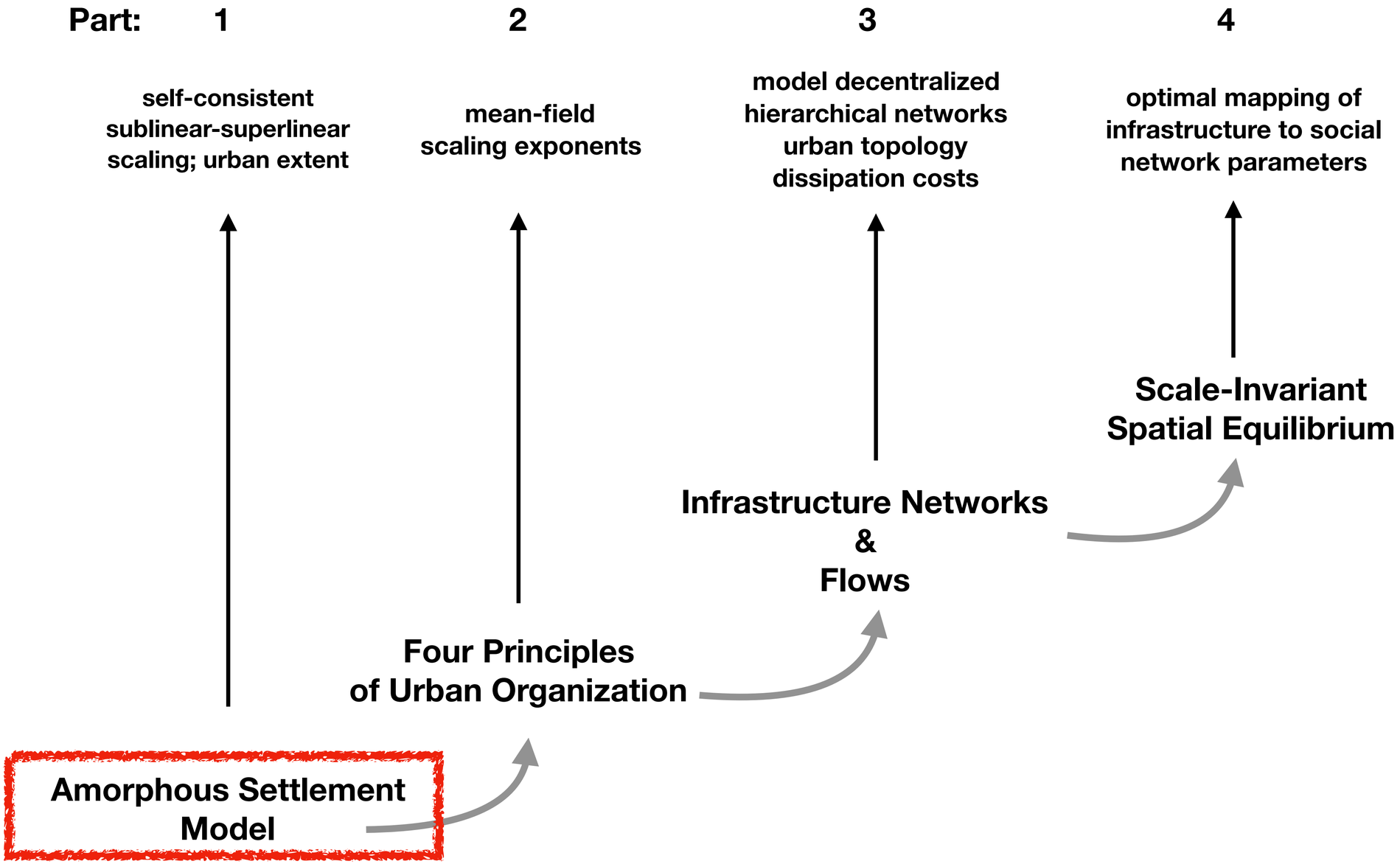
Lecture 6

Network Models of Cities

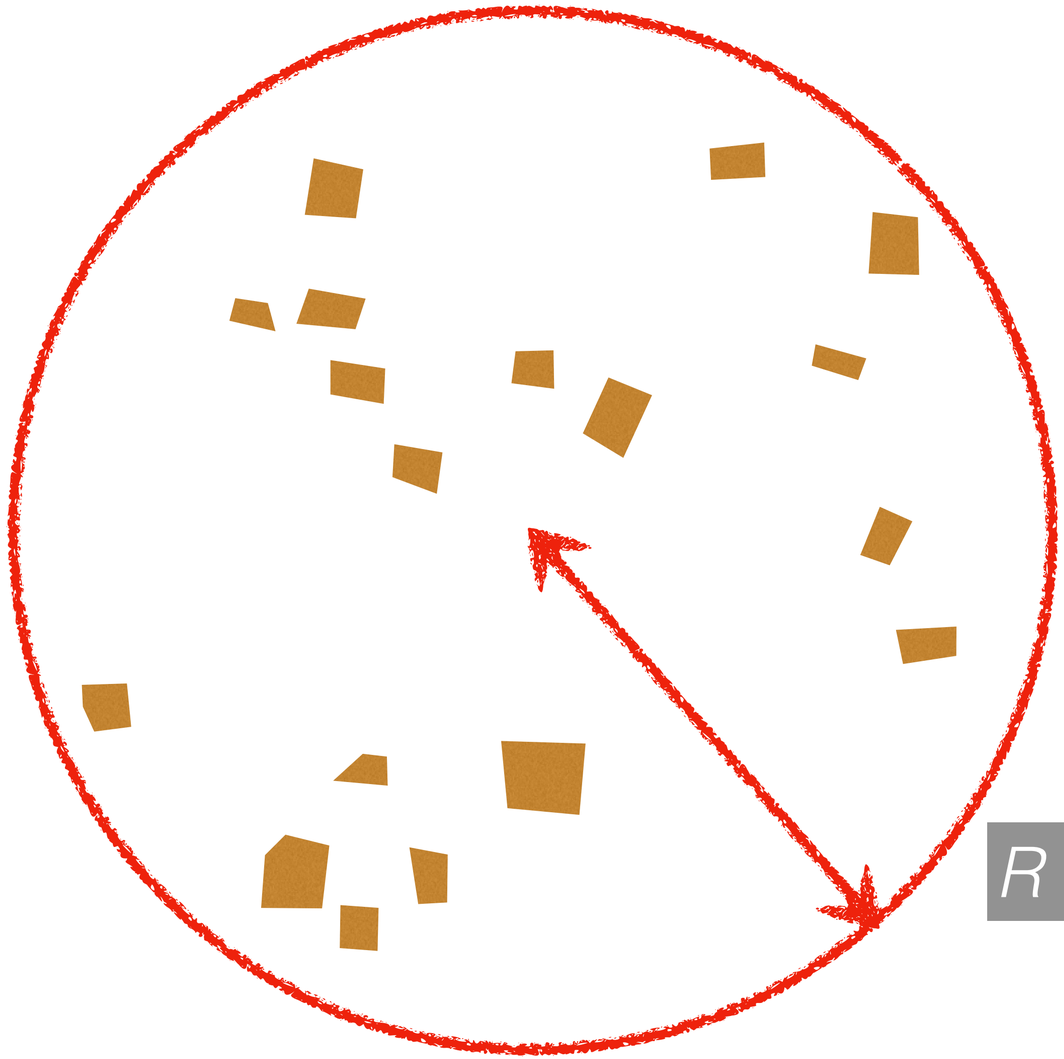
6.2 A First Model: The Amorphous Settlement Model and Urban Scaling

IUS 3.2.2

Urban Scaling Theory

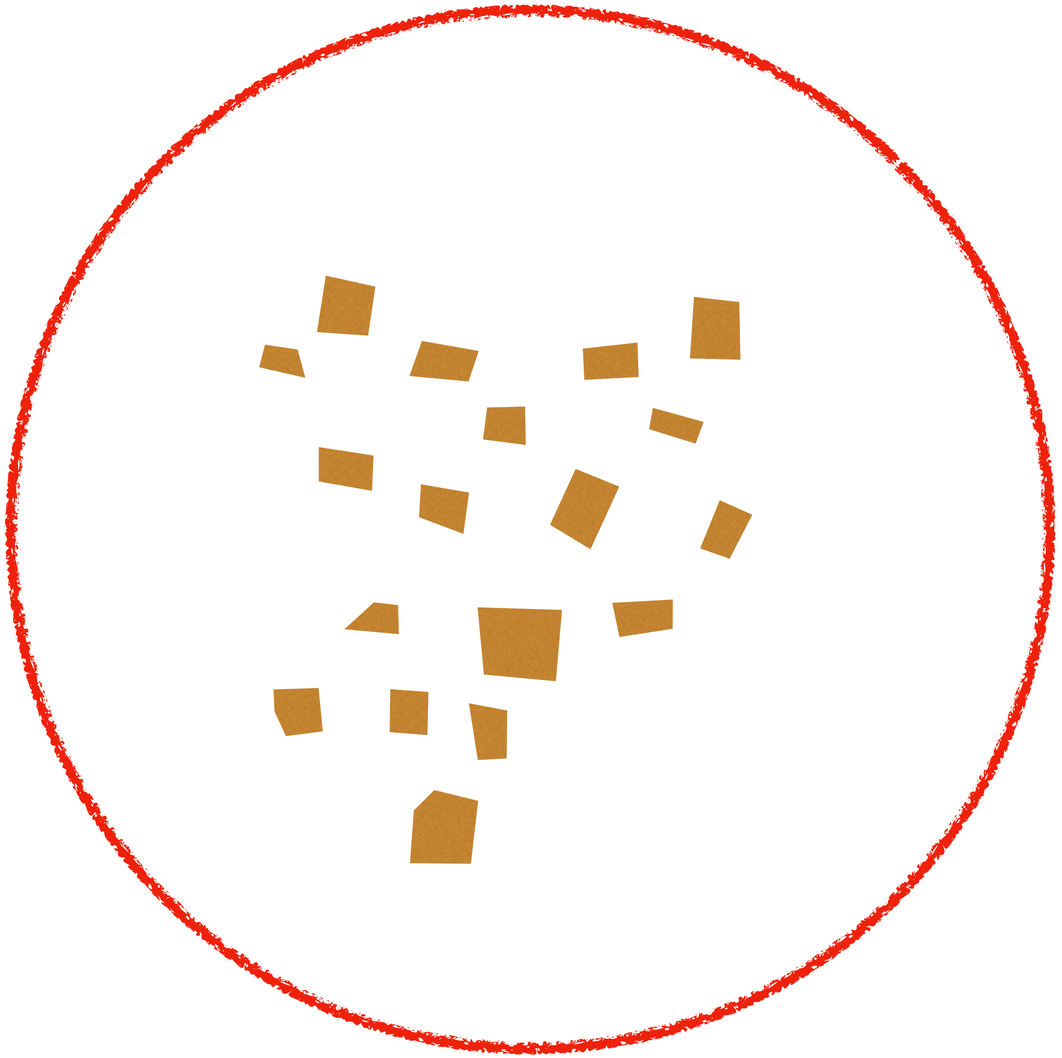


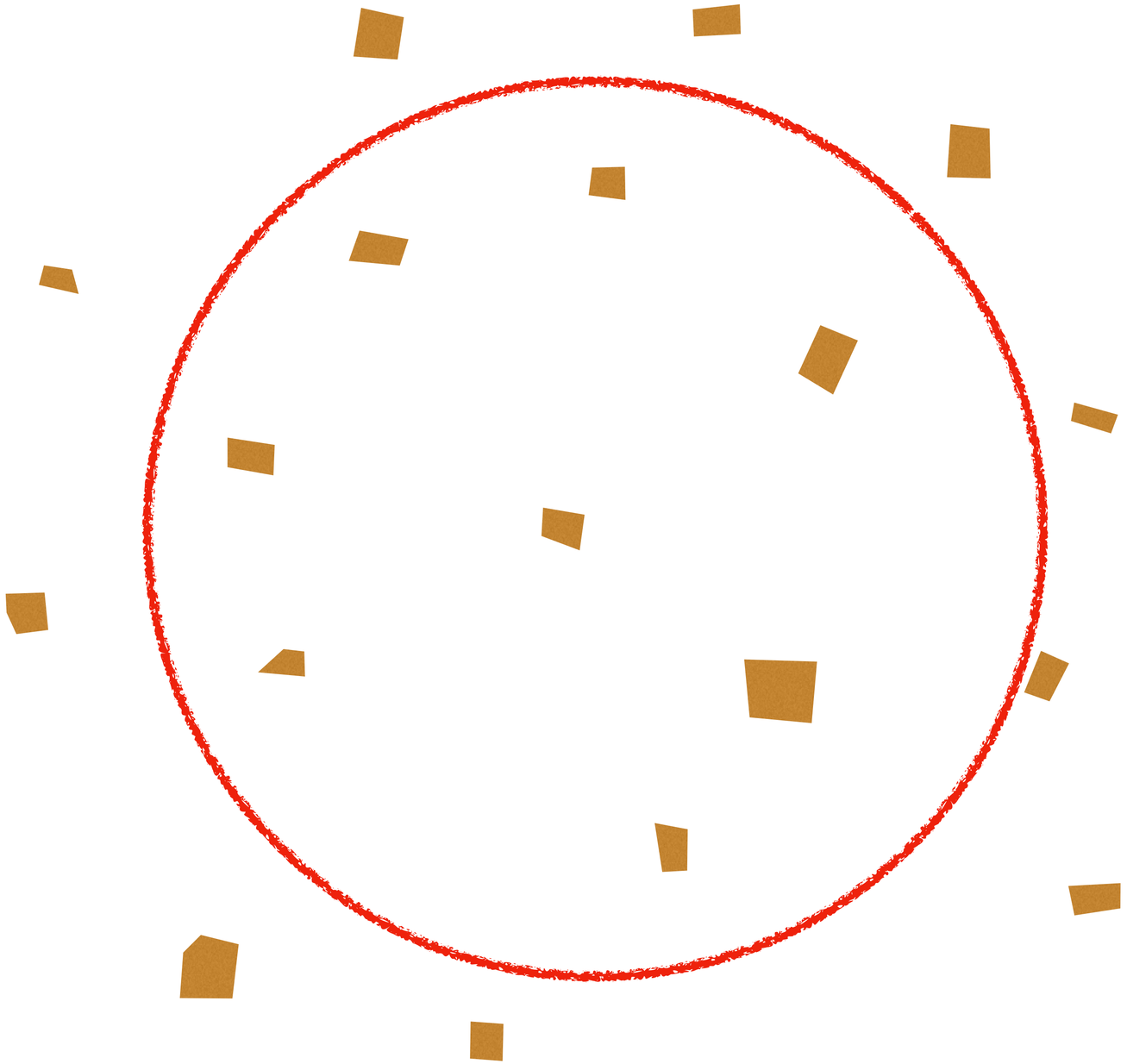
IUS Figure 3.5

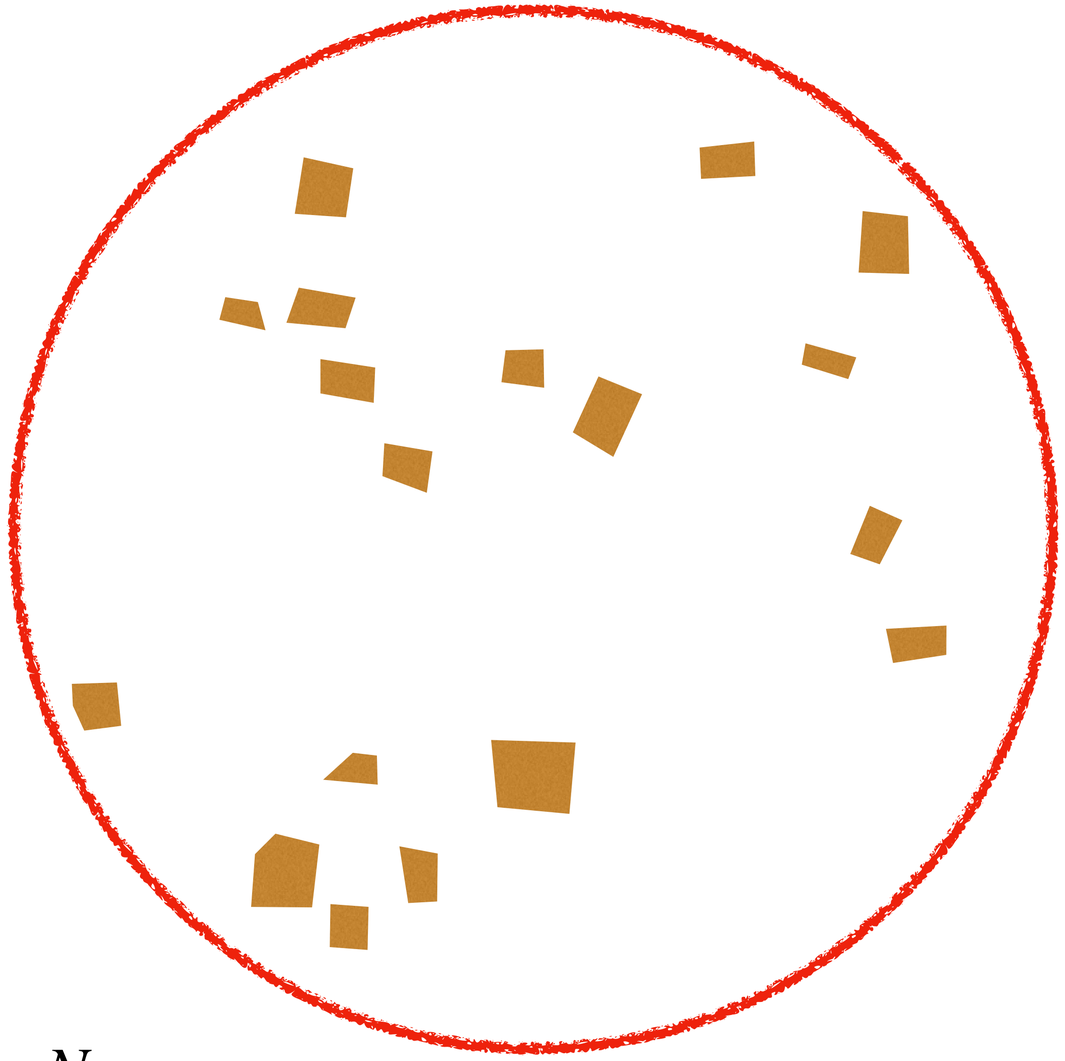


R = Radius

Capilco
Aztec Rural Village







$$A^{3/2} = \frac{\sqrt{\pi}G}{c_{T_0}}N$$

$$G\frac{N}{A} = c_{T_0}(\frac{A}{\pi})^{1/2}$$

For Circle:

$$A = \pi R^2$$

$$G\frac{N}{A} = \textbf{benefit} \sim \textbf{cost} = c_{T_0}R = \frac{c_{T_0}}{\sqrt{\pi}}A^{\frac{1}{2}}$$

social interactions **movement**

This gets us two good things:

"City" Area: $$A(N) = \left(\frac{\sqrt{\pi}G}{c_{T_0}}\right)^{\frac{2}{3}} N^{\frac{2}{3}}$$ sublinear $$1 - \delta = \frac{2}{3}$$

Total Socioeconomic Outputs: $$Y(N) = G\frac{N^2}{A} = \left(\frac{G^{\frac{1}{2}}c_{T_0}}{\sqrt{\pi}}\right)^{\frac{2}{3}} N^{\frac{4}{3}}$$ superlinear $$1 + \delta = \frac{4}{3}$$

Note also different G, c_{T_0} dependences

quality of interactions

transportation costs

Results in: $$\delta = \frac{1}{3}$$ too big !

c. f. income in econ models

To get closer to the right answer need:

A better model of social interactions over built space

To understand the general characteristics of urban spaces

To better compute costs of transportation and land rents (better than in economics)

To understand fundamental constraints on human interactions