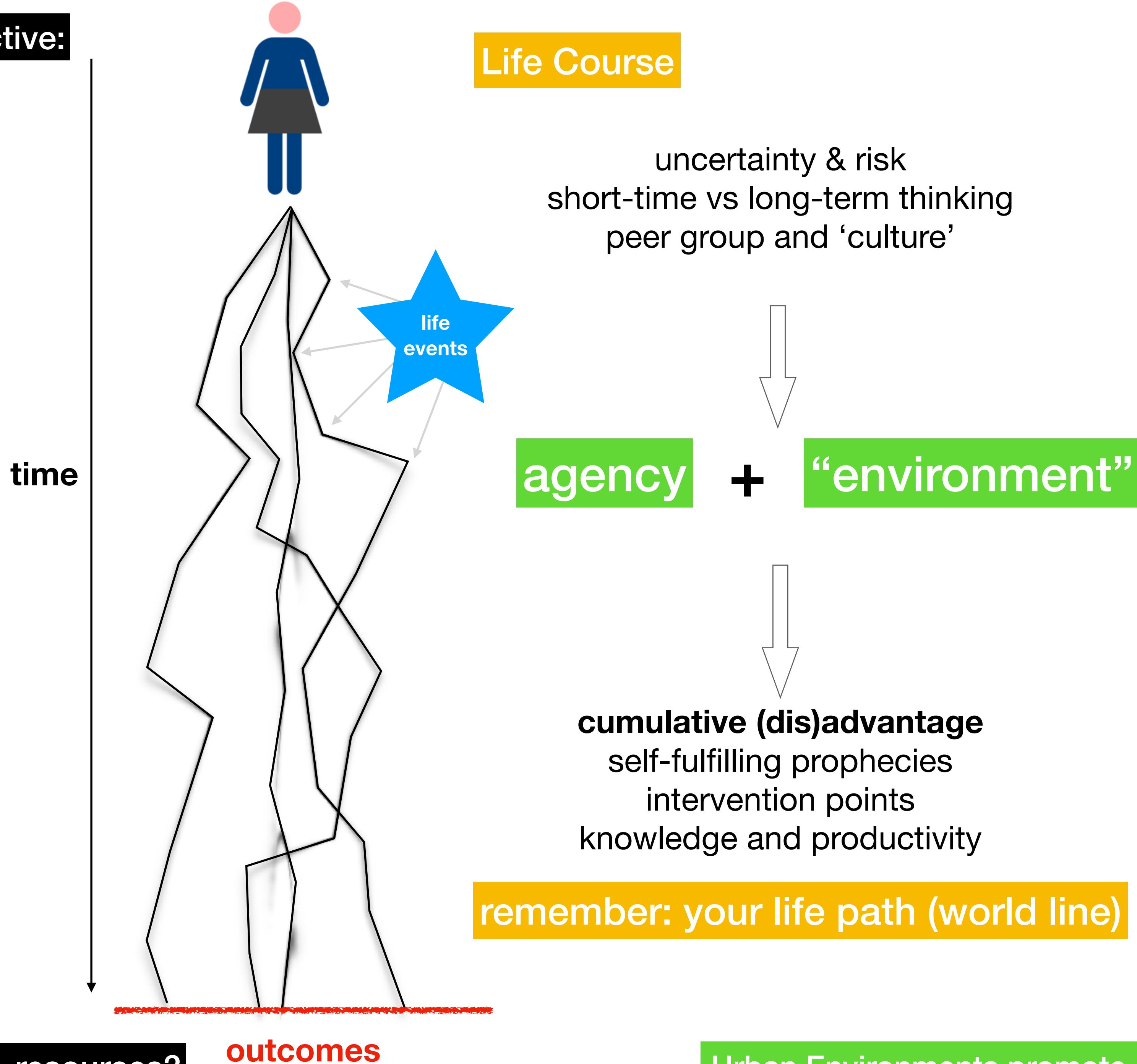
**17.1 Information and Economic Growth: Fortune's Formula and beyond**

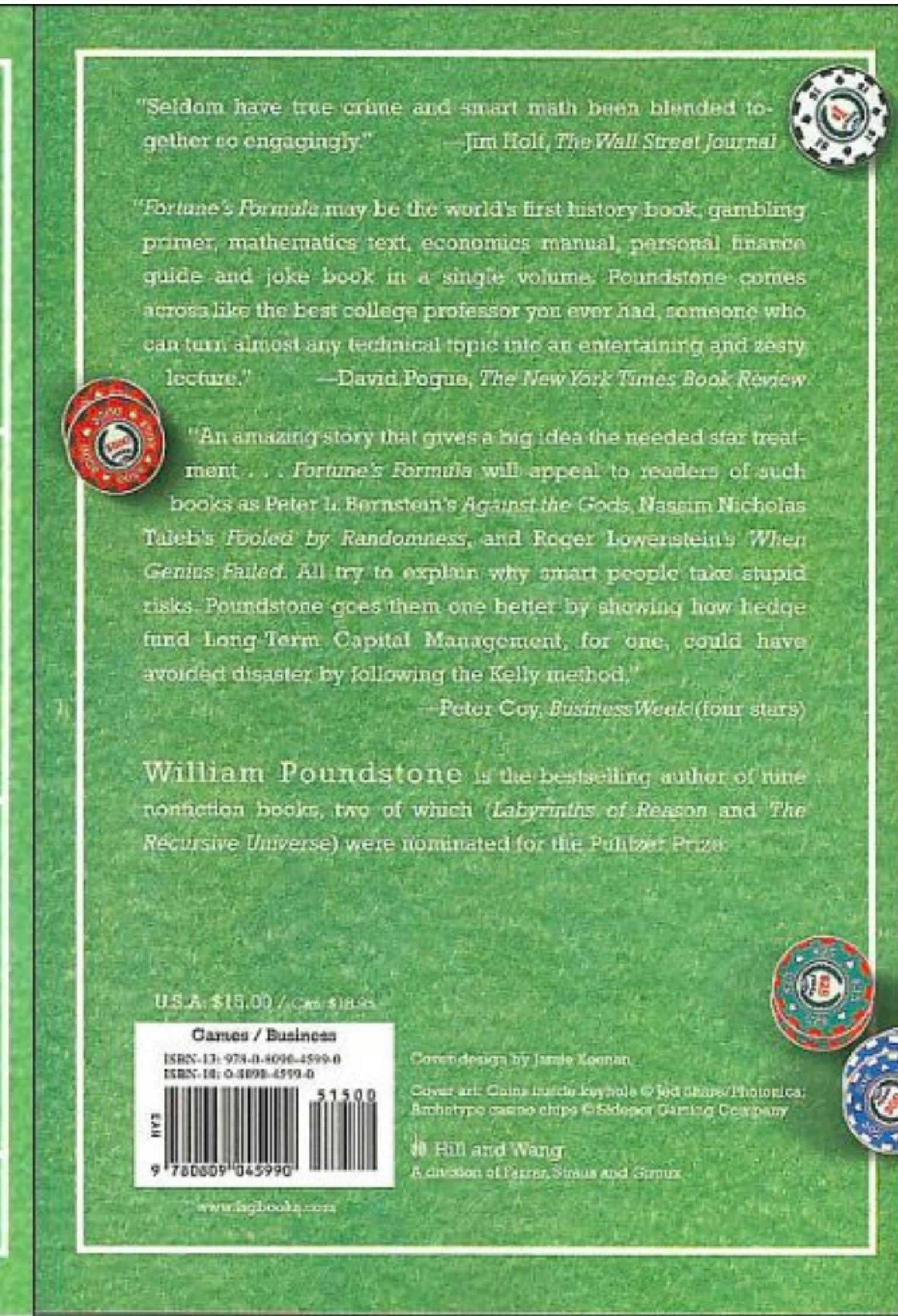
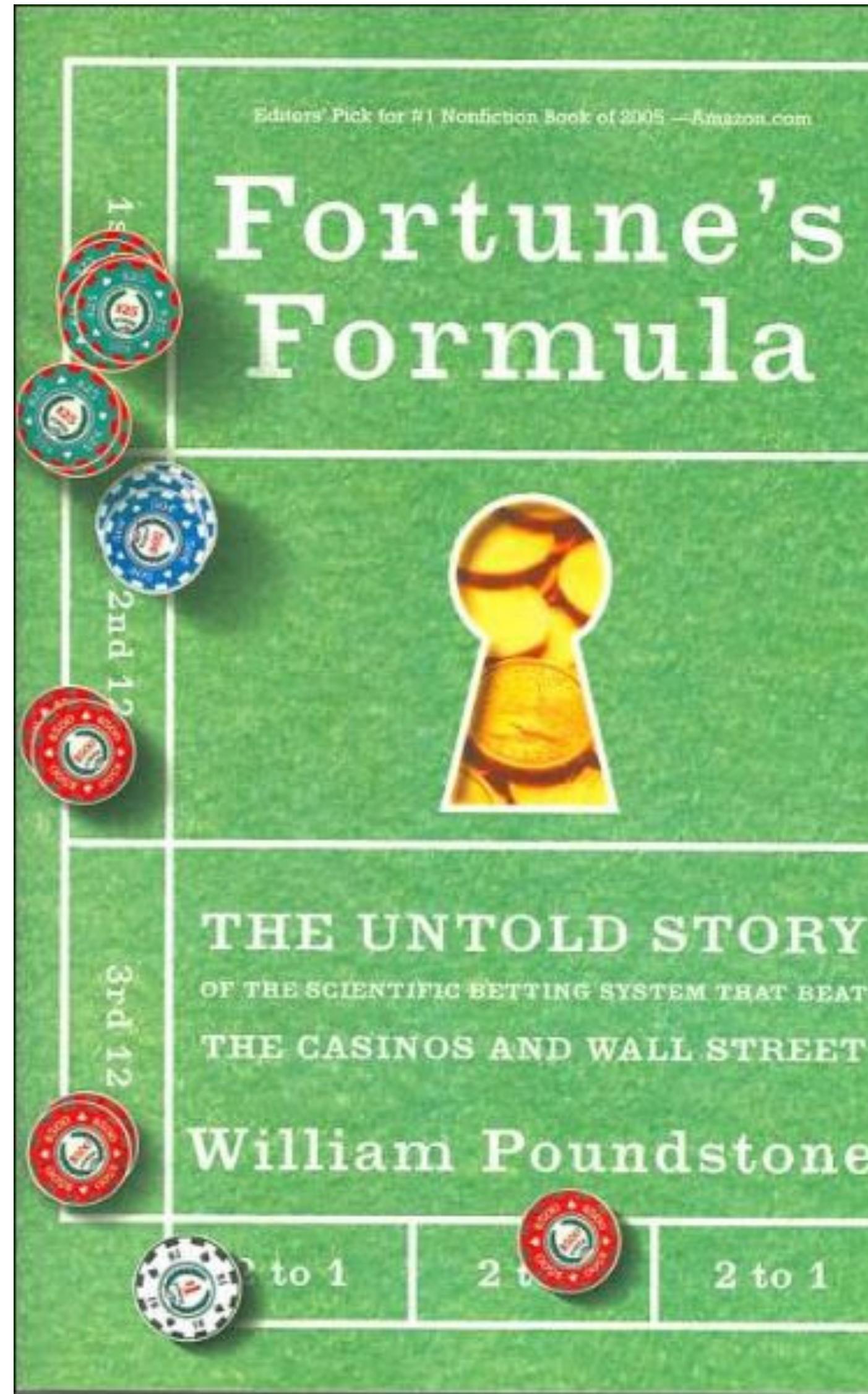
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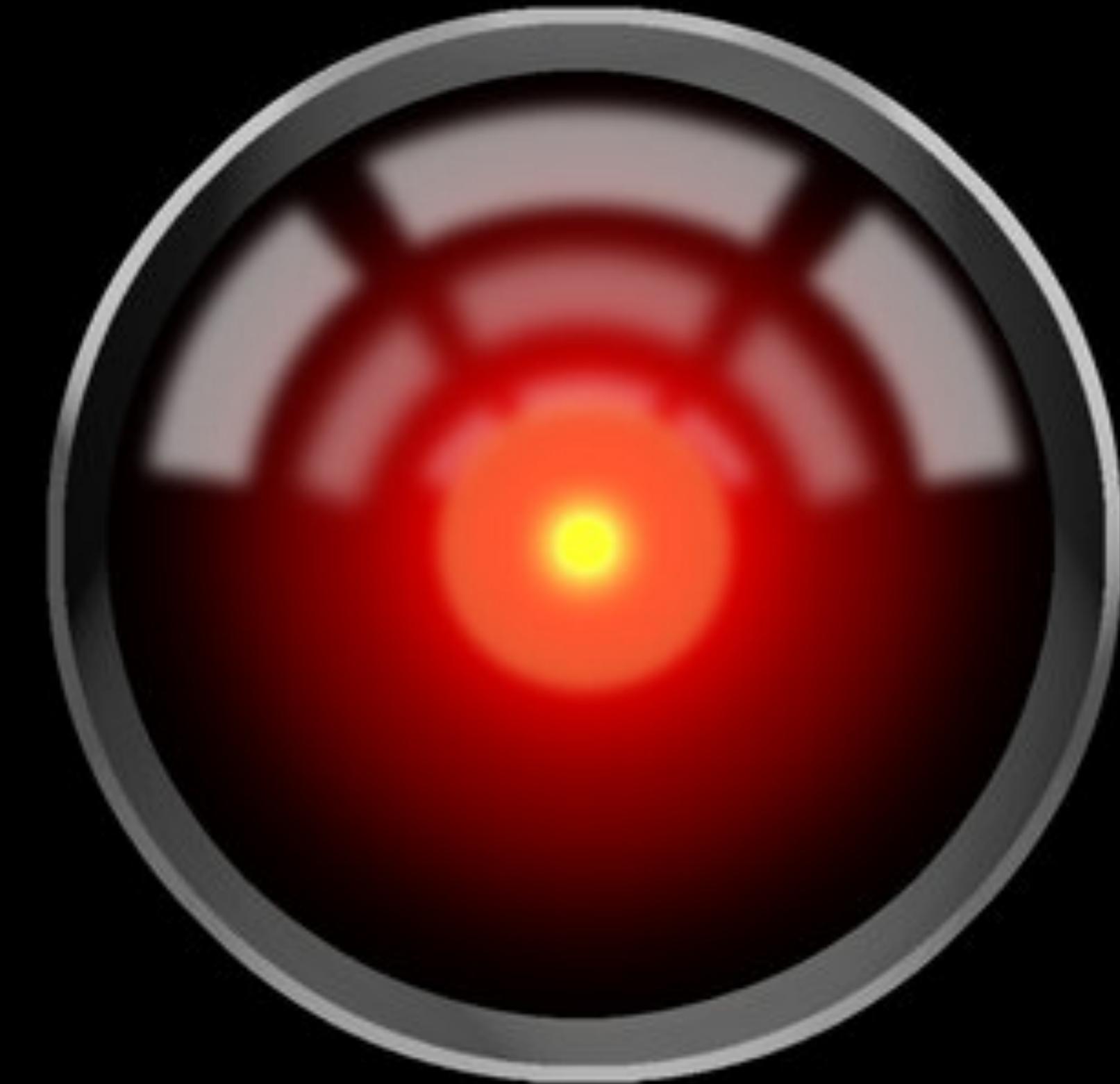
## What creates growth?



From an individual's perspective:







betting on horse races

with math!



# A New Interpretation of Information Rate

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By J. L. KELLY, JR.

(Manuscript received March 21, 1956)

*If the input symbols to a communication channel represent the outcomes of a chance event on which bets are available at odds consistent with their probabilities (i.e., “fair” odds), a gambler can use the knowledge given him by the received symbols to cause his money to grow exponentially. The maximum exponential rate of growth of the gambler’s capital is equal to the rate of transmission of information over the channel. This result is generalized to include the case of arbitrary odds.*

*Thus we find a situation in which the transmission rate is significant even though no coding is contemplated. Previously this quantity was given significance only by a theorem of Shannon’s which asserted that, with suitable encoding, binary digits could be transmitted over the channel at this rate with an arbitrarily small probability of error.*

Simplest case: **binary choice**

heads/tails, red/black, 0/1 etc

Consider a series of events:  $e_1, e_2, \dots, e_i, \dots \in E$

$$p(e = 0) = p(e = 1) = \frac{1}{2}$$

Payoff of guessing right:  $o(e_i) \geq 1.$

what should **b** be?

Get a signal (intuition, friend, experience):  $s_1, s_2, \dots, s_i, \dots \in S$

0 0 1 1 0 0 1 1 0 1

Given the signal, allocate:

$$f(e | s)r$$

$$\sum_e f(e | s) = 1$$

At each step the environment returns state  $e_i$

The agent's resources grow like:

$$r \rightarrow r' = o(e_i)f(e_i | s_j)r$$

proportional to  $r$

**After  $n$  steps:**

$$r(n) = \prod_{j=1}^n o(e^j) f(e^j) r$$

**state occurring at each time**

**The average growth rate is**

$$\nu = n/T$$

velocity of investments

$$\eta = \lim_{n \rightarrow \infty} \frac{\nu}{n} \ln \frac{r(n)}{r} = \nu \sum_{i=1}^E P(e_i) \ln o(e_i) f(e_i).$$

**What is the best allocation?**

**maximizes growth rate**

$$f(e_i) = P(e_i)$$

**proportional betting**

$$\eta = \nu(\overline{\ln o} - H(E))$$

$$\overline{\ln o} = \sum_{i=1}^E P(e_i) \ln o(e_i)$$

**odds need to be good enough; complex environments are worse**

Fair odds:

$$o(e_i) = 1/P(e_i)$$

average “belief”

$$\overline{\ln o} = \sum_{i=1}^E P(e_i) \ln o(e_i) \rightarrow H(E)$$

**For fair odds, the growth rate vanishes without a private signal (unique private information):**

$$\eta = v(\ln o - H(E)) \rightarrow 0.$$

**This reflects a situation when the agent does not have *inside* information**

Need “better” information to beat odds or “market” (average belief)

This is the basic argument for economic markets and not (fully) planned economies

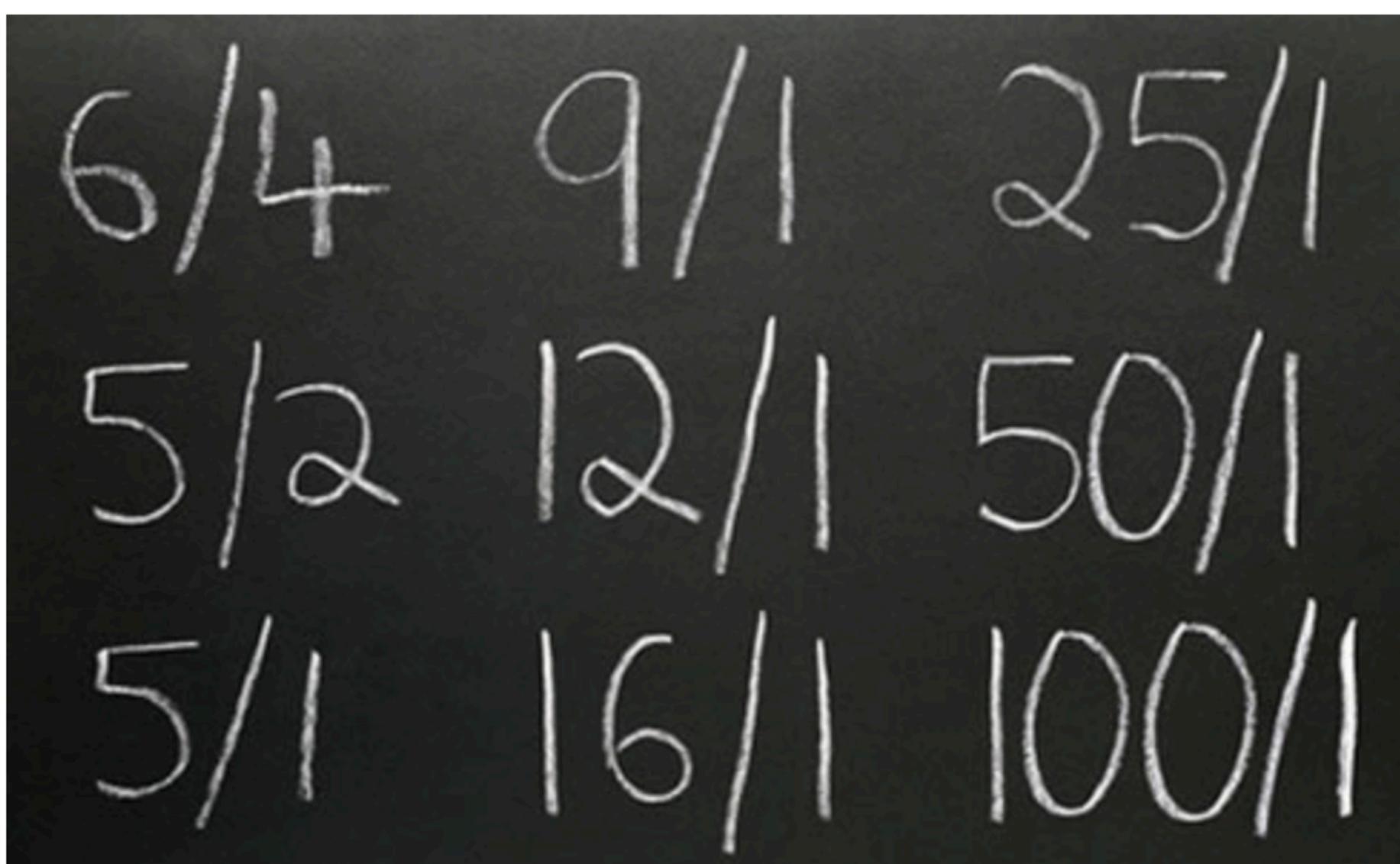
Hayek 1945

<https://www.jstor.org/stable/1809376>

# How To Make A Fair Odds Line

By Derek Simon

<https://www.usracing.com/news/horse-betting-101/making-a-fair-odds-line>



Recently, I've been getting quite a few inquiries about fair odds lines — mainly, how does a horseplayer go about making one and what are they good for?

Let's kick off the discussion with the last question — after all, what's the use in creating something without first knowing what it does (trust me, I've seen enough science fiction movies to know this is a terrible idea)? Simply put, a fair odds line provides gamblers with a means of making rational wagering decisions.

For example, most players know that betting to win on a horse that is 2-5 or less doesn't make a lot of sense. To make any money on such steeds, a gambler would need to cash at least 71 percent of the time, which is extremely unlikely (not to mention the fact that the place and show payoffs would probably be just as high if not higher than the win return, making a win bet look that much more foolish).

<https://www.usracing.com/news/horse-betting-101/making-fair-odds-line>

**Generalize:**

$$o(e_i) = 1/P_m(e_i) \quad \text{guess (estimation) from market, or "crowd", or bookie}$$

**(simplify:  $\nu = 1$ )**

$$\eta = \sum_{i=1}^E P(e_i) \ln \frac{f(e_i)}{P(e_i)} \frac{P(e_i)}{P_m(e_i)} = D_{KL}(P || P_m) - D_{KL}(P || f)$$

**can benefit from imperfect “markets” if we know the actual probability (better)**

But what if we had special private information (edge)?

$$\eta = \sum_{i,j} P(e_i, s_j) \ln o(e_i) f(e_i | s_j)$$

“signal” = memory, knowledge, private channel

## For optimal wealth generation:

Maximize

$$\eta = \sum_{i,j} P(e_i, s_j) \ln o(e_i) f(e_i | s_j)$$

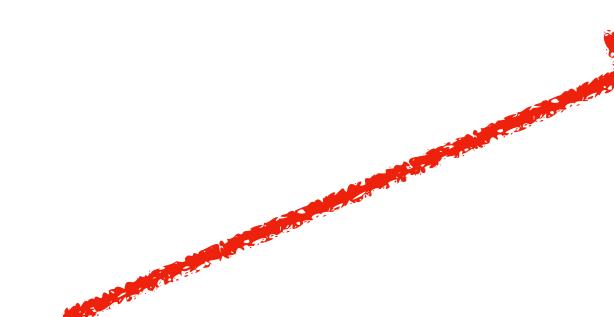
to get the best allocation  $f(e_i | s_j)$

This leads to

$$f(e_i | s_j) = \frac{P(e_i, s_j)}{P(s_j)} = P(e_i | s_j)$$

Fortune's Formula !!

conditional probability of specific event given private signal



$$\Delta\eta = \sum_{i,j} P(e_i, s_j) \ln \frac{P(e_i, s_j)}{P(e_i)P(s_j)} = i(E, S)$$

Mutual Information !!

between environment and private signal

improvement in rate from using the private signal

resources will grow exponentially!!

with a rate given by the information of the signal on the environment

$$\Delta\eta = i(E, S) - D[P(e | s) || f(e | s)]$$

Either information is given (“friend”)

> 0, if estimate is imperfect

Or it must be *learned*:

$$f(e | s^{n+1}) = \frac{p(e | s)}{p(e)} f(e | s^n)$$

Bayesian learning by observation/experience is **Optimal**

$$D[P(e | s) || f(e | s^n)] \sim 1/n$$

but learning from experience is **VERY slow**

**But observed growth rates are much smaller than in a game!**

at the “frontier”

**The meaning of an annual 2% growth rate:**

**1% = doubling (of capital) every 72 years**

“rule of 72”

**2% per year=doubling every 36 years**

~ human generation

**why so “slow”?**

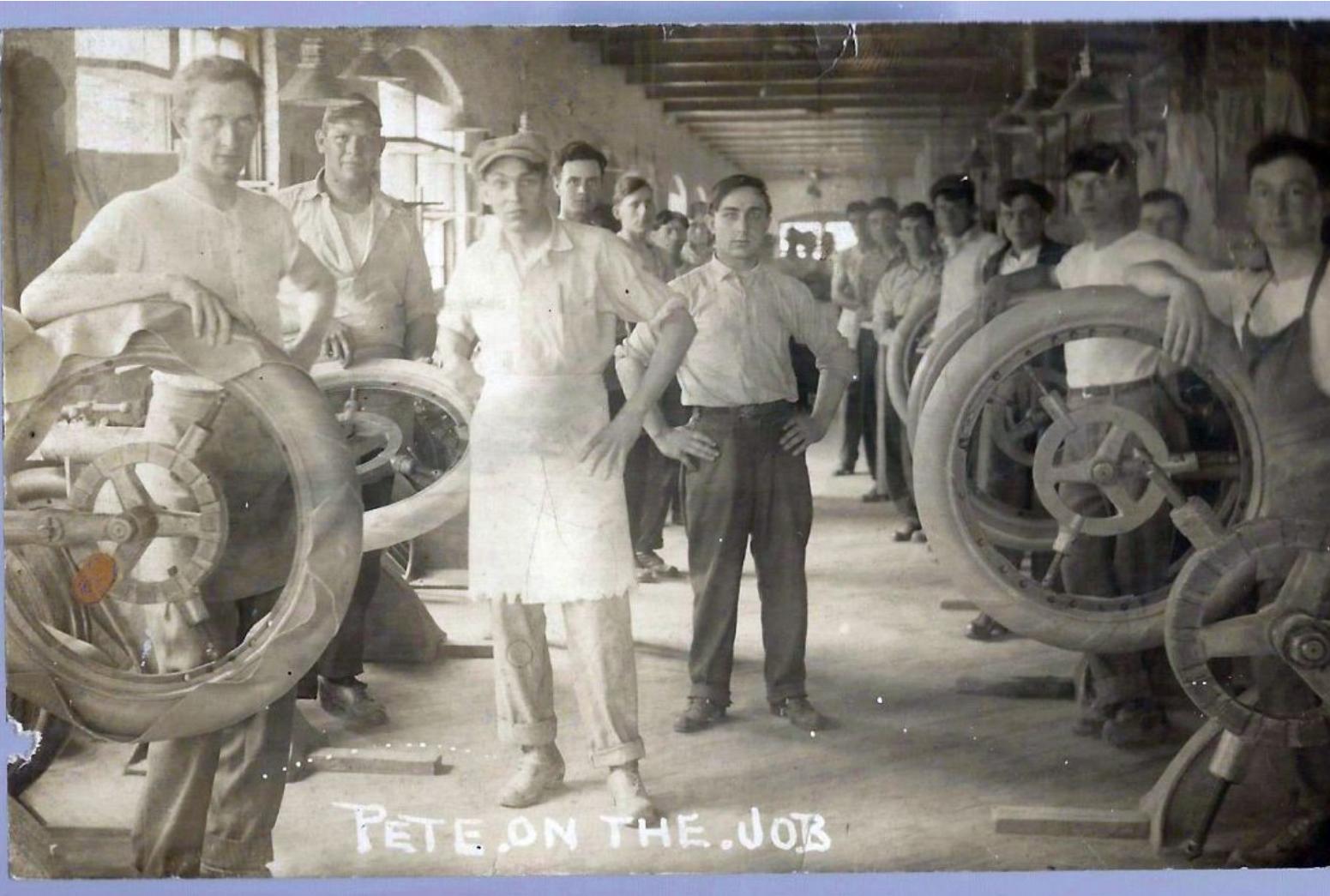
it takes a life time to create new productive information

opportunities that can be learned, education, training, predictability all matter a lot

## But Whose Knowledge (in the production of complex things)?



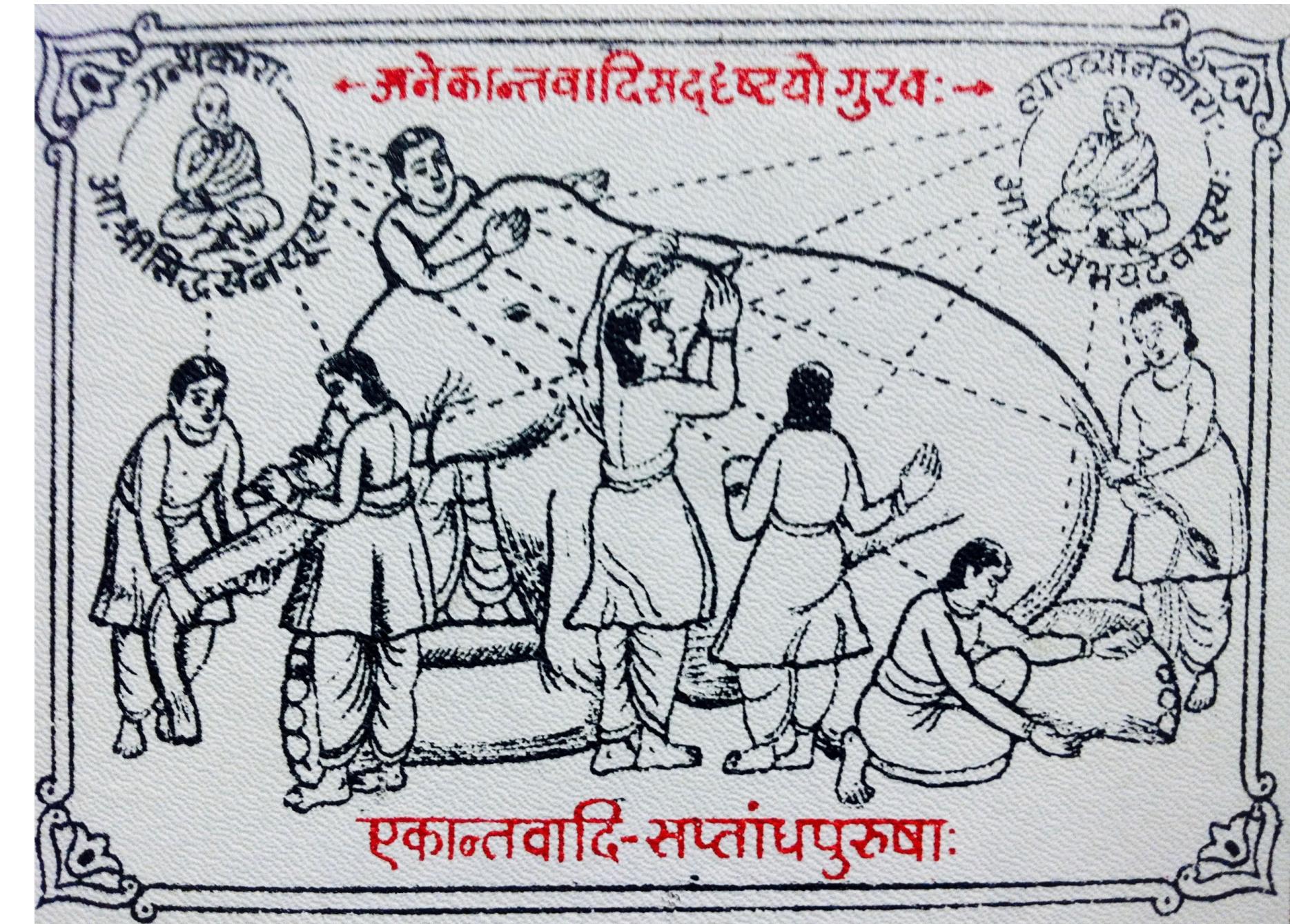
## Whose Resources?



Organizations have **synergistic** information

information in the organization is more than sum of the parts!

$$I(\{Y\}; X) > \sum_j I(Y_j, X)$$



how should earned resources be distributed among agents?

### Team Production Problem

Division of knowledge and Labor:  
and putting it back together fairly, so that it can be repeated and elaborated

## Firms and Organizations

### synergy and redundancy

**1) Organizations have synergistic information**

coordination of people behavior towards a common goal

**2) Information can also be redundant**

two people can know the same thing

**3) Diversity of Information (maximal synergy) is necessary for growth:**

maximum information requires “maximum synergy”=maximum “diversity” !

not for efficiency! not even for justice: For fastest *collective* growth and knowledge

## Consequences:

- knowledge always wins over initial capital\*
  - knowledge has an enormous value, for others and into the future
    - positive externalities, spillovers
  - knowledge producers are not able to capture its full value
    - public investment is necessary, “public good”
  - knowledge can attract capital (for a rent)
    - start-ups, grants, finance, loans
  - \*but capital can also buy knowledge (licensing, hiring)
    - patents

environments that promote collective knowledge, learning and assembly are key for growth:

Cities !!

It is in such environments you will see: diversity, interdependence, connectivity, learning, turnover, public institutions resulting in experimentation and fast change.