Lecture 4

How to think about Cities: Economic Models

4.1 Models of Urban Economics

IUS 2.2.3 + 2.2.5

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Economics Models about Cities

Urban Economics

Monocentric City

Remember Alonso

Economic Geography

Core-Periphery Model

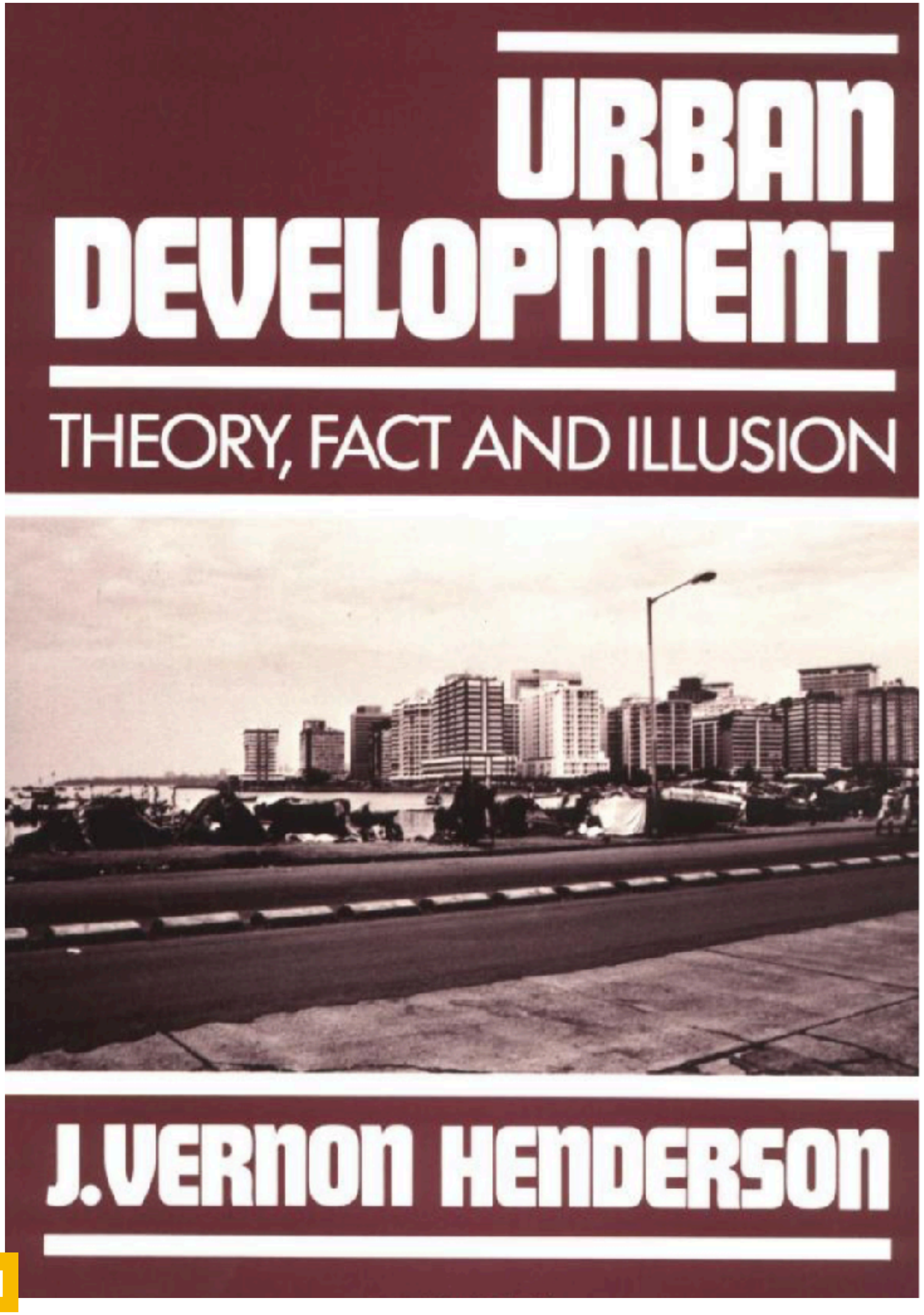
tries to explain

how land rents and transportation costs shape the city **internally**

how sector concentration, trade and transportation costs vary between spatial **regions**

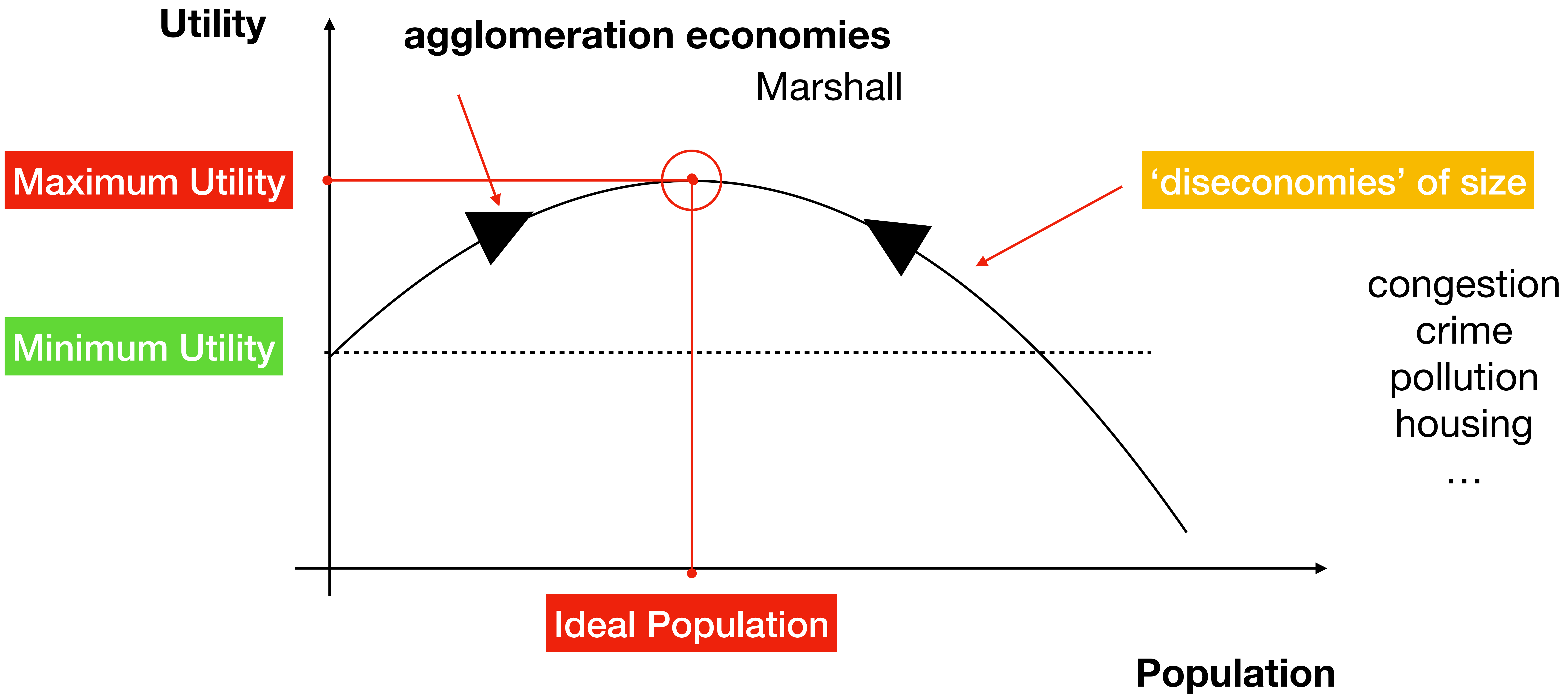
different scales: city versus regions (rural areas + cities)

Models of Urban Economics



1991

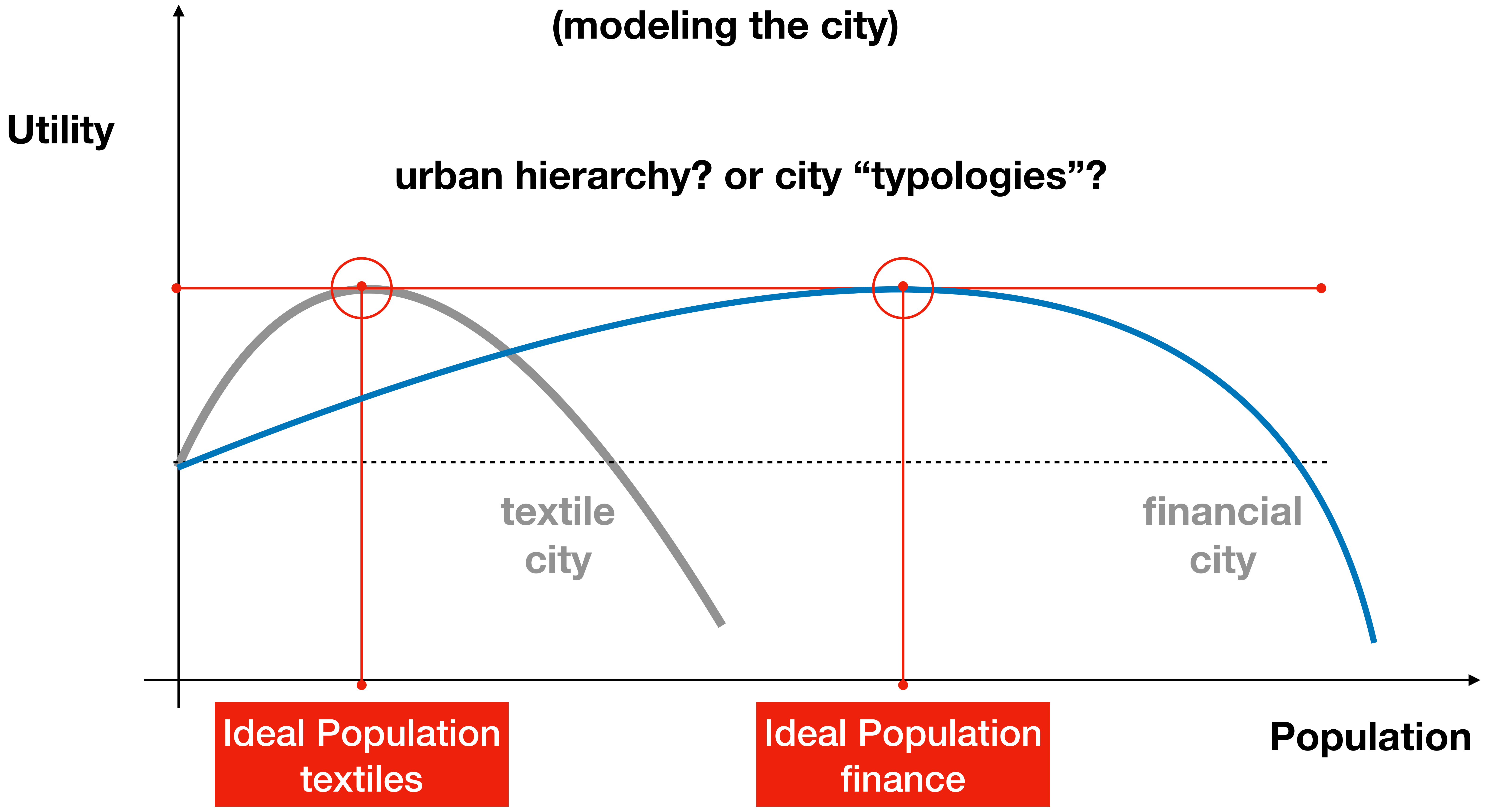
Urban Economics
dis-economies of scale



Henderson 1974, 1988

But cities come in wide range of population sizes!! ["Zipf's law"]

Urban Economics
(modeling the city)



Henderson 1974, 1988

Criticism: “Black box”, too many assumptions, get what you put in

Urban Economics and the Internal Structure of Cities

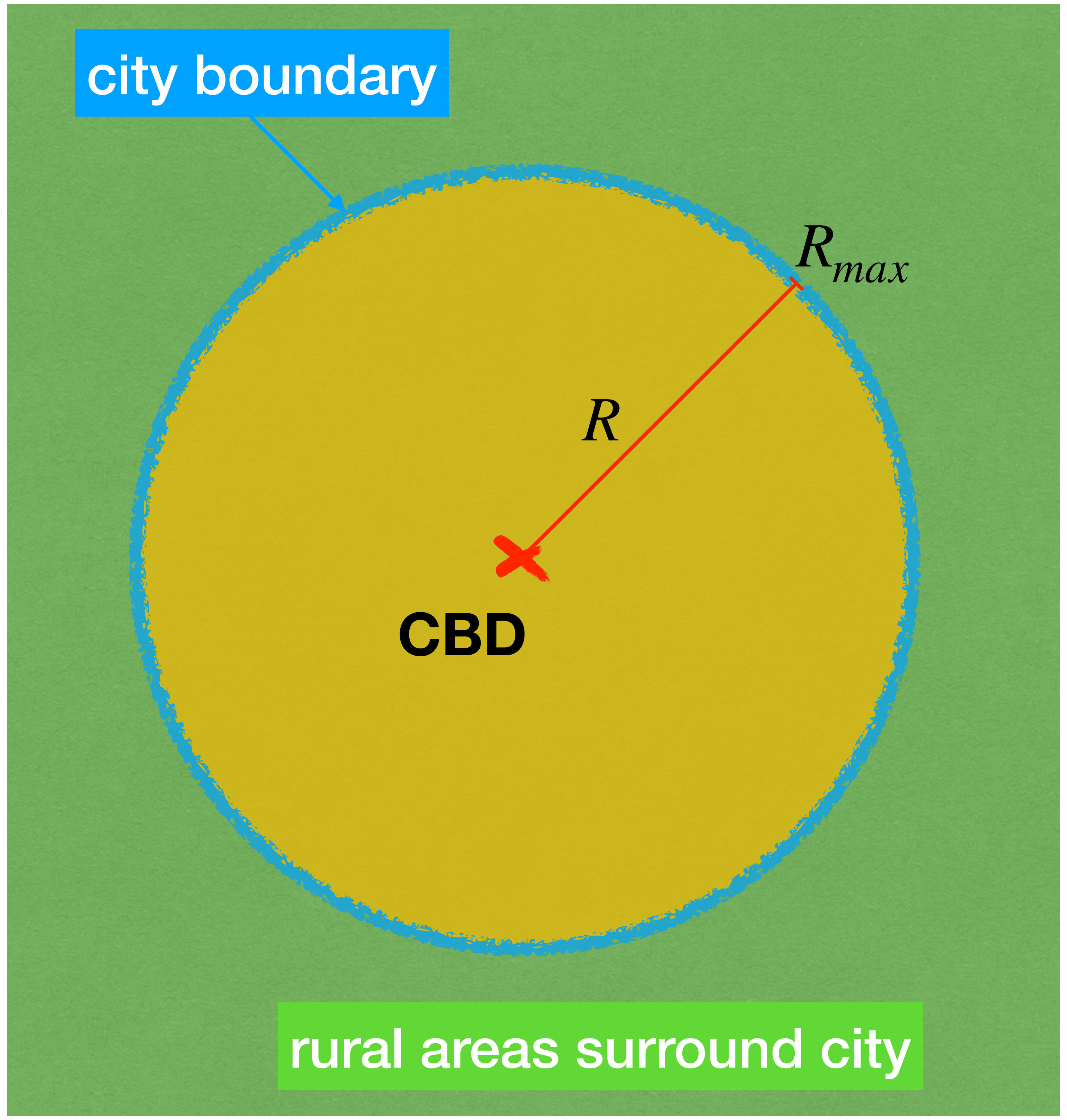
IUS 2.2.5

(optional Reading) Brueckner (1987) The structure of urban equilibria

Alonso Model of the Monocentric city

Central Market for Labor

Warning: 2 averages (time and population) !



Extent of City is determined by Budget Constraint

Idea: each person has a budget:

$$y = c_r(R) + c_T(R)$$

net income (from work at CBD)

land rent expense (at home)

commuting costs (home ⟷ work)

At CBD:

$c_T(R=0) = 0$ minimum commuting costs

$c_r(R=0) = c_{r_{max}}$ maximum rent

$$\boxed{y = c_{r_{max}}}$$

At the city boundary:

$c_T(R=R_{max}) = c_{T_{max}}$ maximum c. costs

$c_r(R=R_{max}) = c_{r_{rural}} = c_{r_{min}}$ minimum rent

$$\boxed{y = c_{r_{min}} + c_{T_{max}}}$$

$c_T(R) = c_{T_0}R, \quad c_{T_0} = \dfrac{c_{T_{max}}}{R_{max}}$ **cost/time/distance travelled**

$c_r(R) = y - c_T(R)$

Start with Alonso Model

1. Consumer behavior

budget constraint:

floor area

$$y - c_{T_0}R = c + p_f(R)a_f\,.$$

consumption of goods

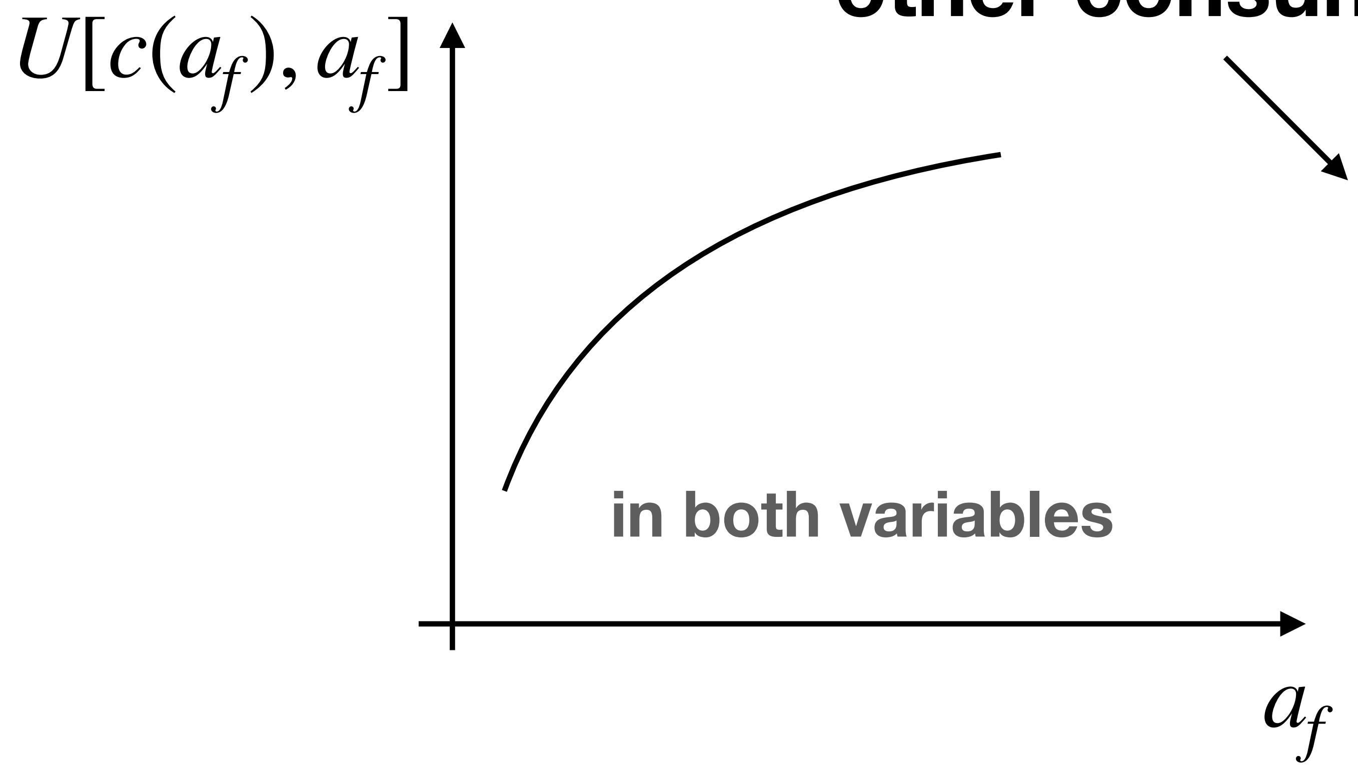
price of land

Maximize Utility:

$$\max_{a_f} U(c, a_f) = U_{\max} \qquad c = y - c_{T_0}R - p_f(R)a_f\,.$$

other consumption

land "consumption"



$$\frac{dU}{dc}\frac{dc}{da_f} + \frac{dU}{da_f} = 0 \rightarrow p_f = \frac{dU/da_f}{dU/dc}$$

maximizing sets price of housing

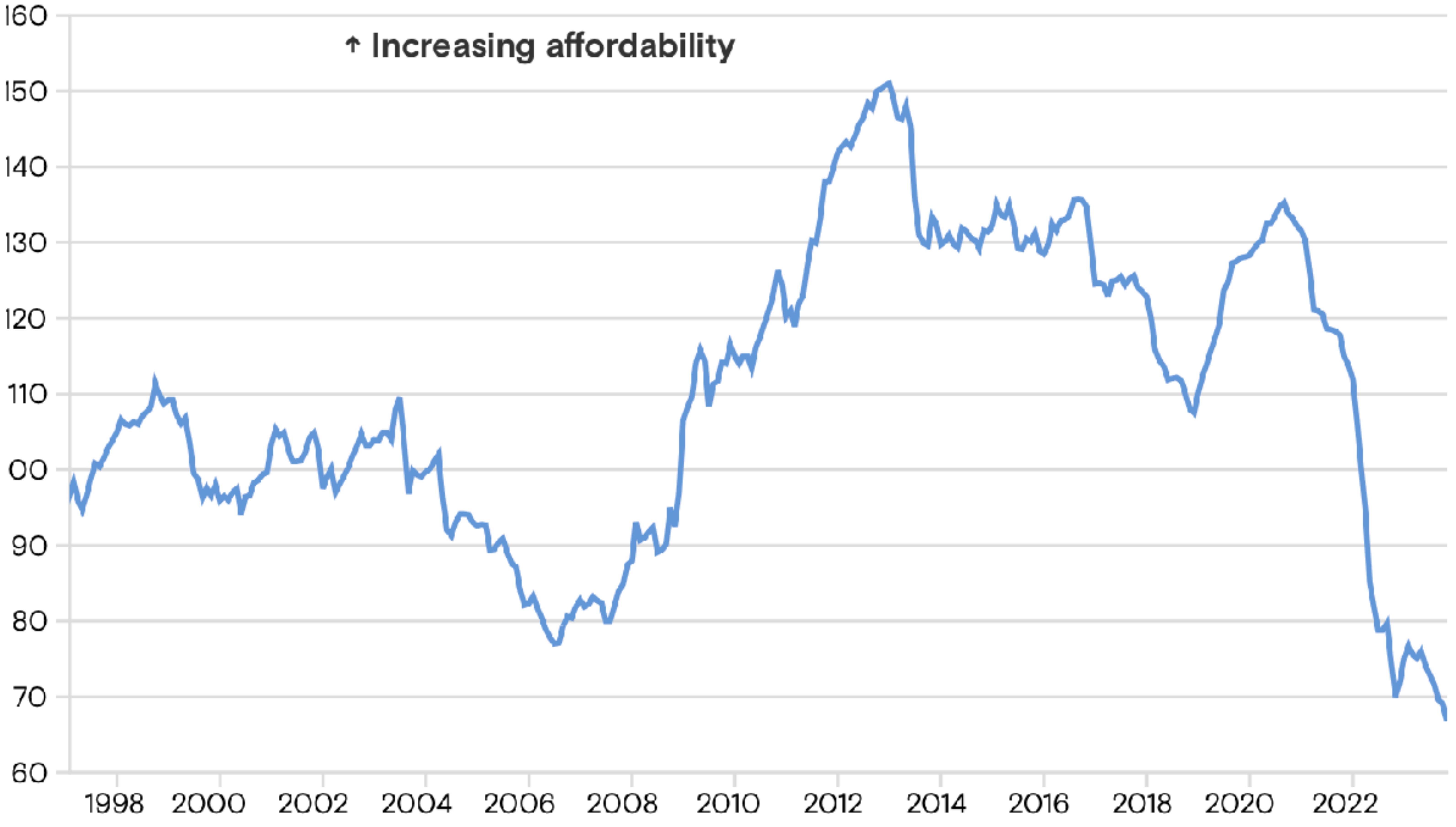
land rents (in units of goods' prices)

$$\frac{y}{p_f a_f}$$

Measured as income and mortgage payment/month

US housing affordability hits record lows

GS Housing Affordability Index



Source: Goldman Sachs Research

Goldman Sachs

https://www.goldmansachs.com/insights/articles/us-mortgage-rates-are-forecast-to-stay-above-7-percent

how do land rents depend on distance from city center?

$$c = y - c_{T_0}R - p_f(R)a_f\,.$$

$$\frac{dU}{dR} = \frac{dU}{dc}\Big(\underline{-c_{T_0}} - \frac{dp_f}{dR}a_f - p_f\frac{da_f}{dR}\Big) + \frac{dU}{da_f}\frac{da_f}{dR} = 0$$

You can take all kinds of variations of utility…

cancel

$$a_f\frac{dp_f}{dR} = -\,c_{T_0} \rightarrow \frac{dp_f}{dR} = -\,\frac{c_{T_0}}{a_f} < 0.$$

so rents go down with distance to CBD

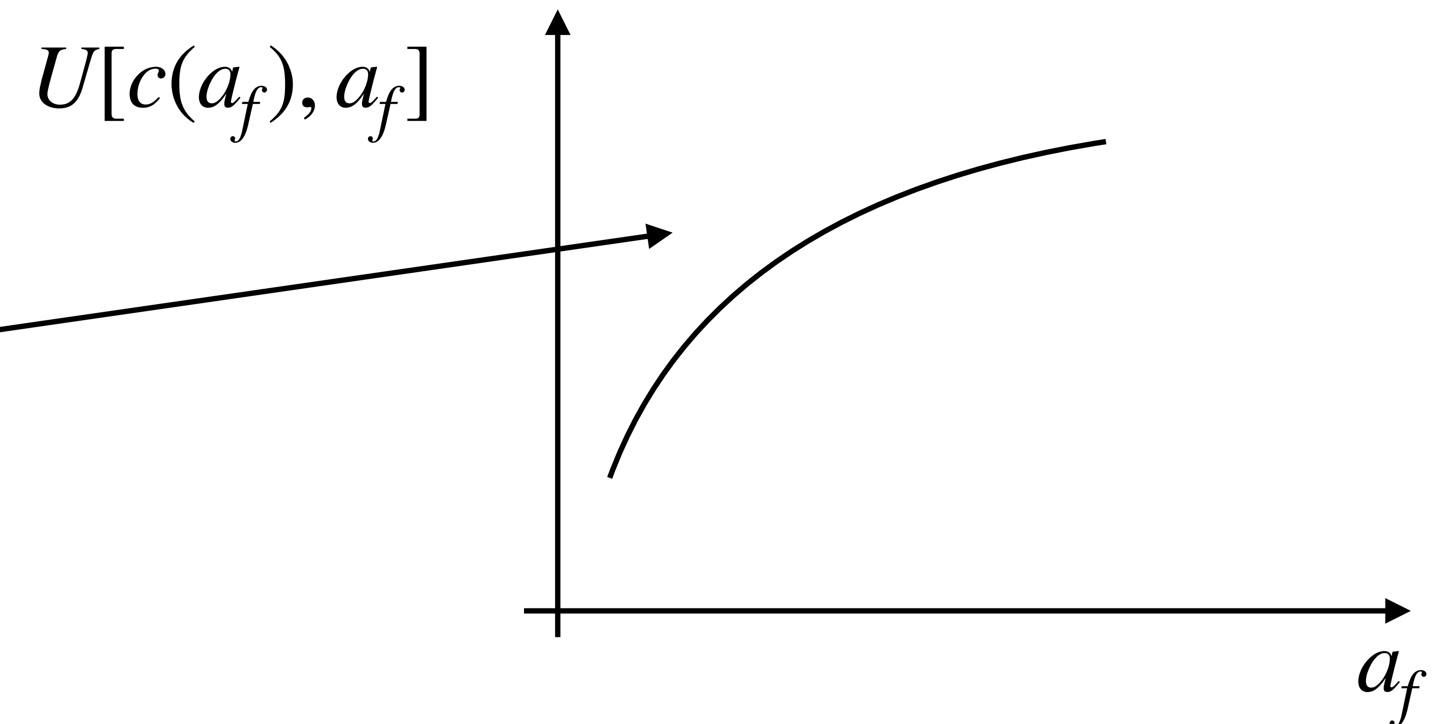
$$\frac{da_f}{dR} = \frac{da_f}{dp_f}\frac{dp_f}{dR} > 0.$$

so the amount of land consumed increases with distance

Conclusion: Suburbs have cheaper, bigger houses

second derivative is convex

$$\frac{d^2U}{da_f^2} < 0$$



Repeat the same reasoning to get:

Because of fixed budget: more money in transportation is less in rent!

land rents...

increase with income

decrease with transportation costs

utility decreases with land rents

$$\frac{dp_f}{dy} = \frac{1}{a_f} > 0, \quad \frac{dp_f}{dc_{T_0}} = -\frac{R}{a_f} < 0, \quad \frac{dU}{dp_f} = -a_f \frac{dU}{dc} < 0.$$

house size...

Varies in the opposite direction to p_f

$$\frac{da_f}{dy} = \frac{da_f}{dp_f}\frac{1}{a_f} < 0, \quad \frac{da_f}{dc_{T_0}} = -\frac{da_f}{dp_f}\frac{R}{a_f} > 0, \quad \frac{dU}{da_f} = -a_f \frac{dU}{dc}\frac{dp_f}{da_f} > 0.$$

decreases with income

increases with transportation costs

utility increases with bigger housing

recall that: $\frac{dU}{dc} > 0, \quad \frac{d^2U}{da_f^2} < 0 \rightarrow \frac{da_f}{dp_f} < 0$

This gives a bunch of qualitative expectations for the structure of the city

Quantitative expectations require a particular Utility !

2. Housing Producers behavior

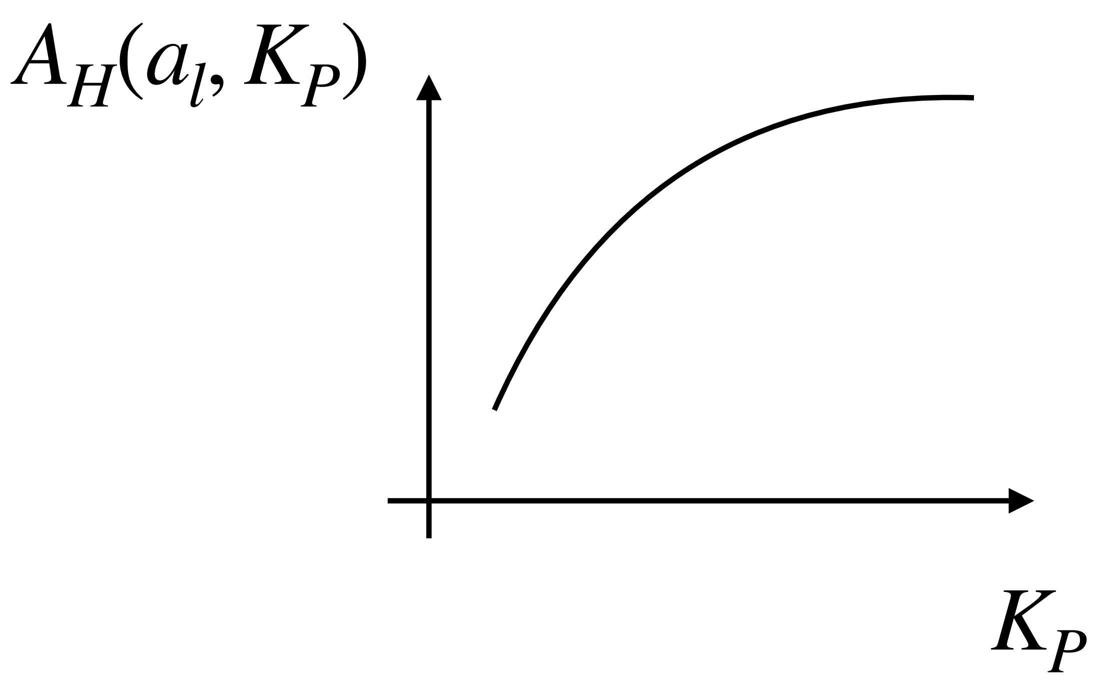
land developers

Production Function

$$A_H(a_l, K_P)$$

= area of housing developed

area of land input **capital**



decreasing returns
to capital

Developer's Profits and Free Entry condition:

$$p_f A_H - a_l p_l - K_P p_K = 0 \rightarrow a_l(p_f a_H - p_l - p_K K_{P_l}) = 0$$

$$a_H = \frac{A_H}{a_l}, \quad K_{P_l} = \frac{K_P}{a_l}$$

building density

area of housing per area of land

capital density

amount of capital per area of land
"building height"

3. Housing Producers behavior

How does the price of land and intensity of development depend on distance from center?

Maximizing Developer's Profit + "Zero Profit" condition

$$K_{P_l} = \frac{p_f}{p_K} a_H - \frac{p_l}{p_K}, \qquad \frac{da_H}{dK_{P_l}} = \frac{p_K}{p_f}$$

from zero profit condition

from maximizing profits

optimal amount of capital invested / land area

optimal amount of capital of building, or price for land

How do these vary with distance to CBD? Recall that p_f depends on all the other variables:

$$\frac{dp_l}{d\phi} = a_H \frac{dp_f}{d\phi}; \qquad \frac{dK_{P_l}}{d\phi} = -\frac{\frac{da_H}{dK_{P_l}}}{\frac{d^2 a_H}{dK_{P_l}^2}} \frac{1}{p_f} \frac{dp_f}{d\phi}, \qquad \phi = y, c_{T_0}, R, U$$

> 0

land prices and capital density increase proportionally to rents payed

decrease with *R* (distance to CBD)

How about population density?

Population Density $n_A = \frac{N}{A} \sim \frac{a_H}{a_f}$ because $A = a_l, \quad N = \frac{A_H}{a_f} = a_l \frac{a_H}{a_f}$

$$\frac{dn_A}{d\phi} = \frac{1}{a_f}\frac{da_H}{dK_{P_l}}\frac{dK_{P_l}}{d\phi} - \frac{a_H}{a_f^2}\frac{da_f}{d\phi} \sim \frac{dp_f}{d\phi}$$

density decreases with distance to CBD

Global Constraints on Population and City Area

$$N = \int_0^{R_{max}} RdR\, n_A(R) = \int_0^{R_{max}} RdR\, \frac{a_H(R)}{a_f}$$

$$p_l(R_{\max}, y, c_{T_0}, U) = p_{l_r}$$

population

city area: $A = \pi R_{max}^2$

rent at city's edge=rent for agriculture

$$\frac{dA}{dN} > 0, \frac{dU}{dN} < 0, \frac{dp_f}{dN} > 0, \frac{da_f}{dN} < 0, \frac{dK_{P_l}}{dN} > 0, \frac{dn_A}{dN} > 0.$$

How about the extent of the city?

$$\frac{dA}{dp_{l_r}} < 0, \frac{dA}{dy} > 0, \frac{dA}{dc_{T_0}} < 0.$$

city area increases with population, decreases with agricultural rents or transportation costs

And if we work a bit harder we recover Alonso's result

cost of housing/person at city's edge

income

$$R_{max} = \frac{y - p_{l_r}\frac{a_f}{a_H}}{c_{T_0}}$$

size of the city (radius)

transportation costs / length travelled

Models of Urban Economics

Reasonable outcomes from desire for consuming housing + transportation costs

Many variants and elaborations.

Features and limitations:

- no time
- just a bit of space (radial, mono centric)
- no social structure

representative agents (no heterogeneity): same utility, same income, …

maximize same utility (via maximizing consumption) within the same budget constraint

no inequality, no diversity…

firms maximize profits, but profits are zero

free entry condition

all firms are the same, make no profit

spatial equilibrium

no change over time

no change or development, no knowledge

Equilibrium market with a fixed budget and a utility maximizing agents