Lecture 14

The Structure of the Urban Systems and the Laws of Geography

14.2 Demographic Dynamics and the Structure of Urban Systems

IUS 8.2

Fundamental Dynamics of Cities

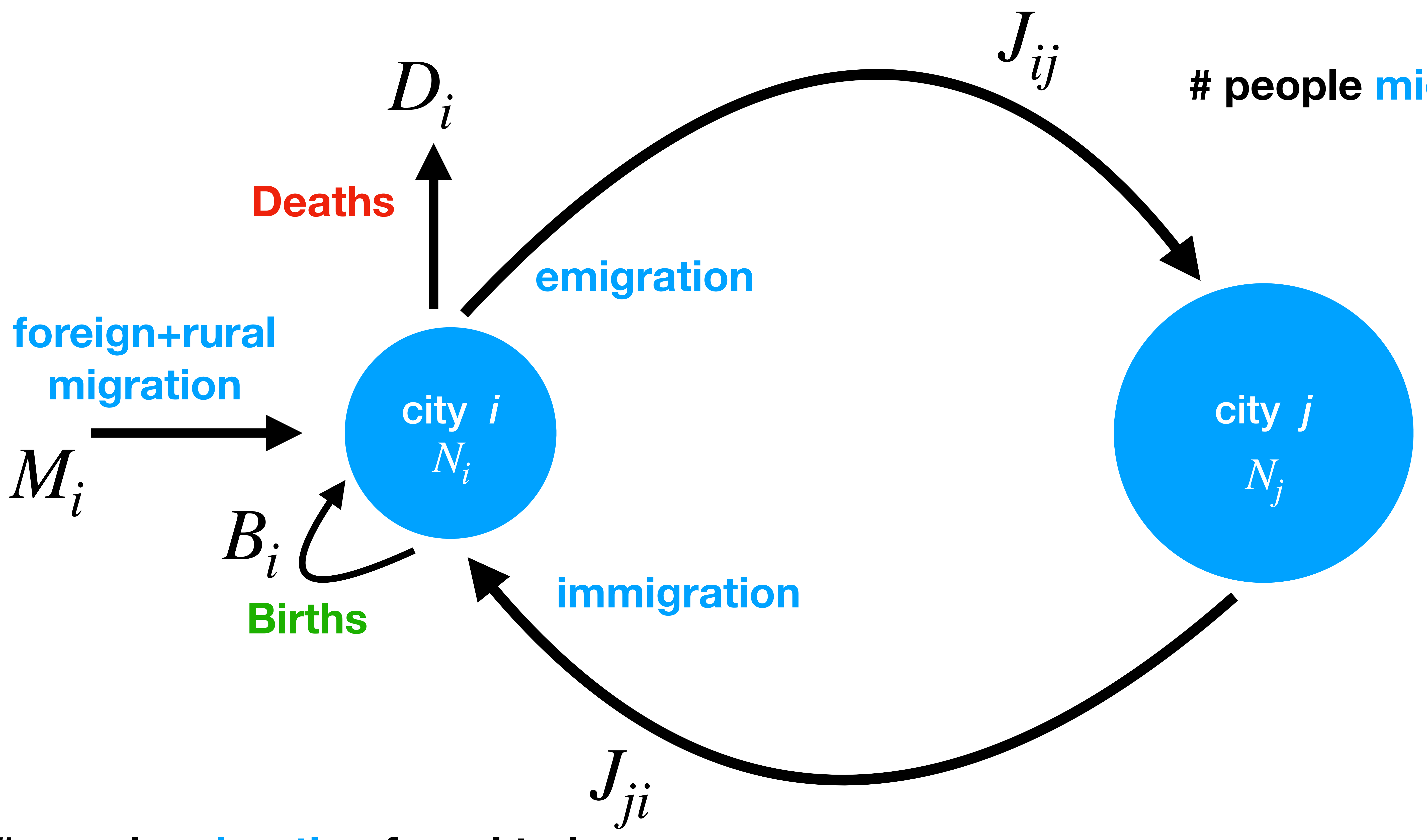
Start with demography

The population change of a city can be decomposed into:

Births **Deaths** — **vital rates**

Migration — **between cities**, **from rural areas**, **from abroad**

Population Growth Accounting



people migrating from i to j

over a period of time
e.g. year

people migrating from j to i

$$N_i(t+1) = N_i(t) + B_i - D_i + M_i + \sum_{j}^{N_c} (J_{ij} - J_{ij})$$

sum over cities

Population Growth Accounting

Introduce rates (per capita):

birth rate $$b_i = \frac{B_i}{N_i}$$

death rate $$d_i = \frac{D_i}{N_i}$$

foreign and rural migration rate $$m_i^{F+R} = \frac{M_i}{N_i}$$

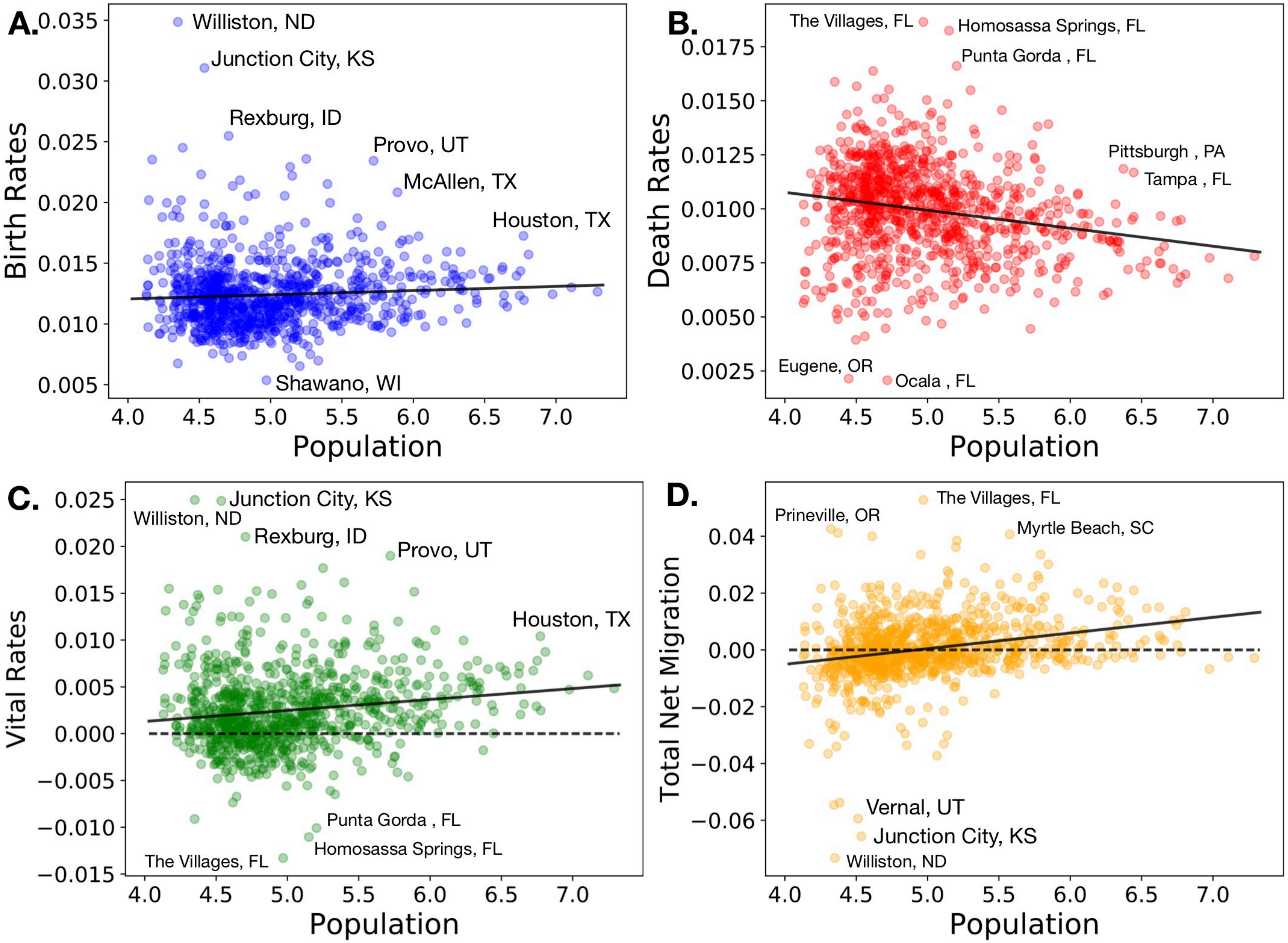
$$v_i = b_i - d_i + m_i^{F+R}$$

“vital” rate

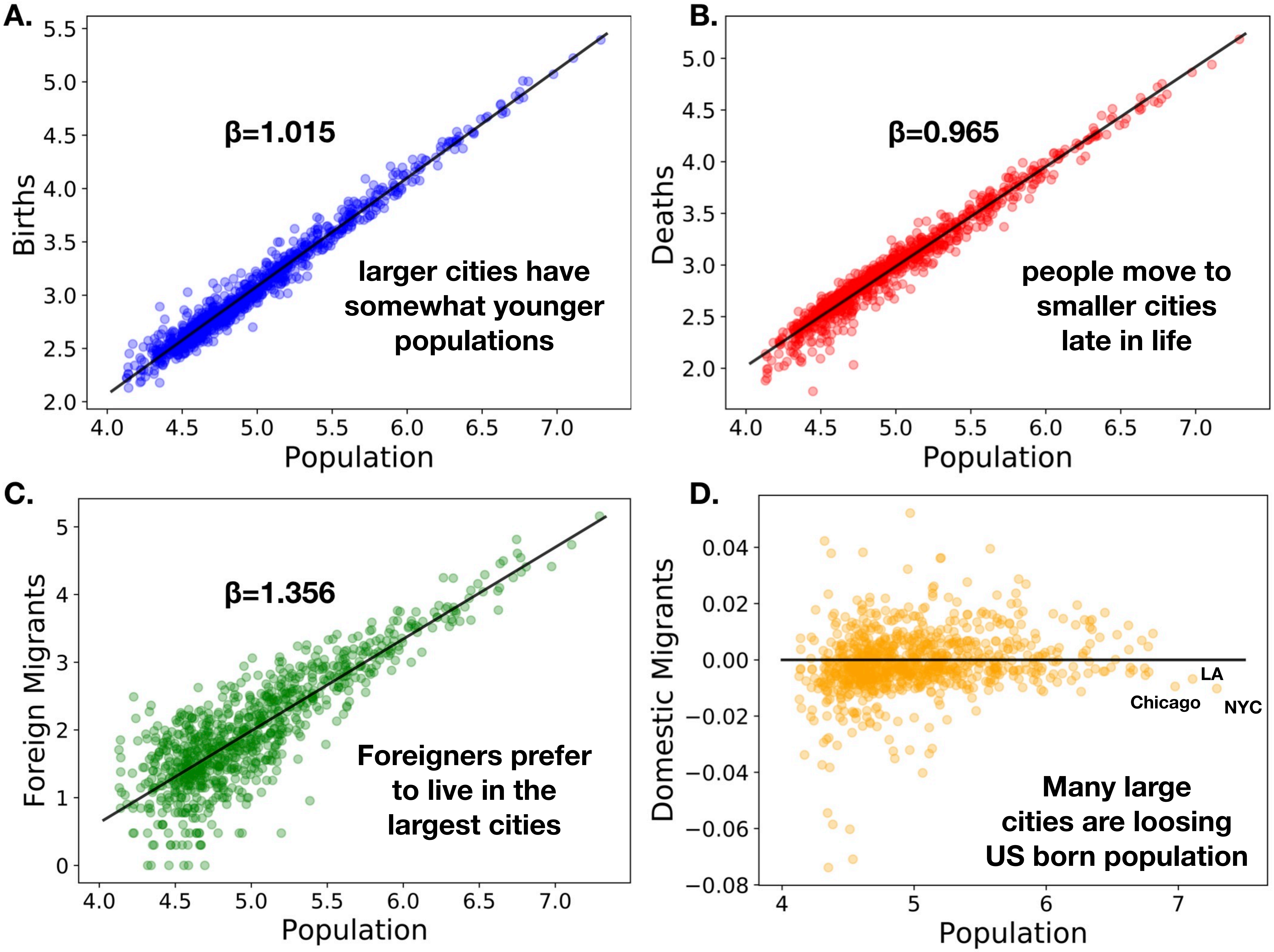
$$N_i(t+1) = (1+v_i)N_i(t) + \sum_{j}^{N_c} (J_{ij} - J_{ij})$$

Simpler!

what to do w/ migration flows?



A.
Birth Rates
Population
Williston, ND
Junction City, KS
Rexburg, ID
Provo, UT
McAllen, TX
Houston, TX
Shawano, WI
B.
Death Rates
Population
The Villages, FL
Homosassa Springs, FL
Punta Gorda , FL
Pittsburgh , PA
Tampa , FL
Eugene, OR
Ocala , FL
C.
Vital Rates
Population
Junction City, KS
Williston, ND
Rexburg, ID
Provo, UT
Houston, TX
Punta Gorda , FL
Homosassa Springs, FL
The Villages, FL
D.
Total Net Migration
Population
The Villages, FL
Prineville, OR
Myrtle Beach, SC
Vernal, UT
Junction City, KS
Williston, ND


A.
Births
Population
β=1.015
larger cities have somewhat younger populations
B.
Deaths
Population
β=0.965
people move to smaller cities late in life
C.
Foreign Migrants
Population
β=1.356
Foreigners prefer to live in the largest cities
D.
Domestic Migrants
Population
LA
Chicago
NYC
Many large cities are loosing US born population

Matrix Models

Trick:

$$\mathbf{N}(t+1) = \mathbf{A}(t)\mathbf{N}(t)$$

try to write population change as

Then solution is simple:

$$\mathbf{N}(t+1) = \mathbf{A}(t)\mathbf{A}(t-1)\mathbf{A}(t-2)\ldots\mathbf{A}(1)\mathbf{N}(0)$$

If *A* does not depend on time, it is VERY simple:

$$\mathbf{N}(t+1) = \mathbf{A}^t\mathbf{N}(0)$$

what is $\mathbf{A}$ **?**

what is $\mathbf{A}^t$ **?**

Matrix Models

We can write the migration current as

$$J_{ij} = m_{ij} N_i$$

probability per person that
someone in city *i* choses to move to city *j*

$$N_i(t+1) = \sum_{i=1}^{N_c} A_{ij}(t) N_j(t)$$

if these are independent of time
then *A* is fixed

$$A_{ij} = (1 + v_i - m_i^{\text{out}}) 1_{ij} + m_{ji}$$

$$m_i^{\text{out}} = \sum_{j=1}^{N_c} m_{ij} < 1$$

is the probability per individual to leave city i

Matrix Solution

The solution to $\mathbf{N}(t+1) = \mathbf{A}(t)\mathbf{N}(t)$

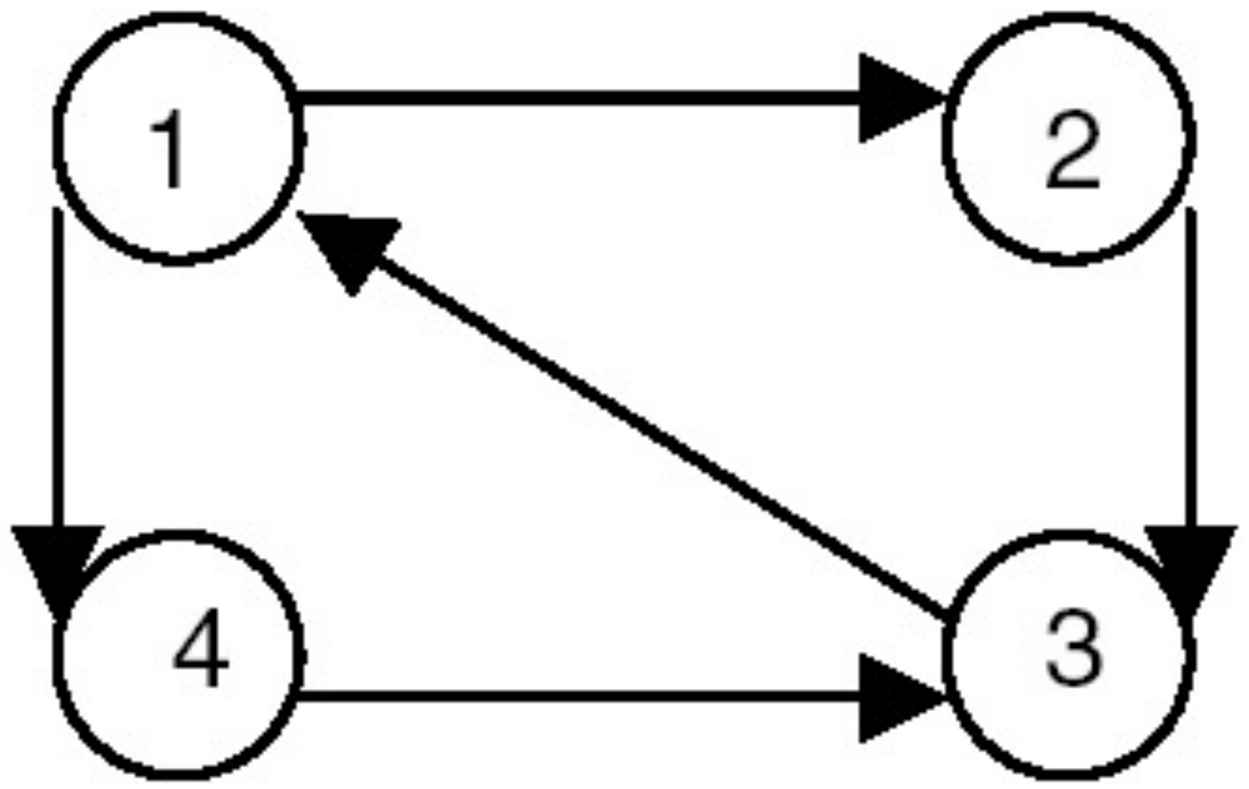
follows from the eigenvalues of *A*: $\mathbf{A}\ e_k = \lambda_k \mathbf{e}_k, \qquad k = 0,1,\ldots,N_c - 1$

The full solution is: $\mathbf{N}(t) = \sum_{k=0}^{N_c-1} \lambda_k^t c_k \mathbf{e}_k$ with the c_k such that: $\mathbf{N}(0) = \sum_{j=1}^{N_c} c_k \mathbf{e}_k$

It all depends now on the eigenvalues and eigenvectors

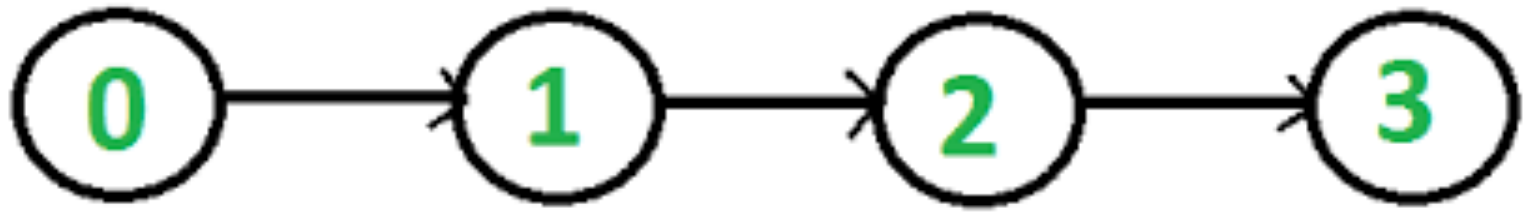
Strongly Connected Graphs

Strongly Connected Graphs



strongly connected

any node can be reached from any other node



NOT strongly connected

some nodes cannot be reached from other nodes

Migration Flows form a Strongly Connected Graph

any city can be reached from any other following migration flows

Strongly Connected Graphs

Perron-Frobenius Theorem

https://en.wikipedia.org/wiki/Perron–Frobenius_theorem

1) The largest eigenvalue λ_0 **is a positive real number**

2) The corresponding eigenvector, $\mathbf{e}_0$**, is made of all positive numbers:**

PageRank

3) All other eigenvalues are smaller (real part)

So, the solution will look like

$$\mathbf{N}(t) = \lambda_0^t \left(c_0 \mathbf{e}_0 + \left(\frac{\lambda_1}{\lambda_0}\right)^t c_1 \mathbf{e}_1 + \left(\frac{\lambda_2}{\lambda_0}\right)^t c_2 \mathbf{e}_2 + \dots \right) \to \lambda_0^t c_0 \mathbf{e}_0$$

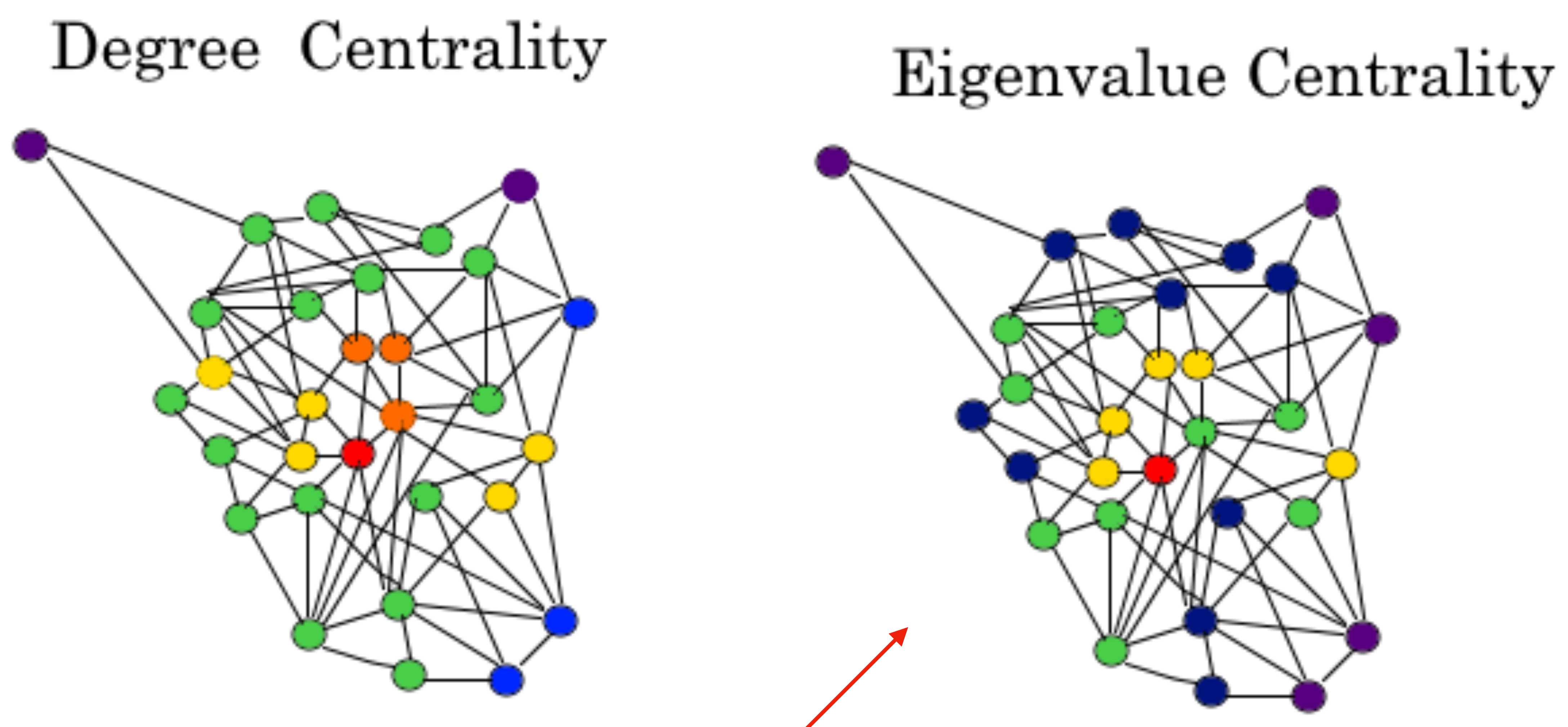
Gibrat's law
(not statistical)

$$e^{-\left(\ln \frac{\lambda_0}{\lambda_1},\right)t} \qquad e^{-\left(\ln \frac{\lambda_0}{\lambda_2}\right)t} \to 0$$

all cities grow at same rate

Power and Influence in Networks

Centrality



emphasizes nodes that have high degree
and
are connected to others of high degree
in recursive manner

A node with high eigenvalue centrality controls not only more other nodes but also those who can control more and so on…

Power and Influence in Networks

Power and Centrality: A Family of Measures[1]

Phillip Bonacich
University of California, Los Angeles

1987

Although network centrality is generally assumed to produce power, recent research shows that this is not the case in exchange networks. This paper proposes a generalization of the concept of centrality that accounts for both the usual positive relationship between power and centrality and Cook et al.'s recent exceptional results.

pioneered by sociologists and geographers

Cook et al. (1983) have shown that power does not equal centrality in exchange networks. In a set of experimental and simulation studies, those who were the most central were not the most successful in exercising bargaining power. This seems to contradict much social network research, especially in the area of interlocking directorates (Mizruchi 1982; Mintz and Schwartz 1985), that assumes that centrality is equivalent to power. Moreover, there is an extensive social psychological literature showing that, in experimentally restricted communication networks, the leadership role typically devolves upon the individual in the most central position (Leavitt 1951; Berkowitz 1956; Shaw 1964).

https://www.jstor.org/stable/2780000

PageRank

The PageRank Citation Ranking: Bringing Order to the Web

Page and Brin

January 29, 1998

Abstract

The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively about the relative importance of Web pages. This paper describes PageRank, a method for rating Web pages objectively and mechanically, effectively measuring the human interest and attention devoted to them.

We compare PageRank to an idealized random Web surfer. We show how to efficiently compute PageRank for large numbers of pages. And, we show how to apply PageRank to search and to user navigation.

http://ilpubs.stanford.edu:8090/422/

Web Page	PageRank (average is 1.0)
Download Netscape Software	11589.00
http://www.w3.org/	10717.70
Welcome to Netscape	8673.51
Point: It's What You're Searching For	7930.92
Web-Counter Home Page	7254.97
The Blue Ribbon Campaign for Online Free Speech	7010.39
CERN Welcome	6562.49
Yahoo!	6561.80
Welcome to Netscape	6203.47
Wusage 4.1: A Usage Statistics System For Web Servers	5963.27
The World Wide Web Consortium (W3C)	5672.21
Lycos, Inc. Home Page	4683.31
Starting Point	4501.98
Welcome to Magellan!	3866.82
Oracle Corporation	3587.63

Table 1: Top 15 Page Ranks: July 1996

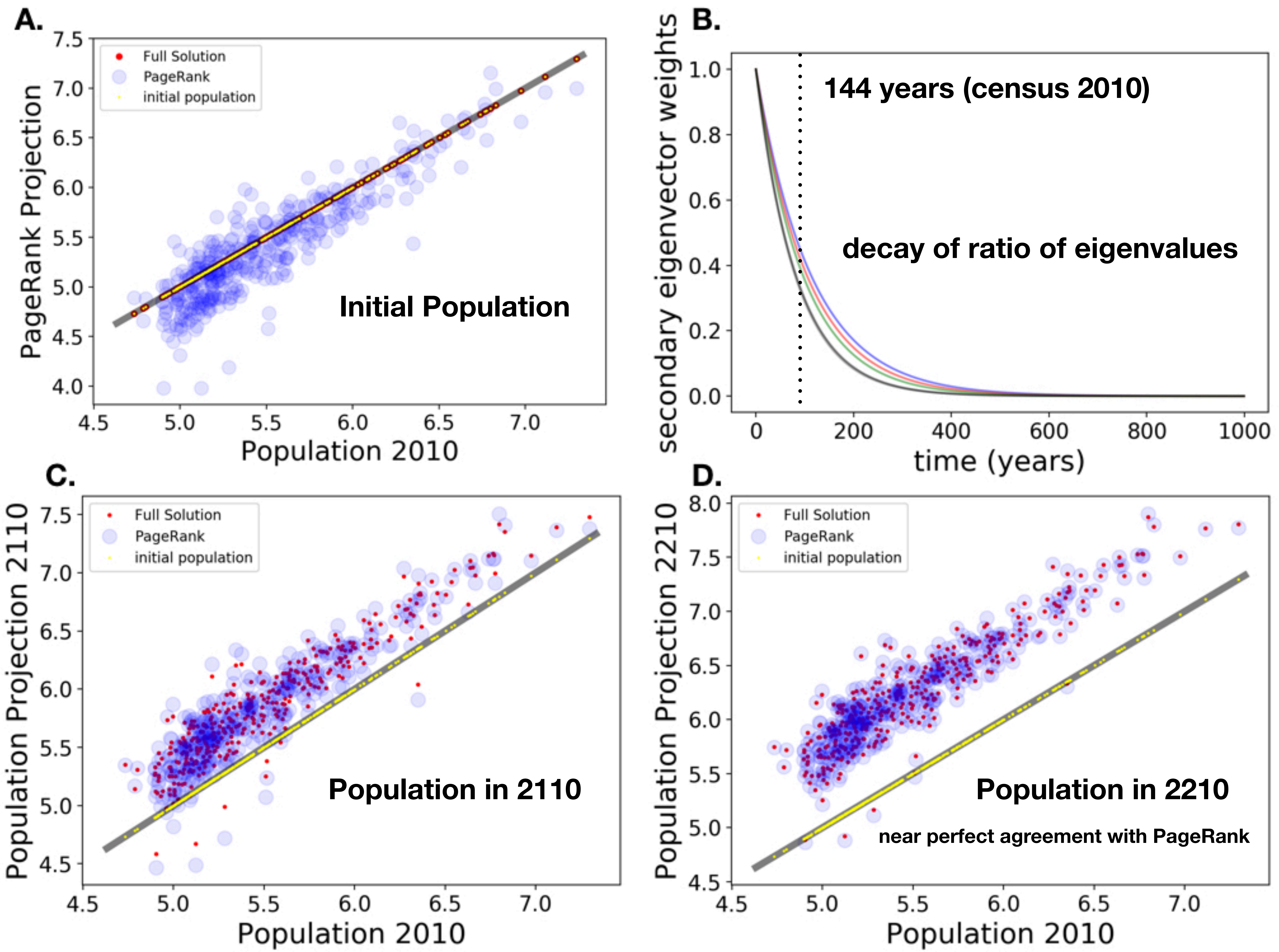
http://ilpubs.stanford.edu:8090/422/

City Population Size Projections

migration from US Census 2010

Page Rank	Rank 2110	Rank 2010	MSA Name (main city)	Projected Population 2110	Present Population
1	2	5	Houston	26,107,974	6,260,171
2	4	4	Dallas-F. Worth	22,415,885	6,741,942
3	1	1	New York City	30,160,729	19,748,581
4	3	2	Los Angeles	24,658,051	12,999,512
5	8	9	Atlanta	13,995,430	5,468,366
6	6	7	Washington DC	14,250,130	5,872,661
7	5	8	Miami	14,631,123	5,798,818
8	9	12	Phoenix	12,692,948	4,352,661
9	7	3	Chicago	14,099,901	9,420,194
10	15	35	Austin	9,327,568	1,865,084
11	12	15	Seattle	10,648,859	3,570,470
12	10	11	San Francisco	11,164,725	4,478,883

largest city in US by 2210



A.
Full Solution
PageRank
initial population
Initial Population
PageRank Projection
Population 2010
B.
144 years (census 2010)
decay of ratio of eigenvalues
secondary eigenvector weights
time (years)
C.
Full Solution
PageRank
initial population
Population in 2110
Population Projection 2110
Population 2010
D.
Full Solution
PageRank
initial population
Population in 2210
near perfect agreement with PageRank
Population Projection 2210
Population 2010

This may well describe what happens if the environment doesn't change.

But this is not Zipf's law for the size distribution of cities:

What are we missing? What could get us there? Or closer?

write migration flows as

$$J_{ij} = \frac{s_{ij} + \delta_{ij}}{2} \frac{N_i N_j}{N_T}$$

total population

symmetric $s_{ij} = s_{ji}$

anti-symmetric $\delta_{ij} = -\delta_{ji}$

gravity

$$N_i(t+1) = (1 + \gamma_{N_i}) N_i(t), \quad \gamma_{N_i} = v_i + \sum_{j=1}^{N_c} \delta_{ij} \frac{N_j}{N_T}$$

This looks like simple random growth:

$$N_i(t+1) = (1+\gamma_{N_i})N_i(t), \qquad \gamma_{N_i} = v_i + \sum_{j=1}^{N_c} \delta_{ij}\frac{N_j}{N_T}$$

define the structure vector: $x_i = \dfrac{N_i}{N_T}$

fraction of total population in city i

$$x_i(t+1) = (1+\epsilon_i)x_i(t), \qquad \epsilon_i = v_i - \bar{v} + \sum_{j=1}^{N_c} \delta_{ij} x_j$$

zero average, just fluctuations $\sum_i \epsilon_i = 0$

This now behaves like random geometric growth !

The noisy growth equation:

$$x_i(t+1) = (1+\epsilon_i)x_i(t), \qquad \epsilon_i = v_i - \bar{v} + \sum_{j=1}^{N_c} \delta_{ij} x_j$$

Leads to an equation for the probability of x (derivation not shown)**:**

$$\frac{d}{dt} P[x, t \;\; x_0, t_0] = \frac{d^2}{dx^2} \sigma_\epsilon^2 x^2 P[x, t \;\; x_0, t_0] \qquad \sigma_\epsilon^2 = \langle \epsilon^2 \rangle$$

probability of finding x at time t,
given we started with x_0 at time t_0

One time-independent solution is:

$$P[x] = \frac{A}{\sigma_\epsilon^2 x^2} \rightarrow P[N] = \frac{A'}{N^2}$$

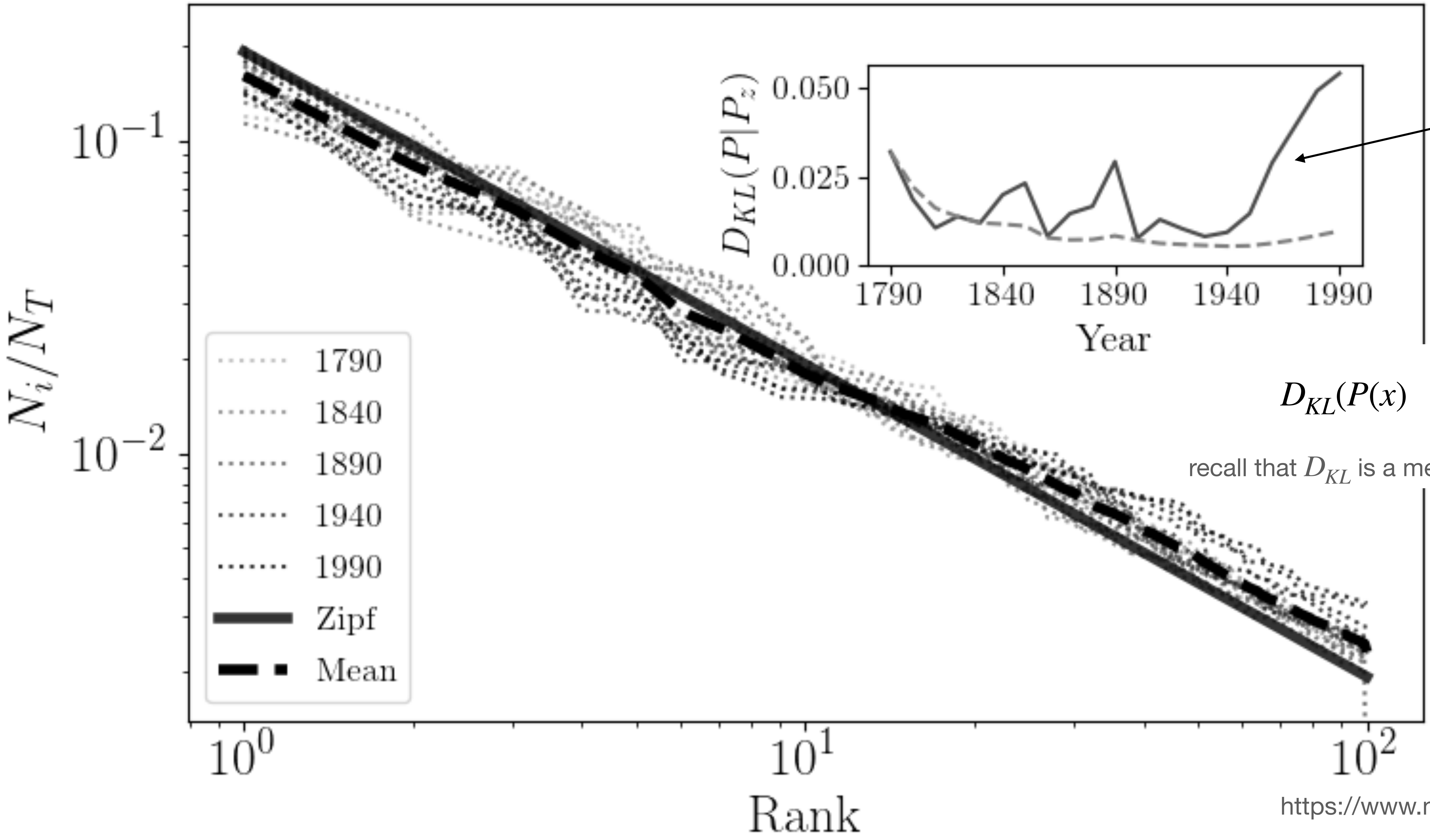
Zipf's Law !!

For this to be the solution we need that:

$$J_x = \frac{d}{dx} \sigma_\epsilon^2 x^2 P[x, t \;\; x_0, t_0] = 0.$$

which imposes boundary conditions for large and small cities

US Urban System Since 1790 (top 100 cities)



How close is it to Zipf's law?

measured as information

$$D_{KL}(P(x) \quad P_{zipf}(x)) = \sum_i P(x_i)\log\frac{P(x_i)}{P_{zipf}(x_i)}$$

recall that D_{KL} is a measure of distance between 2 distributions

https://www.nature.com/articles/s41467-020-18205-1

US City Size distribution has been deviating from Zipf's law since 1940

People prefer to go to the West and Southwest, Texas

Summary

The “laws” of Geography are contained in Demography

in the dynamics of births, deaths and migration for cities

But they require special conditions

and time to emerge

Demographic “Equilibrium”

They are NOT automatic or exact

we do see how decision made by individuals
are interconnected to create the structure of the urban system