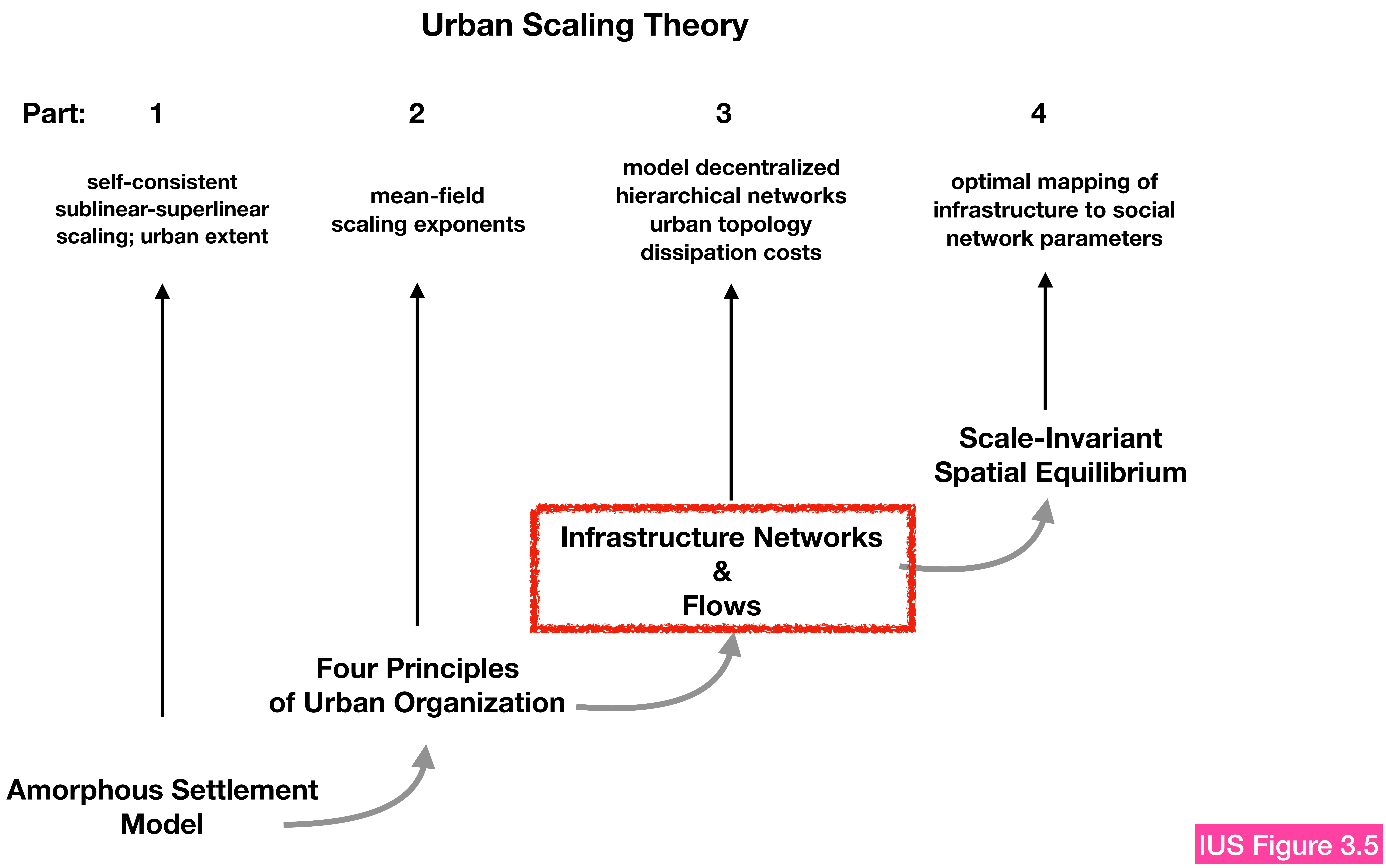
Lecture 7

Network Models of Cities

7.3 Urban Infrastructure, Energy Costs of Movement, Spatial Equilibrium

IUS 3.2.2 pp 82-89


Urban Scaling Theory
Part: 1 2 3 4
self-consistent sublinear-superlinear scaling; urban extent
mean-field scaling exponents
model decentralized hierarchical networks urban topology dissipation costs
optimal mapping of infrastructure to social network parameters
Scale-Invariant Spatial Equilibrium
Infrastructure Networks & Flows
Four Principles of Urban Organization
Amorphous Settlement Model
IUS Figure 3.5

To get closer to the right answer need:

To understand fundamental constraints on human interactions

To understand the general characteristics of urban spaces

A better model of social interactions over built space

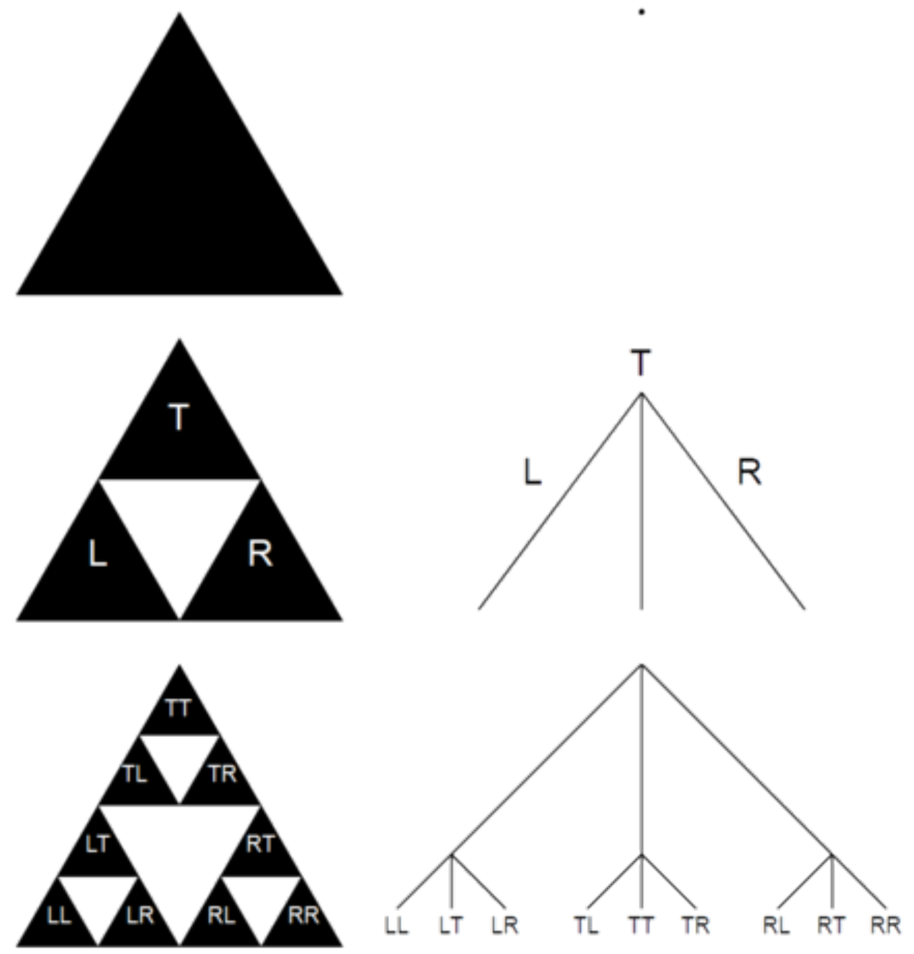
To better compute costs of transportation and land rents

The Scale-Independence of City Size

in the presence of increasing returns and transportation costs

1. Detailed Model of **Urban Infrastructure**
2. General Model of **Cost of Transportation in Cities**
3. The Properties of **Scale-Independent Equilibrium**

Infrastructure Networks in the City have a Hierarchy



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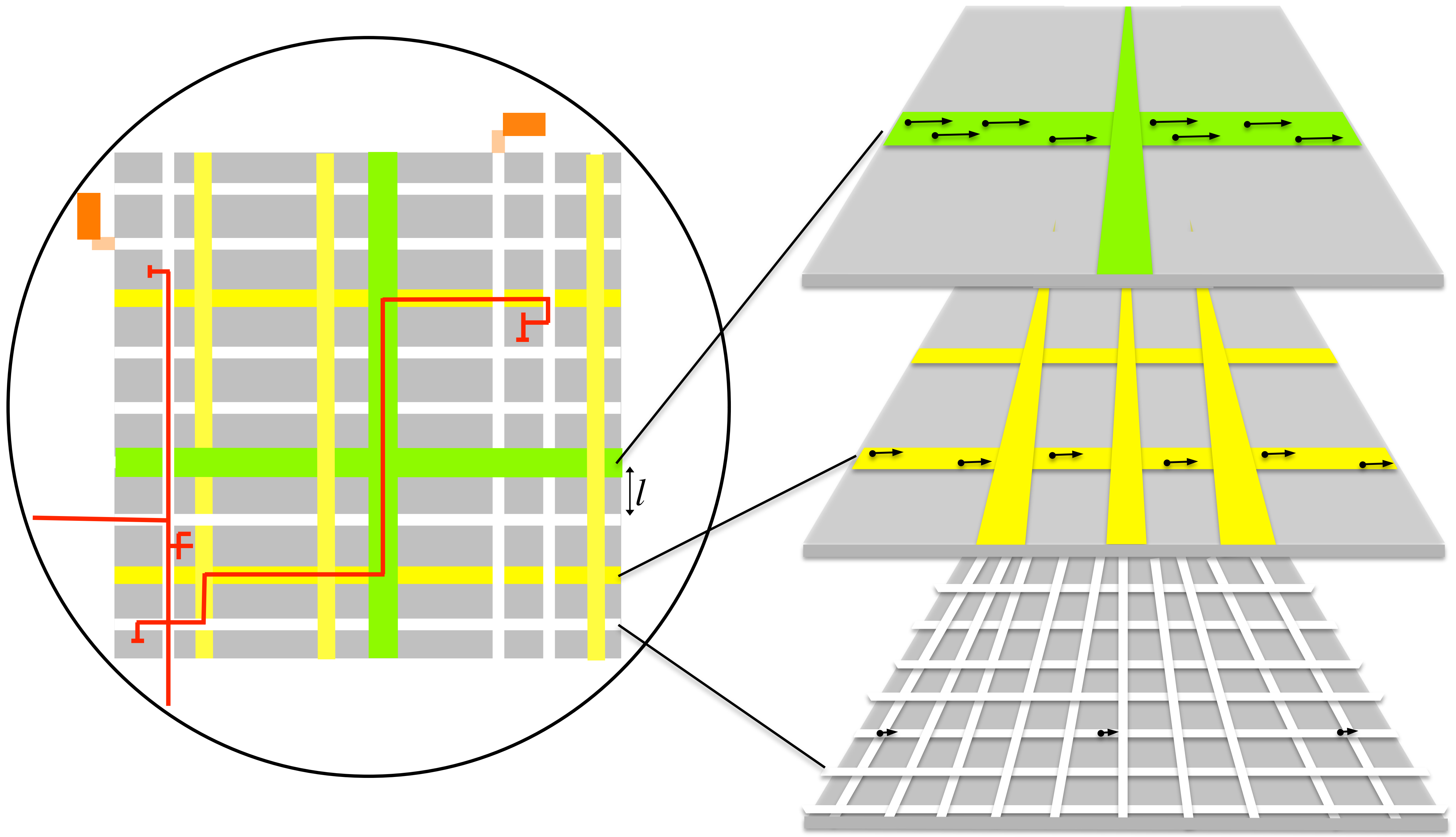


$i = 0$ **Highways: wider + faster**

infrastructure hierarchy levels

$N_i = b^i$ **Main Roads**

units of infrastructure at level i

$i = h$ $\quad h = \dfrac{\ln N}{\ln b}$ **Local Roads**

$N = b^h$

$$s_i = s_* b^{(1-\delta)(h-i)}$$ width segments

$$s_0 = s_* b^{(1-\delta)h} \gg s_h = s_*$$

width highways — keeps increasing with city size (and individual flows)

width doorways — same everywhere

$$a_i = ab^{(\alpha-1)i}$$ land area segments

$$a_h = ab^{(\alpha-1)h} = aN^{\alpha-1}$$

land area per person

$$l_i = \frac{a_i}{l}$$ infrastructure length segments

$$l_h = \frac{a}{l} N^{\alpha-1}$$

minimal distance between people

We will need this math trick to sum over levels of hierarchies:

Sum of geometric series:

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \sum_{k=0}^{n-1} ar^k = a\left(\frac{1-r^n}{1-r}\right),$$

hierarchical networks (exponentials)

number of nodes at level 0 (root)

$N_0 = 2^0 = 1$

branching ratio: b=2

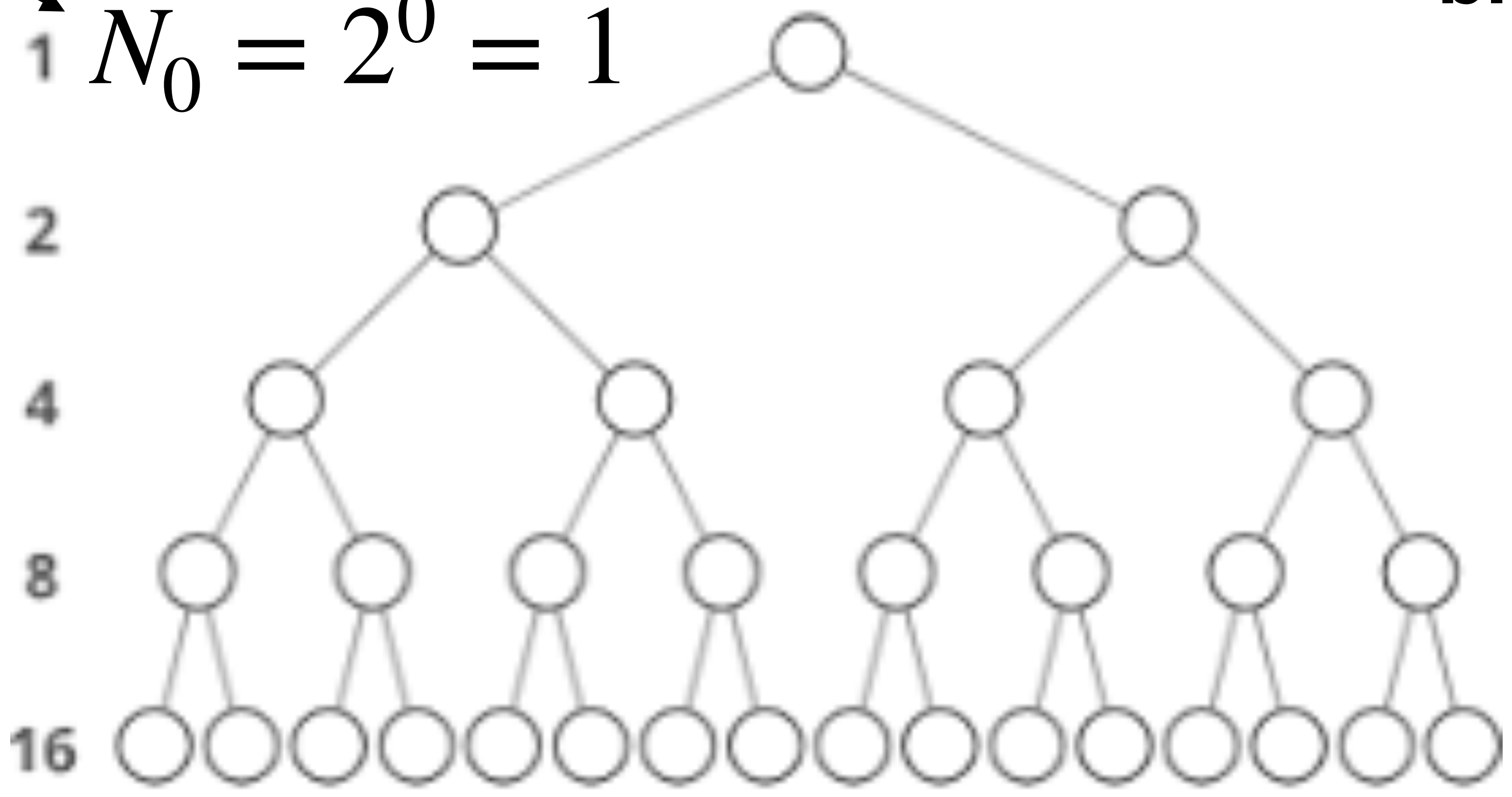
i=0

i=1

i=2

i=3

i=h=4



This is a form of self-similarity (~fractals)

$N_h = b^h = 2^4 = 16$

number of nodes at level h (leaves)

Sum of geometric series

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \sum_{k=0}^{n-1} ar^k = a\left(\frac{1-r^n}{1-r}\right),$$

Total Length and Area of Infrastructure Networks

$$L_n = \sum_{i=0}^{h} l_i N_i = \frac{a}{l}\sum_{i=0}^{h} b^{\alpha i} = \frac{a}{l}\frac{b^{\alpha(h+1)}-1}{b^{\alpha}-1} \simeq L_0 N^{\alpha}, \quad L_0 = a/l,$$

$$A_n = \sum_{i=0}^{h} s_i l_i N_i = s_* \frac{a}{l} b^{(1-\delta)h} \sum_{i=0}^{h} b^{(\alpha+\delta-1)i} \simeq A_0 N^{1-\delta}, \quad A_0 = \frac{s_* a}{l(1-b^{\alpha+\delta-1})},$$

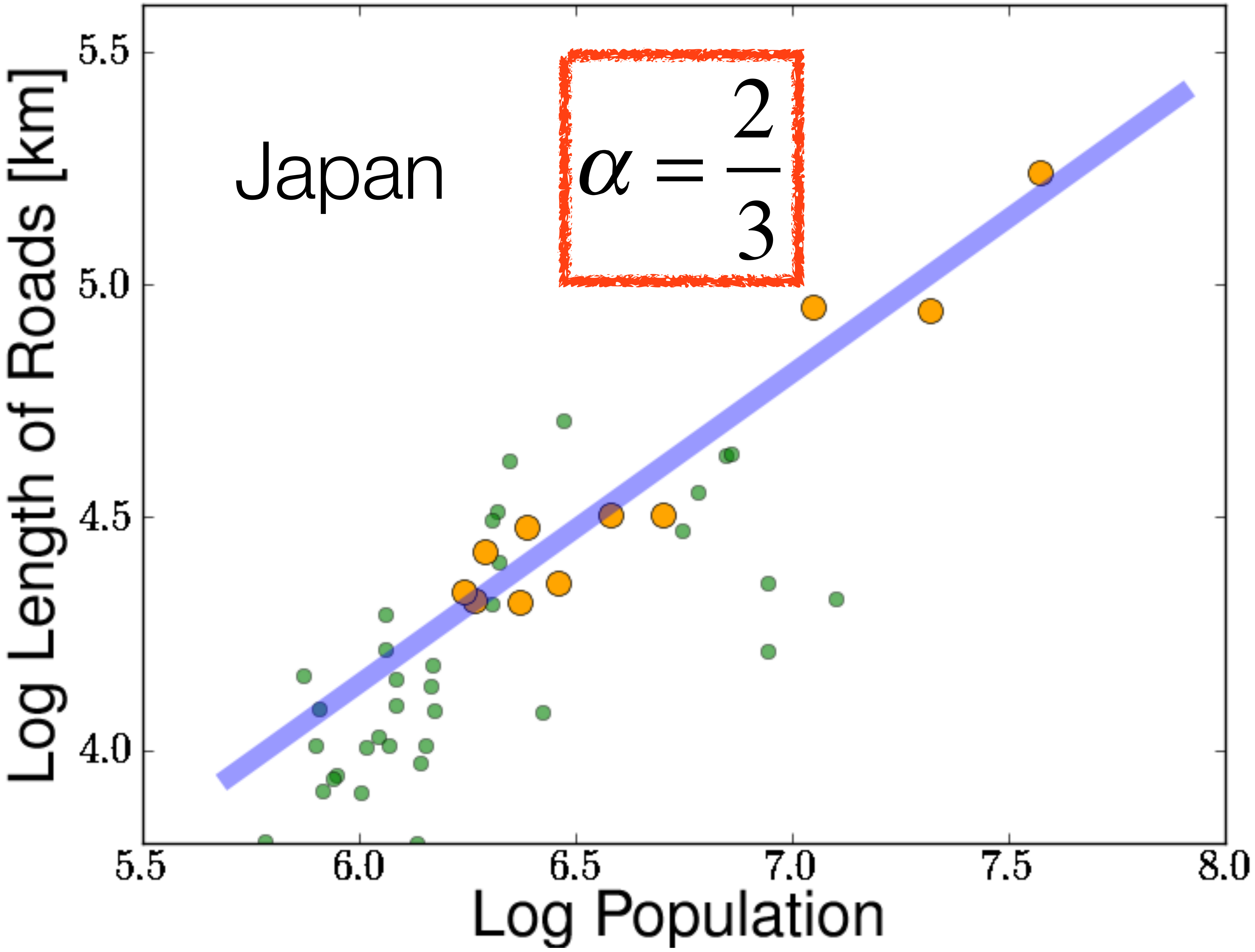
Length is area filling
L ~ A

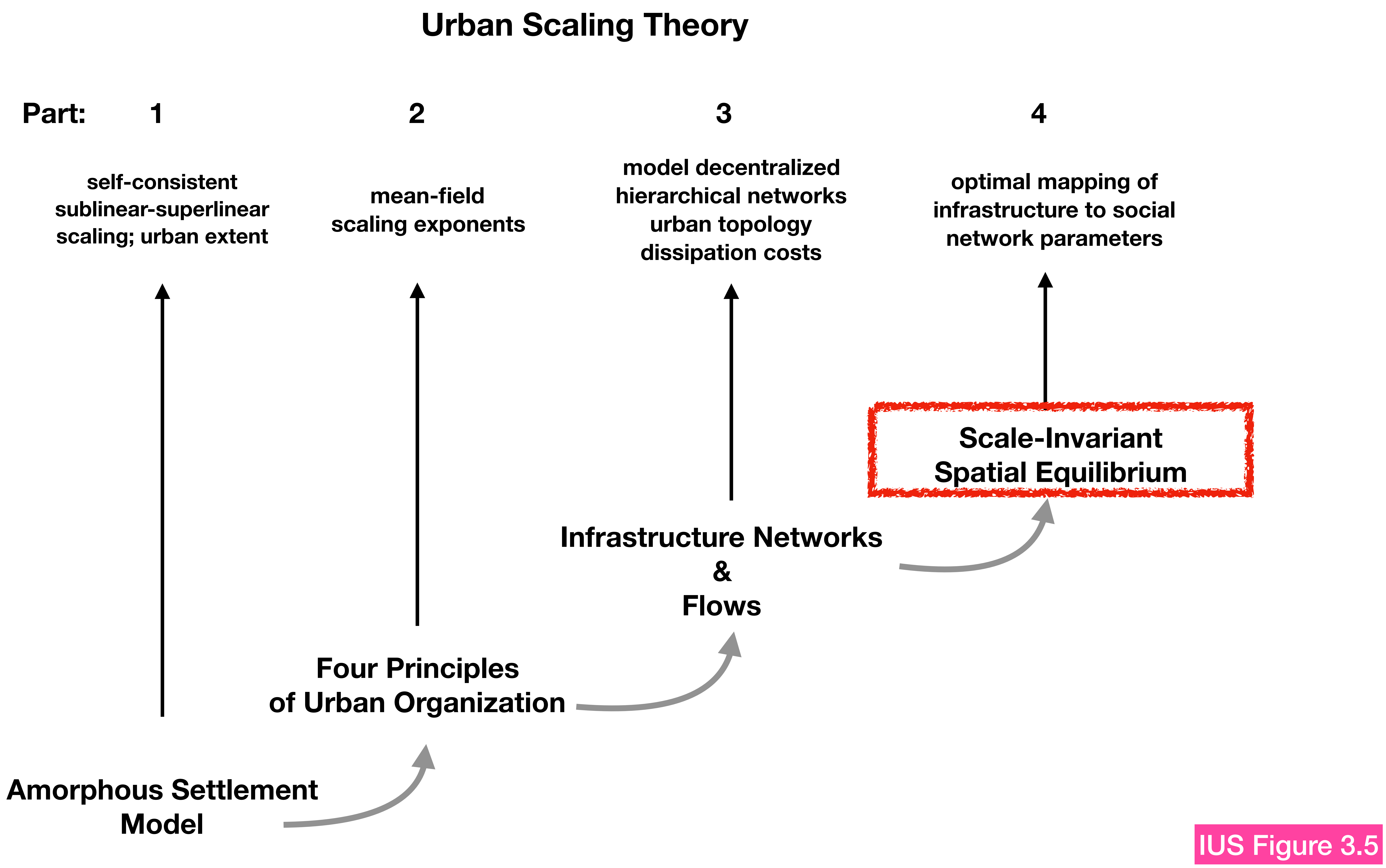
width of network

This gives us back our empirical observations with

$$\alpha = \frac{2}{3}; \quad \delta = \frac{1}{6}$$

but now also gives us a theory of the entire infrastructure of cities


Japan
$\boxed{\alpha = \frac{2}{3}}$
Log Length of Roads [km]
Log Population
5.5
5.0
4.5
4.0
5.5
6.0
6.5
7.0
7.5
8.0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The Cost of Socializing in the City

Conservation of Current across infrastructural levels

$$J_i = s_i \rho_i v_i N_i = s_{i-1} \rho_{i-1} v_{i-1} N_{i-1} = J_{i-1}$$

$$\rho_0 v_0 \gg \rho_h v_h$$

highways faster and more densely packed

doorways the same everywhere

$$\rho_i v_i = b^{\delta(h-i)} \rho_* v_*$$

flow per unit area

$$J_i = J = J_0 N, \text{ with } J_0 = s_* \rho_* v_*$$

Resistance accounts for Cost of Movement:

$$r_i = r \frac{l_i}{s_i} \qquad R_i = \frac{r_i}{N_i} = \frac{ar}{l s_*} b^{-(1-\alpha+\delta)i-(1-\delta)h}$$

Parallel resistance because flow can take alternate routes (decentralized networks)

$$\boxed{W = J^2 \sum_{i=1}^{h} R_i = J^2 \frac{ar}{l s_*} b^{-(1-\delta)h} \frac{1-b^{-(1-\alpha+\delta)(h+1)}}{1-b^{-1+\alpha-\delta}} \simeq W_0 N^{1+\delta}, \quad W_0 = \frac{arJ_0^2}{l s_* (1-b^{-1+\alpha-\delta})},}$$

Cost of Transportation scales super linearly
like Social Benefits -> Spatial Equilibrium independent of City population size

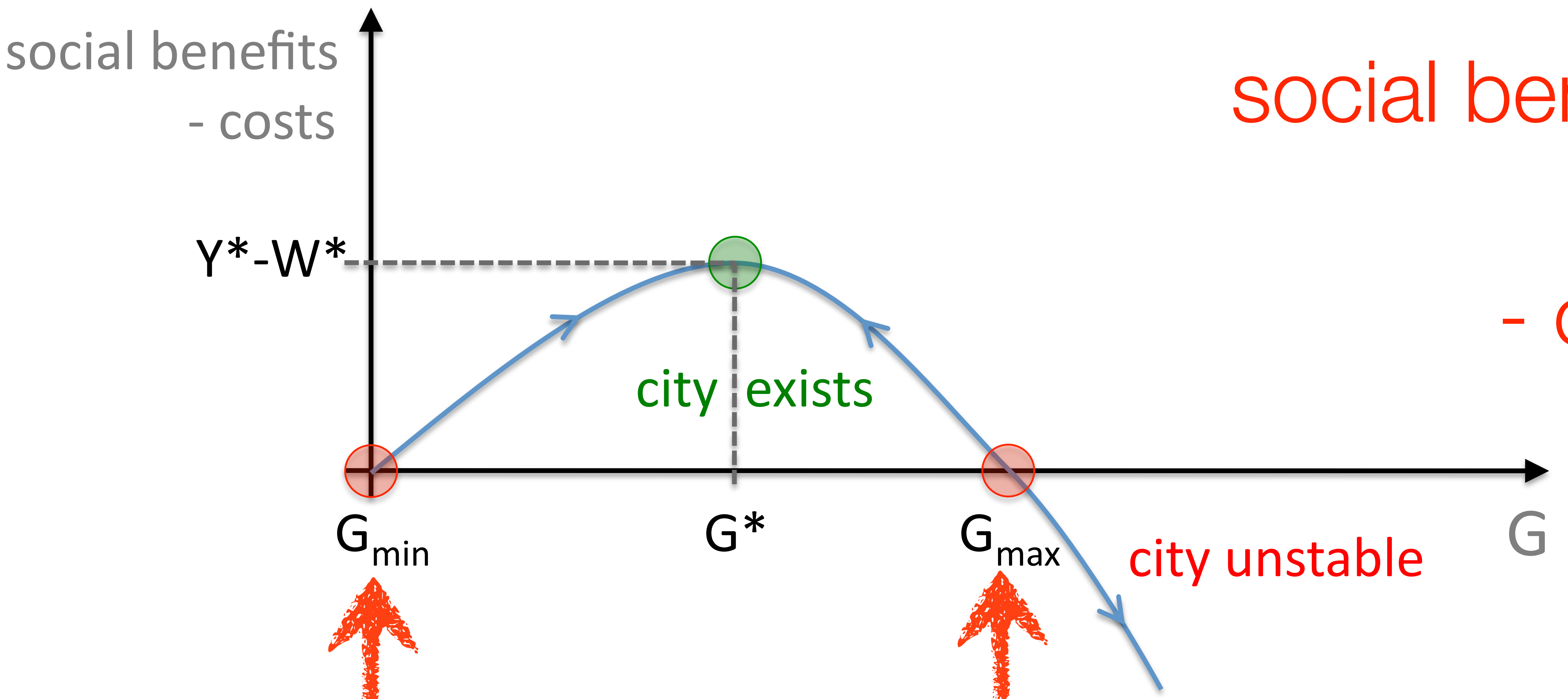
New (scale-invariant) Spatial Equilibrium

Spatial equilibrium between networked social benefits and costs

Scale invariance: cities can exist at “any” population size

Different from Urban Economics: V Henderson

Real “Income”: $Y(N) - W(N)$



social benefits: $Y(N) \sim G \dfrac{N^2}{A_n(N)} \sim N^{7/6}$

- costs: $W(N) \sim W_0 \dfrac{I^2}{A_n(N)} \sim N^{7/6}$

$$\boxed{Y_0 \sim G^{1-\alpha} \text{ and } W_0 \sim G^{\alpha}}$$

calculation in IUS pp 88-89

Congestion/Danger

High Costs/Dispersion

~Lagos

~”Florida”

Poor, congested, intense, dangerous

Diffuse, sprawling, uneventful

Human Effort is conserved: **Estimating G**

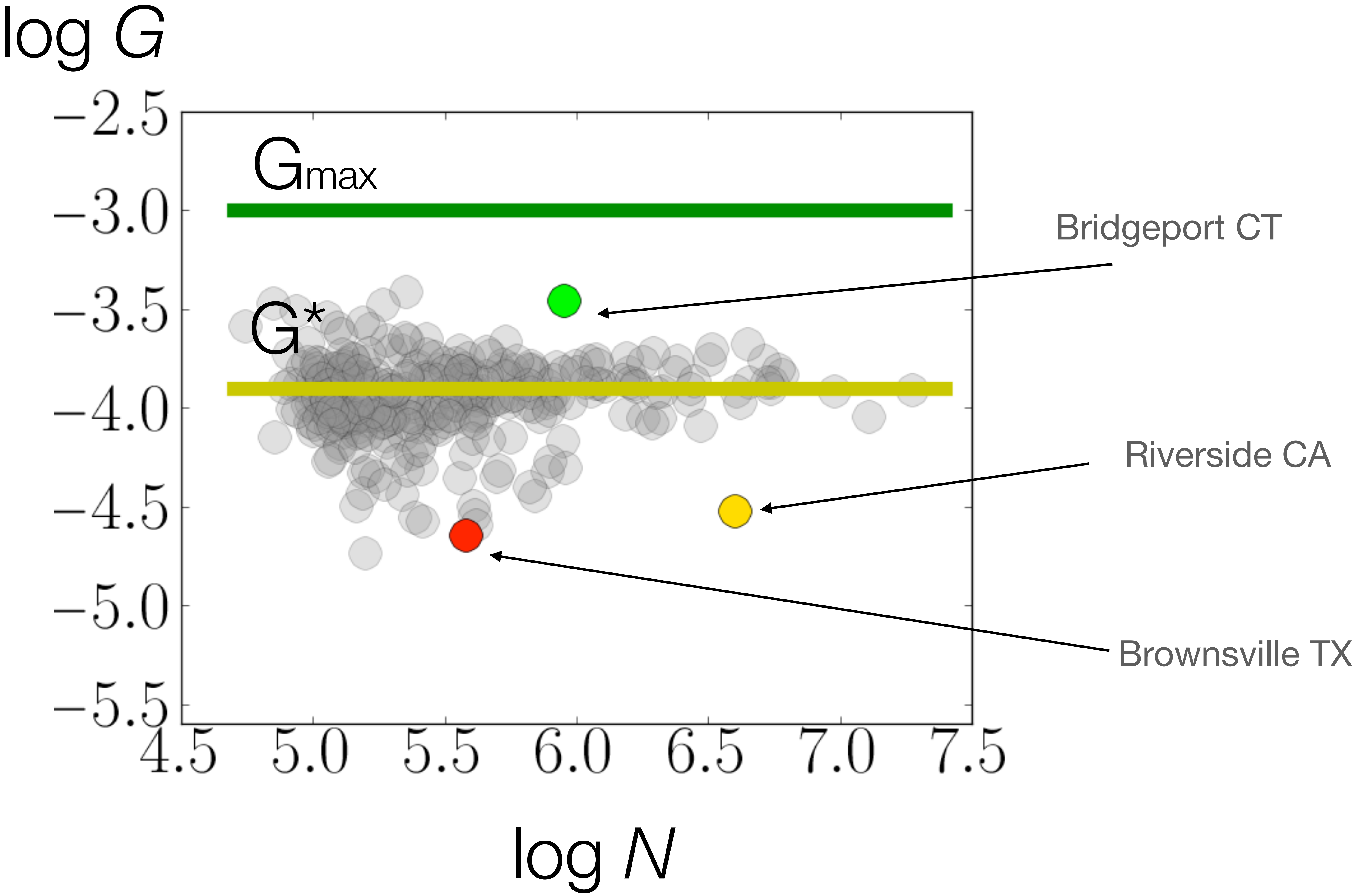
The parameter G measures the strength of social interactions over a life path.

For a city it is:

$$G = \left(\frac{Y}{N}\right)\left(\frac{A_n}{N}\right) = a_0 \ell \bar{g}$$

average income x average area per capita

Total "freedom" = social x spatial freedom



Tradeoff between more Social Space versus more Physical Space

larger cities

smaller towns

Many quantitative predictions + general consequences

Urban scaling relation	Exponent prediction $D = 2, H_m = 1$	Exponent prediction General D, H_m
Land area $A = aN^{\alpha}$	$\alpha = 2/3$	$\alpha = \frac{D}{D + H_m}$
Network volume $A_n = A_0 N^{\nu}$	$\nu = 5/6$	$\nu = 1 - \delta$
Network Length $L_n = L_0 N^{\lambda}$	$\lambda = 2/3$	$\lambda = \alpha$
Interactions/capita $k = k_0 N^{\delta}$	$\delta = 1/6$	$\delta = \frac{H}{D(D + H_m)}$
Social outputs $Y = Y_0 N^{\beta}$	$\beta = 7/6$	$\beta = 1 + \delta$
Power dissipation $W = W_0 N^{\omega}$	$\omega = 7/6$	$\omega = 1 + \delta$
Land rents ($\$/m^2$) $P_L = P_0 N^{\beta_L}$	$\beta_L = 4/3$	$\beta_L = 1 + 2\,\delta$

Summary of Urban Scaling relations and exponent predictions for various important quantities. Note that agglomeration effects vanish when $H_m \rightarrow 0$ because then people remain spatially separated social networks fail to emerge (we will look at internet quantities later).