Lecture 10

The differences between cities, inequality and generating processes

10.1 Statistical Distributions of Urban Quantities, Inequality, stochastic processes

IUS 4.2

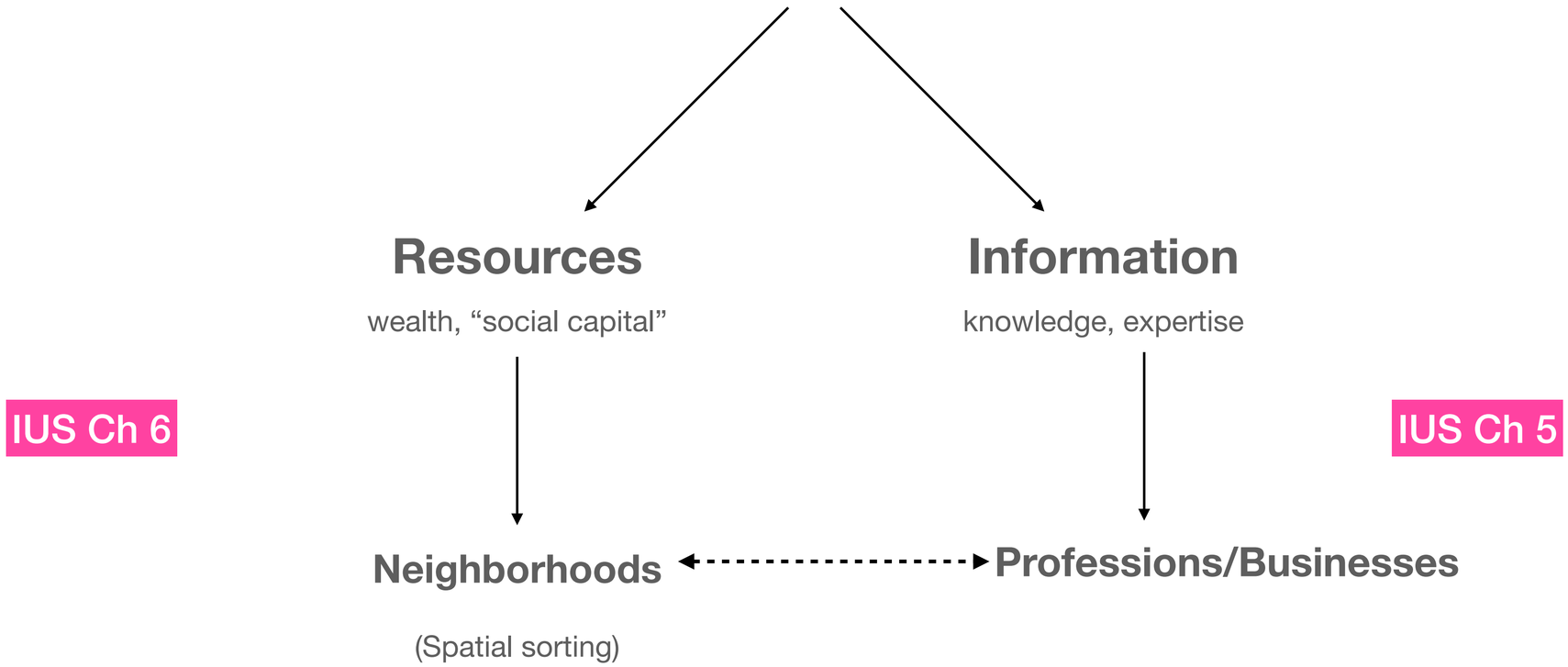
We want to explain why deviations from scaling for each city

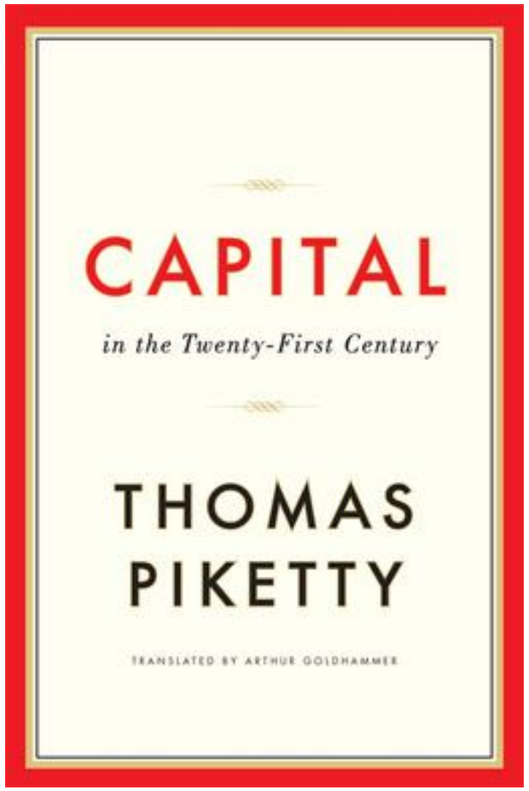
have the observed distribution (lognormal)

This is connected to how they **use resources and information** at the individual and firm levels

and brings in the problems of **growth and inequality**

Inequality and statistics of variation in cities

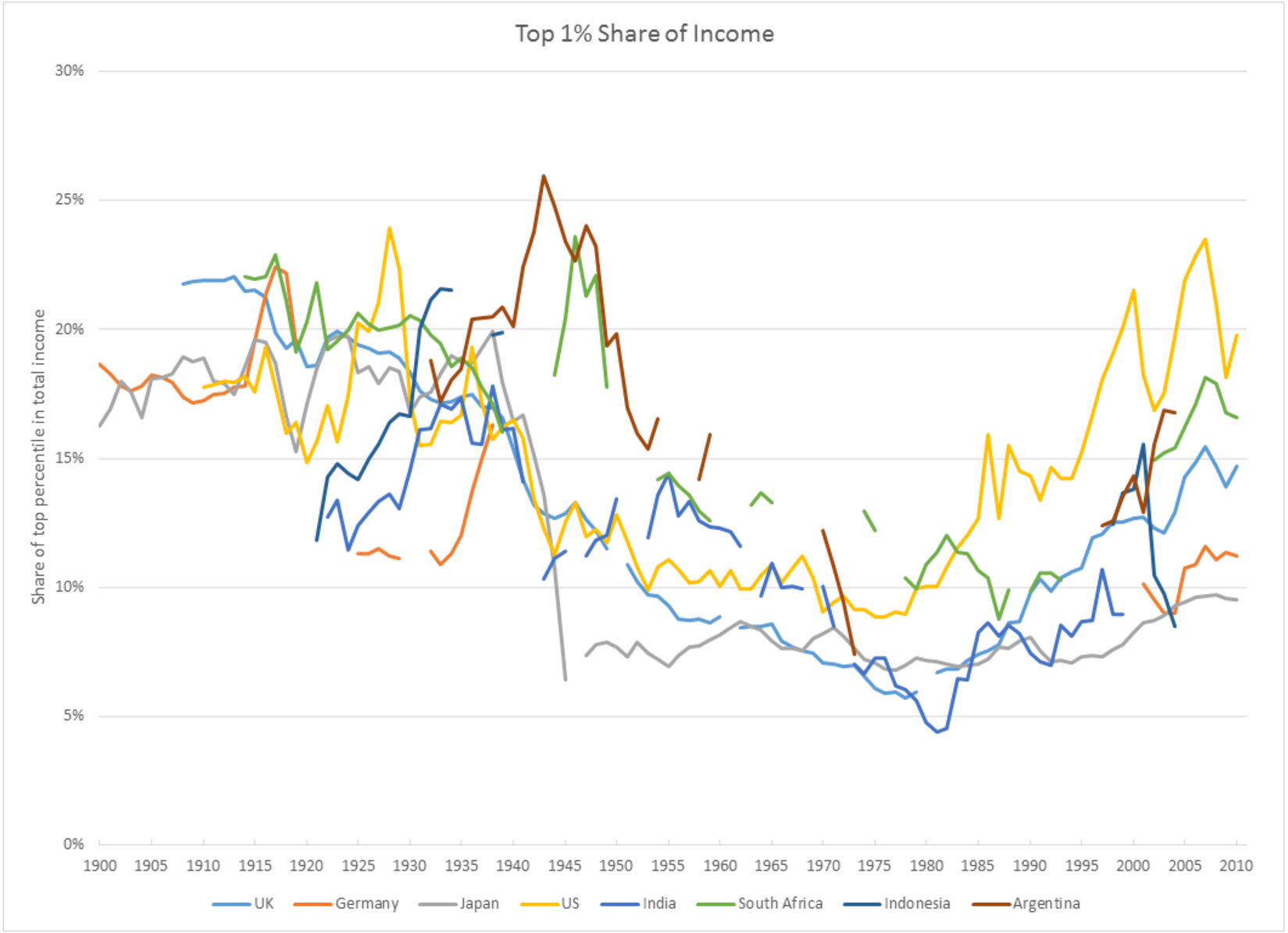




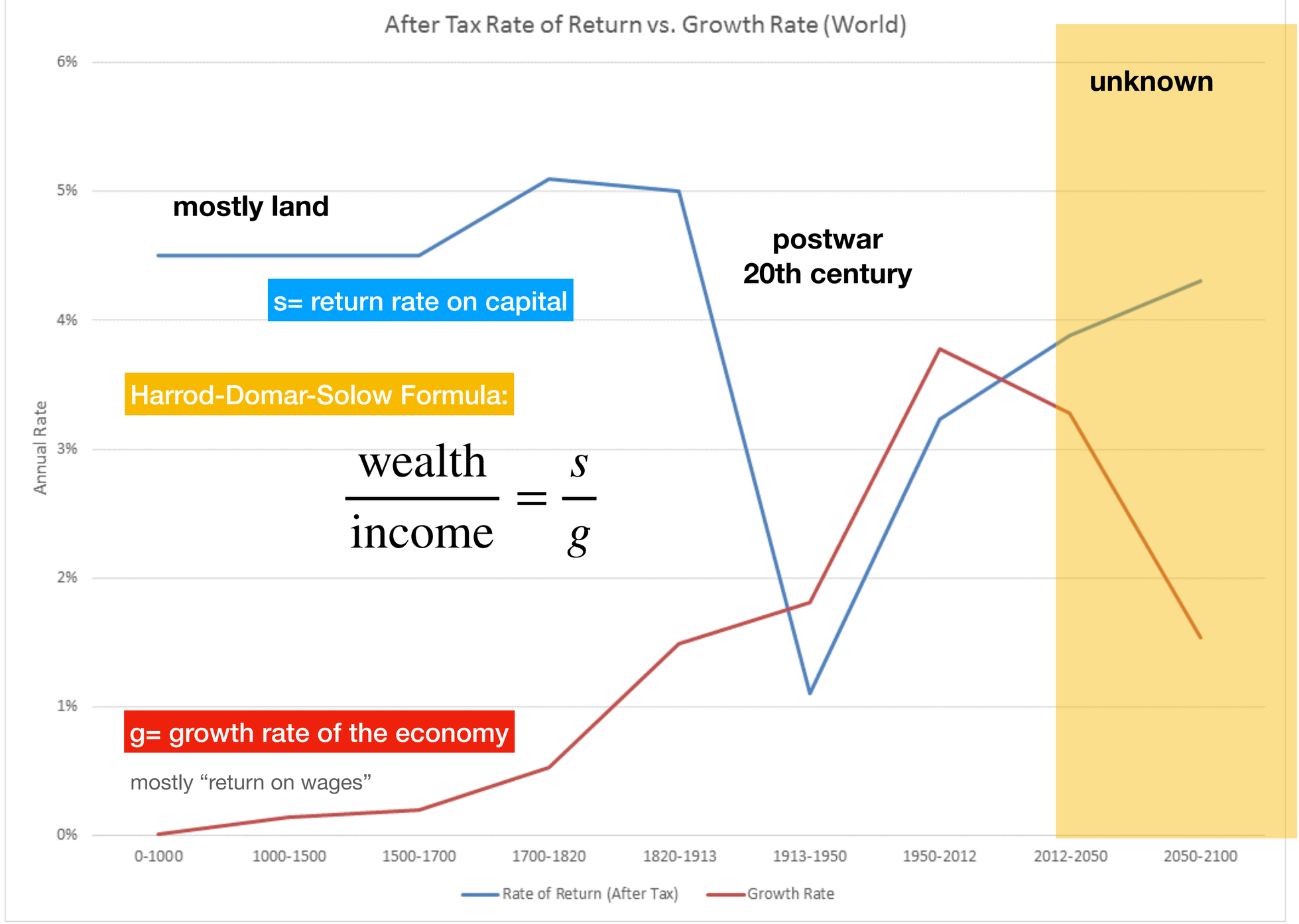
2013

The Classic Approach to Economic Inequality

nations



Thomas Picketty


After Tax Rate of Return vs. Growth Rate (World)
6%
5%
4%
3%
2%
1%
0%
Annual Rate
unknown
mostly land
postwar
20th century
s= return rate on capital
Harrod-Domar-Solow Formula:
$\frac{\text{wealth}}{\text{income}} = \frac{s}{g}$
g= growth rate of the economy
mostly “return on wages”
0-1000
1000-1500
1500-1700
1700-1820
1820-1913
1913-1950
1950-2012
2012-2050
2050-2100
Rate of Return (After Tax)
Growth Rate

But many sources and forms of inequality…

SCIENTIFIC AMERICAN®

POLICY & ETHICS

The American Economy Is Rigged

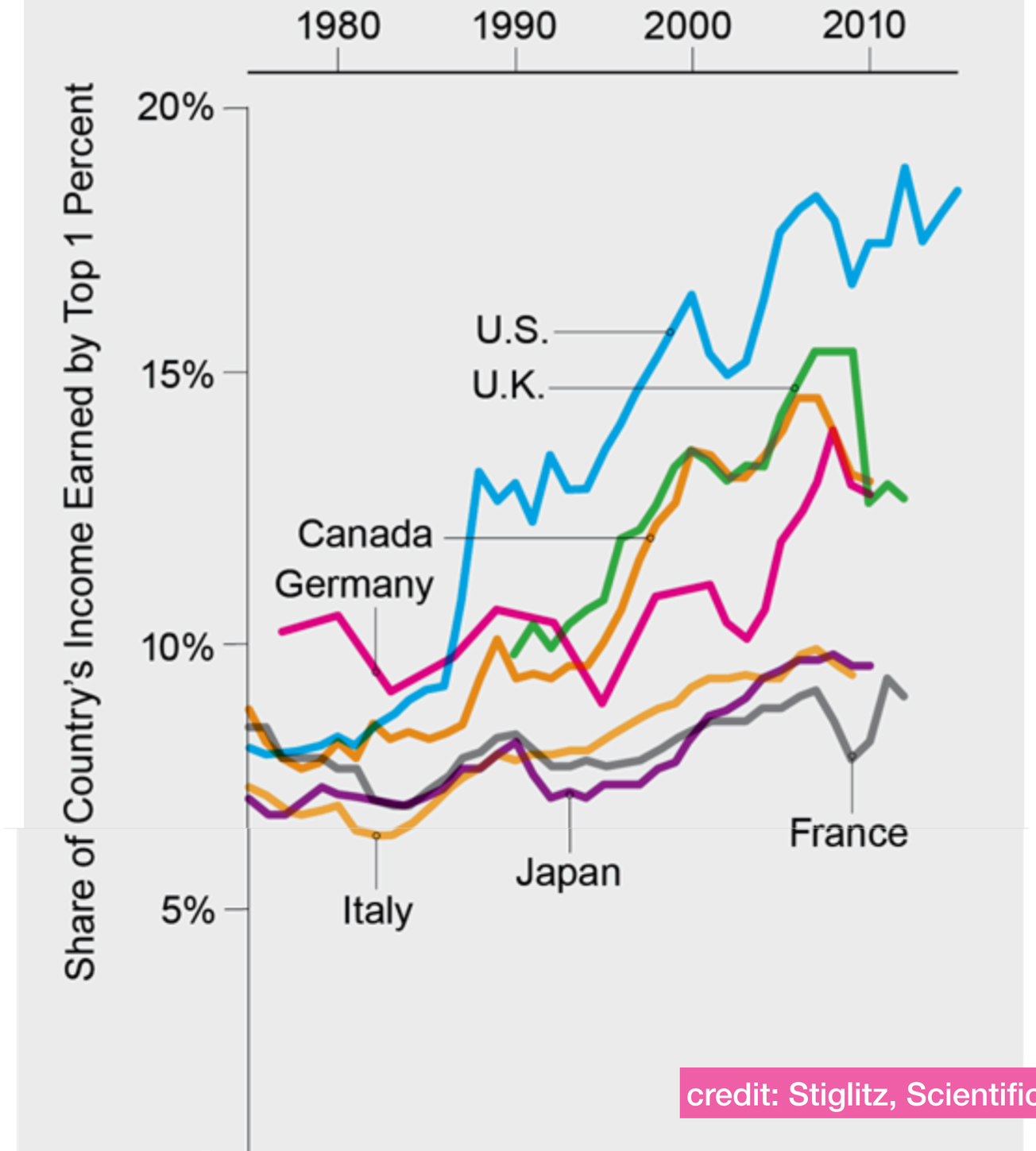
And what we can do about it

By Joseph E. Stiglitz | Scientific American November 2018 Issue

Joseph E. Stiglitz

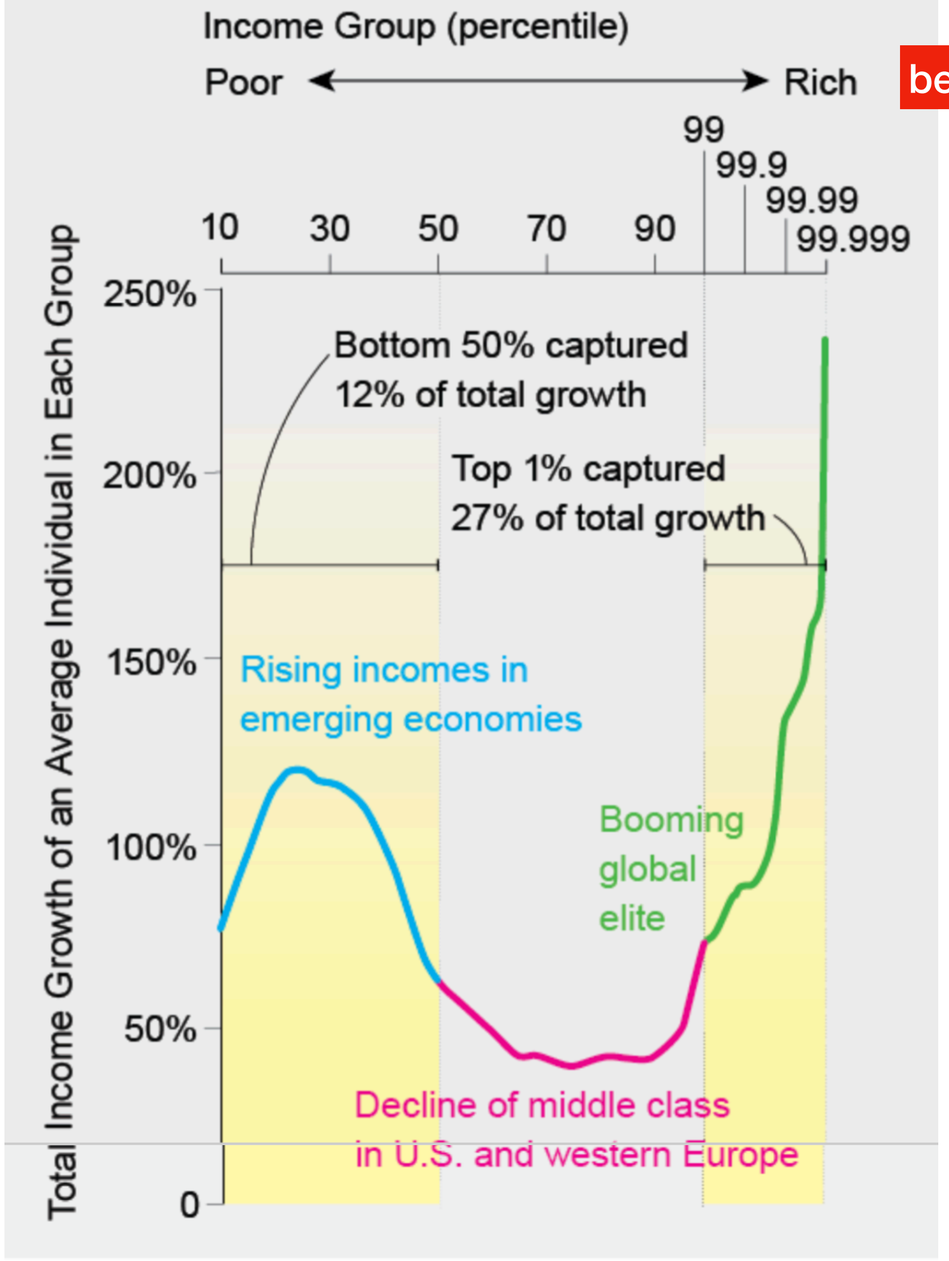
Joseph E. Stiglitz is a University Professor at Columbia University and Chief Economist at the Roosevelt Institute. He received the Nobel prize in economics in 2001. Stiglitz chaired the Council of Economic Advisers from 1995–1997, during the Clinton administration, and served as the chief economist and senior vice president of the World Bank from 1997–2000. He chaired the United Nations commission on reforms of the international financial system in 2008–2009. His latest authored book is *Globalization and Its Discontents Revisited* (2017).

Credit: Nick Higgins



credit: Stiglitz, Scientific American (2018)

between 1980-2016



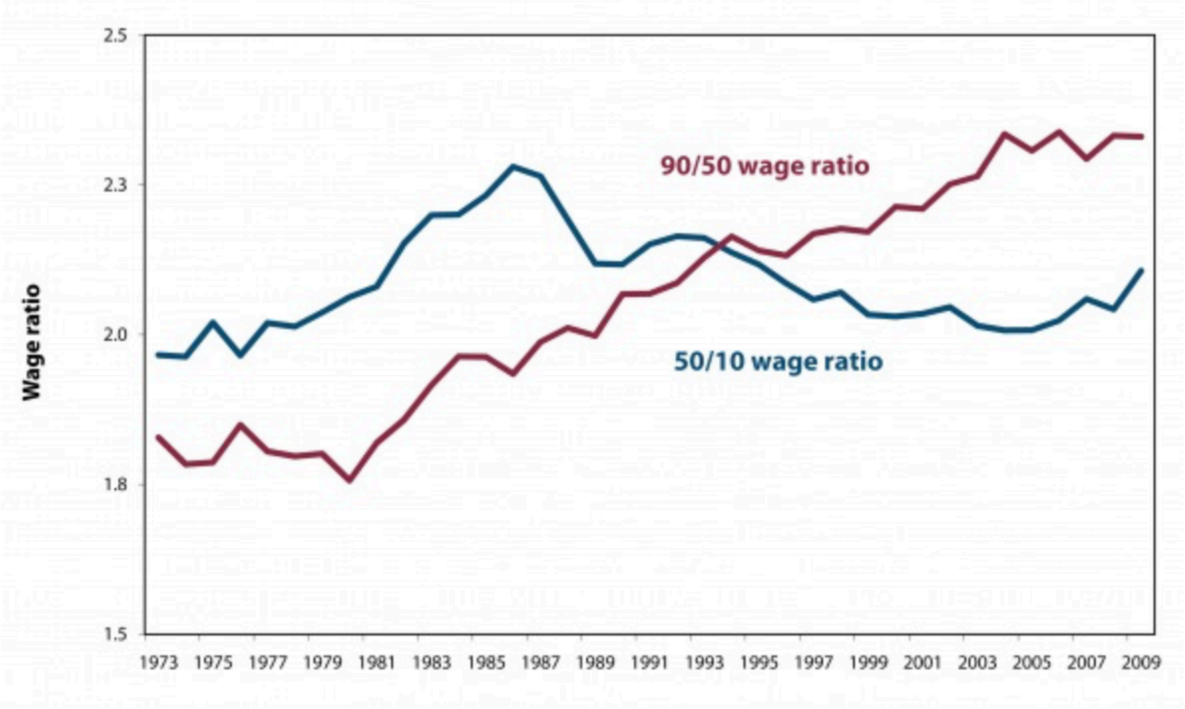
income = wages + capital rents

credit: Stiglitz, Scientific American (2018)

1. Wage Inequality

Over the last 30 years, wage inequality in the United States has increased substantially, with the overall level of inequality now approaching the extreme level that prevailed prior to the Great Depression. This general characterization of the inequality trend oversimplifies, though, the actual pattern of change: The chart below shows that the trend at the top of the income distribution (the "upper tail") is not exactly the same as the trend at the bottom of the distribution (the "lower tail"). "Lower-tail" inequality is measured here by taking the ratio of wages at the middle of the income distribution (i.e., the 50th percentile) to those near the bottom of the distribution (i.e., the 10th percentile); "upper-tail" inequality is measured by taking the ratio of wages near the top of the distribution (i.e., the 90th percentile) to those at the middle of the distribution (i.e., the 50th percentile of workers). We find that lower-tail inequality rose sharply in the 1980s and contracted somewhat thereafter, while upper-tail inequality has increased steadily since 1980.

Men's wage inequality



Nation wide

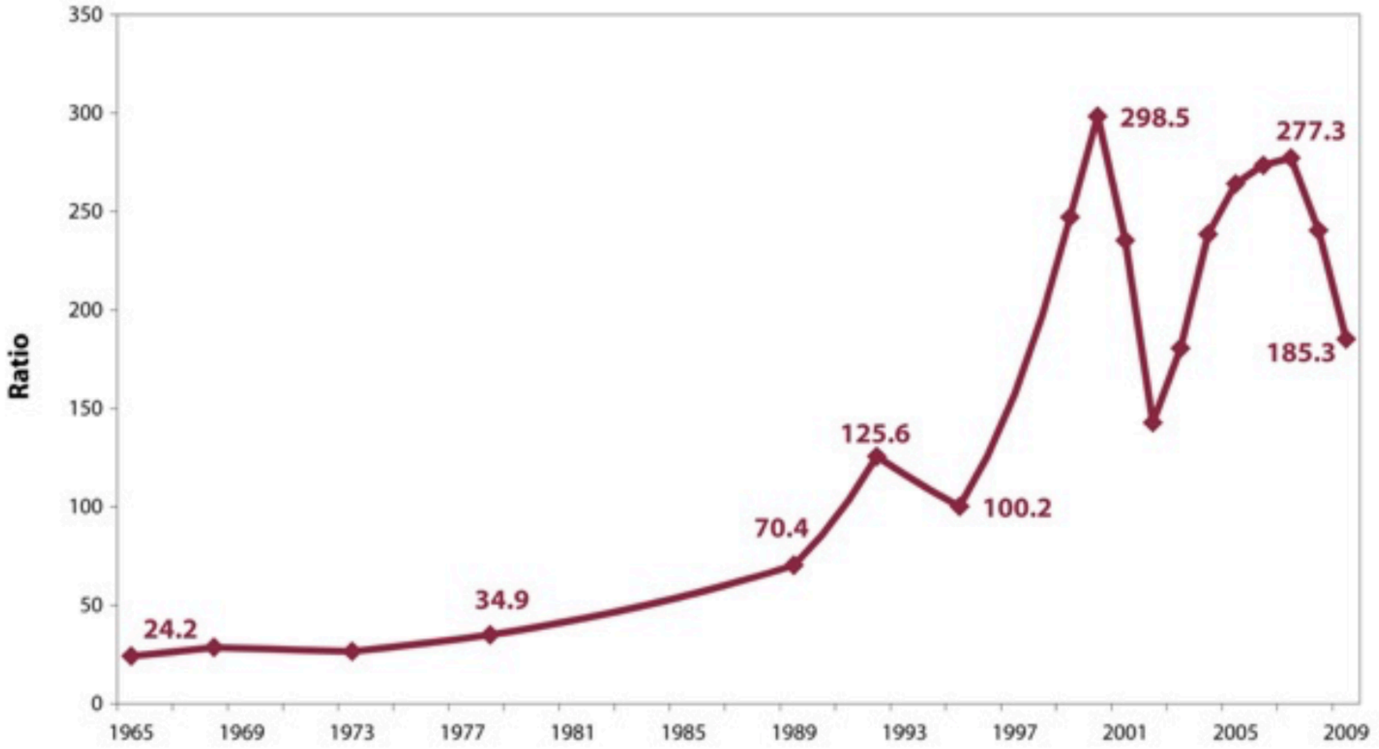
Source: Economic Policy Institute. 2011. "Upper Tail" inequality growing steadily: Men's wage inequality, 1973-2009. Washington, D.C.: Economic Policy Institute. May 11, 2011. <http://www.stateofworkingamerica.org/charts/view/192>.

20 Facts About U.S. Inequality that Everyone Should Know

2. CEO pay

Recent decades have seen a clear increase in the difference between CEO compensation and that of the average worker in manufacturing ("production." CEOs in 1965 made 24 times more than the average production worker, whereas in 2009 they made 185 times more. This cha shows how this ratio between the compensation of CEOs and production workers took off in the 1980s.

U.S. CEO pay in relation to the average production worker's compensation



Within firms

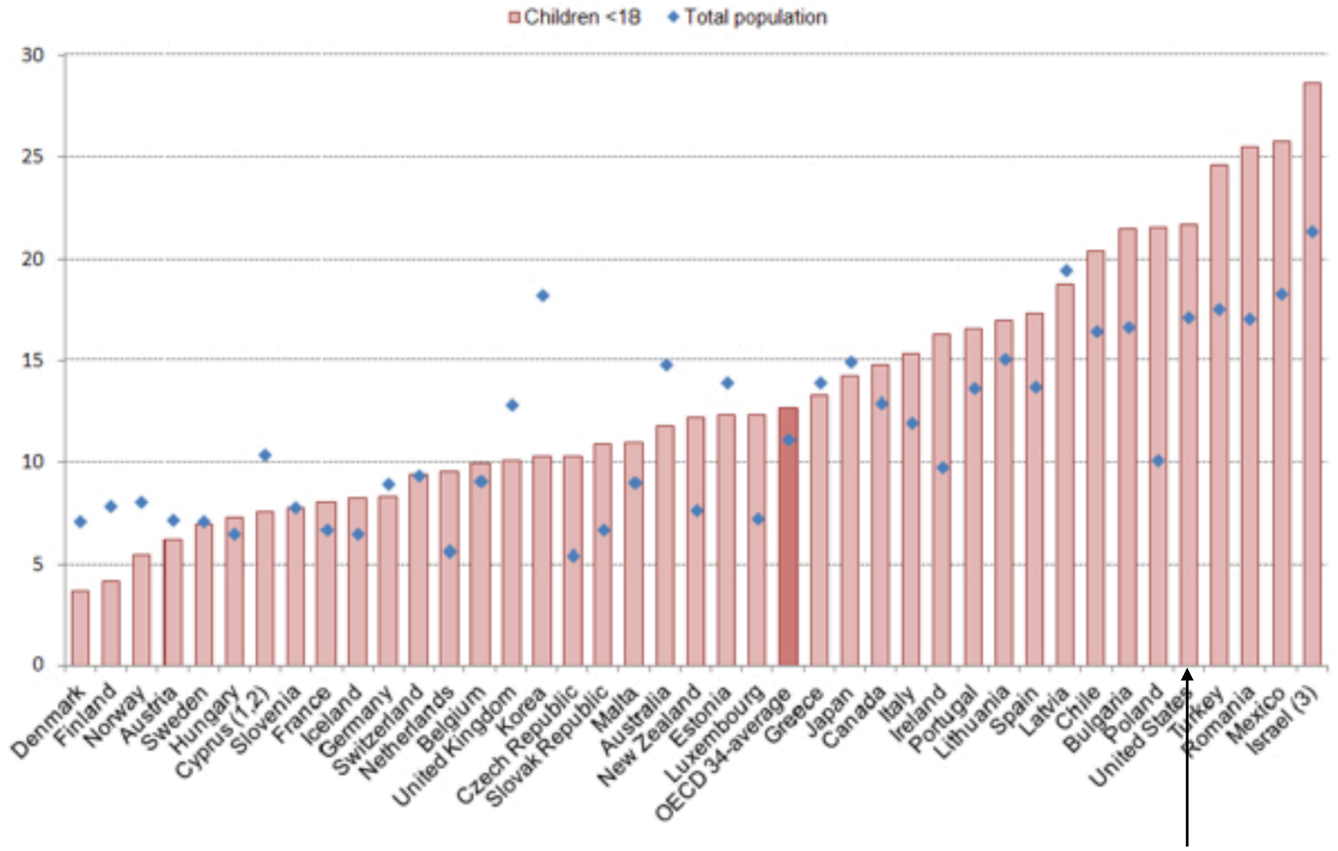
Source: Economic Policy Institute. 2011. More compensation heading to the very top: Ratio of average CEO total direct compensation to average production worker compensation, 1965-2009. Washington, D.C.: Economic Policy Institute. May 16, 2011. <http://www.stateofworkingamerica.org/charts/view/17>.

from: 20 Facts About U.S. Inequality that Everyone Should Know

9. Child Poverty

In the United States, 21 percent of all children are in poverty, a poverty rate higher than what prevails in virtually all other rich nations.

Relative Poverty Rates in Forty Nations in the Mid-to-Late 2000s



Intergenerational

Source: OECD Income Distribution questionnaire, February 2011. Data refer to 2008 for Germany, Israel, Italy, Korea, Mexico, Netherlands, New Zealand, Norway, Sweden and the United States; 2007 for Canada, Denmark and Hungary; 2006 for Chile, Estonia, Japan and Slovenia; 2005 for France, Ireland, Switzerland and the United Kingdom; 2004 for Australia, Austria, Belgium, Czech Republic, Finland, Greece, Iceland, Luxembourg, Poland, Portugal, the Slovak Republic, Spain and Turkey.

from: 20 Facts About U.S. Inequality that Everyone Should Know

If you are interested in the general moral philosophy of inequality, I recommend:

In **1998**, Sen received the Prize in Economic Sciences in Memory of Alfred Nobel for his theoretical, field, and ethics work in welfare economics and for his research advancing the understanding of social-choice theory, poverty, and the measurement of welfare. Jun 3, 2021

Equality of What?

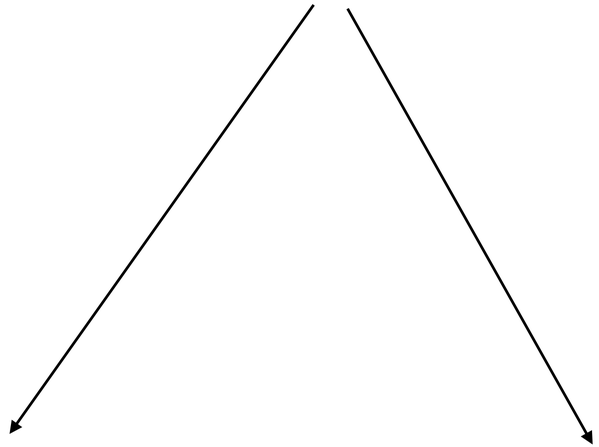
AMARTYA SEN

THE TANNER LECTURE ON HUMAN VALUES

https://www.ophi.org.uk/wp-content/uploads/Sen-1979_Equality-of-What.pdf

Origins of wealth inequality

Two Different Issues:



Piketty, Bowles,…

Marx, Stieglitz, …

Transmission of wealth over time

Allocation of production returns as Income

wealth “dynamics”

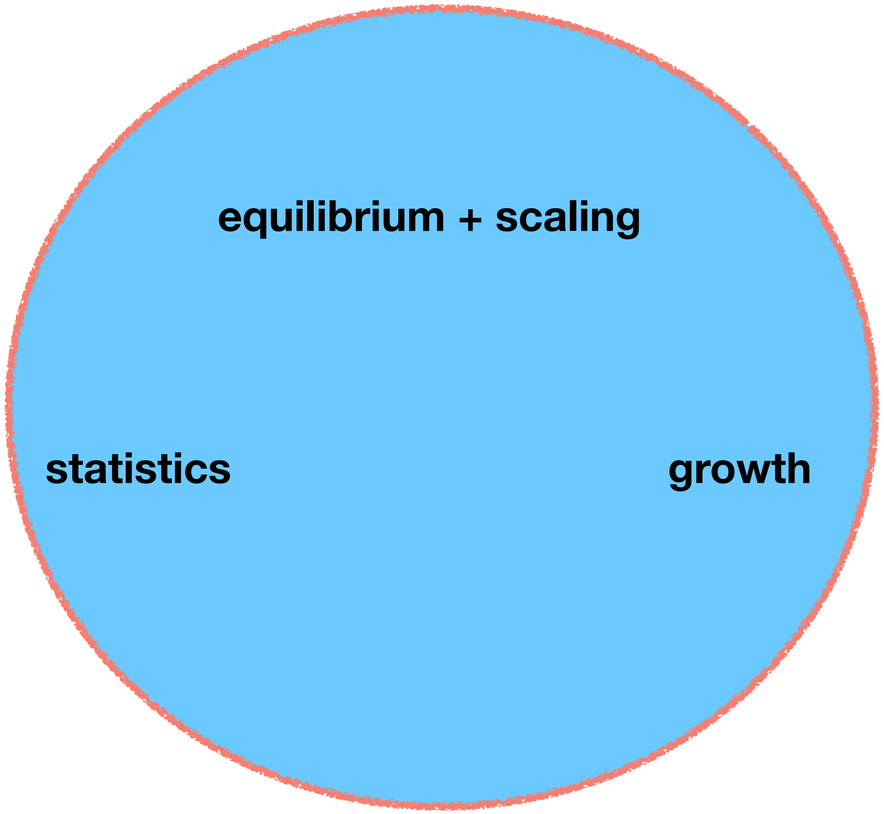
income: labor vs capital

Policy solutions :

depreciation, expropriation
government: inheritance taxes

firms
government: redistribution, social services

Where does this all fit with Cities?



Economic Growth and Cities: Surplus from spatial “equilibrium”

budget: $Y(N) - C(N)$ → Surplus → **growth**

interactions **costs**

transportation, housing

1. The Problem of Statistics

diversity, inequality: broad distributions of wealth

2. The Problem of Growth

cities are not static: they grow at a few percent a year!! good or bad?

Recall:

Economic Growth

where do economic growth rates come from?

$$\frac{dY}{dt} = \eta Y$$

Exponential Growth: 2-3% a year for USA

noisy, scale dependent, variable

inequality

A Simple Model of Resource Dynamics

flows: **income** **costs**

$$\frac{dr(t)}{dt} = y(t) - c(t) = \eta_r(t) r(t)$$

"one-good wealth accumulation model"

in economics

change in *resources*

stock: wealth, energy

"spatial equilibrium"

savings rate = growth rate

growth

$$\eta_r(t) \equiv \frac{y(t)}{r(t)} - \frac{c(t)}{r(t)}.$$

definition

Solution:

$$\ln \frac{r(t)}{r(0)} = \left(\bar{\eta}_r - \frac{\sigma_r^2}{2} \right) t + \Theta(t),$$

noise: $\Theta(t) = \sum_{l=1}^{t} \varepsilon_r(t_l),$ → **Normal "bell curve"**

because of the central limit theorem

The counter intuitive consequences of stochastic geometric growth

Consequences:

zero average, standard deviation: $\sigma_\Theta = \sigma_r\sqrt{t}$

1.

$$\ln\frac{r(t)}{r(0)} = \left(\bar{\eta}_r - \frac{\sigma_r^2}{2}\right)t + \Theta(t), \qquad \Theta(t) = \sum_{l=1}^{t}\varepsilon_r(t_l) \quad \sim \sigma_r\sqrt{t}$$

Central Limit Theorem -> Gaussian

lognormal statistics !!

2.

$$\frac{1}{t}\ln\frac{r(t)}{r(0)} = \gamma_r + \frac{\Theta(t)}{t} \to \gamma_r,$$

The average growth rate over long times becomes a simple number

ergodicity

3.

$$t_* = \frac{\sigma_r^2}{\left(\bar{\eta}_r - \frac{\sigma_r^2}{2}\right)^2},$$

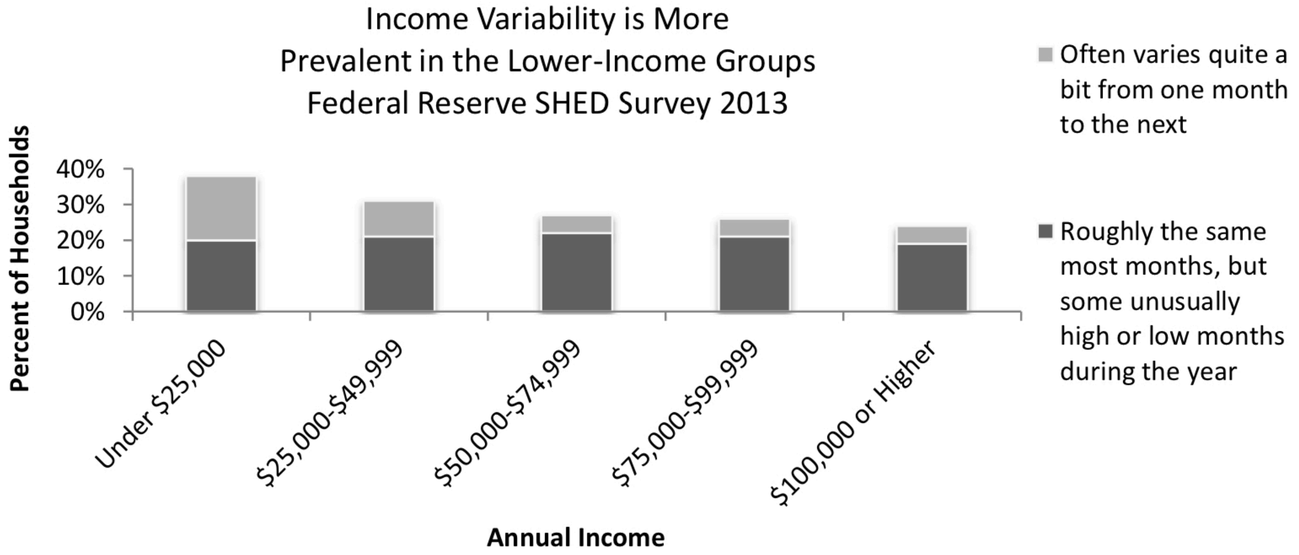
There is a minimum time to see through the "fog" of sort term fluctuations.

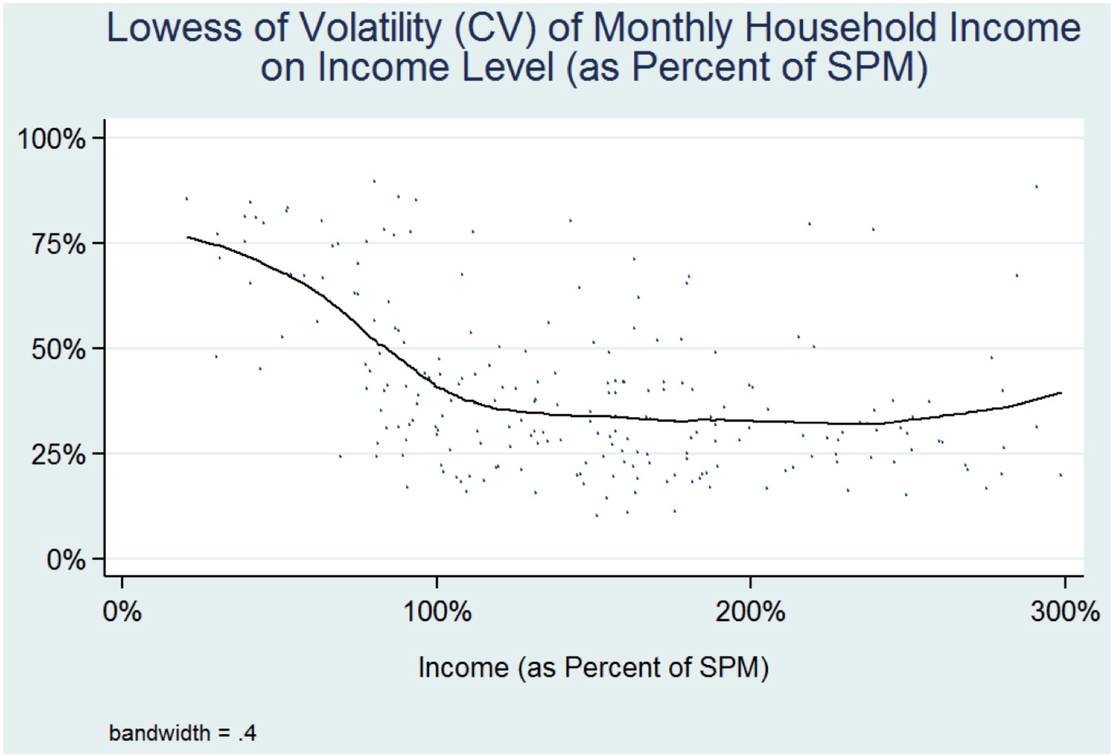
proportional to the uncertainty (volatility, std) between relative income and costs

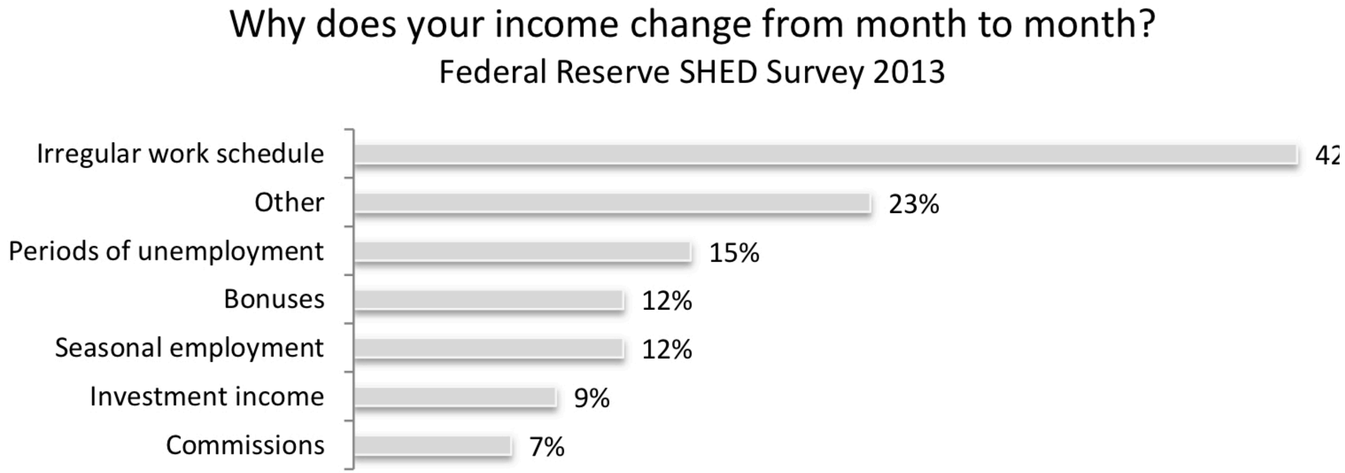
people with a lot if uncertainty do not see growth as a possibility

The working poor experience much more volatility

lower effective growth rates, unclear horizons

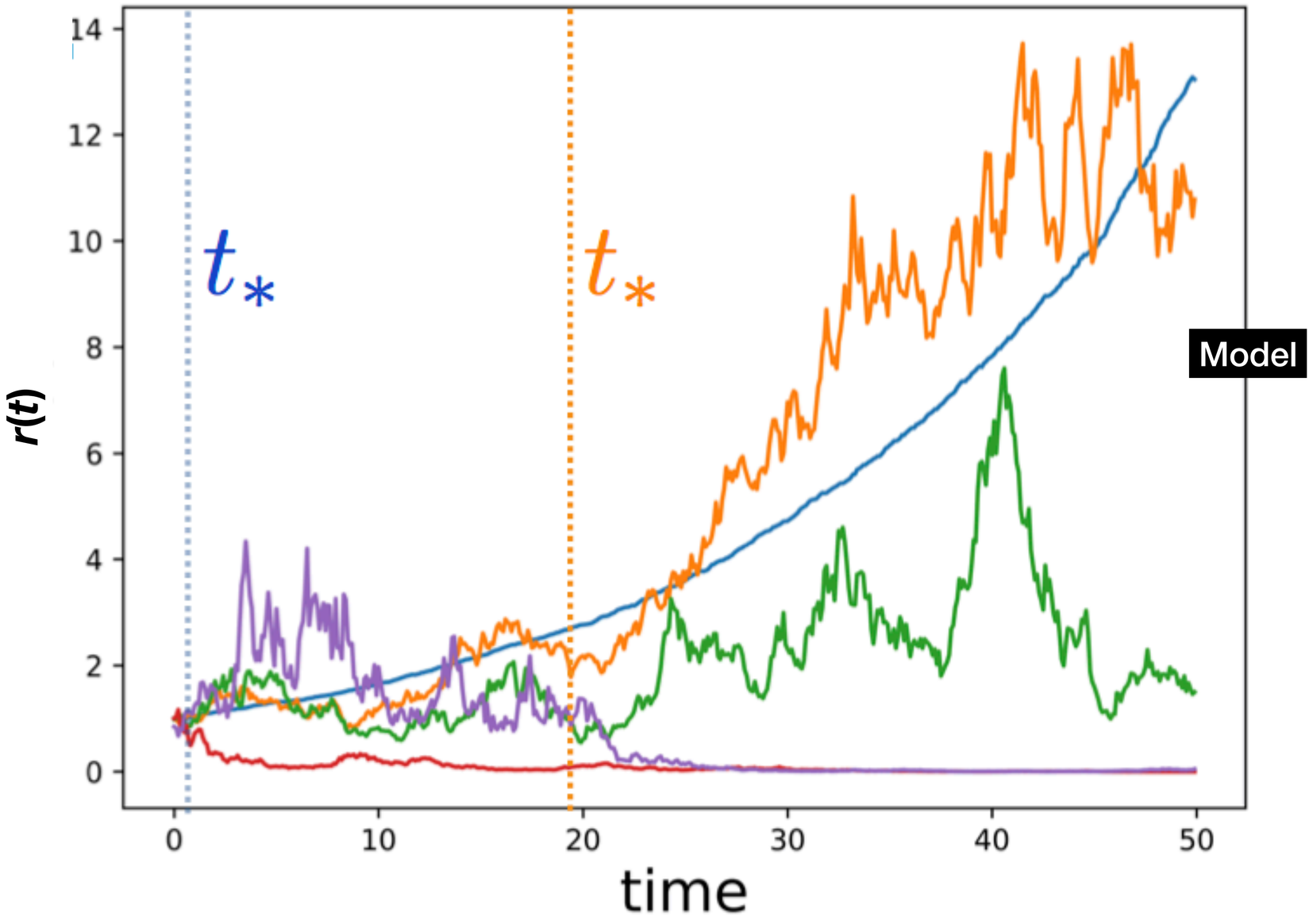




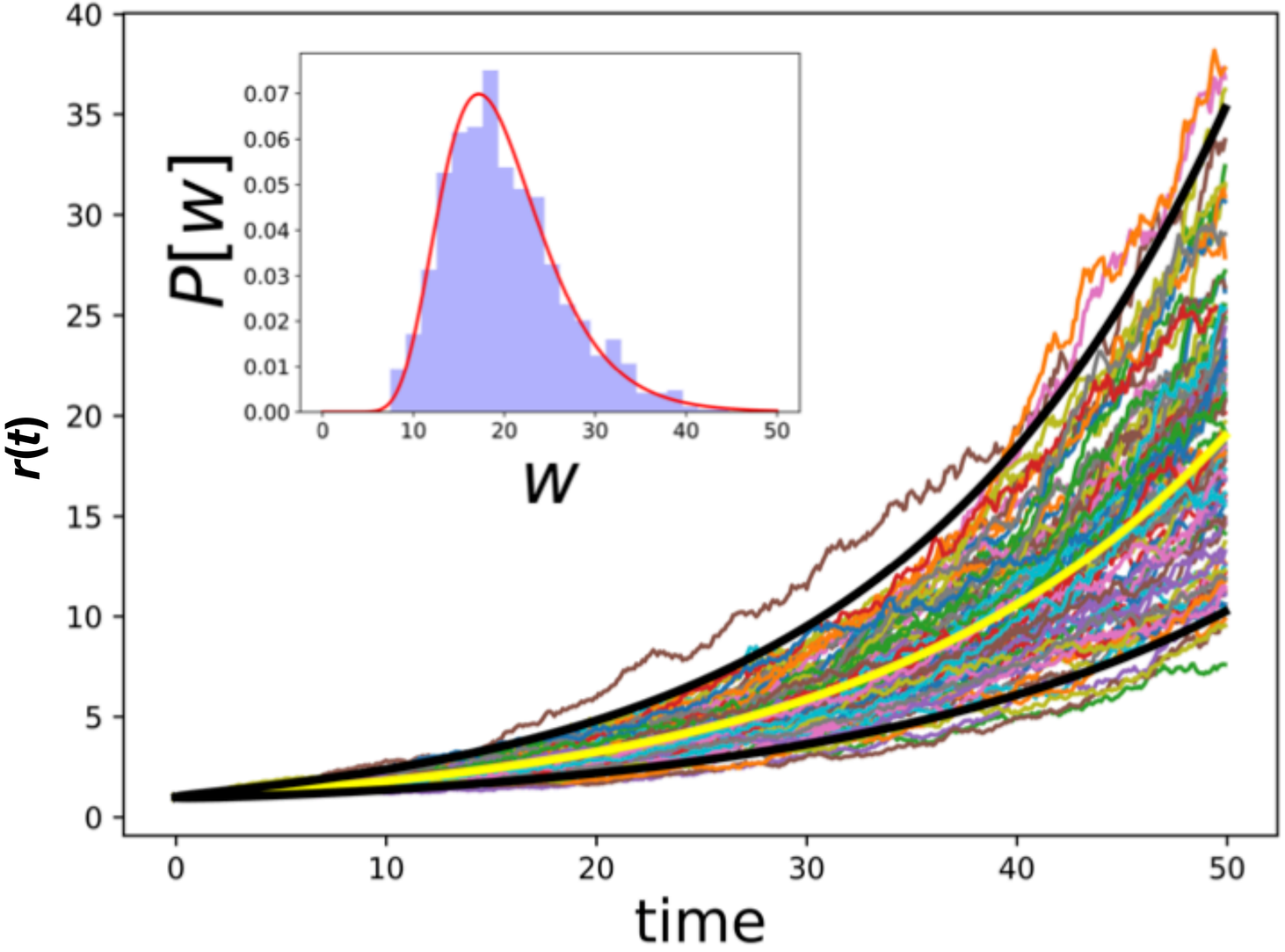


from: Income Gains and Month-to-Month Income Volatility: Household evidence from the US Financial Diaries, Hannagan + Morduch (2015)

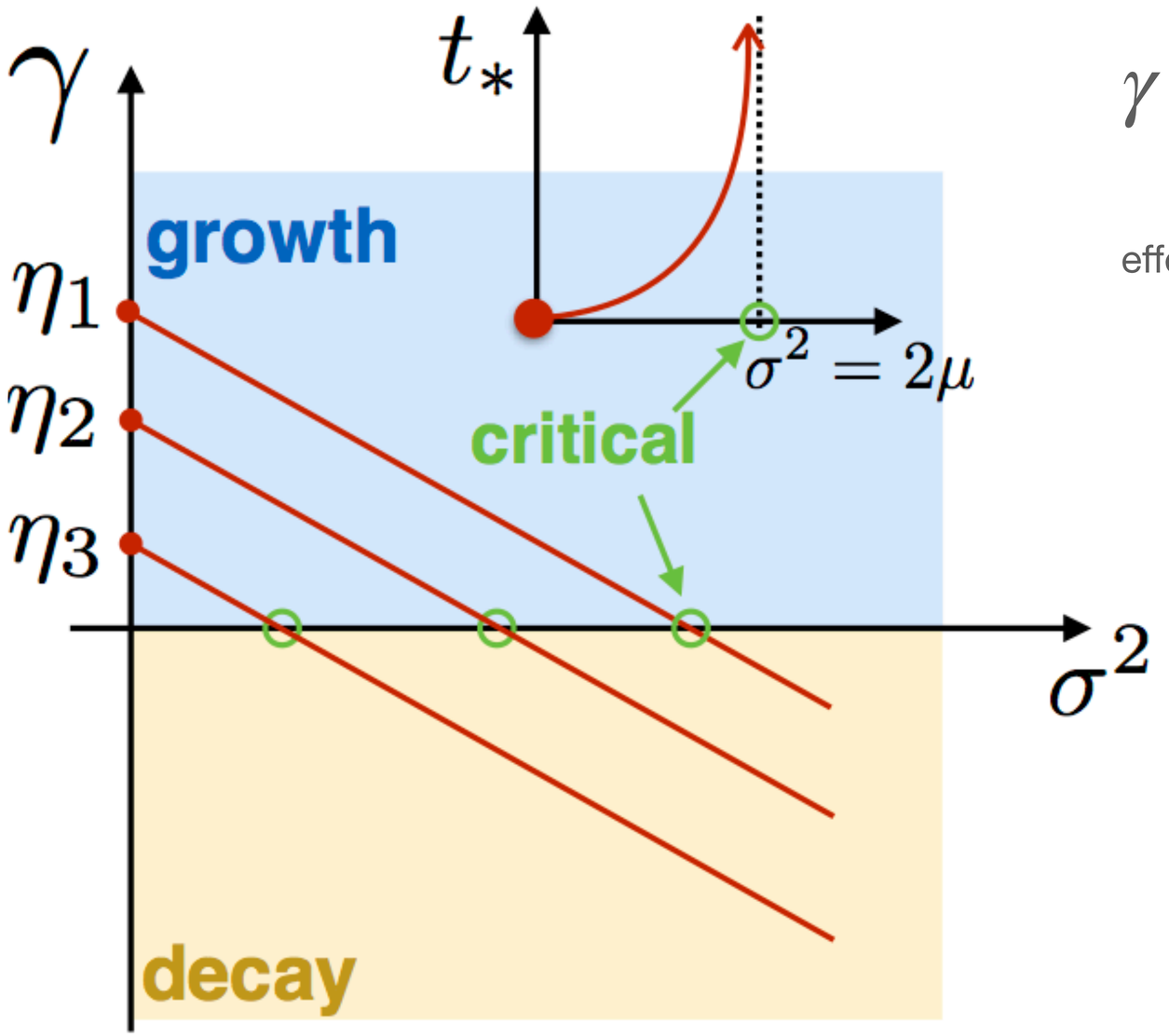
Model Solutions look like this



Log normal statistics



Net Growth Requires Low Fluctuations



$$\gamma = \eta - \frac{\sigma^2}{2}$$

effective growth rate

Want low fluctuations:

$$\eta_r(t) \equiv \frac{y(t)}{r(t)} - \frac{c(t)}{r(t)} = b(t) - a(t) = \bar{\eta} + u(t) - v(t) = \bar{\eta} + \epsilon(t)$$

definition

income fluctuations

choice expenditures

definition

when I have more money can spend more and vice-versa

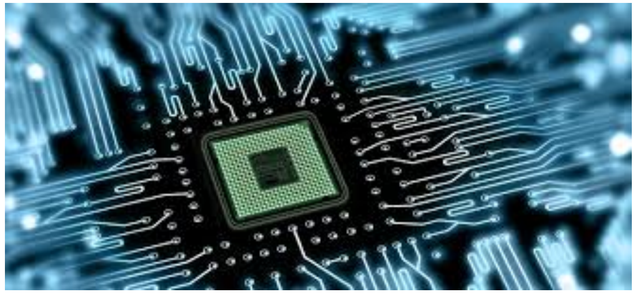
Consumption Smoothing

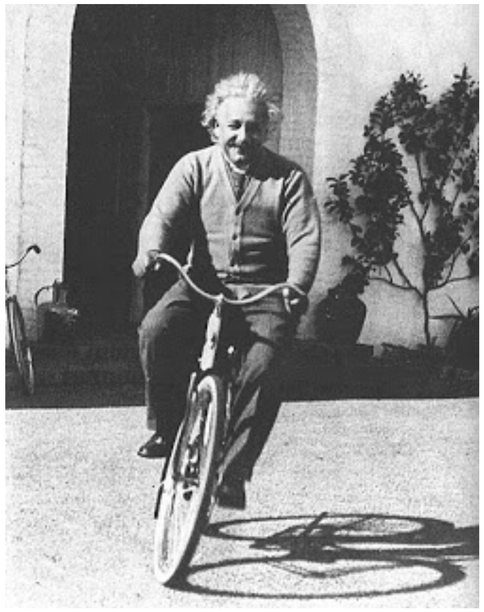
need to average (smoothen) expenditures and incomes over some time
month, season, year

feedback control theory

Feedback Control is a big part of your life

Feedback Control is a big part of your life





How Control theory works:

PID Controller

$$u(t) = k_P \varepsilon_r(t) + k_I \int_0^t \varepsilon_r(t')dt' + k_D \frac{d\varepsilon_r}{dt},$$

Time costs to track income fluctuations

This gives a simple dynamics for errors

$$\frac{d^2\varepsilon_r}{dt^2} + 2\zeta\omega\frac{d\varepsilon_r}{dt} + \omega^2\varepsilon_r = F/m, \qquad m = k_D,\ \omega = \sqrt{k_I/k_D} \text{ and } \zeta = \frac{k_P+1}{2\sqrt{k_D k_I}},$$

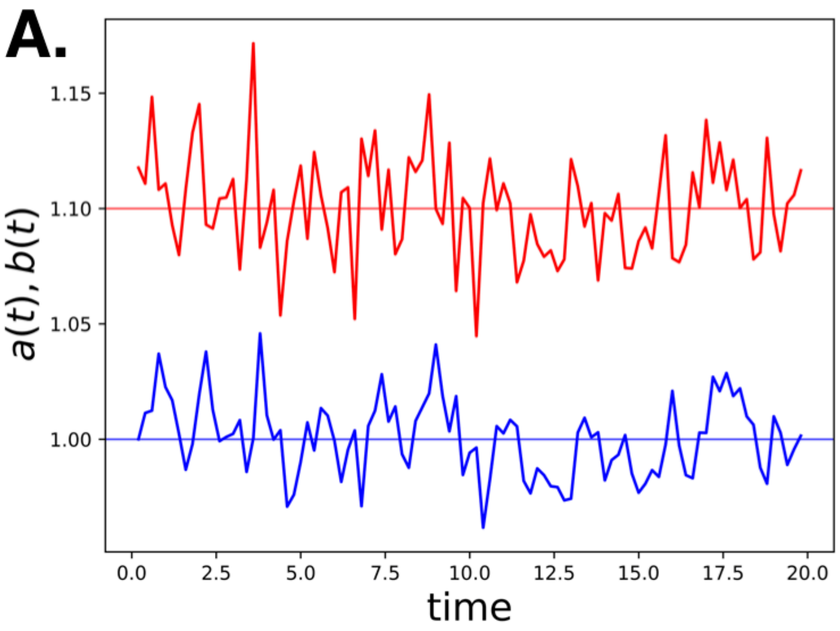
noisy spring

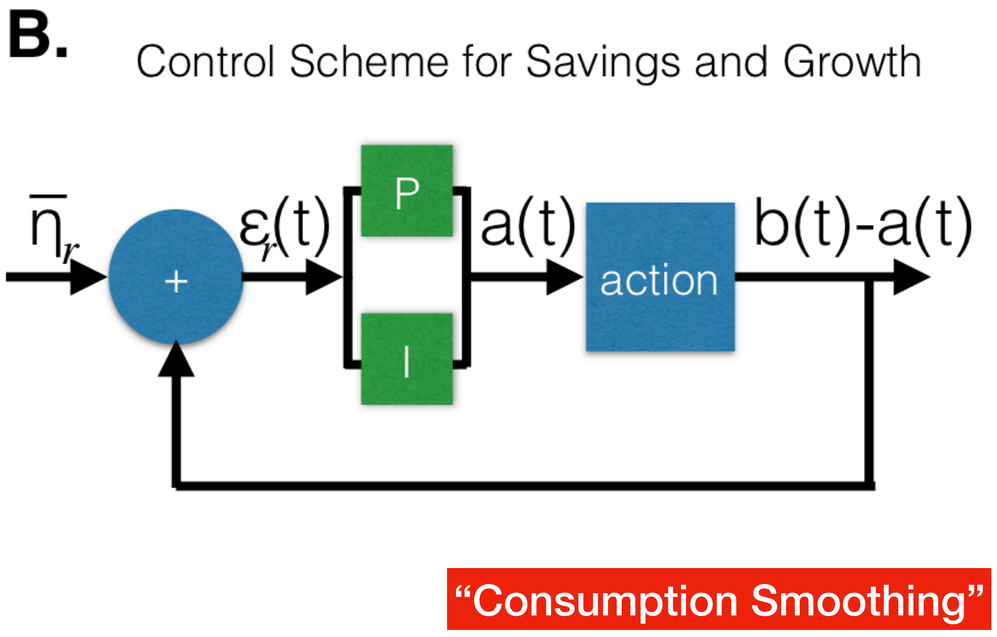
$$d\varepsilon_r = -\frac{M}{2}\varepsilon dt + \Omega dW(t). \qquad M = \omega/\zeta = k_I/[2(k_P+1)]$$

error wants to vanish **noise**

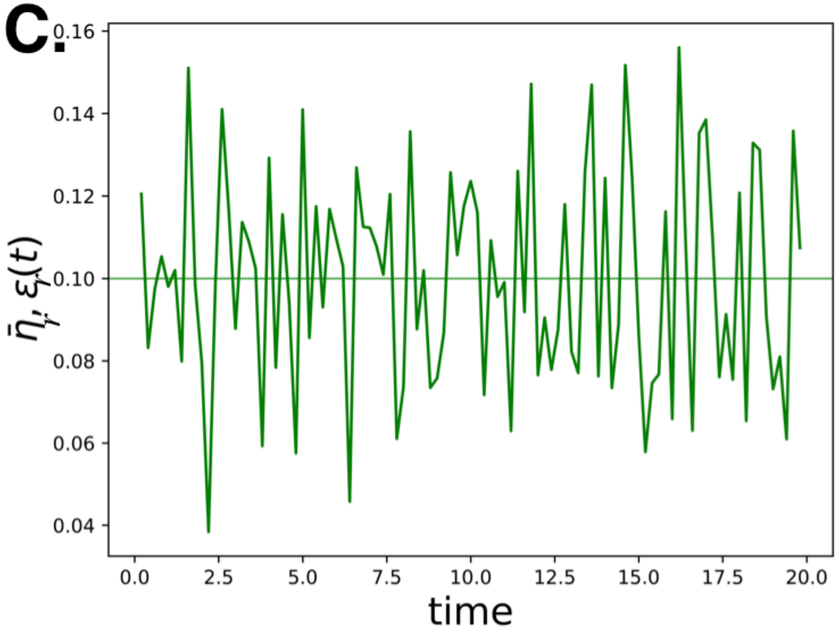
Statistics of error is "under control":

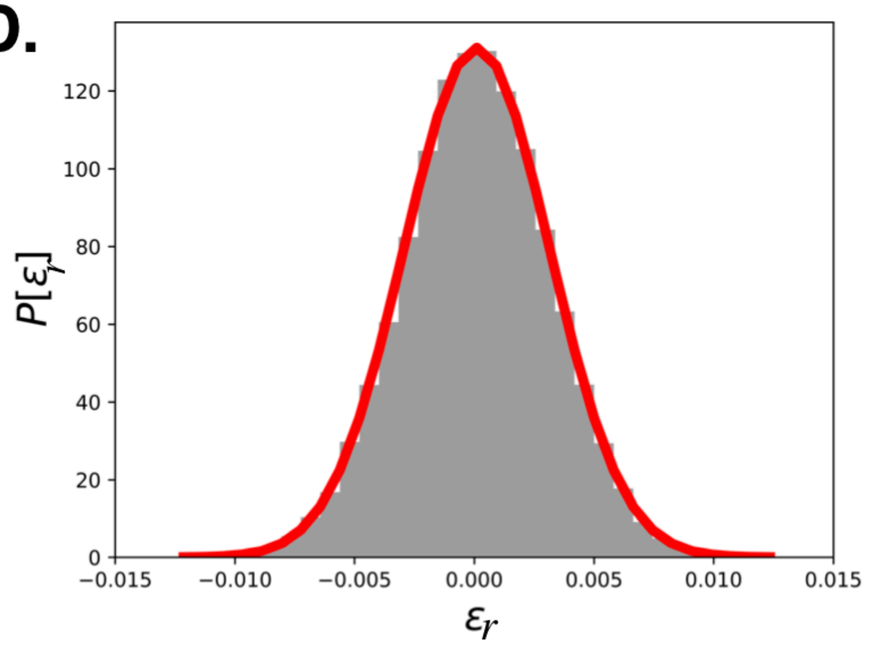
$$P[\varepsilon_r, t|\varepsilon_0] = \sqrt{\frac{M}{2\pi\Omega^2(1-e^{-Mt})}}\, e^{-\frac{M}{2\Omega^2}\left[\frac{(\varepsilon_r - \varepsilon_0 e^{-M/2\ t})^2}{1-e^{-Mt}}\right]} \to \sqrt{\frac{M}{2\pi\Omega^2}}\, e^{-\frac{M}{2\Omega^2}\varepsilon_r^2},$$





"Consumption Smoothing"





Growth rates for individuals and for cities

Is the growth rate for a city the average of the growth rates for individuals?

Not in general:

$$r_G = \frac{1}{G}\sum_{j=1}^{G} r_j$$

average resources

$$\frac{dr_G}{dt} = y_G - c_G = (\eta r)_G,$$

growth of resources per capita in a population

average of product

$$(\eta r)_G = \frac{1}{G}\sum_{j=1}^{G} \eta_j w_j = \eta_G r_G + \mathrm{covar}_G(\eta, r) = [\eta_G + \mathrm{covar}_G(\eta, r/r_G)]\, r_G.$$

average rate

"selection"

Consequences:

$$(\eta r)_G = \frac{1}{G}\sum_{j=1}^{G} \eta_j w_j = \eta_G r_G + \mathrm{covar}_G(\eta, r) = [\eta_G + \mathrm{covar}_G(\eta, r/r_G)]\, r_G.$$

positive: richer people get higher growth rates

correlation between resources and rates

negative: richer people get lower growth rates

anti-correlation between resources and rates

What situation gives the highest overall growth rates?

What situation gives the lowest inequality?

Corrections to scaling exponents

$$Y(N) = Y_0 N^{\beta} e^{\xi} \rightarrow \langle Y(N) \rangle = Y_0 N^{\beta} \langle e^{\xi} \rangle$$

lognormal

$$e^{\frac{\sigma^2(N)}{2}}$$

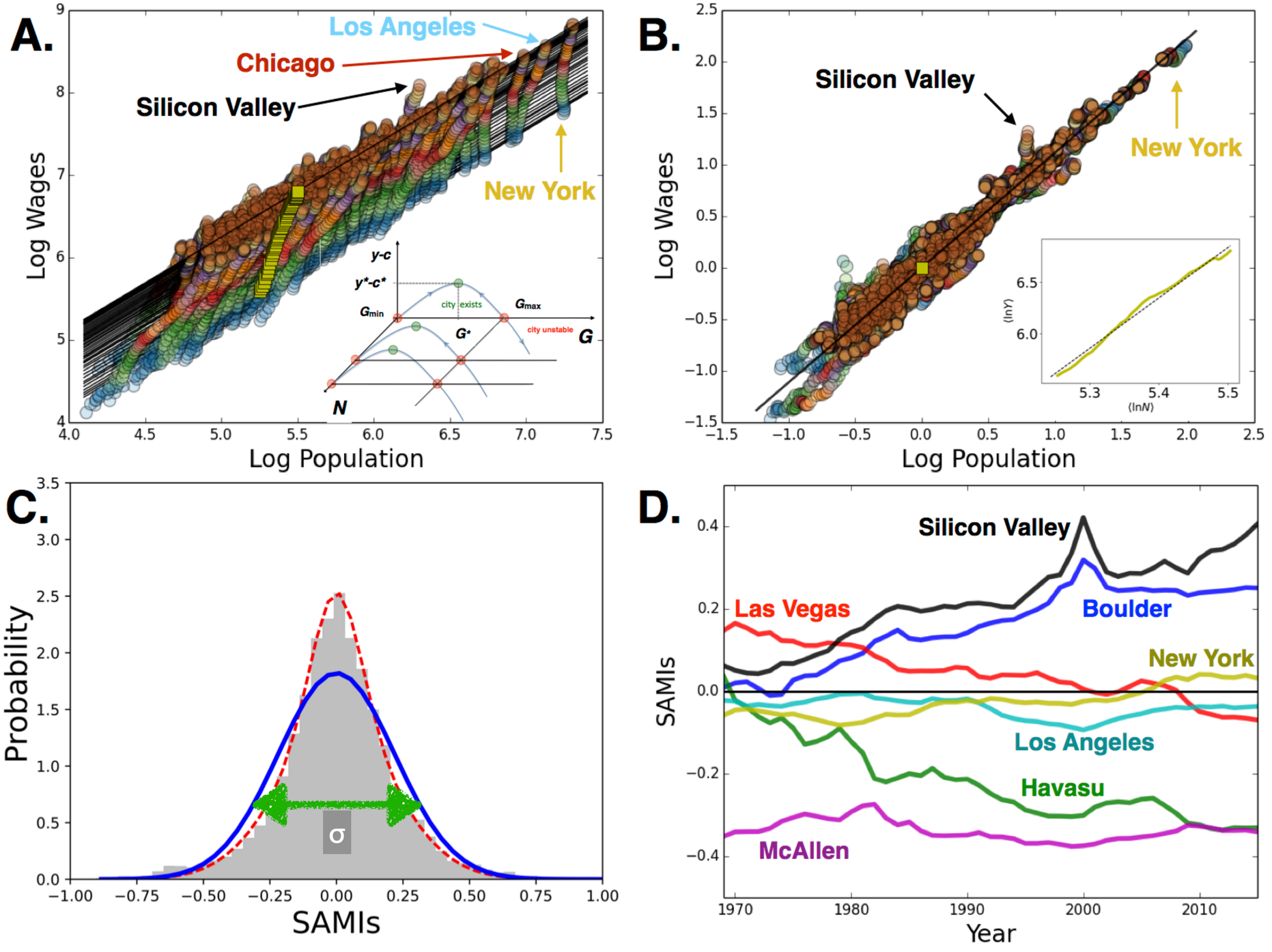
varies with N

scaling exponents become scale dependent

loss of scale invariance

requires a theory for the variances

more complicated statistics


A.
Los Angeles
Chicago
Silicon Valley
New York
Log Wages
Log Population
y-c
y*-c*
G_min
city exists
G_max
city unstable
G*
G
N
B.
Silicon Valley
New York
Log Wages
Log Population
⟨lnY⟩
⟨lnN⟩
C.
Probability
SAMIs
σ
D.
Silicon Valley
Las Vegas
Boulder
New York
Los Angeles
Havasu
McAllen
SAMIs
Year

How Long does it take Cities to Catch Up with Others?

In the US, very small randomness

The typical square displacement of

$$\xi^2(t) = \sigma^2 t \simeq 0.054$$

in 50 years !!

Observe:

$$\langle \xi^2(1969) \rangle = 0.043$$

could have been ~ 0 **40 years before: in the 1920s**

How long does it take to create an equitable society? **About a Generation ~ 20-50 years**

slower with slow economic and demographic growth rates