

Lecture 10

The differences between cities, inequality and dynamical processes

10.1 Statistical Distributions of Urban Quantities, Inequality, stochastic processes

IUS 4.2

Happy “World Cities Day 2024”

Print Edition

DownToEarth

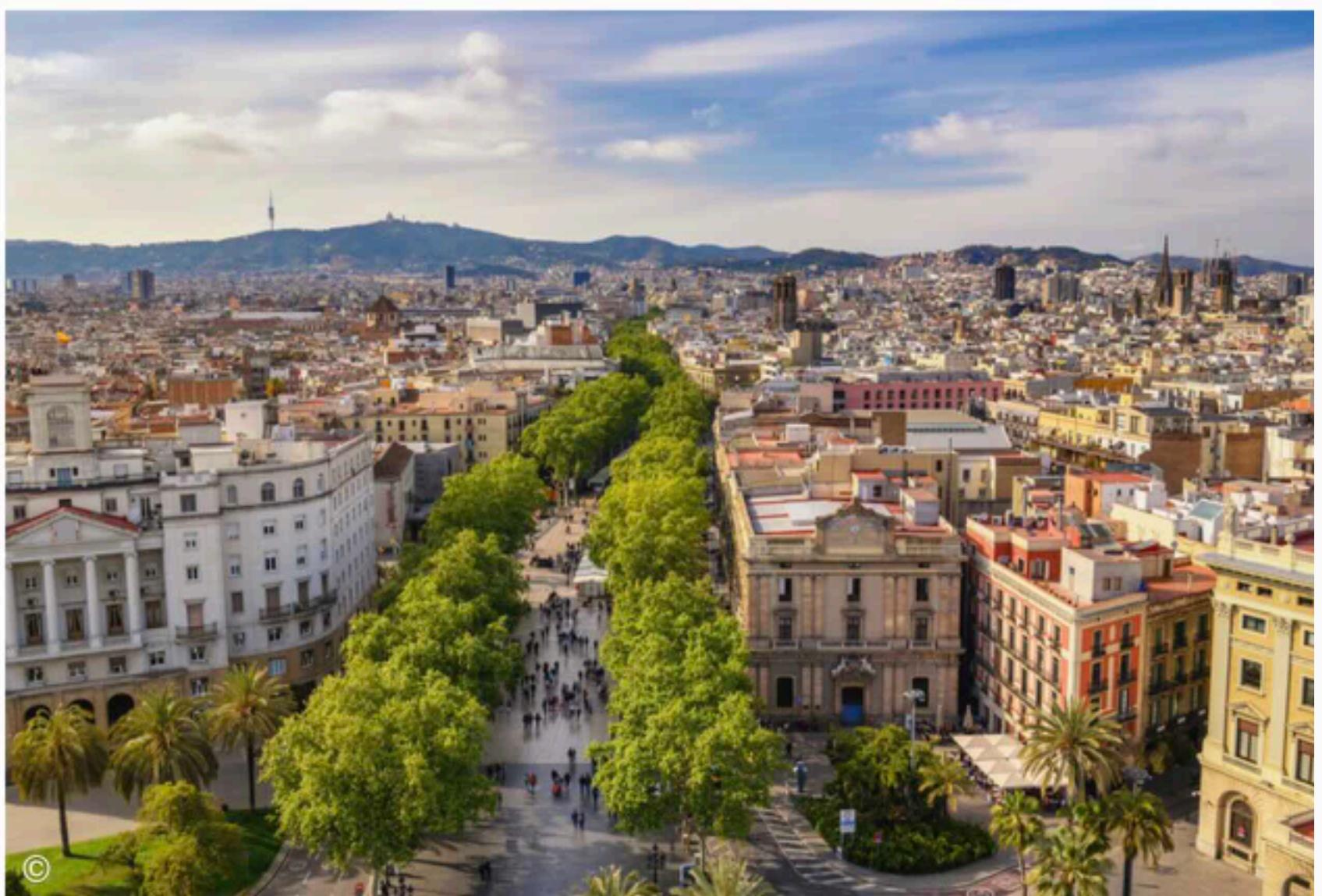
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World Cities Day 2024: Transforming Urban Futures Through Preservation, Innovation, and Resilience



Save



Written by Nour Fakharany

Published on October 31, 2024

In honor of [World Cities Day](#), which concludes Urban October, this year's theme, "[Youth Leading Climate and Local Action for Cities](#)," reflects a growing momentum for sustainable urban solutions

Urbanisation

World Cities Day: Clock ticking for India’s urban areas; act now, say experts

The time has come for active policies to make sustainable modes work for all income levels, they say



The 2024 World Cities Day China Observance (Shanghai) Opens, Unveiling Practices, Indicators for Sustainable Urban Development

Provided by [GlobeNewswire](#)

Oct 31, 2024 10:04am

SHANGHAI, Oct. 31, 2024 (GLOBE NEWSWIRE) -- As an event to mark the World Cities Day, the 2024 World Cities Day China Observance (Shanghai) and 2024 SDG Cities Global Conference, themed "Building People-centered Cities for Better Life," was held on Thursday at the Shanghai World Expo Exhibition and Convention Center.



A Media Snippet accompanying this announcement is available by clicking on [this link](#).

We want to explain why **deviations from scaling** for each city

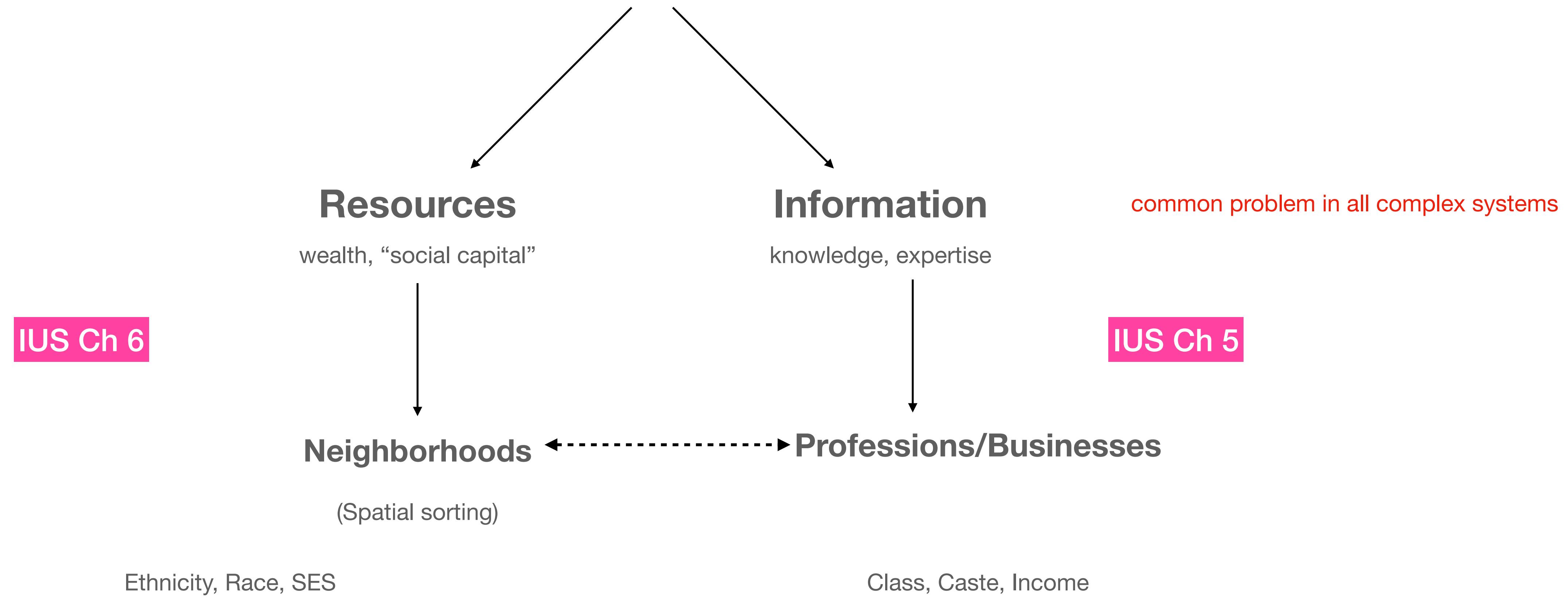
have the **observed distribution (~lognormal)**

Bell-curve in logs

This is connected to how **resources and information** are used, at the individual and firm levels

and brings in the problems of **growth and inequality**

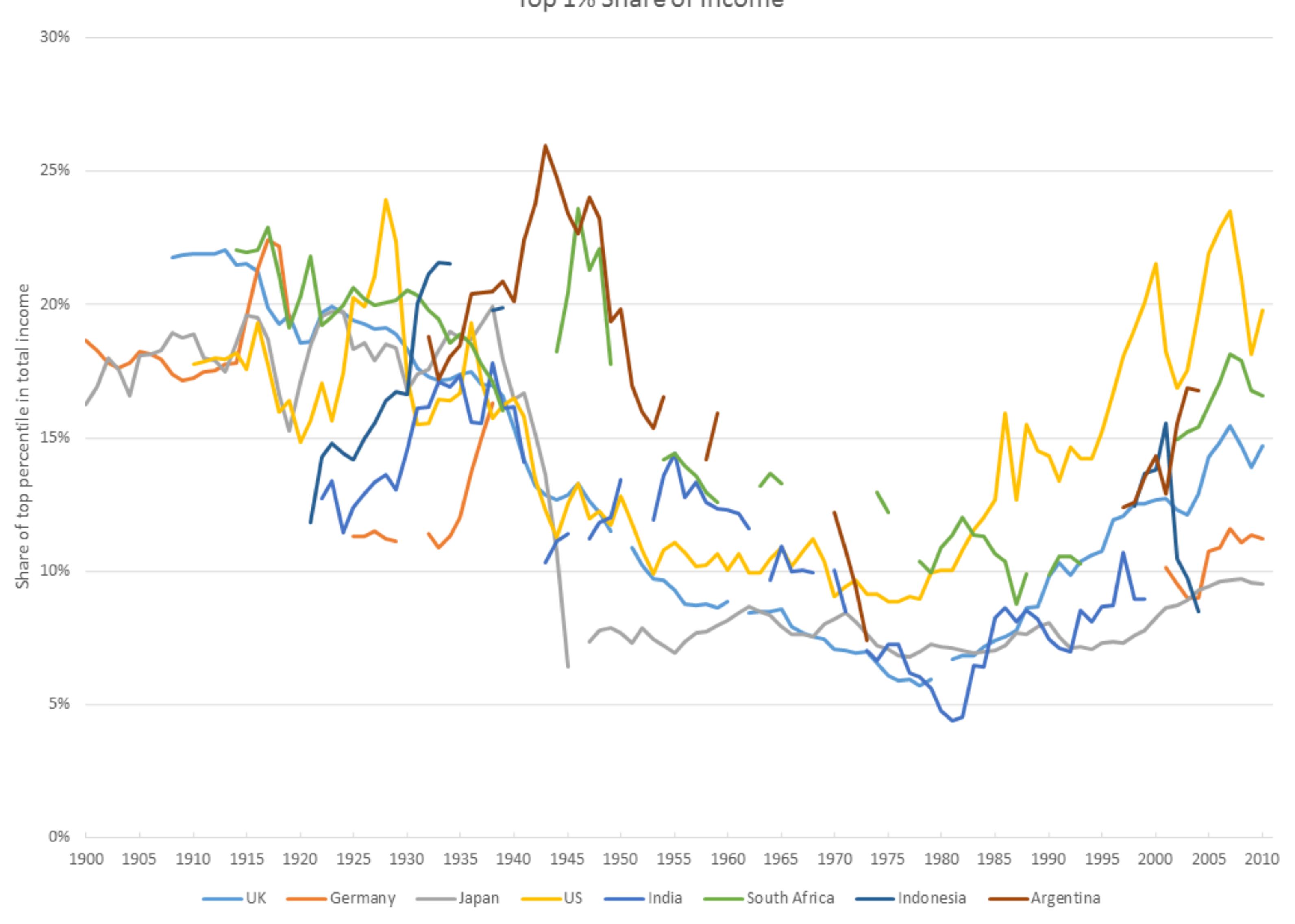
Inequality and statistics of variation in cities



The Classic Approach to Economic Inequality

nations

Top 1% Share of Income



Thomas Picketty

CAPITAL

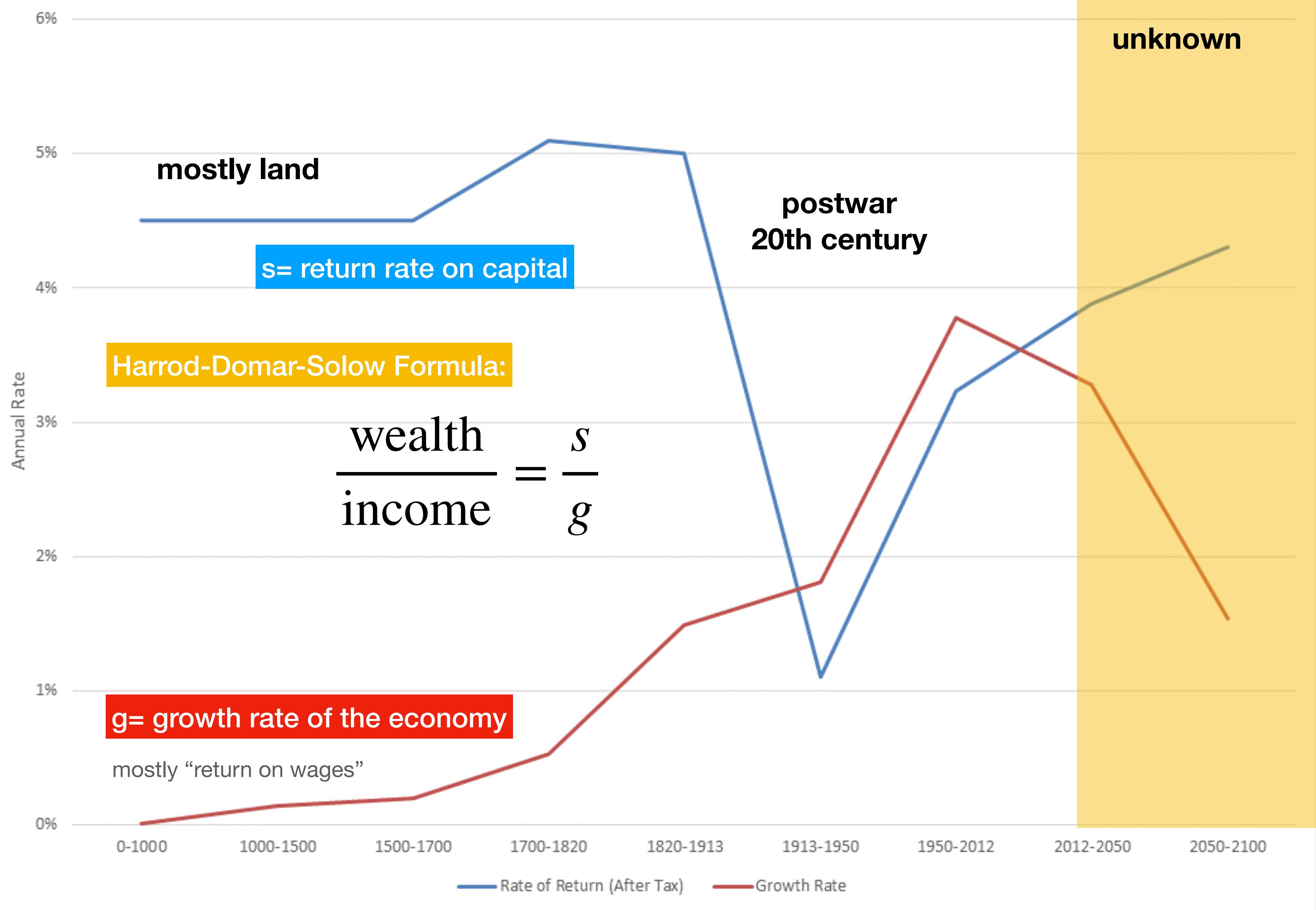
in the Twenty-First Century

THOMAS
PIKETTY

TRANSLATED BY ARTHUR GOLDHAMMER

2013

After Tax Rate of Return vs. Growth Rate (World)



But many sources and forms of inequality...

The American Economy Is Rigged

And what we can do about it

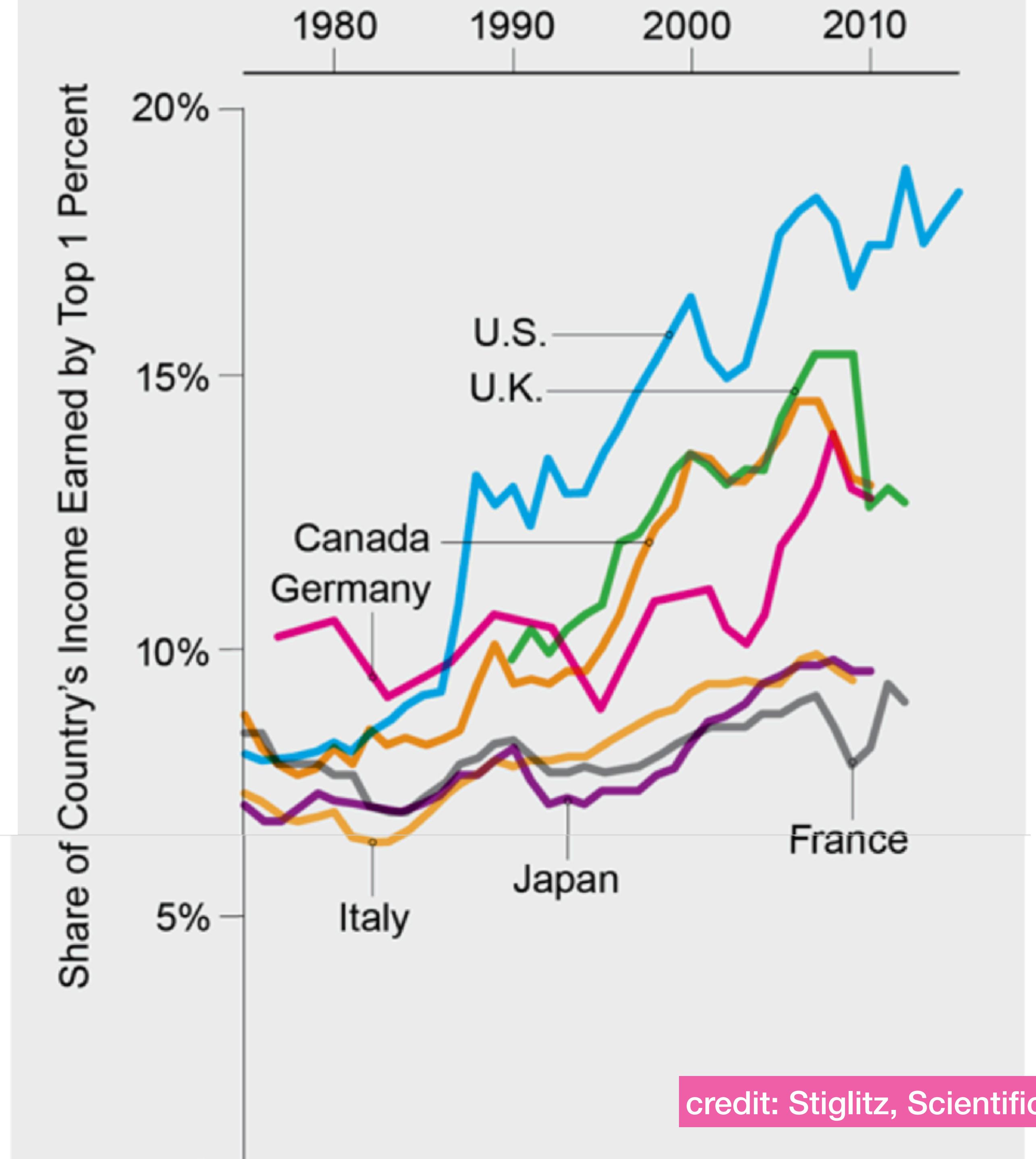
By Joseph E. Stiglitz | Scientific American November 2018 Issue



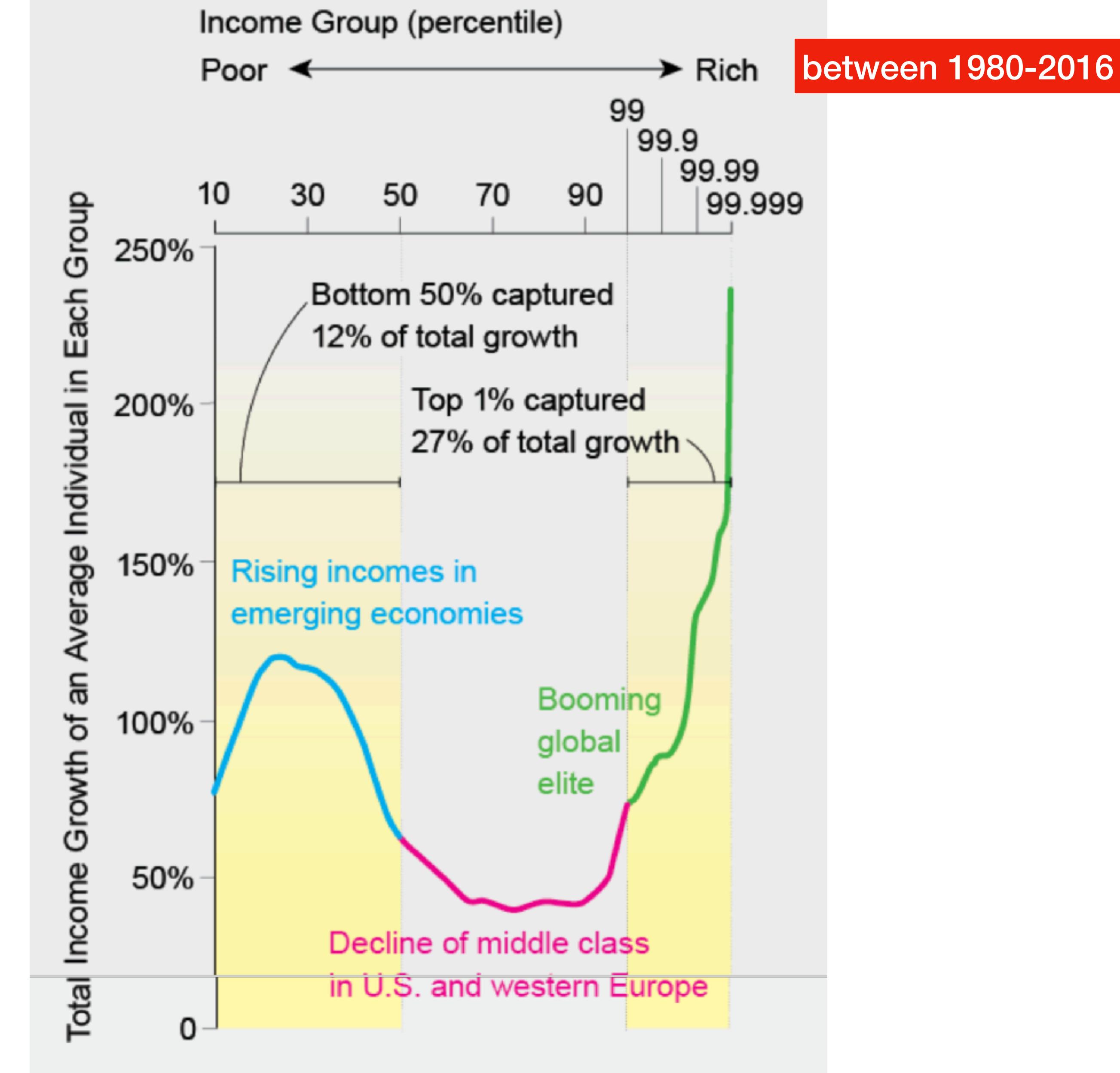
Joseph E. Stiglitz

Joseph E. Stiglitz is a University Professor at Columbia University and Chief Economist at the Roosevelt Institute. He received the Nobel prize in economics in 2001. Stiglitz chaired the Council of Economic Advisers from 1995–1997, during the Clinton administration, and served as the chief economist and senior vice president of the World Bank from 1997–2000. He chaired the United Nations commission on reforms of the international financial system in 2008–2009. His latest authored book is *Globalization and Its Discontents Revisited* (2017).

Credit: Nick Higgins



credit: Stiglitz, Scientific American (2018)



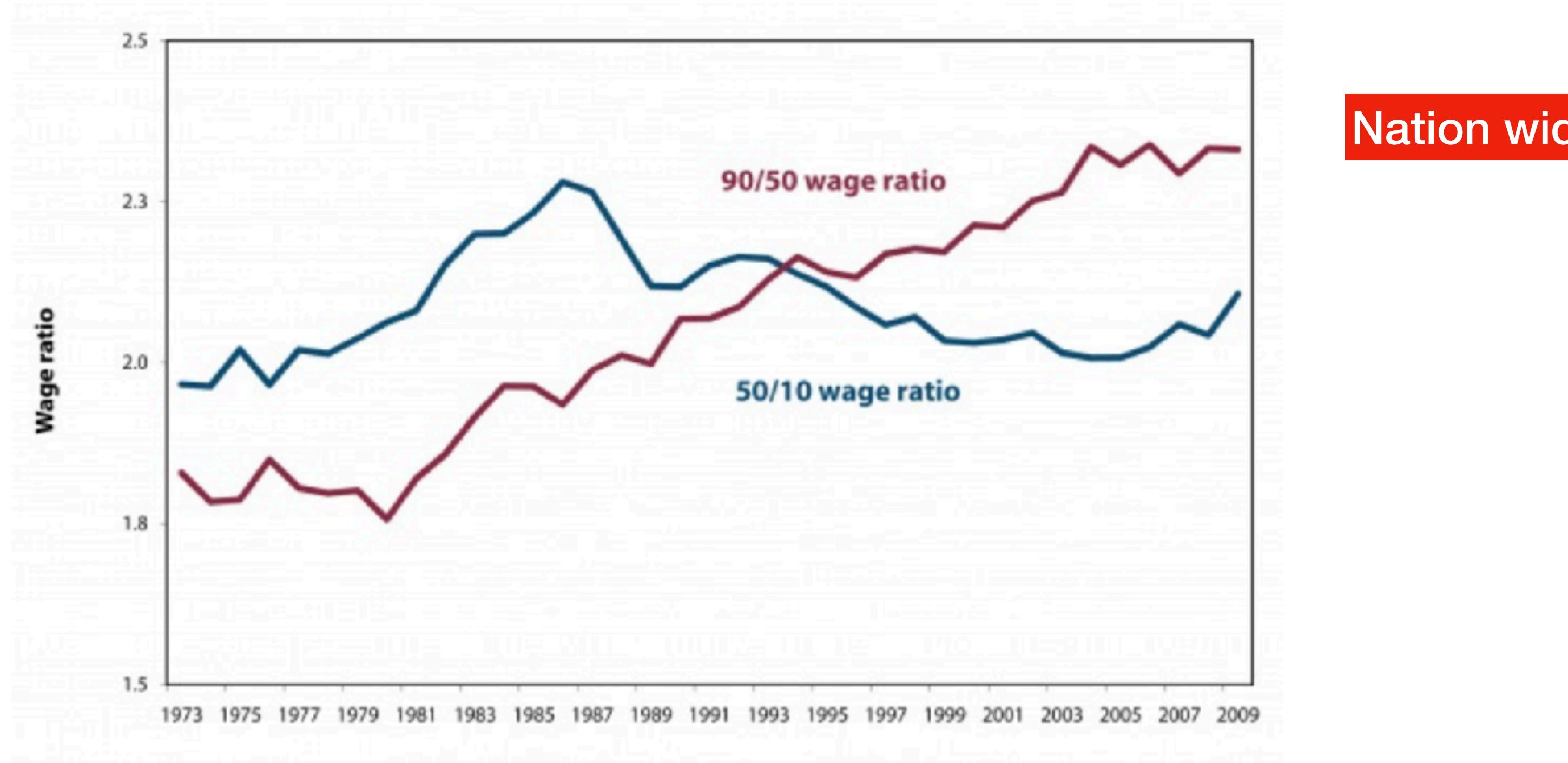
income = wages + capital rents

credit: Stiglitz, Scientific American (2018)

1. Wage Inequality

Over the last 30 years, wage inequality in the United States has increased substantially, with the overall level of inequality now approaching the extreme level that prevailed prior to the Great Depression. This general characterization of the inequality trend oversimplifies, though, the actual pattern of change: The chart below shows that the trend at the top of the income distribution (the “upper tail”) is not exactly the same as the trend at the bottom of the distribution (the “lower tail”). “Lower-tail” inequality is measured here by taking the ratio of wages at the middle of the income distribution (i.e., the 50th percentile) to those near the bottom of the distribution (i.e., the 10th percentile); “upper-tail” inequality is measured by taking the ratio of wages near the top of the distribution (i.e., the 90th percentile) to those at the middle of the distribution (i.e., the 50th percentile of workers). We find that lower-tail inequality rose sharply in the 1980s and contracted somewhat thereafter, while upper-tail inequality has increased steadily since 1980.

Men's wage inequality

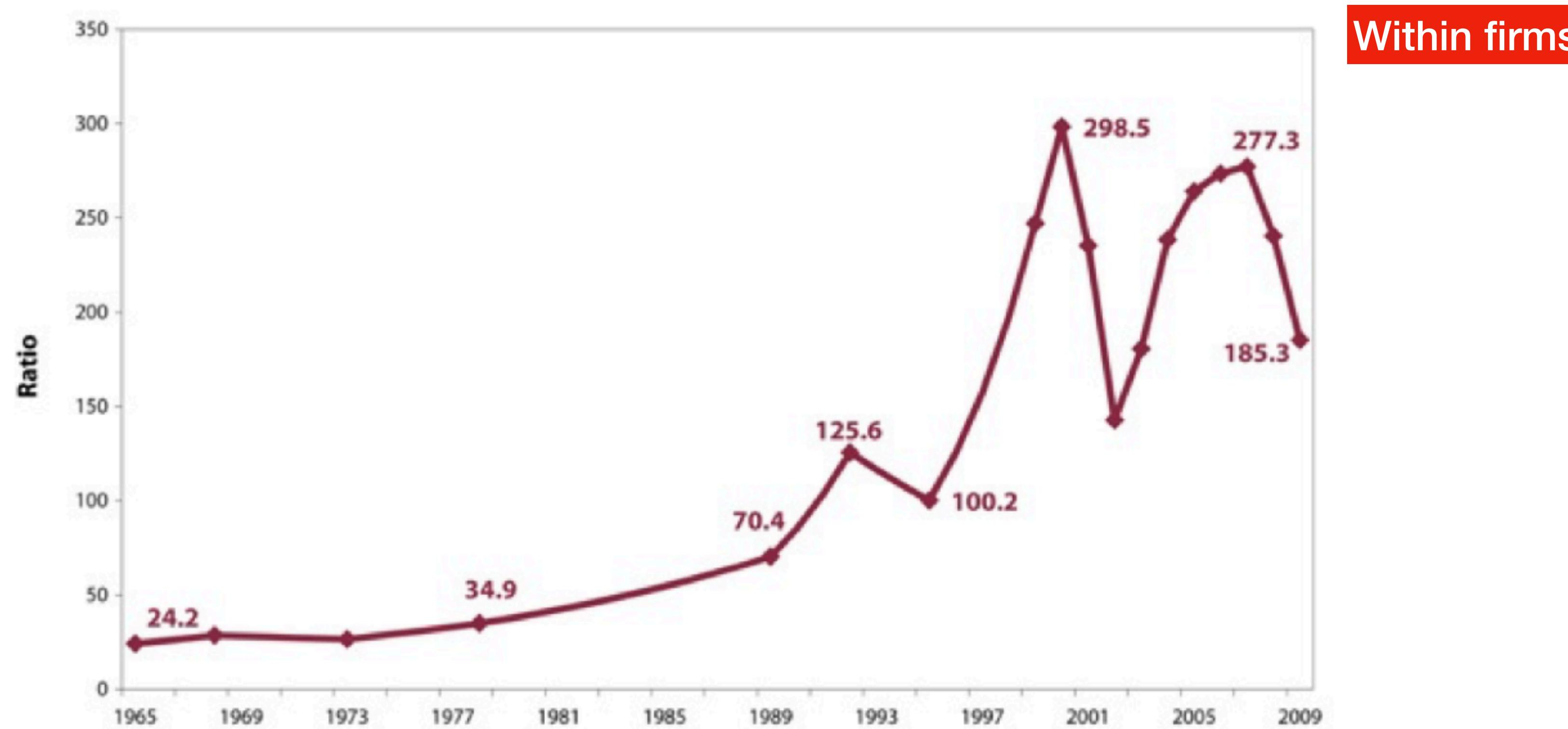


Source: Economic Policy Institute. 2011. “Upper Tail” inequality growing steadily: Men's wage inequality, 1973-2009. Washington, D.C.: Economic Policy Institute. May 11, 2011. <<http://www.stateofworkingamerica.org/charts/view/192>>

2. CEO pay

Recent decades have seen a clear increase in the difference between CEO compensation and that of the average worker in manufacturing ("production." CEOs in 1965 made 24 times more than the average production worker, whereas in 2009 they made 185 times more. This chart shows how this ratio between the compensation of CEOs and production workers took off in the 1980s.

U.S. CEO pay in relation to the average production worker's compensation



*Source: Economic Policy Institute. 2011. More compensation heading to the very top: Ratio of average CEO total direct compensation to average production worker compensation, 1965-2009. Washington, D.C.: Economic Policy Institute. May 16, 2011.
<<http://www.stateofworkingamerica.org/charts/view/17>>.*

FIGURE A

CEOs make 344 times as much as typical workers

CEO-to-worker compensation ratio, 1965–2022

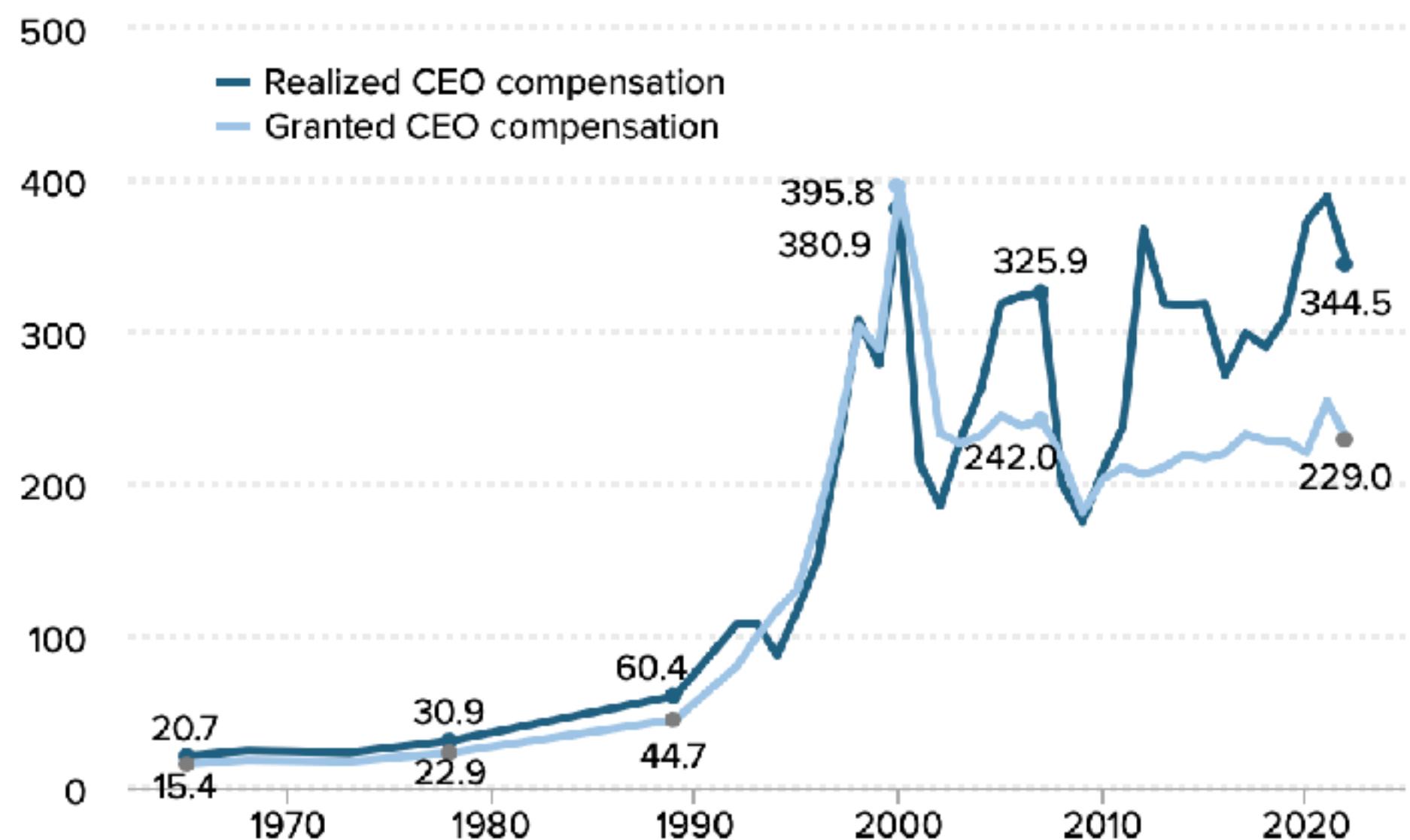


Chart Data

Notes: Average annual compensation for CEOs is for CEOs at the top 350 U.S. firms ranked by sales. Typical worker compensation is the average annual compensation (wages and benefits of a full-time, full-year worker) of production/nonsupervisory workers in the industries that the top 350 firms operate in.

Source: Authors' analysis of data from Compustat's ExecuComp database, the Bureau of Labor Statistics' Current Employment Statistics data series, and the Bureau of Economic Analysis NIPA tables.

Economic Policy Institute

The gap between productivity and a typical worker's compensation has increased dramatically since 1979

Productivity growth and hourly compensation growth, 1948–2024

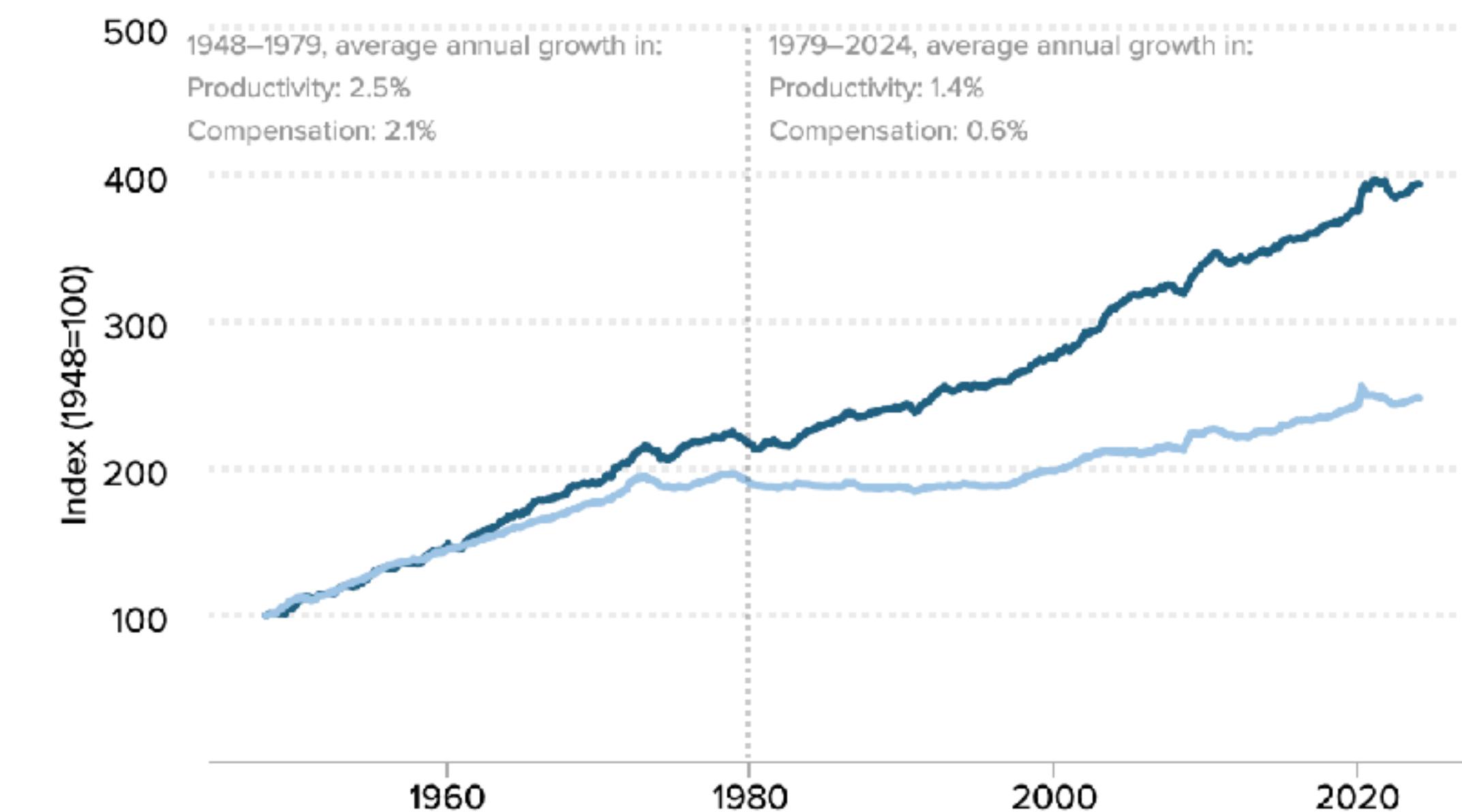


Chart Data

Notes: Data are for compensation (wages and benefits) of production/nonsupervisory workers in the private sector and net productivity of the total economy. "Net productivity" is the growth of output of goods and services less depreciation per hour worked.

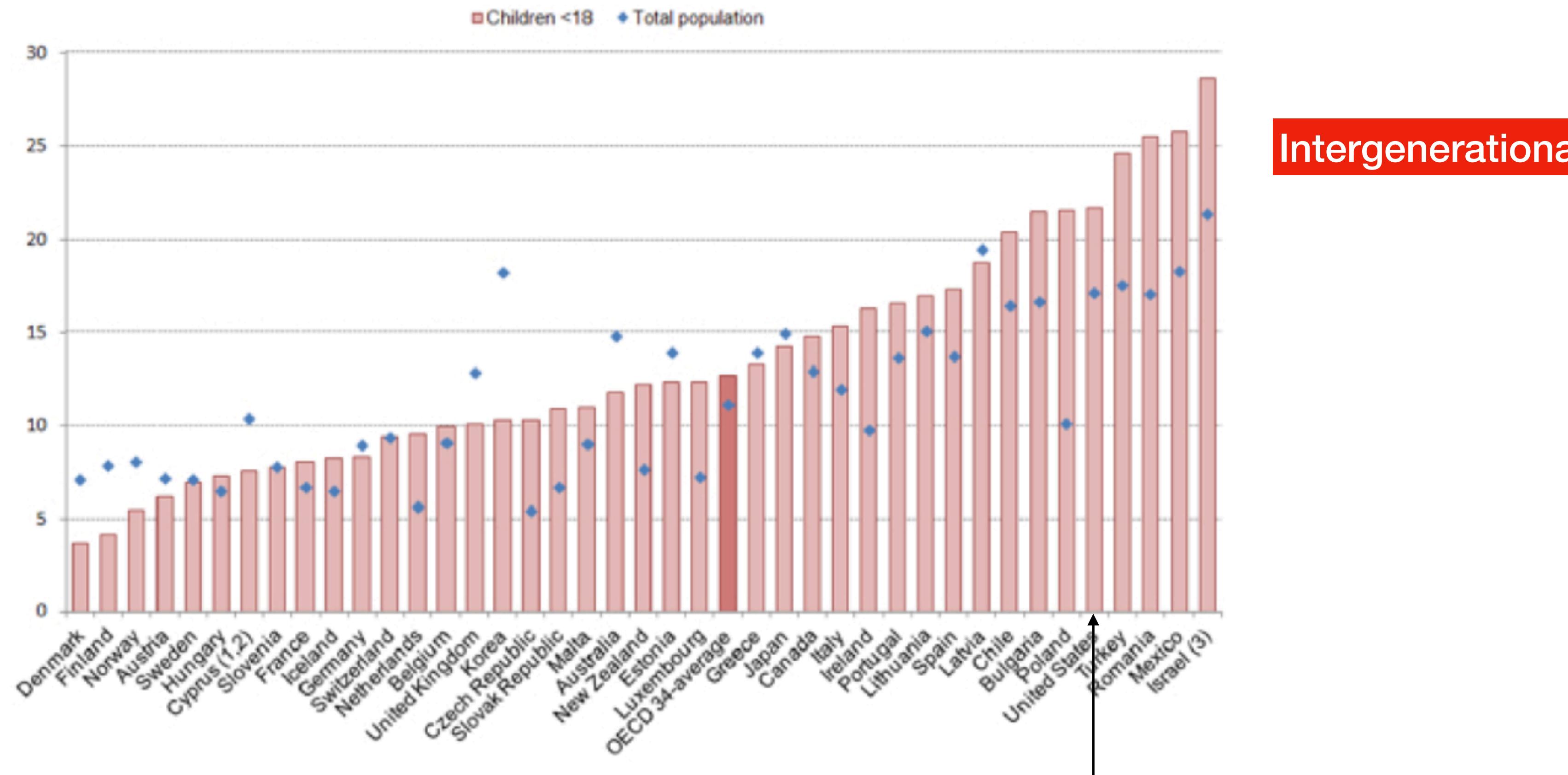
Source: EPI analysis of unpublished Total Economy Productivity data from Bureau of Labor Statistics (BLS) Labor Productivity and Costs program, wage data from the BLS Current Employment Statistics, BLS Employment Cost Trends, BLS Consumer Price Index, and Bureau of Economic Analysis National Income and Product Accounts.

Economic Policy Institute

9. Child Poverty

In the United States, 21 percent of all children are in poverty, a poverty rate higher than what prevails in virtually all other rich nations.

Relative Poverty Rates in Forty Nations in the Mid-to-Late 2000s



Source: *OECD Income Distribution questionnaire, February 2011*. Data refer to 2008 for Germany, Israel, Italy, Korea, Mexico, Netherlands, New Zealand, Norway, Sweden and the United States; 2007 for Canada, Denmark and Hungary; 2006 for Chile, Estonia, Japan and Slovenia; 2005 for France, Ireland, Switzerland and the United Kingdom; 2004 for Australia, Austria, Belgium, Czech Republic, Finland, Greece, Iceland, Luxembourg, Poland, Portugal, the Slovak Republic, Spain and Turkey.

If you are interested in the general moral philosophy of inequality, I recommend:

In 1998, Sen received the Prize in Economic Sciences in Memory of Alfred Nobel for his theoretical, field, and ethics work in welfare economics and for his research advancing the understanding of social-choice theory, poverty, and the measurement of welfare. Jun 3, 2021



Equality of What?

AMARTYA SEN

THE TANNER LECTURE ON HUMAN VALUES

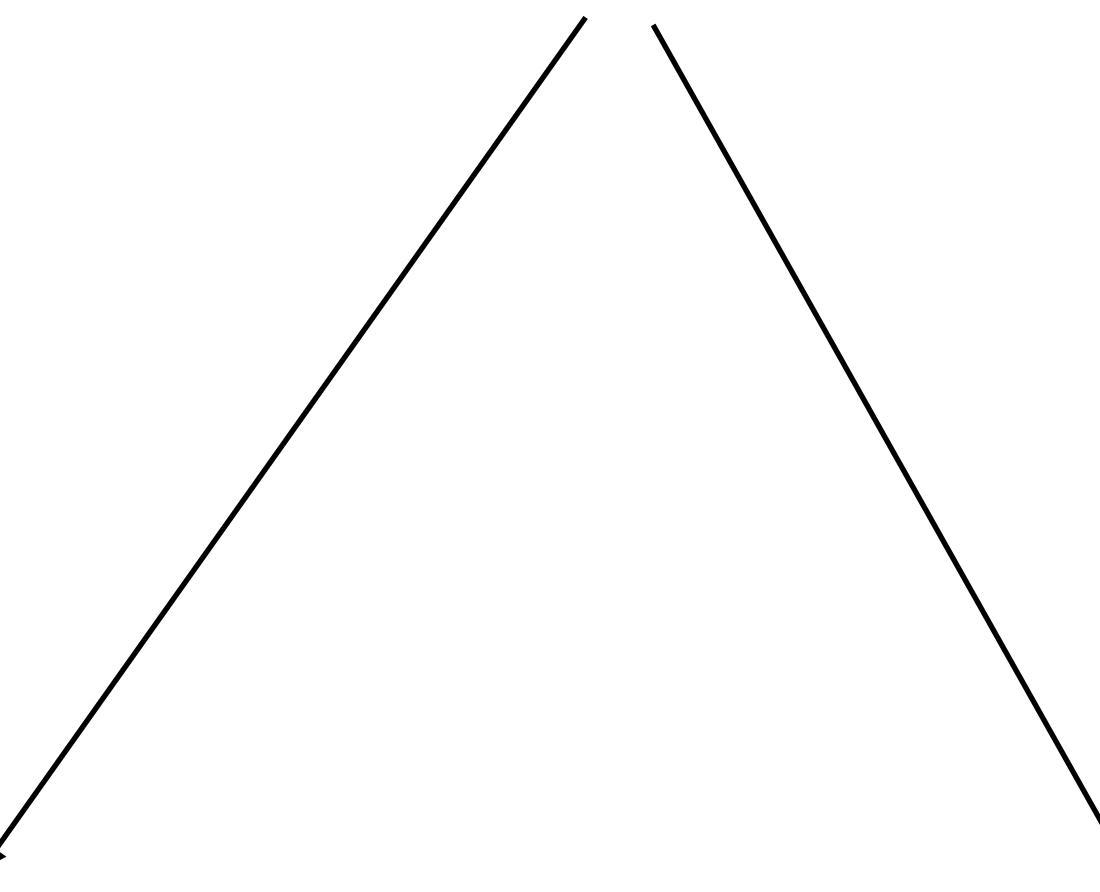
In your reading list (optional)

Origins of wealth inequality

Two Different Issues:

Piketty, Bowles,...

Marx, Stieglitz, ...



Transmission of wealth over time

Allocation of production returns as Income

wealth “dynamics”

income: labor vs capital

Policy solutions :
depreciation, expropriation
government: inheritance taxes

firms
government: redistribution, social services

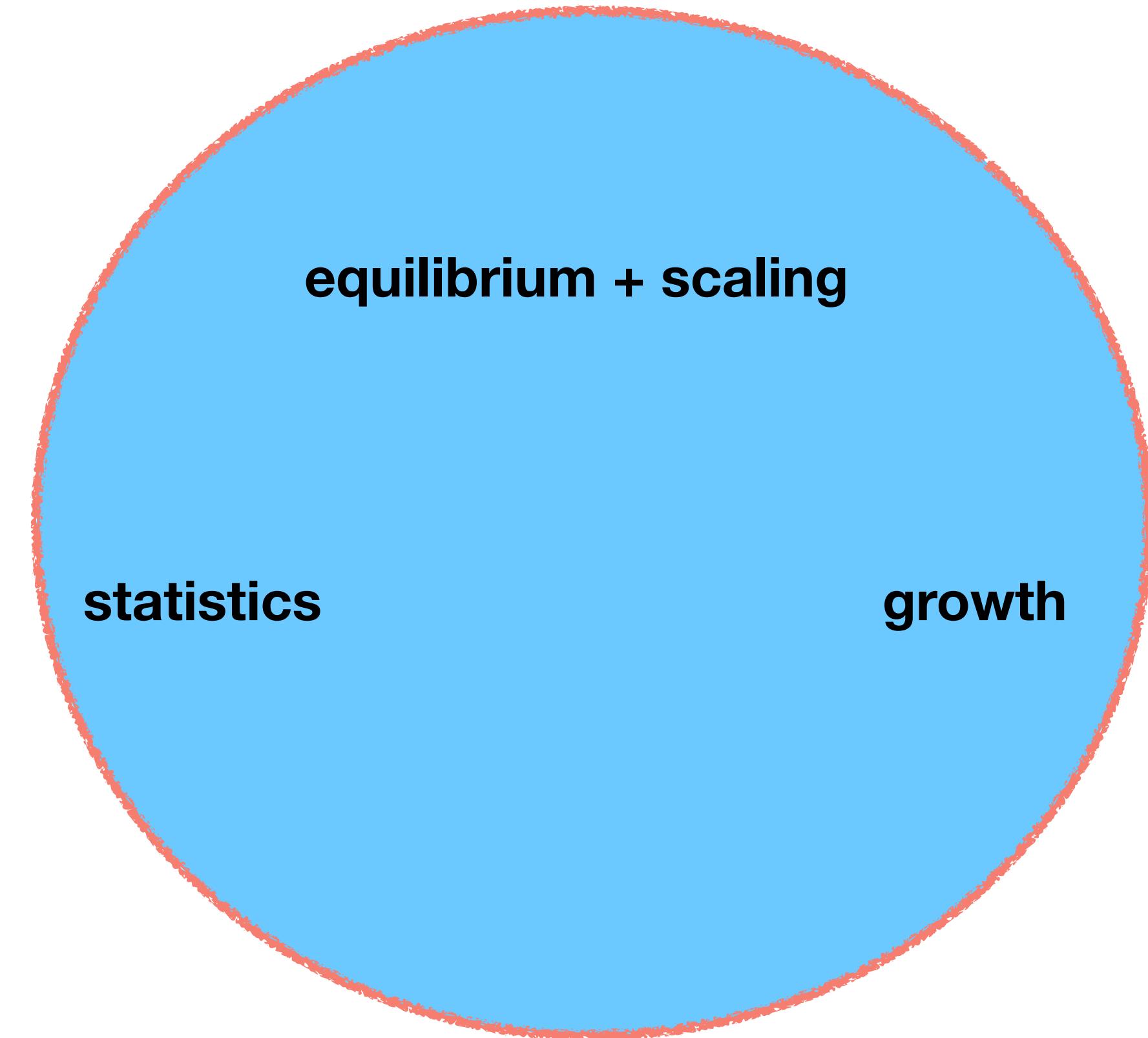


News

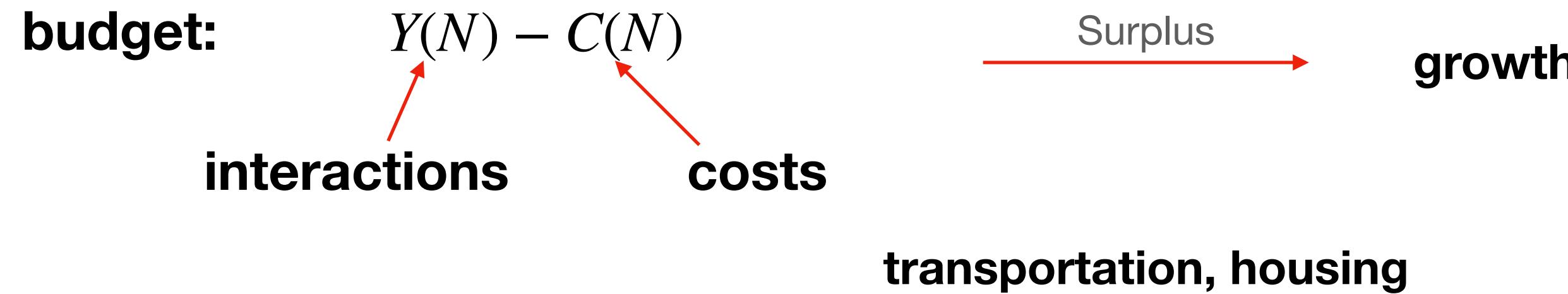
Stone Center Premieres "The Inequality Podcast"

New podcast discusses the causes and consequences of inequality and strategies to promote economic mobility.

Where does this all fit with Cities?



Economic Growth and Cities: Surplus from spatial “equilibrium”



1. The Problem of Statistics

diversity, inequality: broad distributions of wealth

2. The Problem of Growth

cities are not static: they grow at a few percent a year!! good or bad?

Recall:

Economic Growth

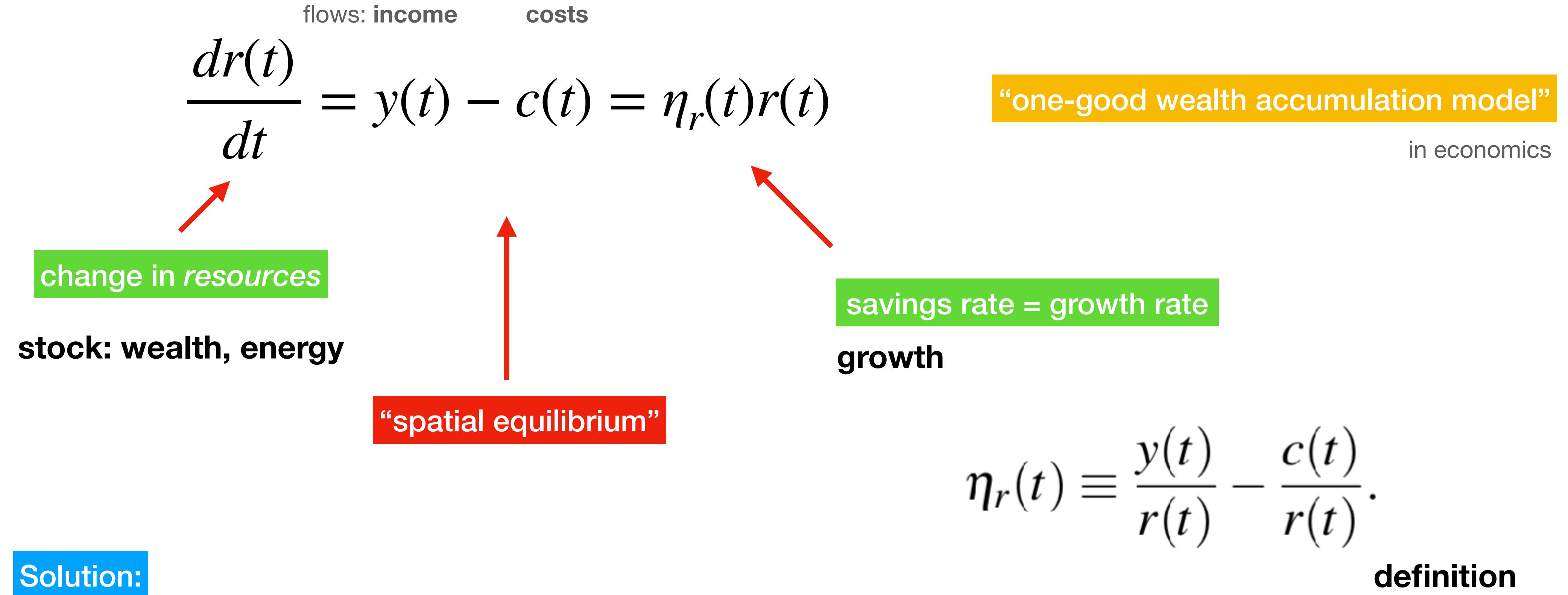
where do economic growth rates come from?

$$\frac{dY}{dt} = \eta Y$$

Exponential Growth: 2-3% a year for USA

noisy, scale dependent, variable
↑
inequality

A Simple Model of Resource Dynamics



$$\ln \frac{r(t)}{r(0)} = \left(\bar{\eta}_r - \frac{\sigma_r^2}{2} \right) t + \Theta(t),$$

noise: $\Theta(t) = \sum_{l=1}^t \varepsilon_r(t_l), \quad \longrightarrow \quad \text{Normal ‘bell curve’}$

because of the central limit theorem

The counter intuitive consequences of stochastic geometric growth

Consequences:

zero average, standard deviation: $\sigma_\Theta = \sigma_r \sqrt{t}$

1.

$$\ln \frac{r(t)}{r(0)} = \left(\bar{\eta}_r - \frac{\sigma_r^2}{2} \right) t + \Theta(t),$$

$$\Theta(t) = \sum_{l=1}^t \varepsilon_r(t_l) \sim \sigma_r \sqrt{t}$$

Central Limit Theorem -> Gaussian

lognormal statistics !!

2.

$$\frac{1}{t} \ln \frac{r(t)}{r(0)} = \gamma_r + \frac{\Theta(t)}{t} \rightarrow \gamma_r,$$

The average growth rate over long times becomes a simple number

ergodicity

3.

$$t_* = \frac{\sigma_r^2}{\left(\bar{\eta}_r - \frac{\sigma_r^2}{2} \right)^2},$$

There is a minimum time to see through the “fog” of short term fluctuations.

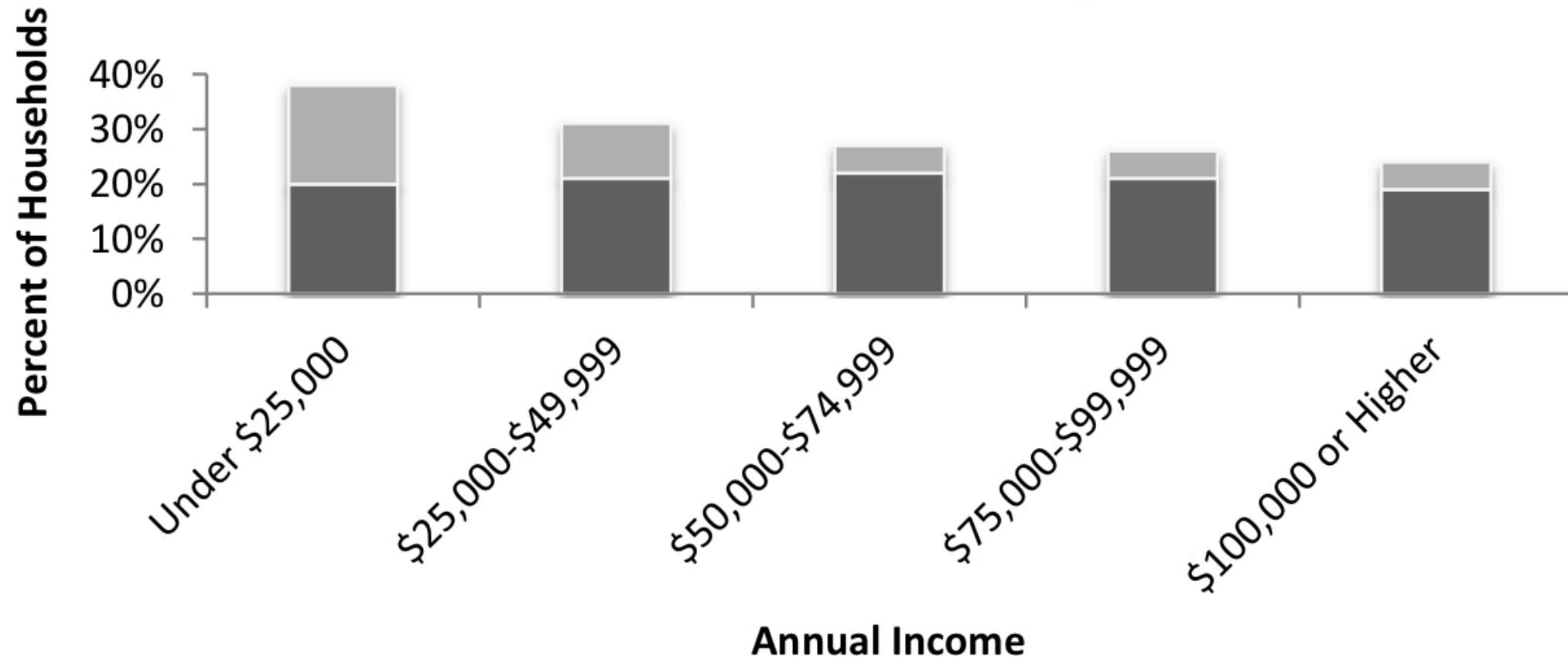
proportional to the uncertainty (volatility, std) between relative income and costs

people with a lot of uncertainty do not see growth as a possibility

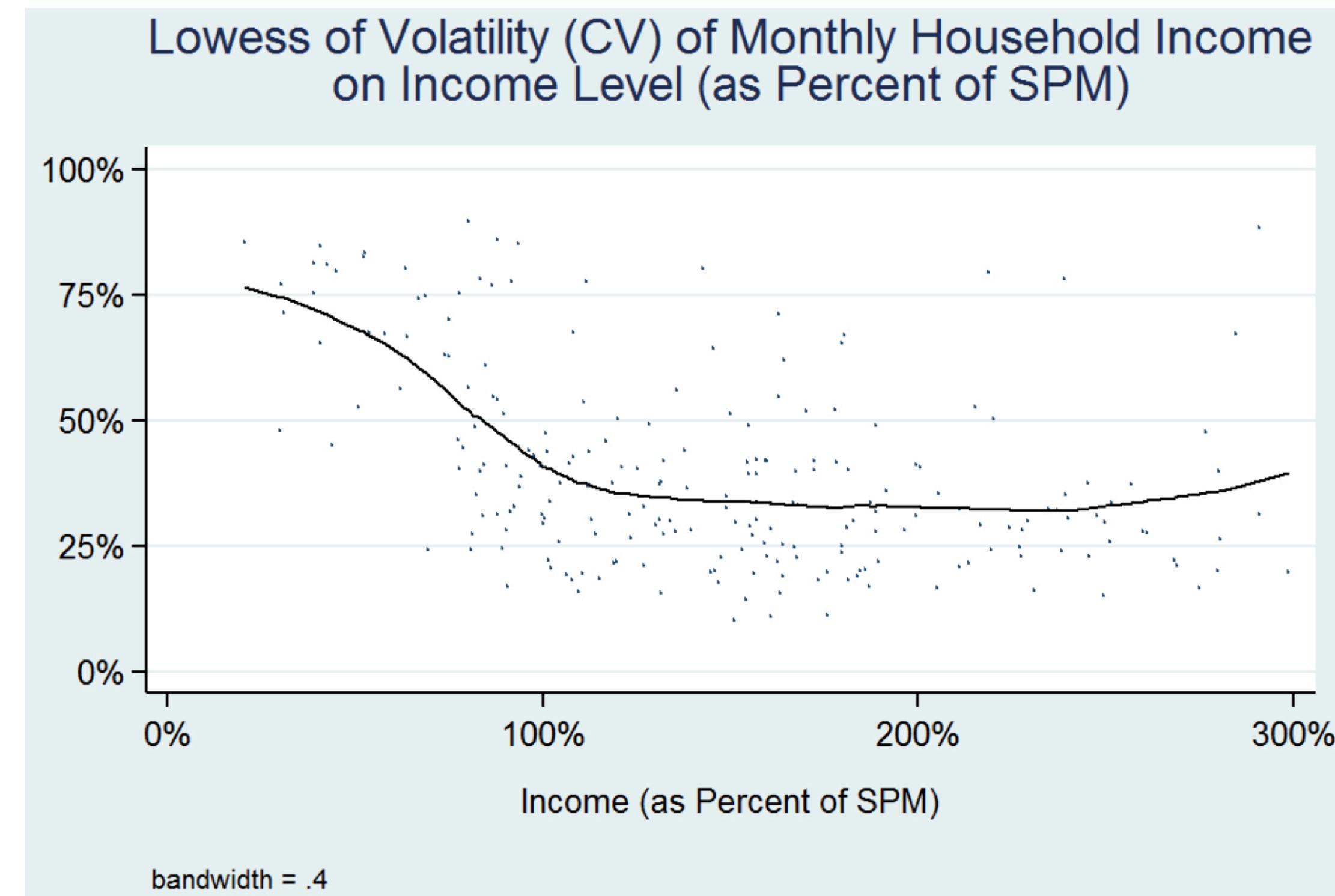
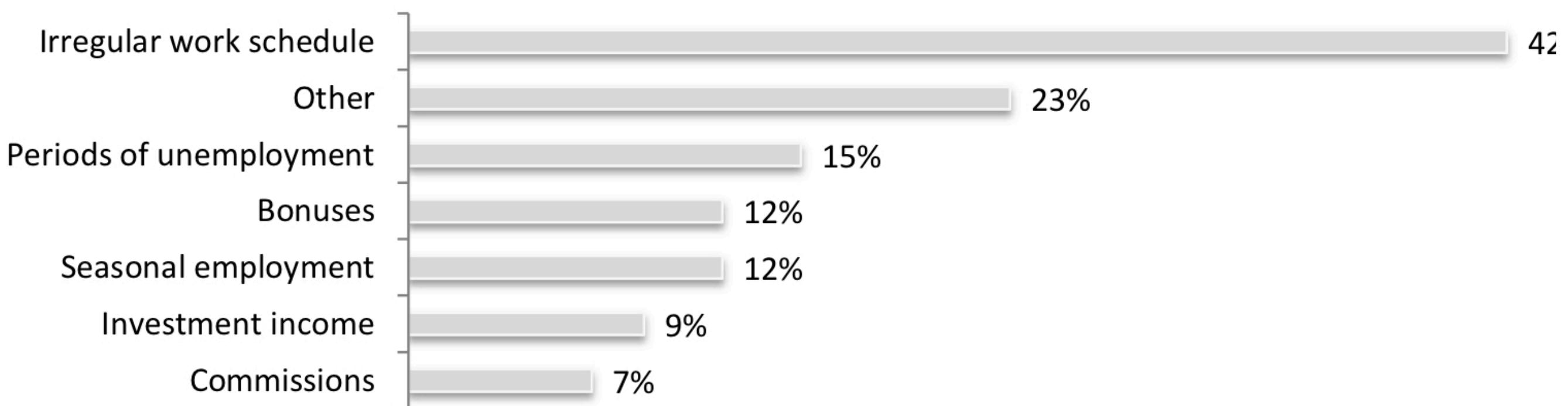
The working poor experience much more volatility

lower effective growth rates, unclear horizons

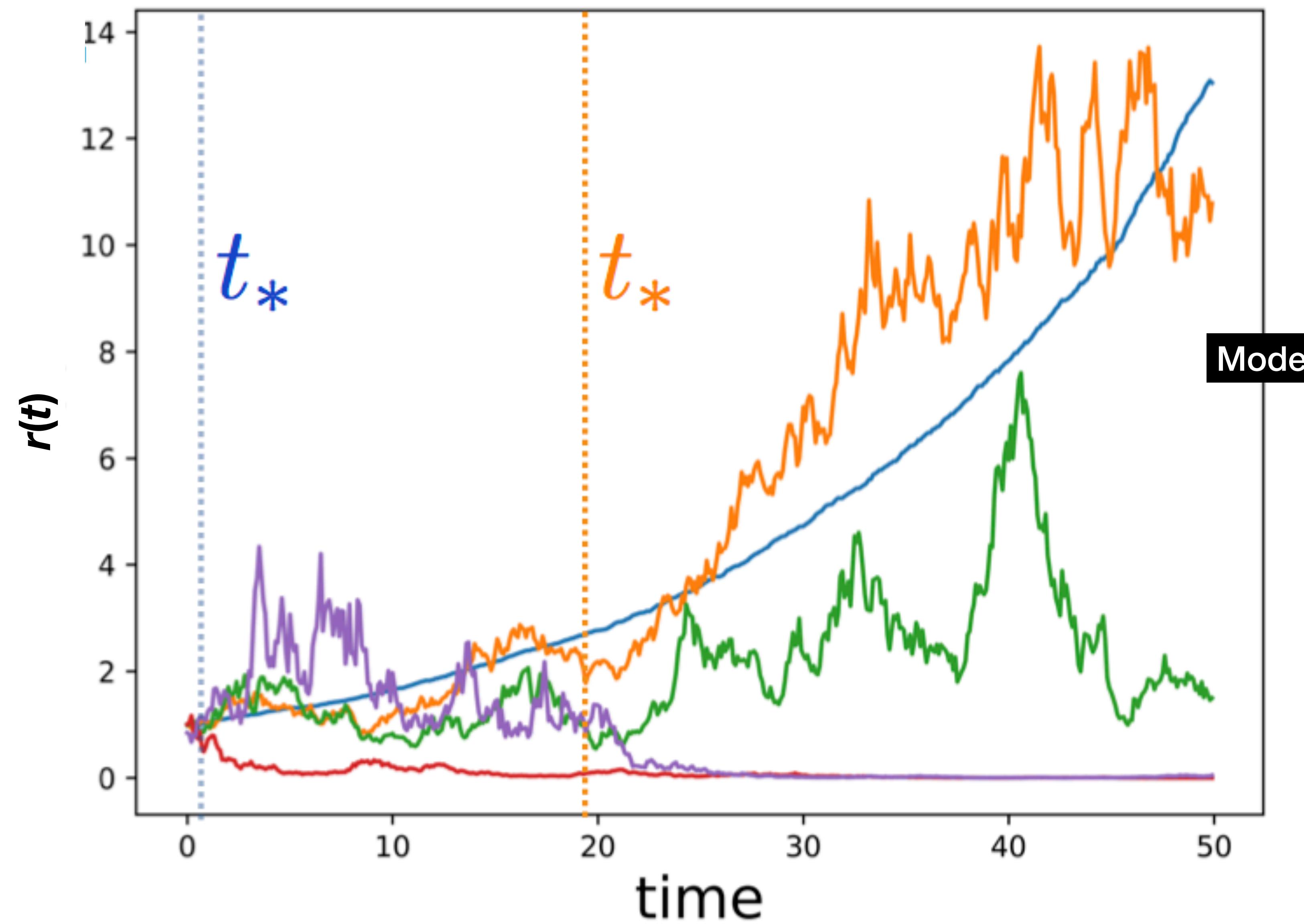
Income Variability is More Prevalent in the Lower-Income Groups
Federal Reserve SHED Survey 2013



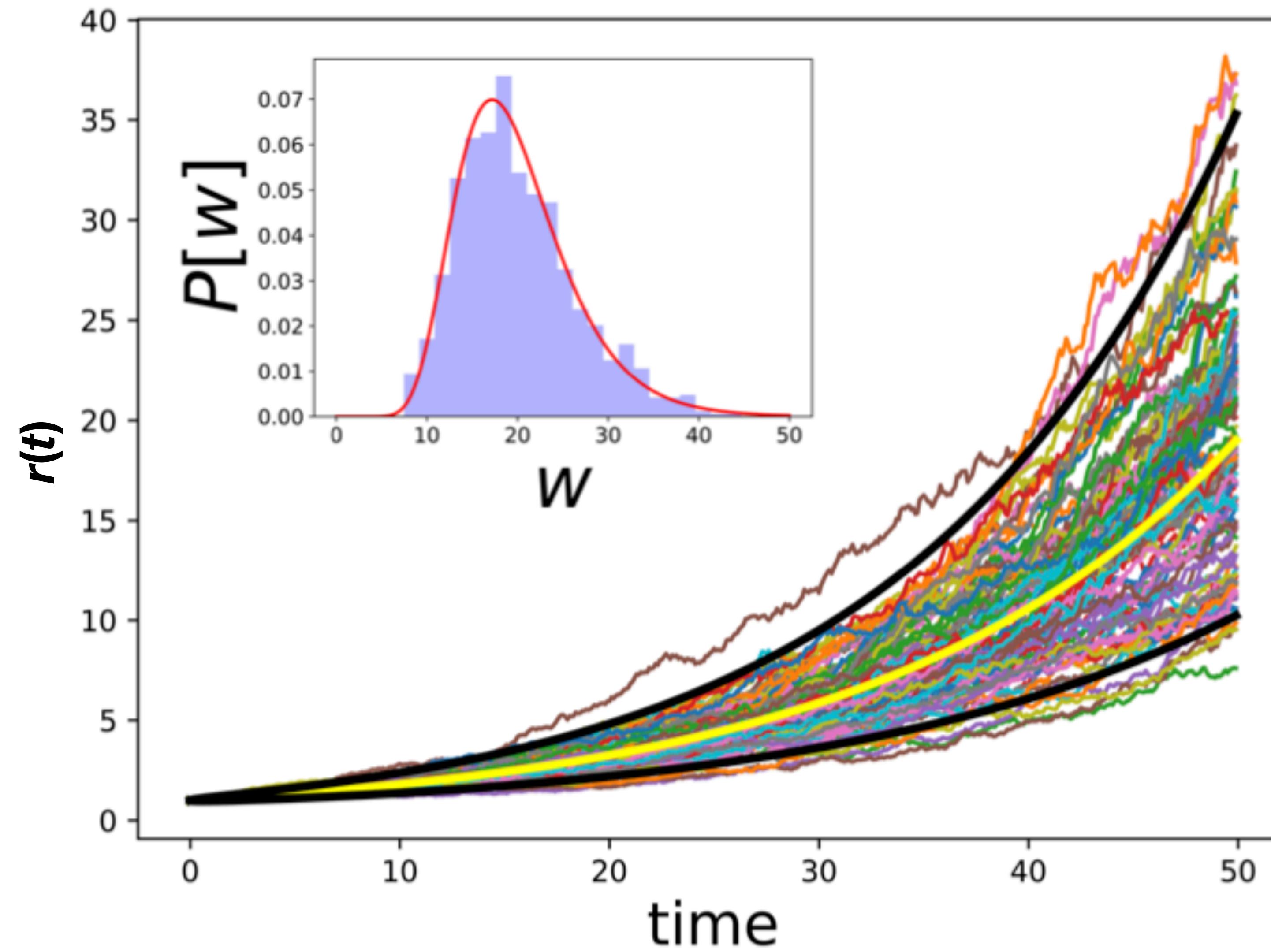
Why does your income change from month to month?
Federal Reserve SHED Survey 2013



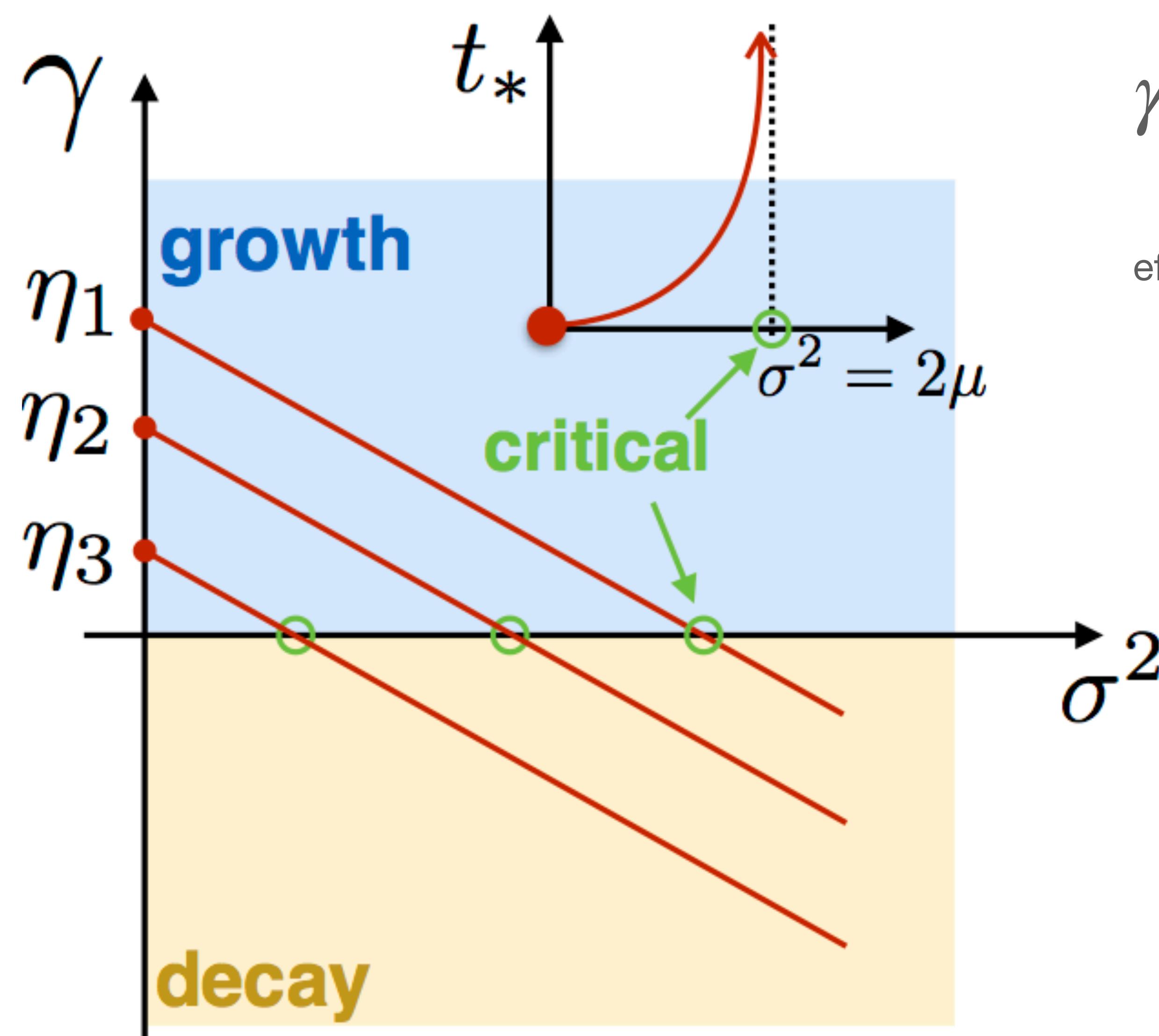
Model Solutions look like this



Log normal statistics



Net Growth Requires Low Fluctuations



$$\gamma = \eta - \frac{\sigma^2}{2}$$

effective growth rate

Want low fluctuations:

$$\eta_r(t) \equiv \frac{y(t)}{r(t)} - \frac{c(t)}{r(t)} = b(t) - a(t) = \bar{\eta} + u(t) - v(t) = \bar{\eta} + \epsilon(t)$$

The diagram illustrates the decomposition of income fluctuations. A red arrow points from the term $\eta_r(t)$ to the right side of the equation. This arrow branches into two paths: one leading to the term $u(t) - v(t)$ and another leading to the term $\bar{\eta}$. The label "definition" is placed under the first path, and "choice" is placed under the second path. The label "income fluctuations" is centered between the two paths.

when I have more money can spend more and vice-versa

Consumption Smoothing

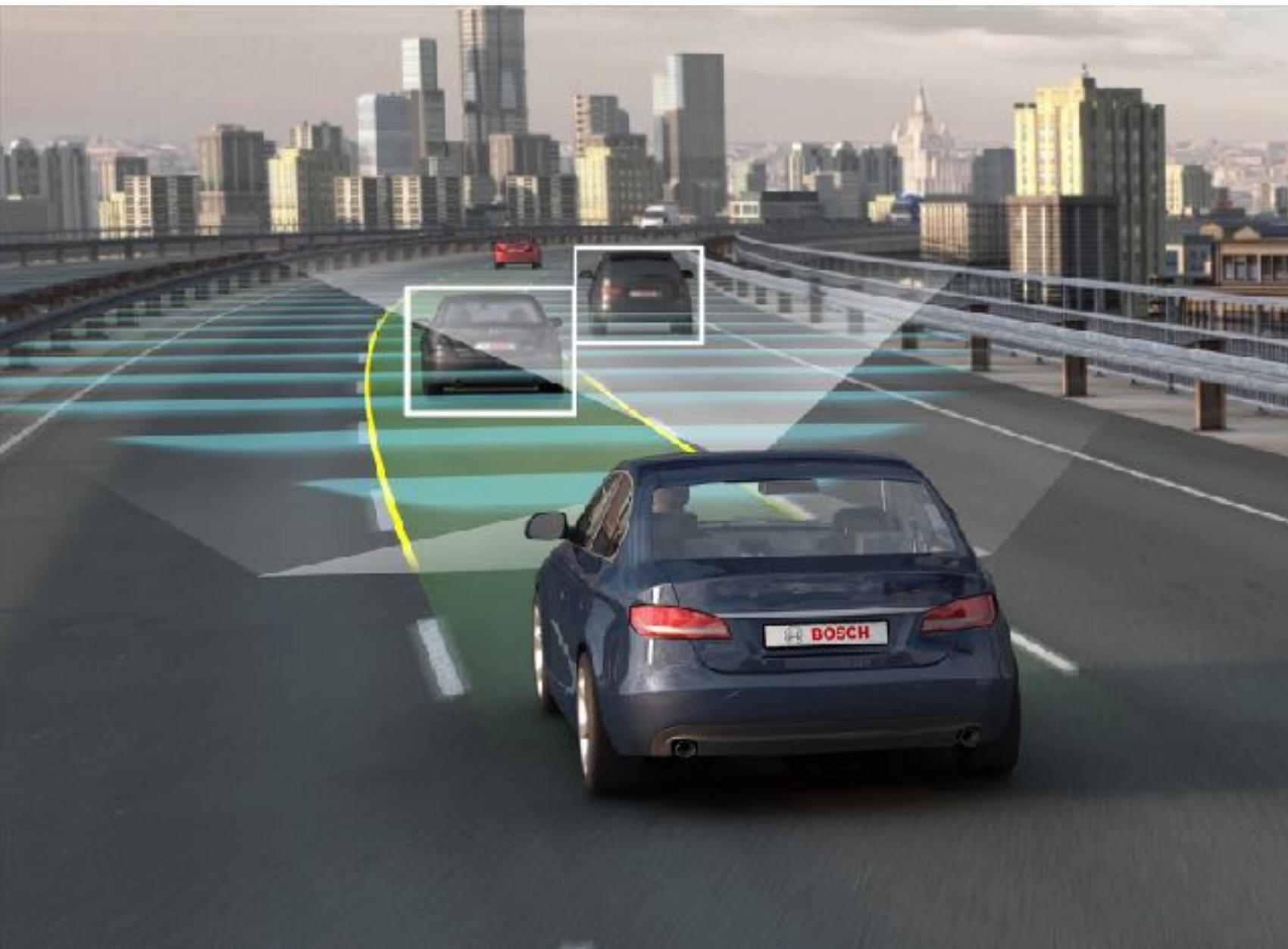
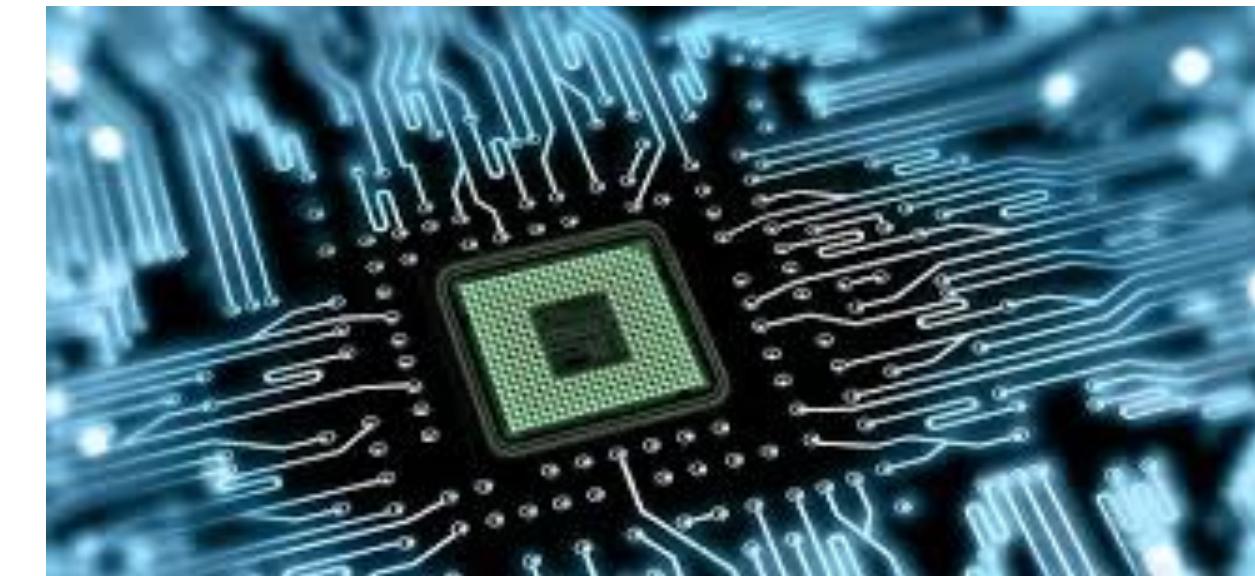
**need to average (smoothen) expenditures and incomes over some time
month, season, year**

feedback control theory

Feedback Control is a big part of your life



Feedback Control is a big part of your life



How Control theory works:

PID Controller

$$u(t) = k_P \varepsilon_r(t) + k_I \int_0^t \varepsilon_r(t') dt' + k_D \frac{d\varepsilon_r}{dt}, \quad \text{Time costs to track income fluctuations}$$

This gives a simple dynamics for errors

$$\cancel{\frac{d^2 \varepsilon_r}{dt^2}} + 2\zeta \omega \frac{d\varepsilon_r}{dt} + \omega^2 \varepsilon_r = F/m, \quad m = k_D, \omega = \sqrt{k_I/k_D} \text{ and } \zeta = \frac{k_P+1}{2\sqrt{k_D k_I}},$$

noisy spring

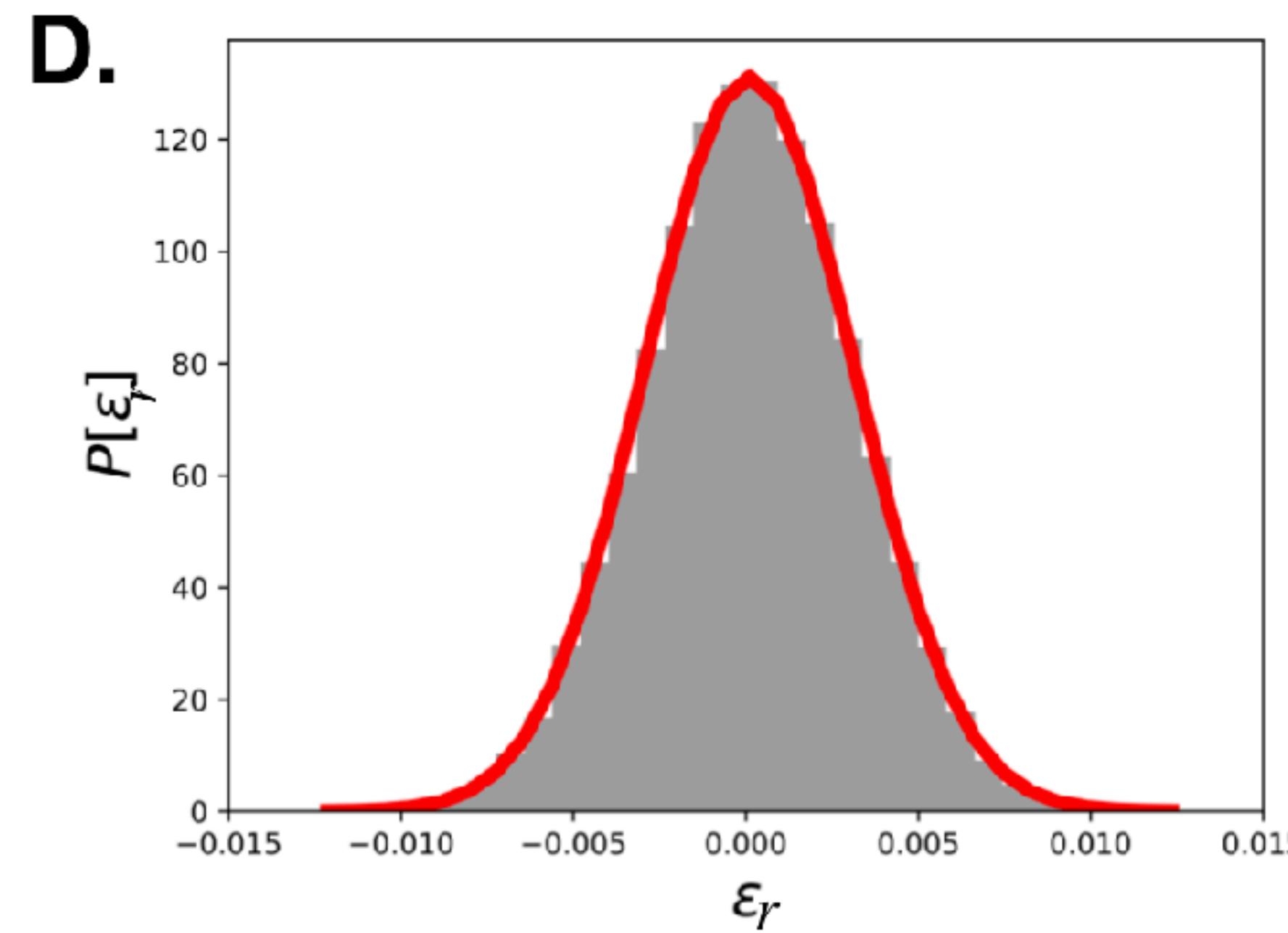
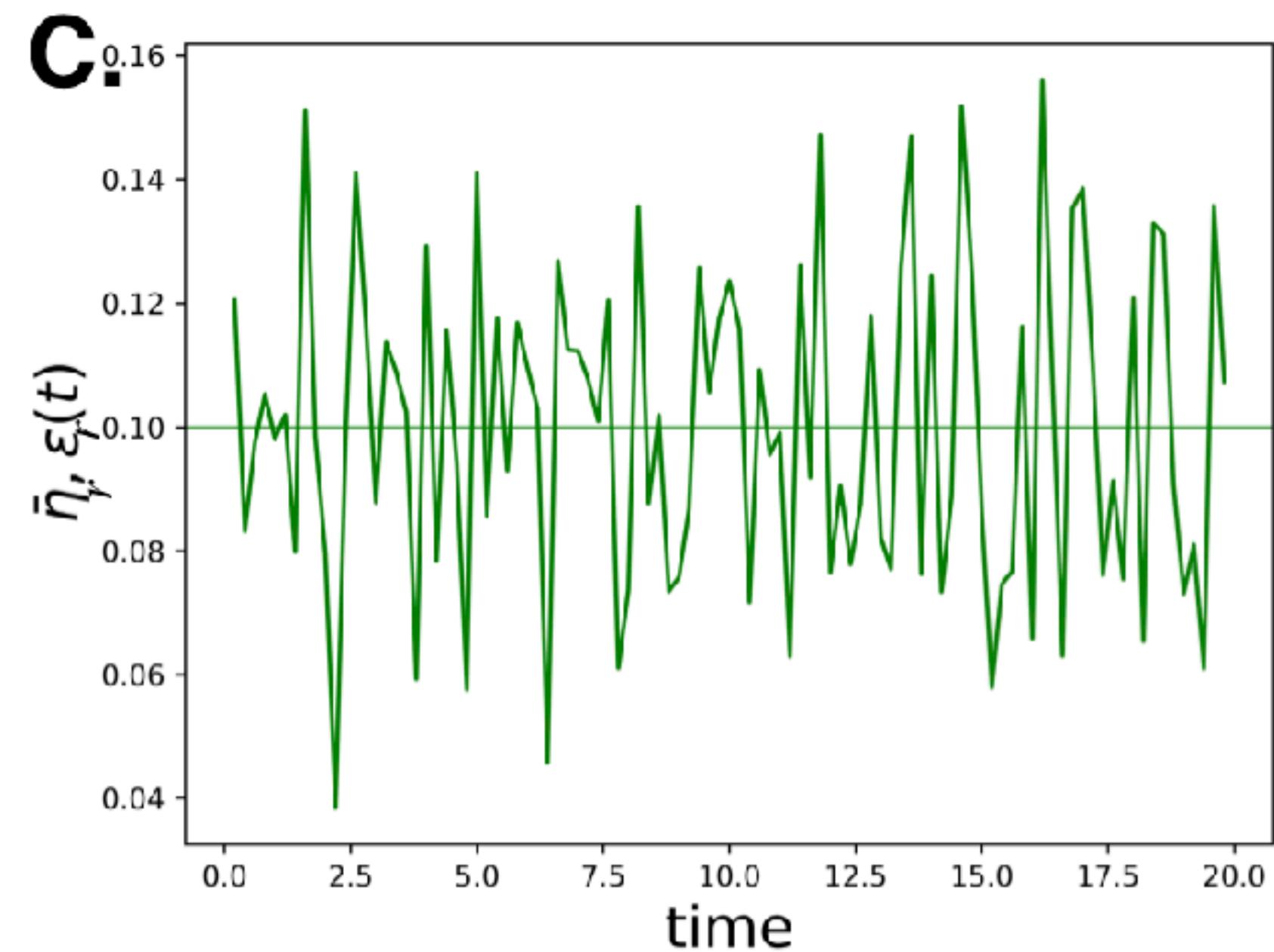
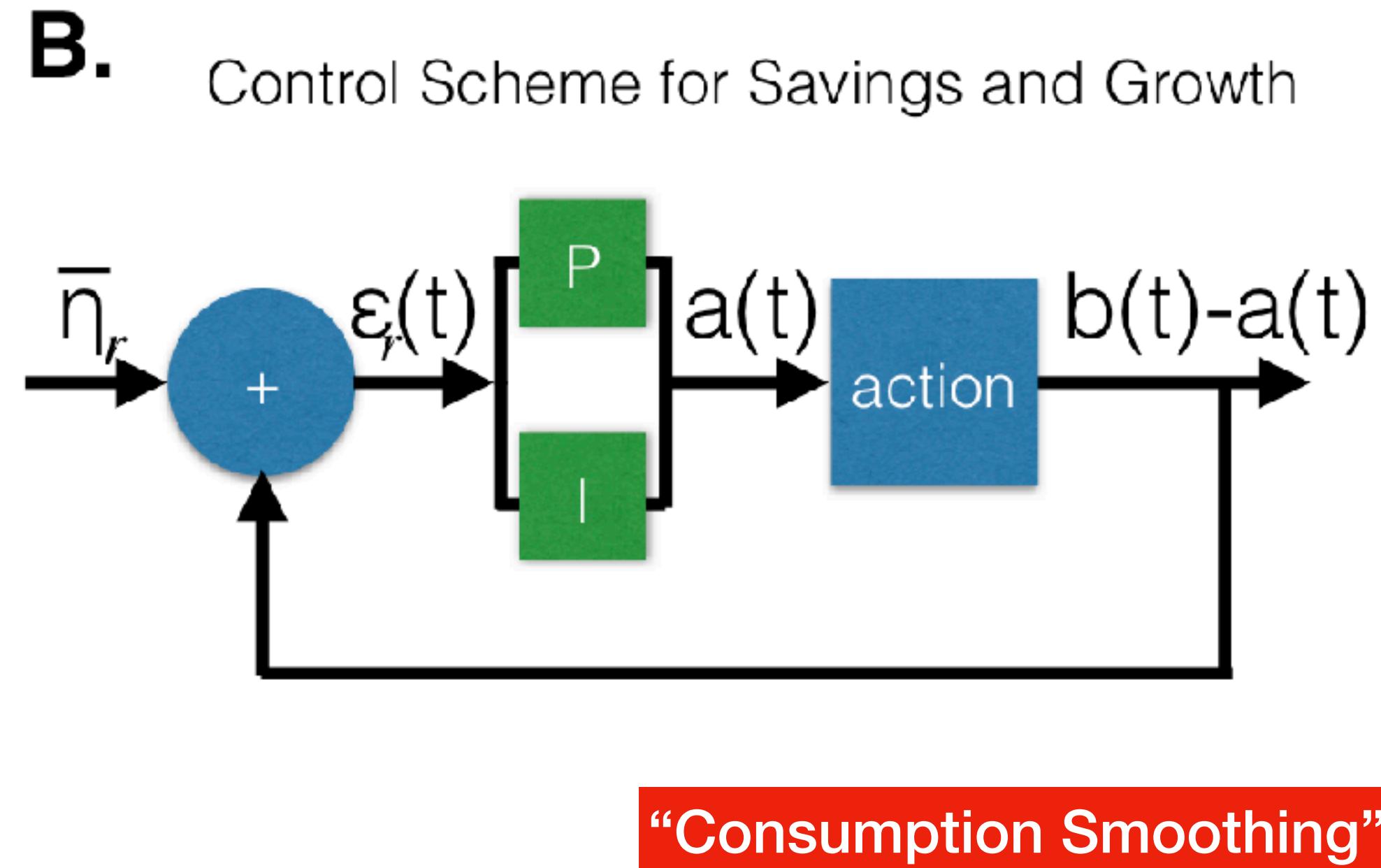
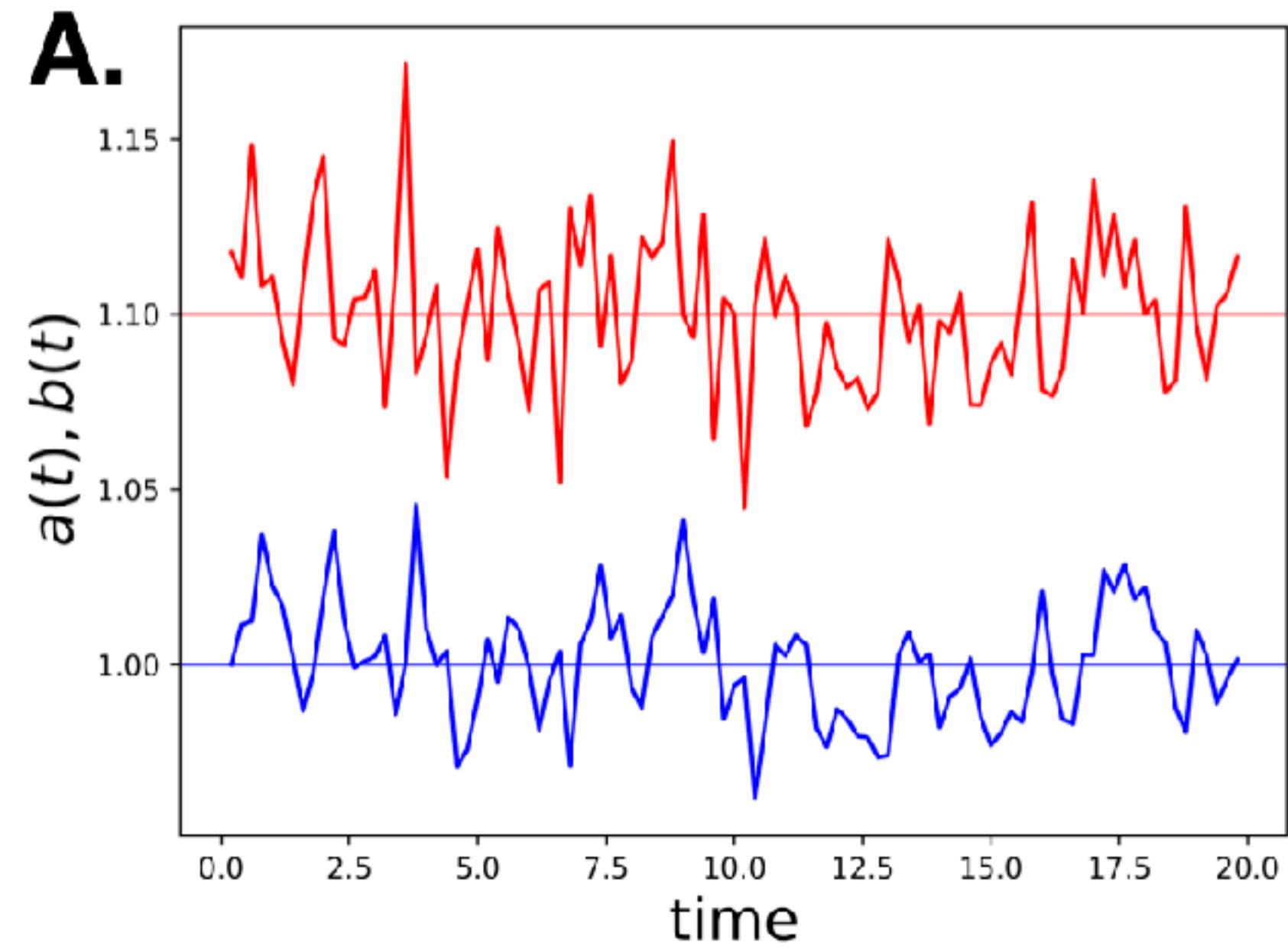
$$d\varepsilon_r = -\frac{M}{2} \varepsilon dt + \Omega dW(t).$$

error wants to vanish noise

$$M = \omega/\zeta = k_I/[2(k_P + 1)]$$

Statistics of error is “under control”:

$$P[\varepsilon_r, t | \varepsilon_0] = \sqrt{\frac{M}{2\pi\Omega^2(1-e^{-Mt})}} e^{-\frac{M}{2\Omega^2} \left[\frac{(\varepsilon_r - \varepsilon_0 e^{-M/2} t)^2}{1-e^{-Mt}} \right]} \rightarrow \sqrt{\frac{M}{2\pi\Omega^2}} e^{-\frac{M}{2\Omega^2} \varepsilon_r^2},$$



Growth rates for individuals and for cities

Is the growth rate for a city the average of the growth rates for individuals?

Not in general:

$$r_G = \frac{1}{G} \sum_{j=1}^G r_j \quad \text{average resources}$$

$$\frac{dr_G}{dt} = y_G - c_G = \underline{(\eta r)_G}, \quad \text{growth of resources per capita in a population}$$

average of product

$$(\eta r)_G = \frac{1}{G} \sum_{j=1}^G \eta_j w_j = \eta_G r_G + \text{covar}_G(\eta, r) = [\eta_G + \text{covar}_G(\eta, r/r_G)] r_G.$$

average rate

“selection”

Consequences:

$$(\eta r)_G = \frac{1}{G} \sum_{j=1}^G \eta_j w_j = \eta_G r_G + \text{covar}_G(\eta, r) = [\eta_G + \text{covar}_G(\eta, r/r_G)] r_G.$$

positive: richer people get higher growth rates

correlation between resources and rates

negative: richer people get lower growth rates

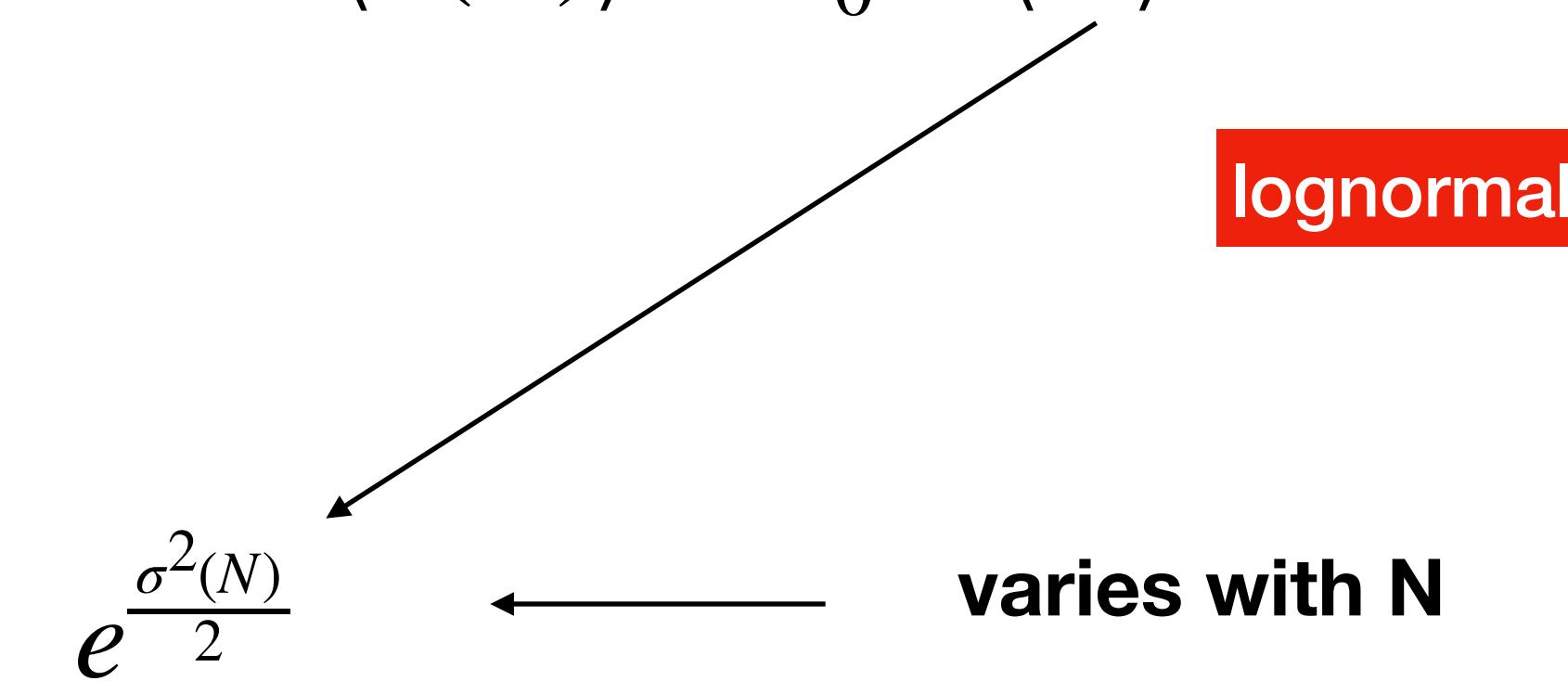
anti-correlation between resources and rates

What situation gives the highest overall growth rates?

What situation gives the lowest inequality?

Corrections to scaling exponents

$$Y(N) = Y_0 N^\beta e^\xi \rightarrow \langle Y(N) \rangle = Y_0 N^\beta \langle e^\xi \rangle$$

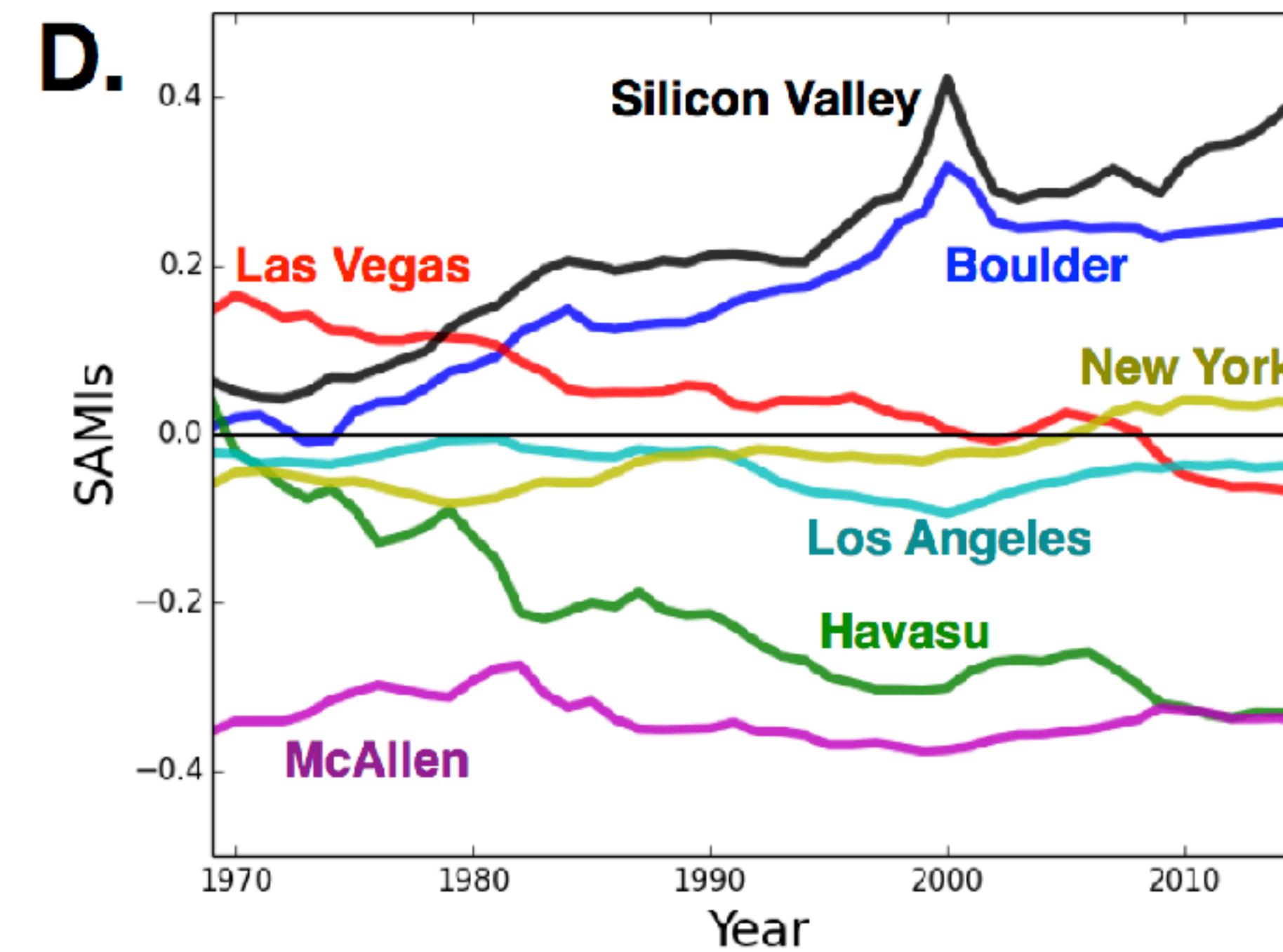
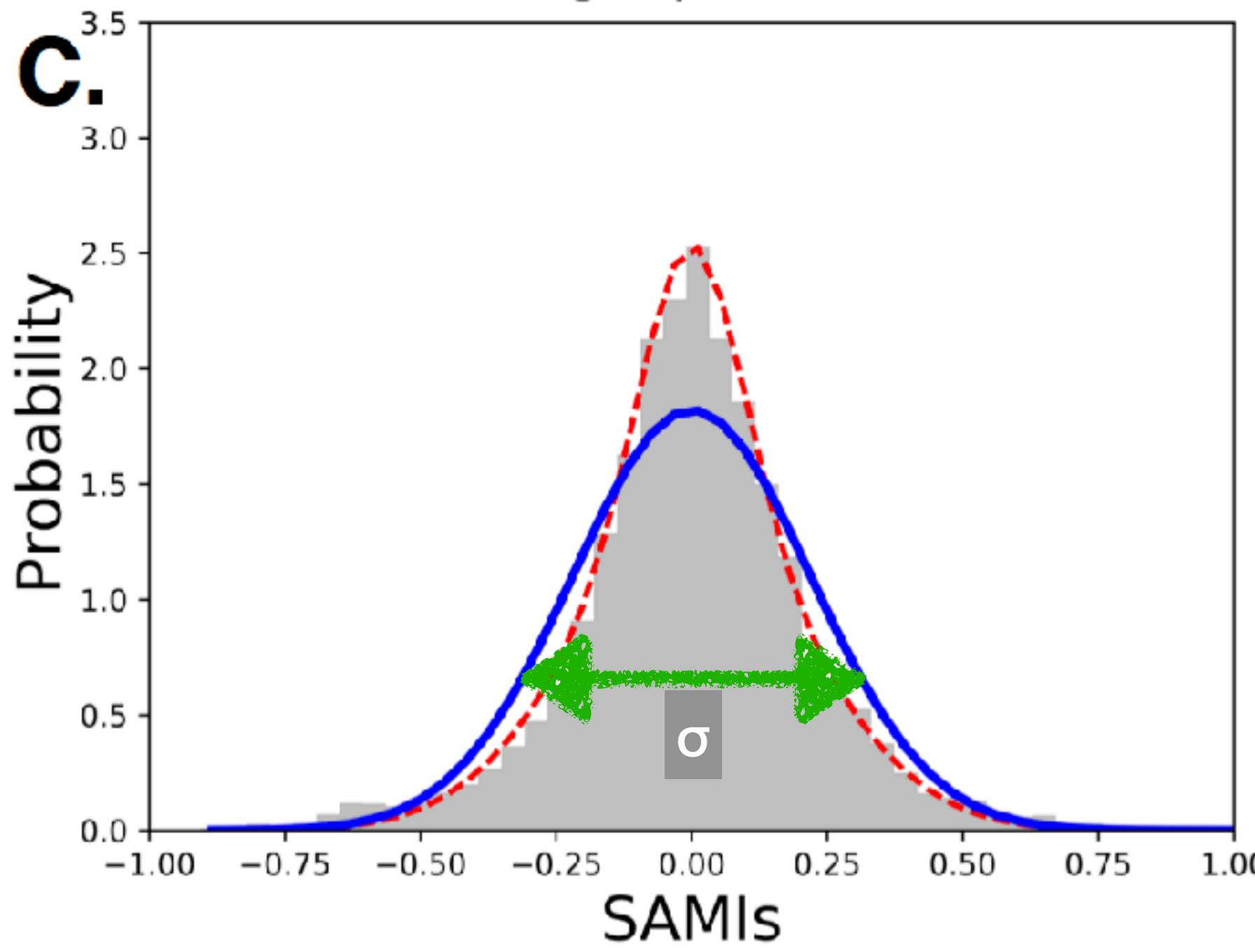
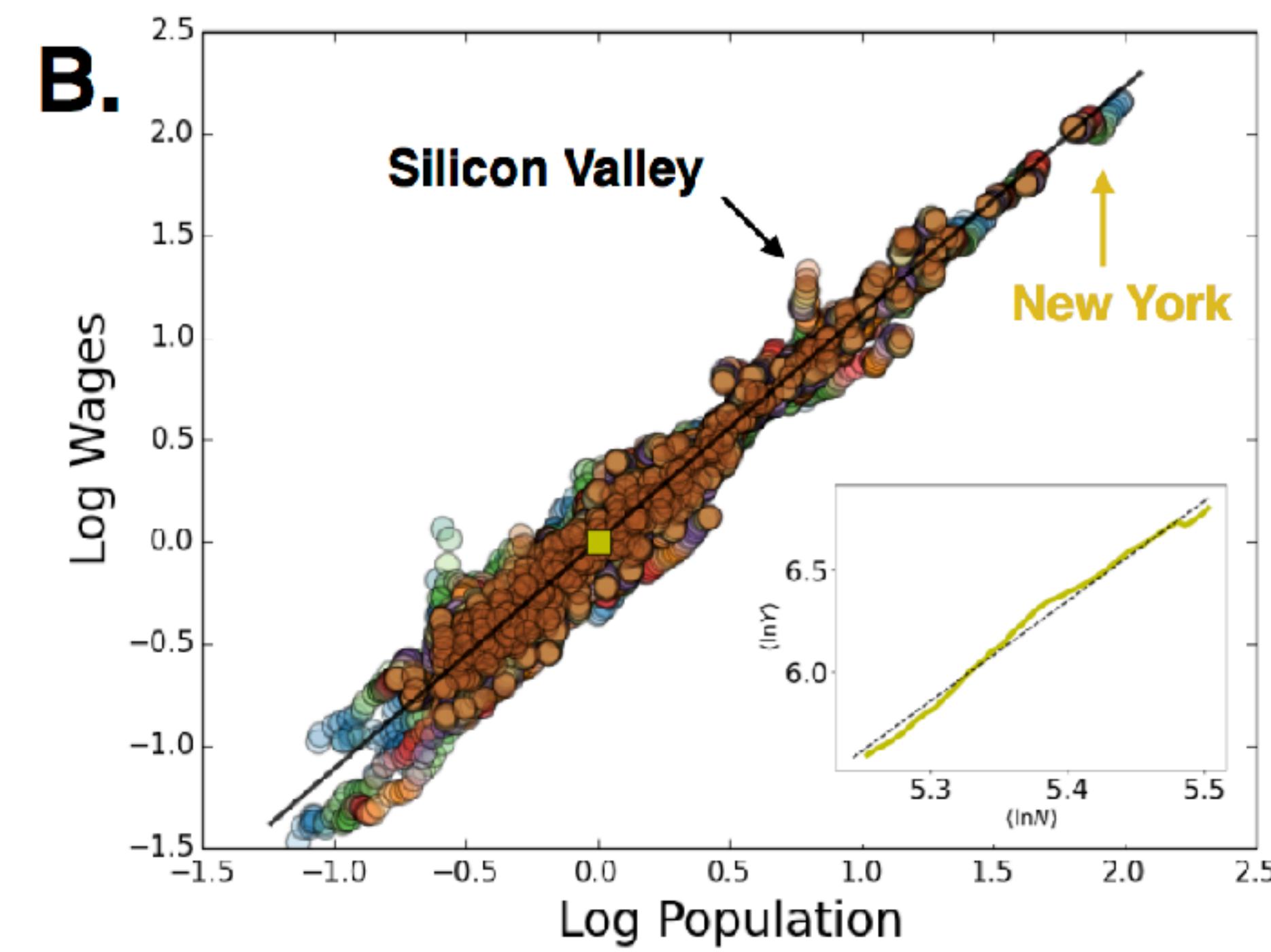
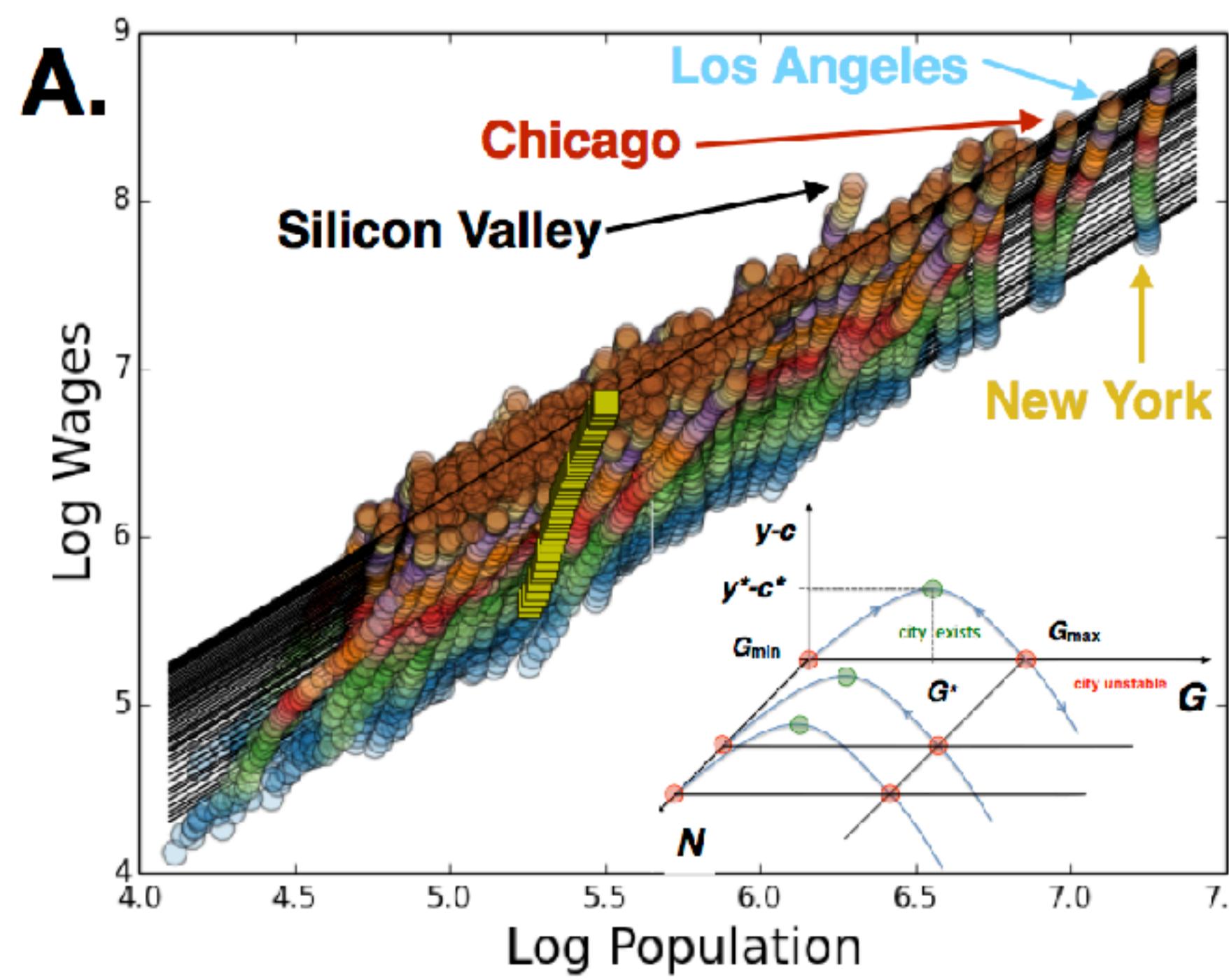


scaling exponents become scale dependent

loss of scale invariance

requires a theory for the variances

more complicated statistics



How Long does it take Cities to Catch Up with Others?

In the US, very small randomness

The typical square displacement of

$$\xi^2(t) = \sigma^2 t \simeq 0.054$$

in 50 years !!

Observe:

$$\langle \xi^2(1969) \rangle = 0.043$$

could have been ~ 0

40 years before: in the 1920s

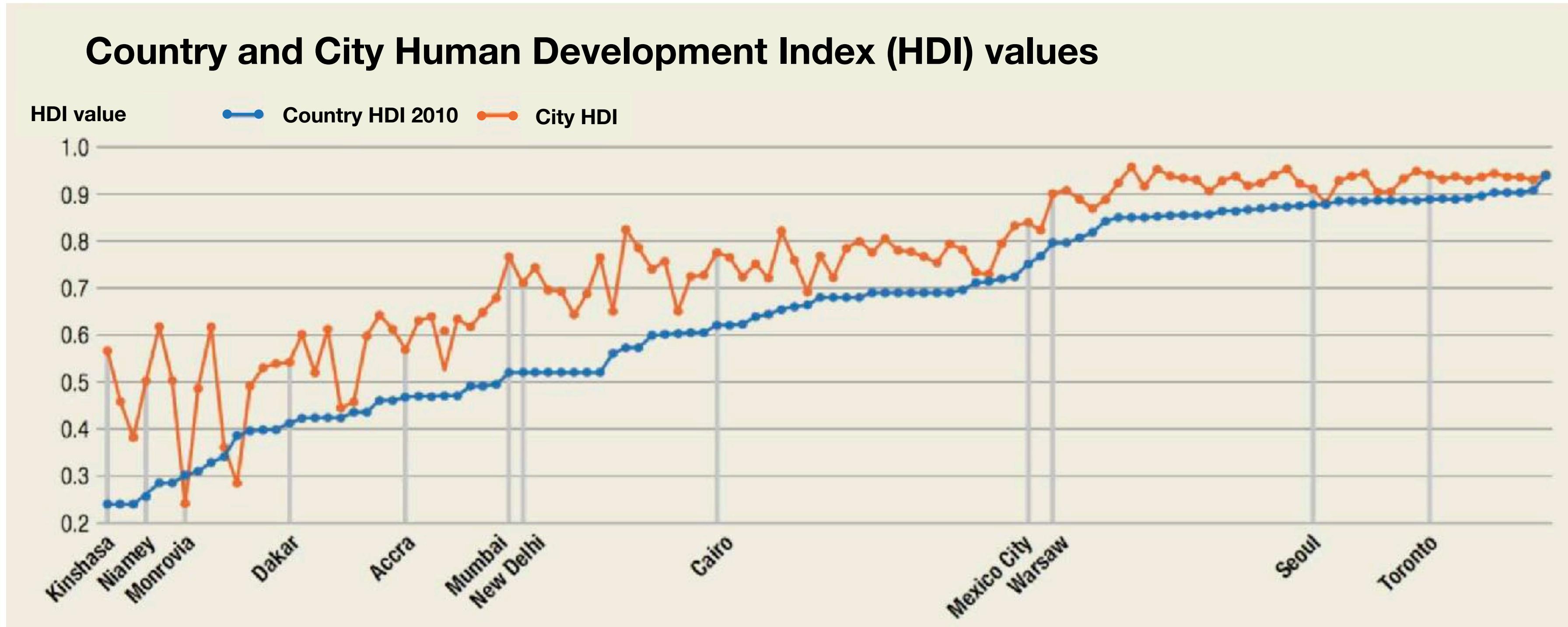
How long does it take to create an equitable society?

About a Generation ~ 20-50 years

slower with slow economic and demographic growth rates

How about human development?

Broad Human Development is a Feature of Larger Cities !



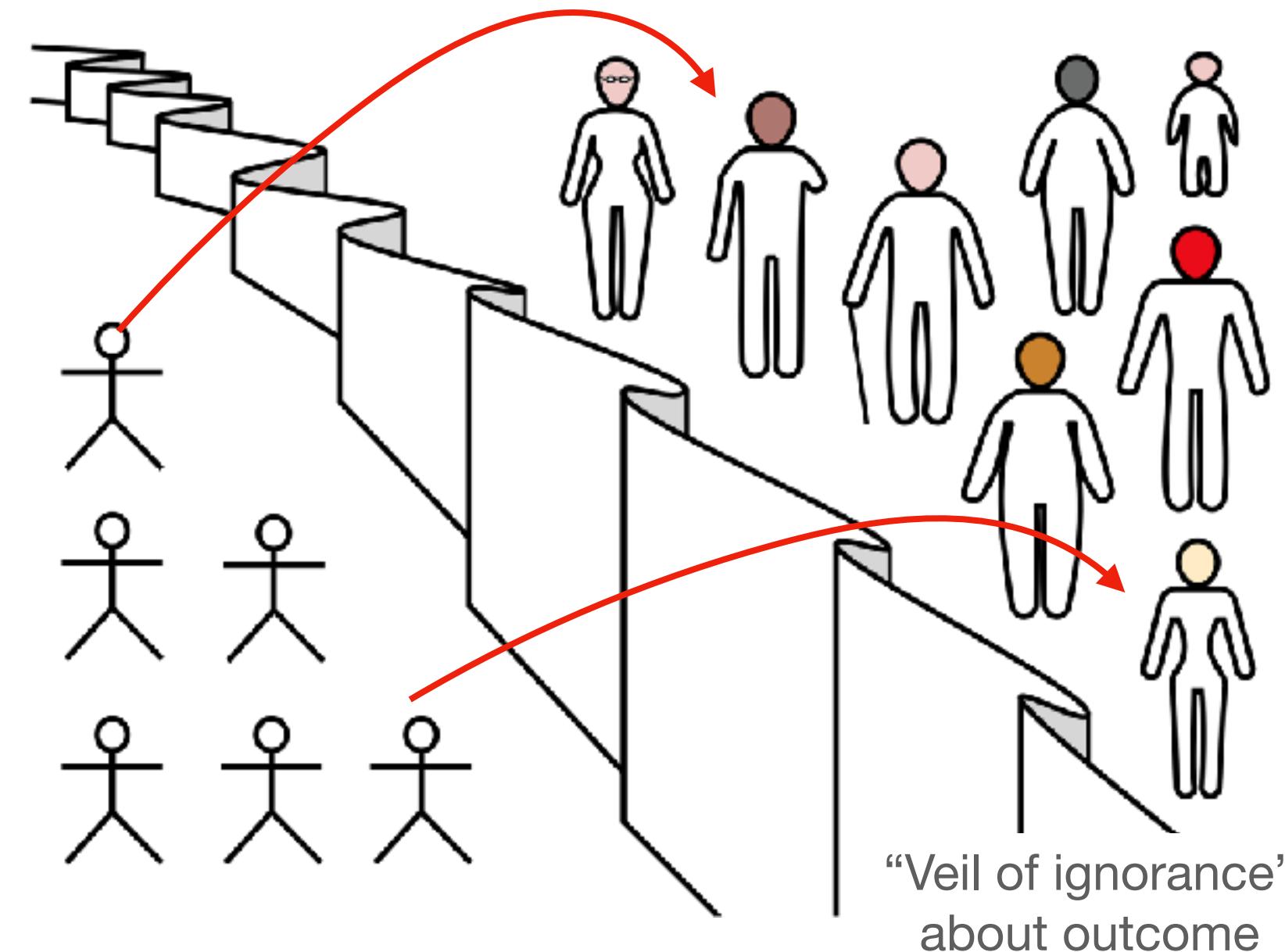
credit: UNDP Human Development Report 2013

A theory of justice

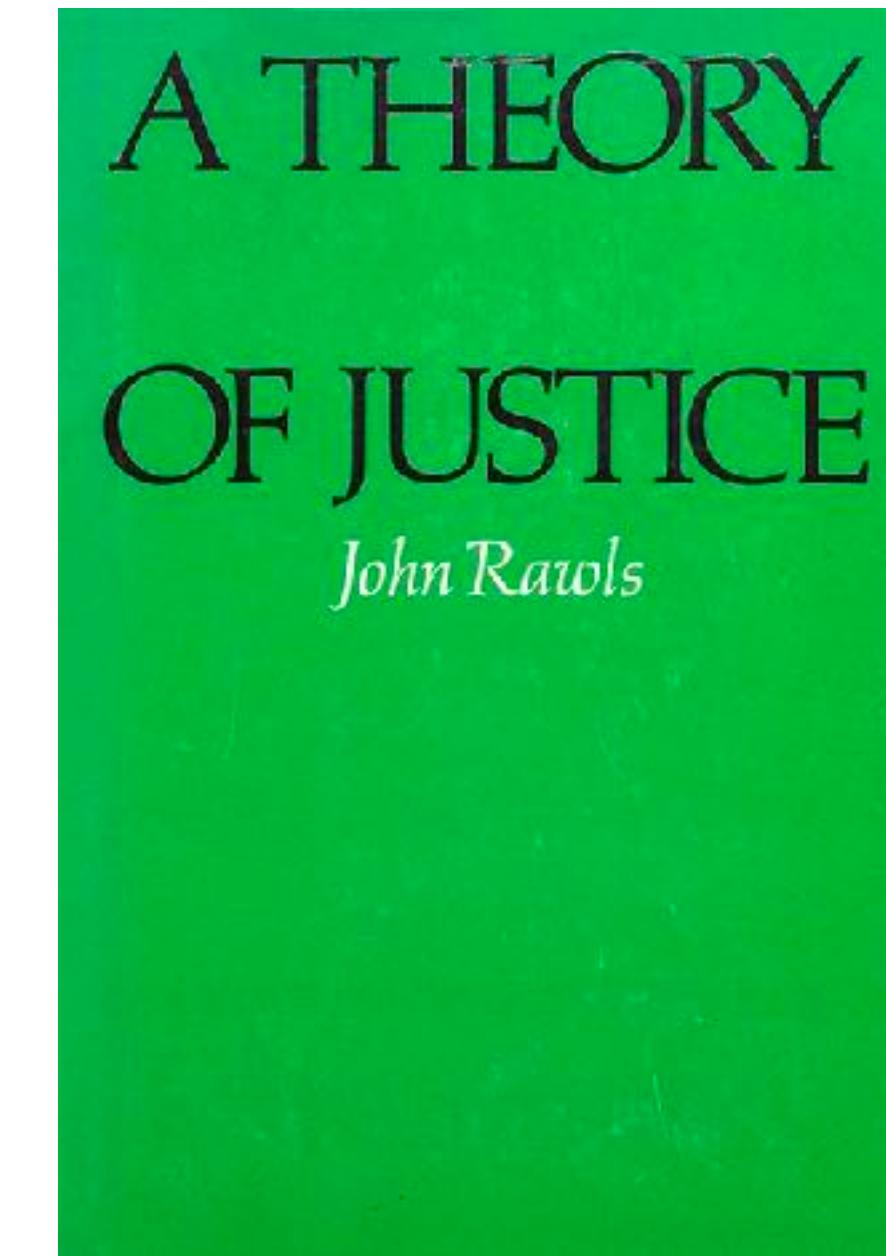
Problem: how to find a socially just distribution of goods in a society?

Thought experiment:

What society would you want if you were to be swapped at random with any other person?



Original Position:

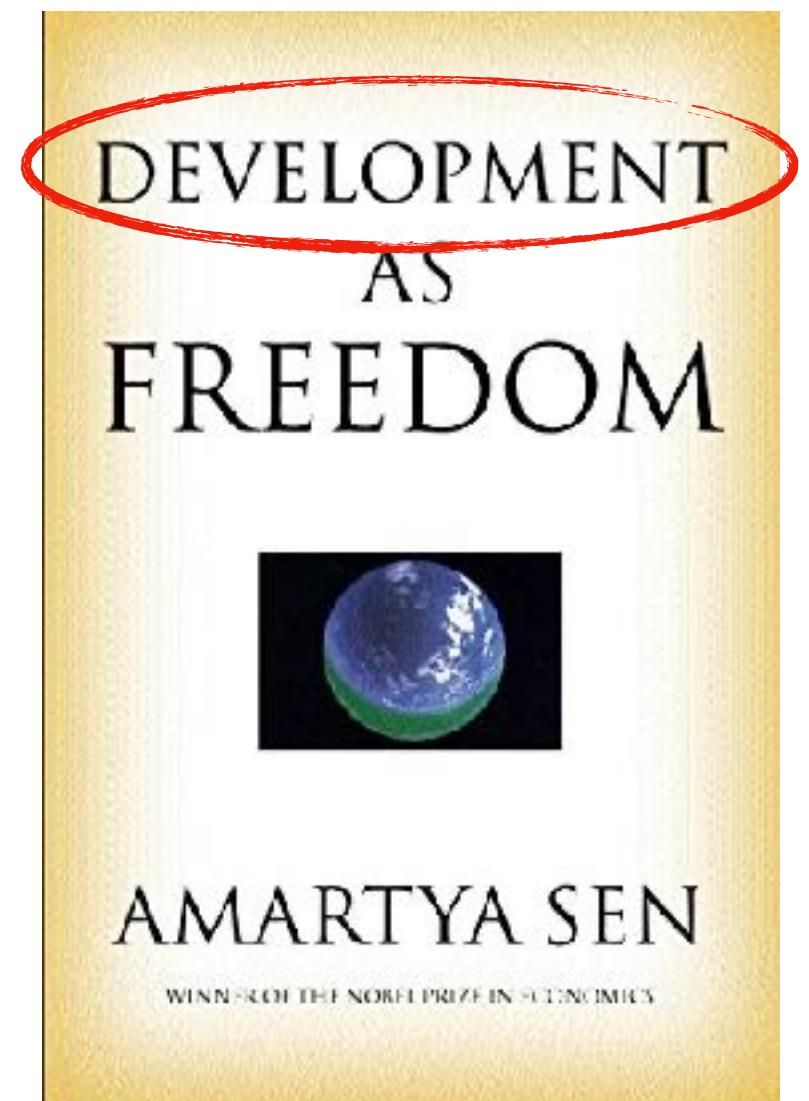
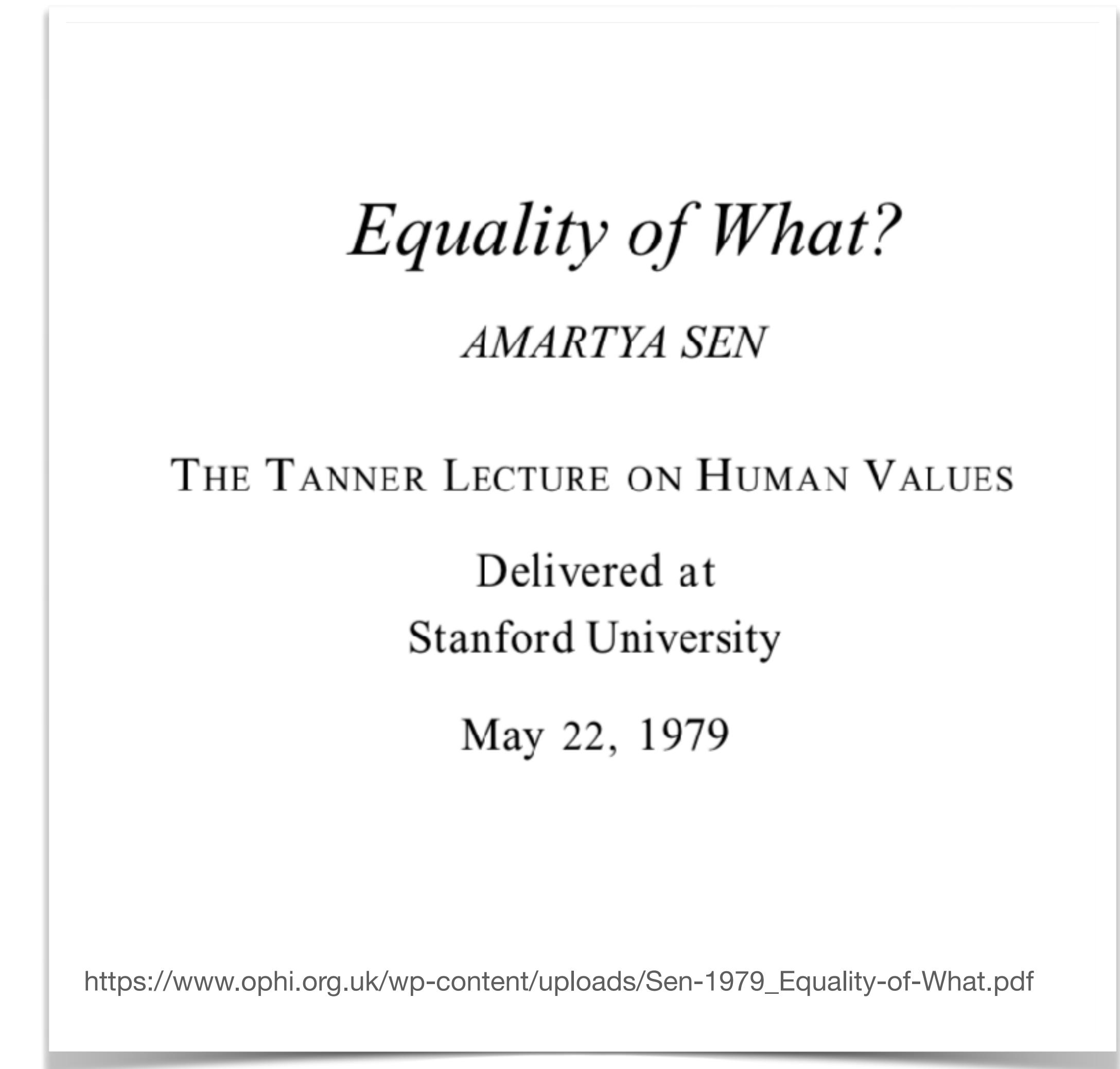


https://en.wikipedia.org/wiki/A_Theory_of_Justice

"Preference symmetry invariance"

1. Each citizen is guaranteed a fully adequate scheme of basic liberties, compatible with the same liberties for all
2. Social and economic inequalities are:
 - to the greatest benefit of the least advantaged,
 - attached to offices and positions open to all under conditions of fair equality of opportunity

But can people “pursue their happiness”? That takes more than goods and liberties



https://www.ophi.org.uk/wp-content/uploads/Sen-1979_Equality-of-What.pdf

“capabilities approach”:

processual equalities

basic capabilities

not just money, goods, freedom,...

agency

empowerment

in context

+ Martha Nussbaum

Human development as a network process (in cities)

Measuring Human Development across scales



[nature](#) > [npj urban sustainability](#) > [articles](#) > [article](#)

Article | [Open Access](#) | [Published: 20 February 2023](#)

Measuring health and human development in cities and neighborhoods in the United States

[Suraj K. Sheth](#)  & [Luís M. A. Bettencourt](#)

[npj Urban Sustainability](#) **3**, Article number: 7 (2023) | [Cite this article](#)

Suraj Sheth

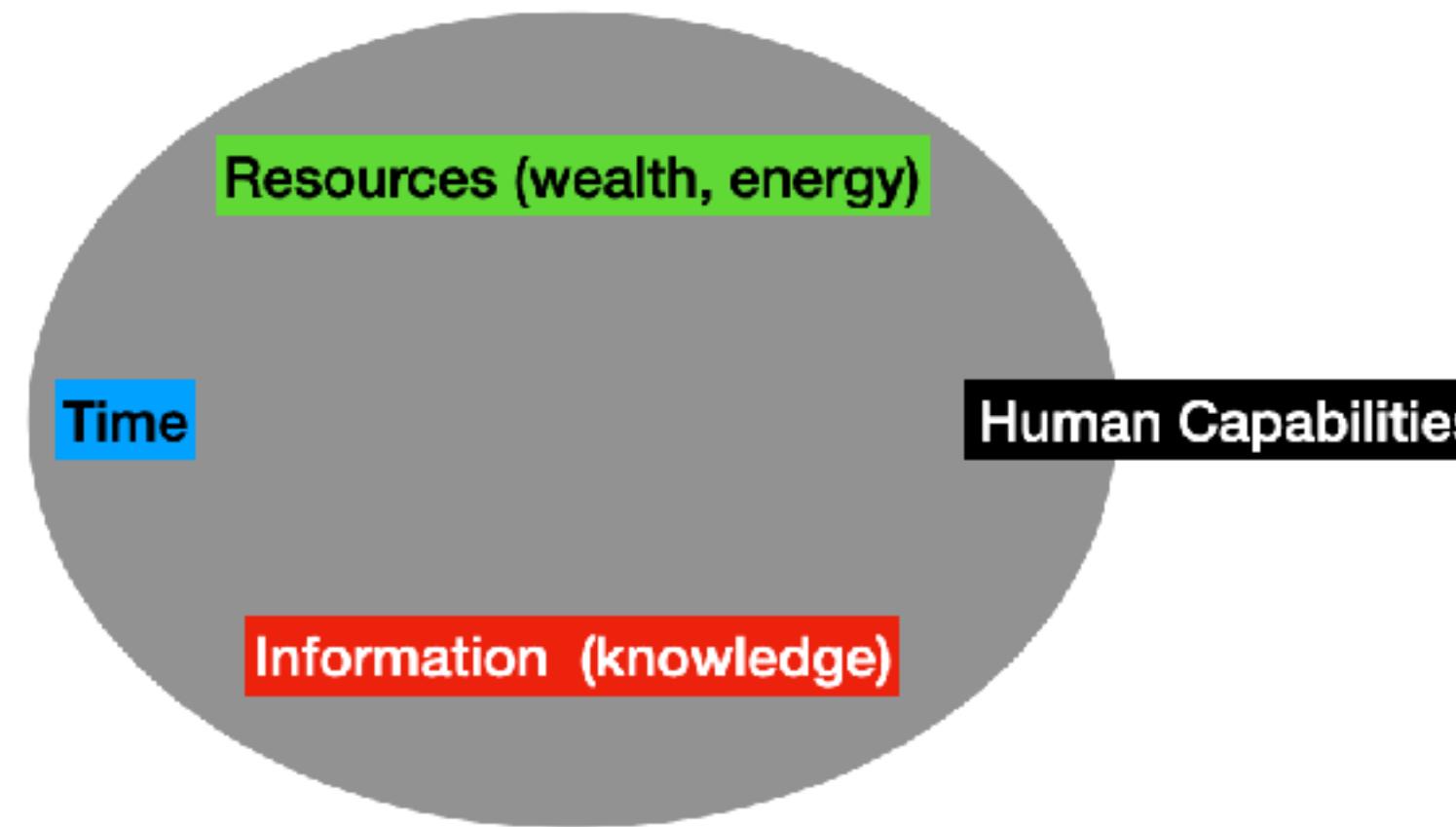
Human Development Index

“measures the ability of human populations to lead long, healthy and fulfilled lives”



Mahbub ul Haq

$$HDI_i = \left(\text{education}_i \cdot \text{life expectancy}_i \cdot \text{real income}_i \right)^{1/3}$$

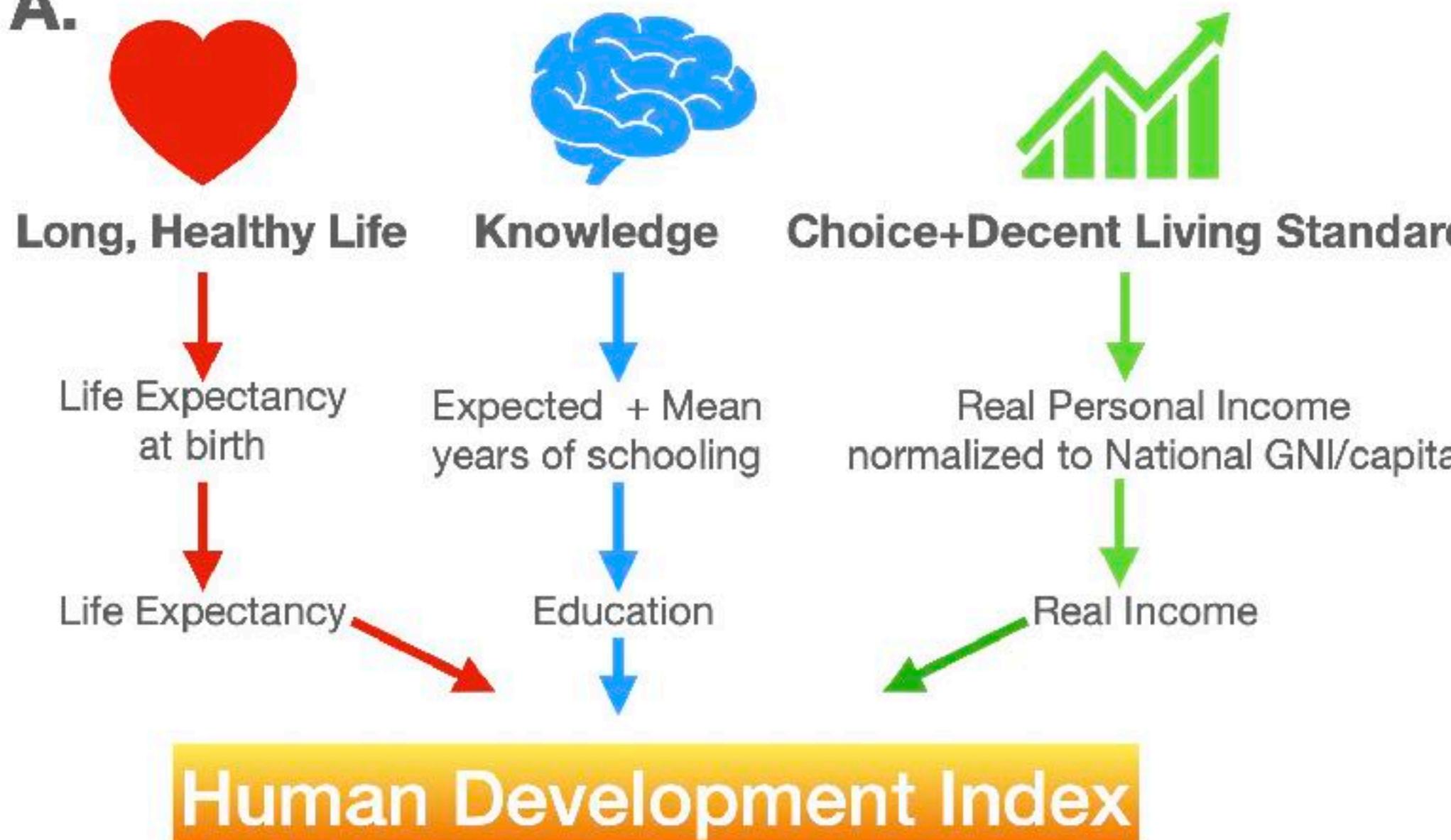


The US is now # 17

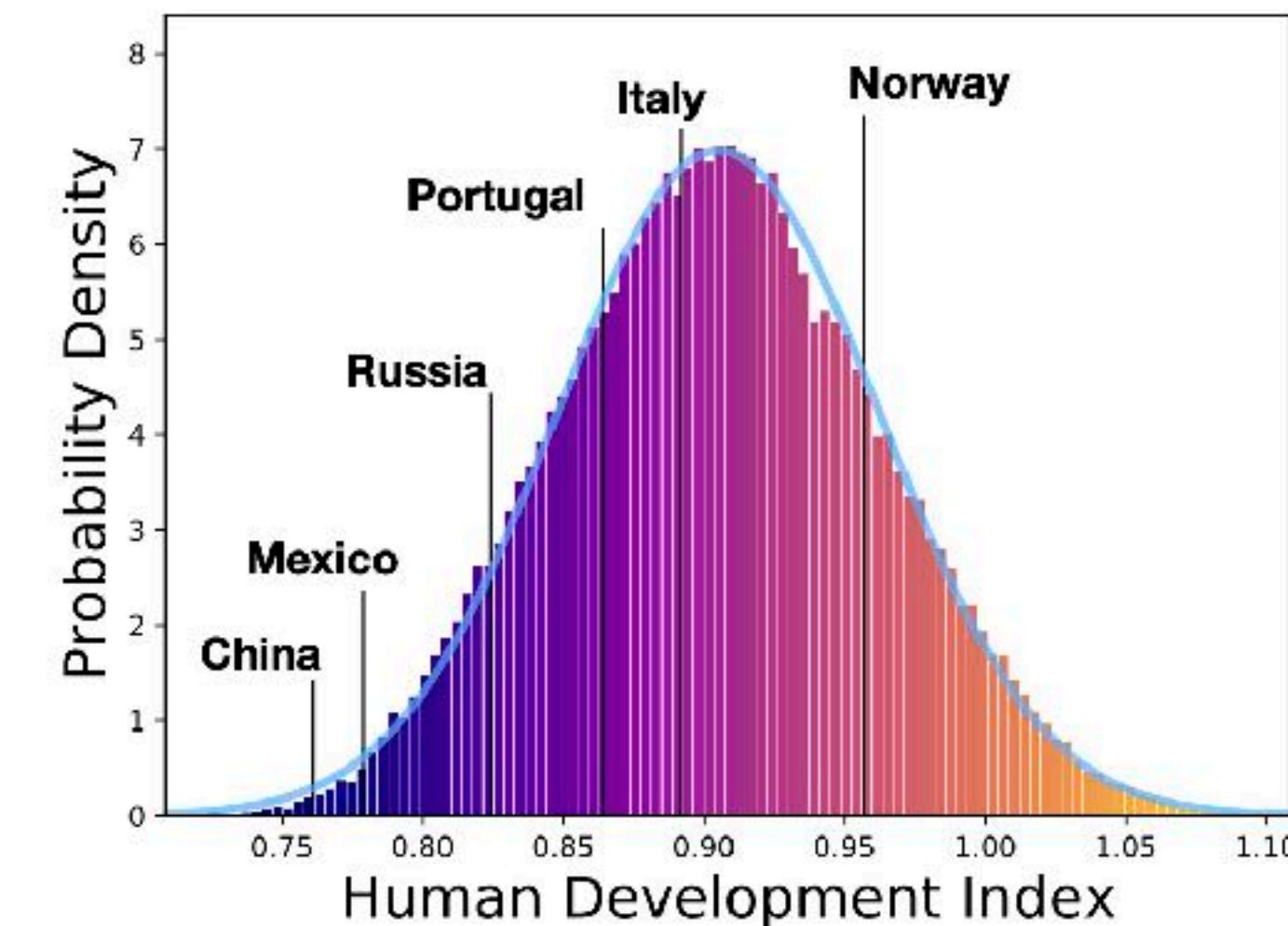
Rank	Country or Territory	HDI	
		2019 data (2020 report) ^[14]	Average annual HDI growth (2010-2019) ^[15]
1	Norway	0.957	▲ 0.20%
2	Ireland	0.955	▲ 0.65%
2	Switzerland	0.955	▲ 0.16%
4	Hong Kong	0.949	▲ 0.54%
4	Iceland	0.949	▲ 0.62%
6	Germany	0.947	▲ 0.24%
7	Sweden	0.945	▲ 0.41%
8	Australia	0.944	▲ 0.17%
8	Netherlands	0.944	▲ 0.32%
10	Denmark	0.940	▲ 0.28%
11	Finland	0.938	▲ 0.26%
11	Singapore	0.938	▲ 0.35%
13	United Kingdom	0.932	▲ 0.24%
14	Belgium	0.931	▲ 0.25%
14	New Zealand	0.931	▲ 0.30%
16	Canada	0.929	▲ 0.34%
17	United States	0.926	▲ 0.12%

because of health and educational inequality

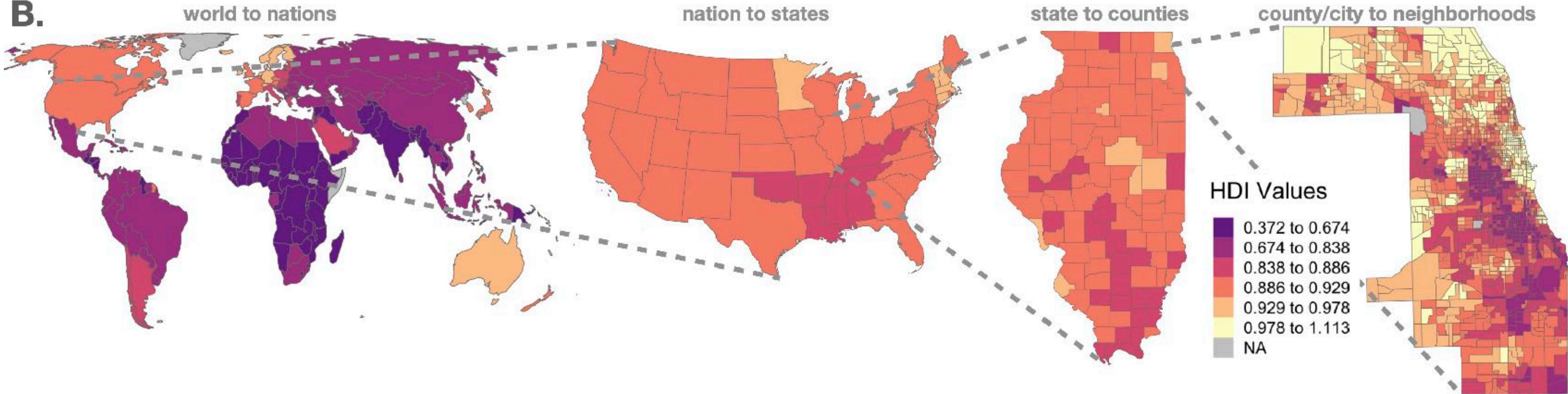
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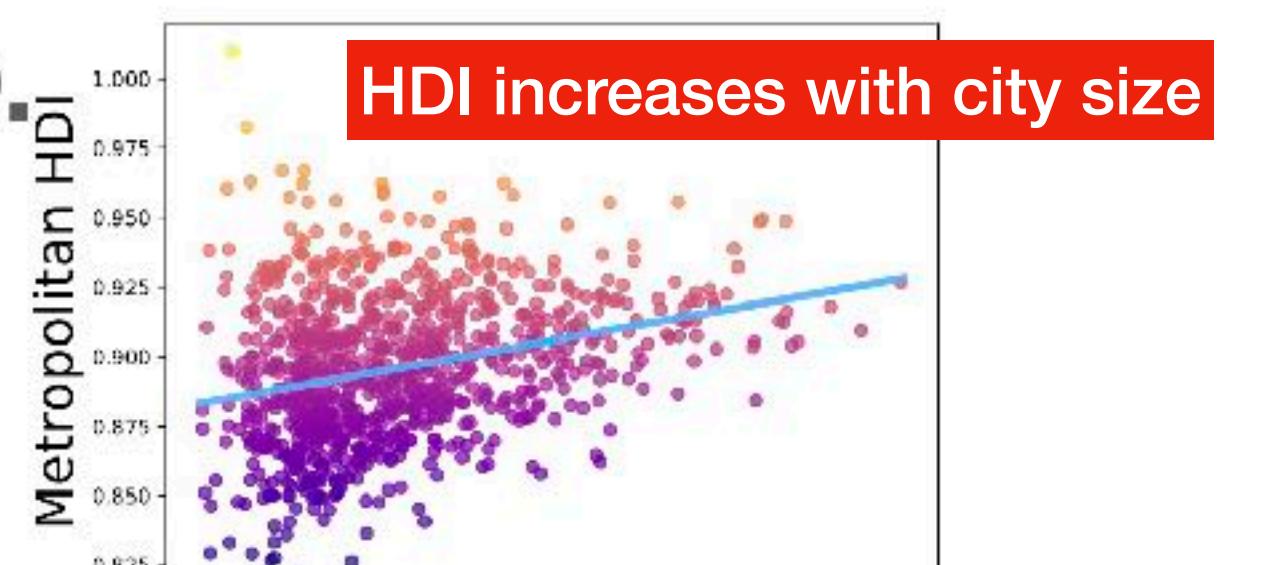
C.



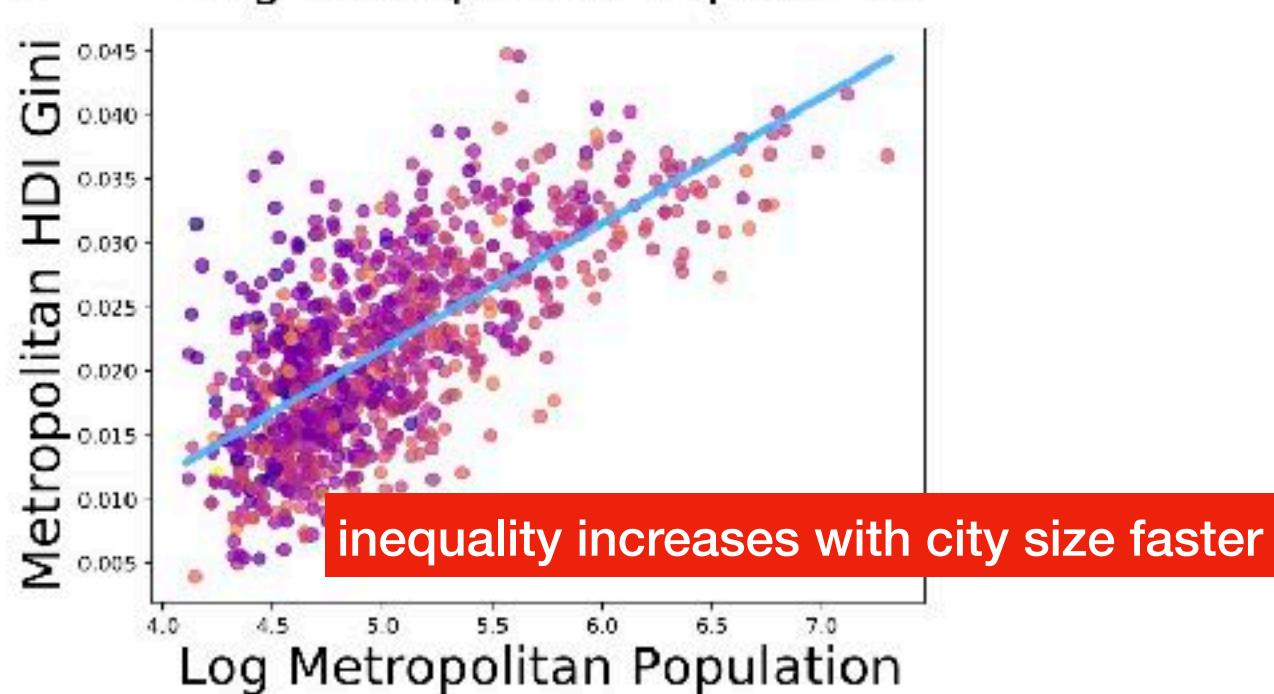
B.



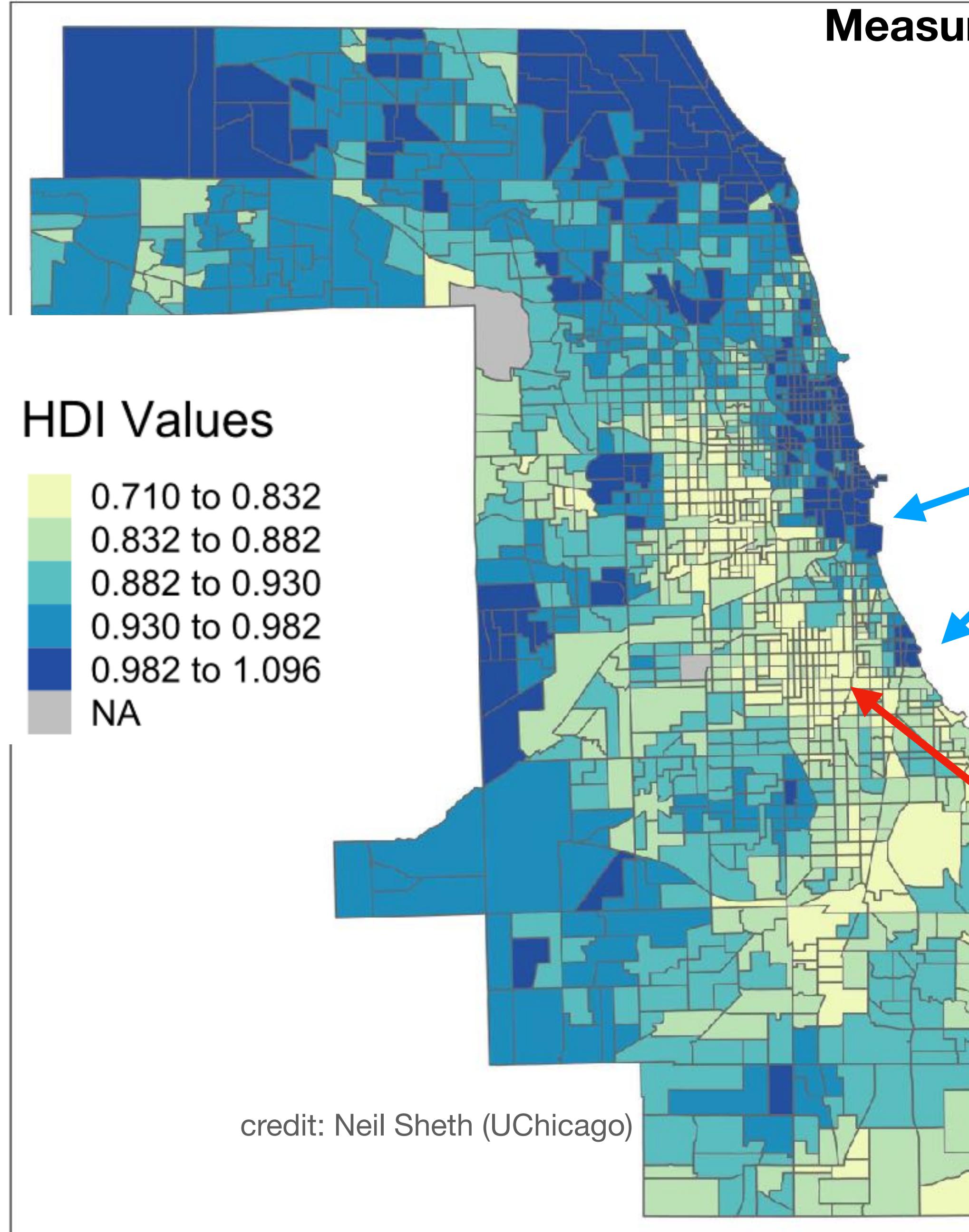
D.



E.



Measuring the Human Development of Chicago neighborhoods



better than Norway

worse than China or Mexico

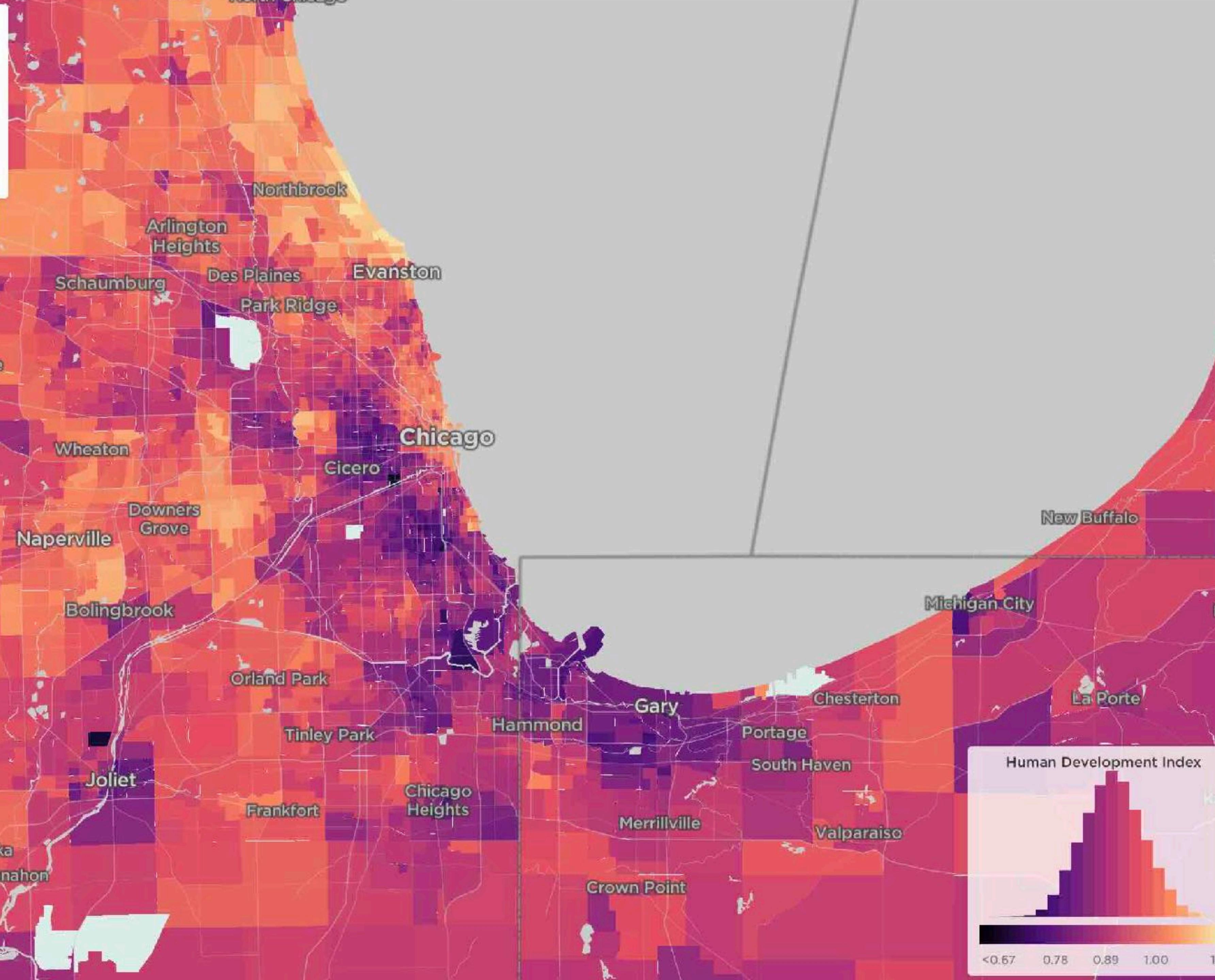
Human Development of US Communities

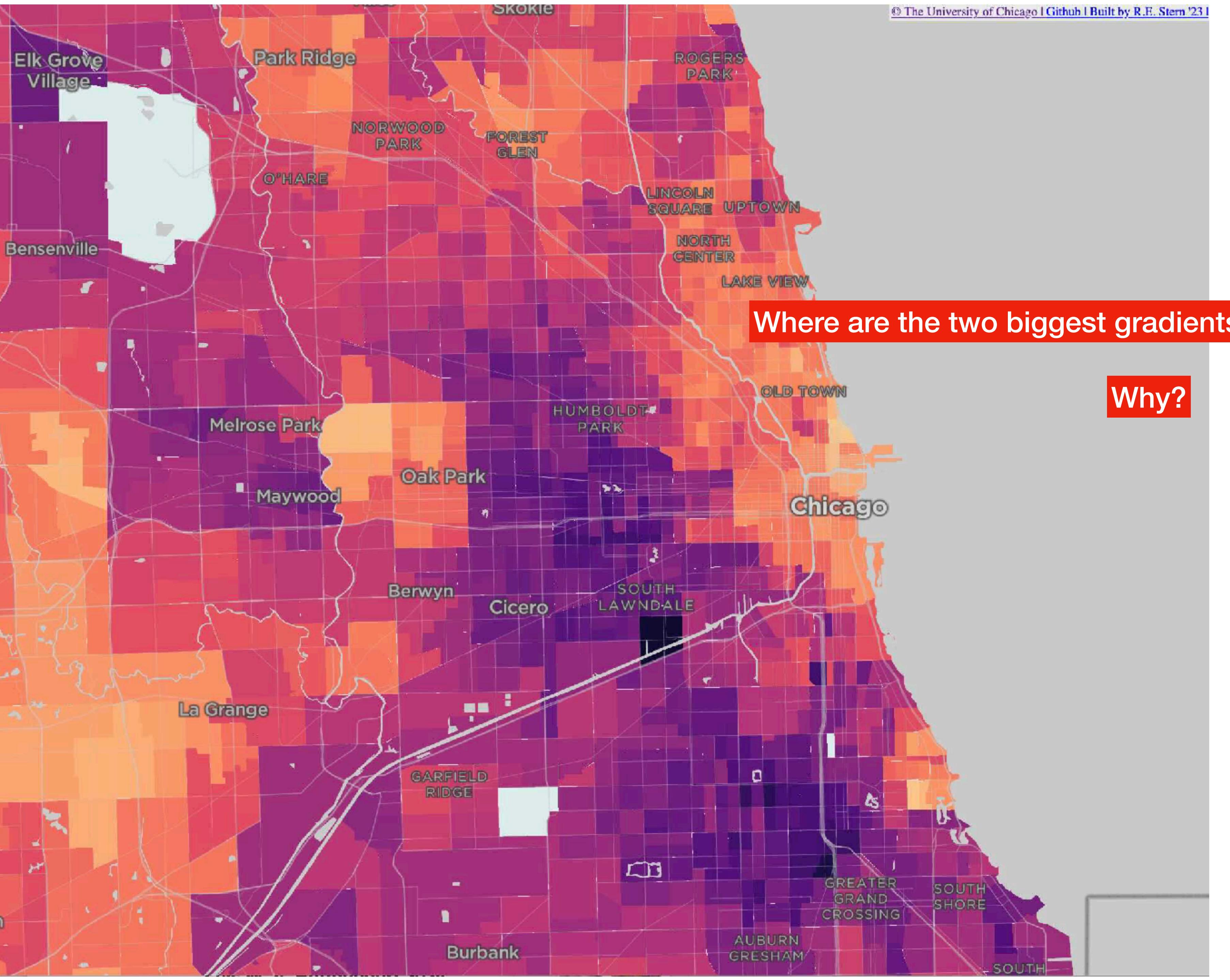
The Human Development Index (HDI) is the international gold standard of a person's ability to lead a long, healthy life with access to knowledge and a decent standard of living. It is a composite of life expectancy, education, and real income.

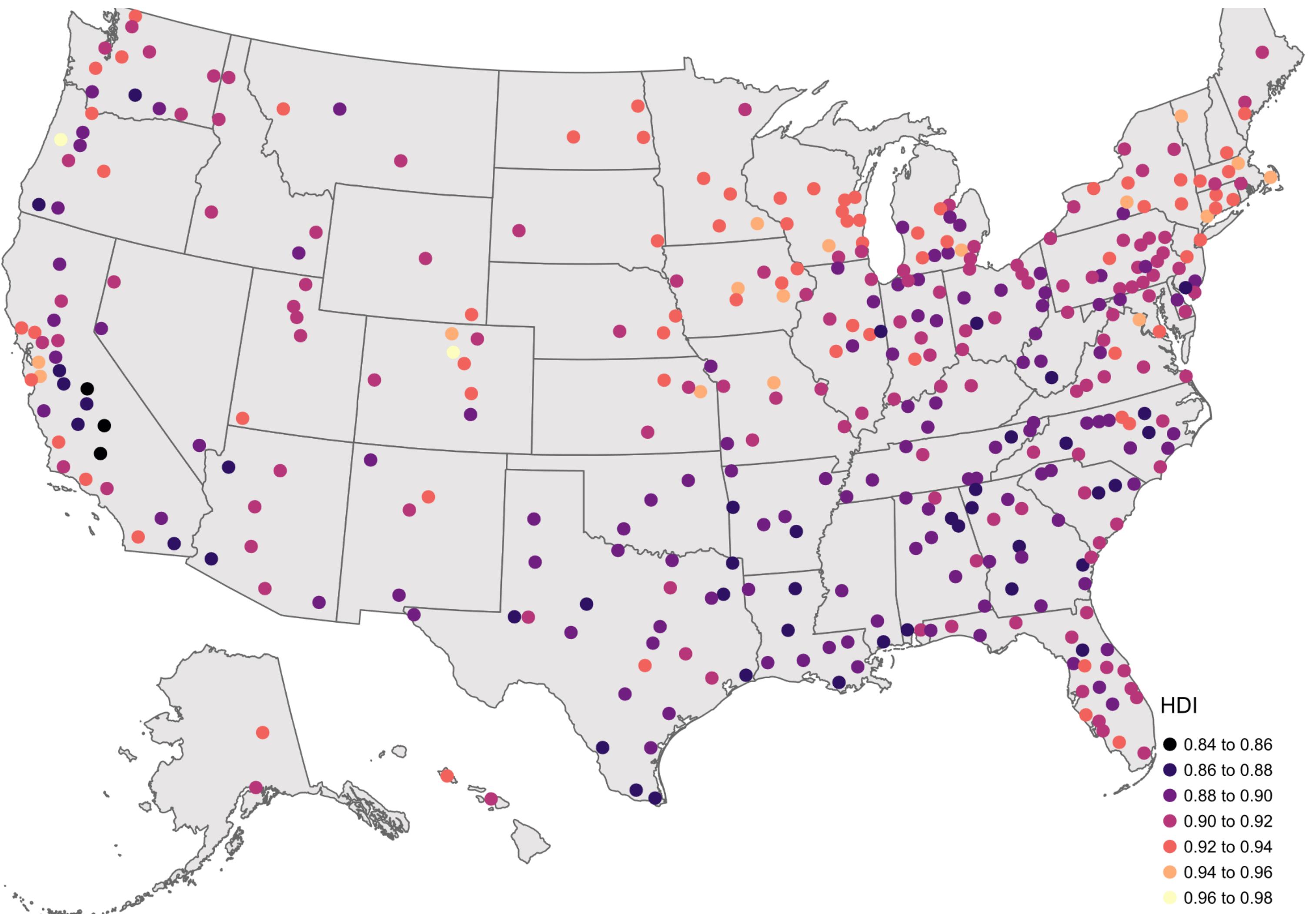
We measured it in every US local community to compare them to international levels of development.

Explore a city

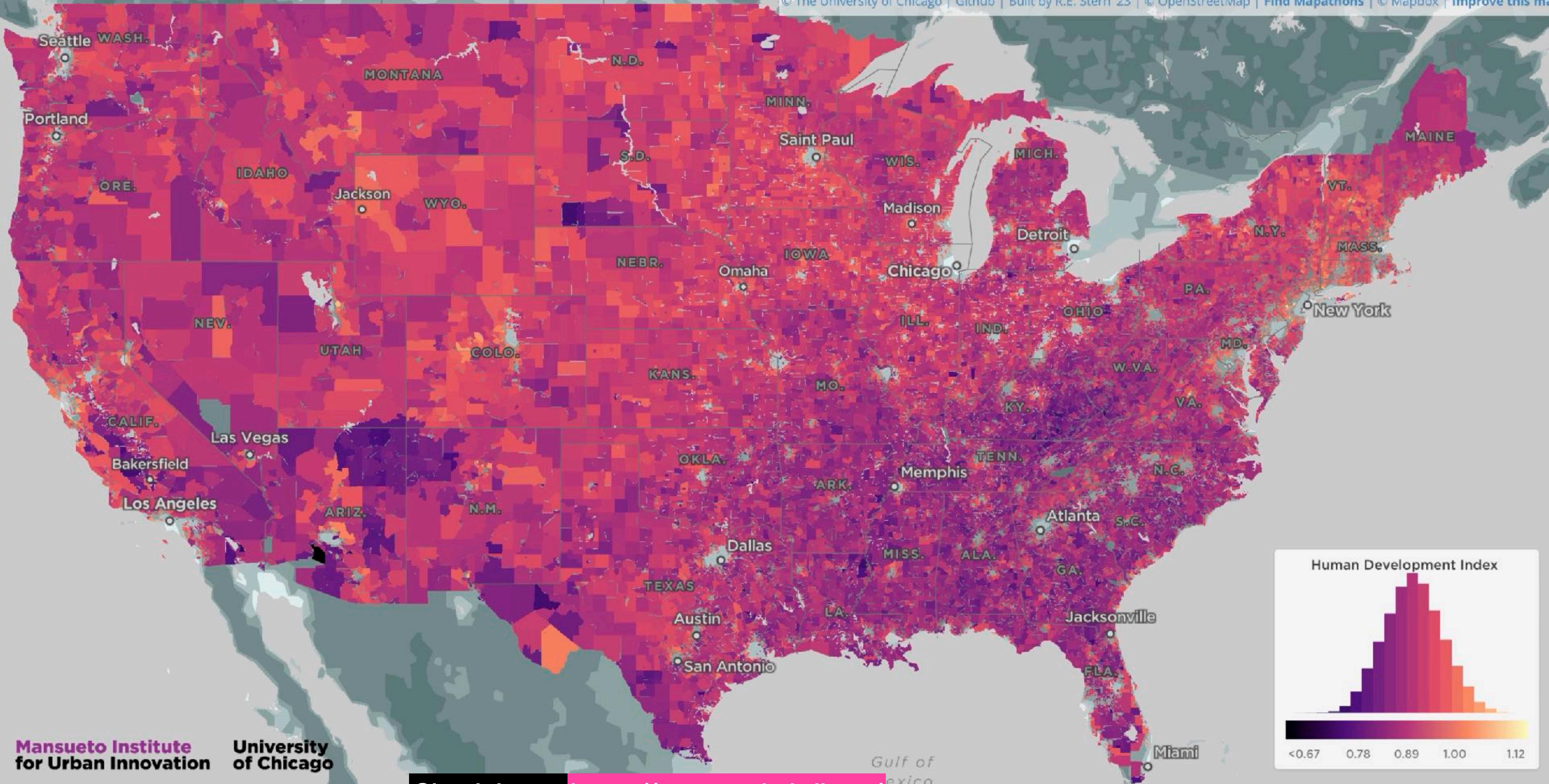
- New York City
- Los Angeles
- Chicago
- Houston
- Phoenix
- DeKalb
- Philadelphia
- San Antonio
- San Diego
- Dallas
- San Jose
- Austin
- Jacksonville
- Fort Worth
- Columbus
- San Francisco
- Charlotte
- Indianapolis
- Seattle
- Denver
- Washington

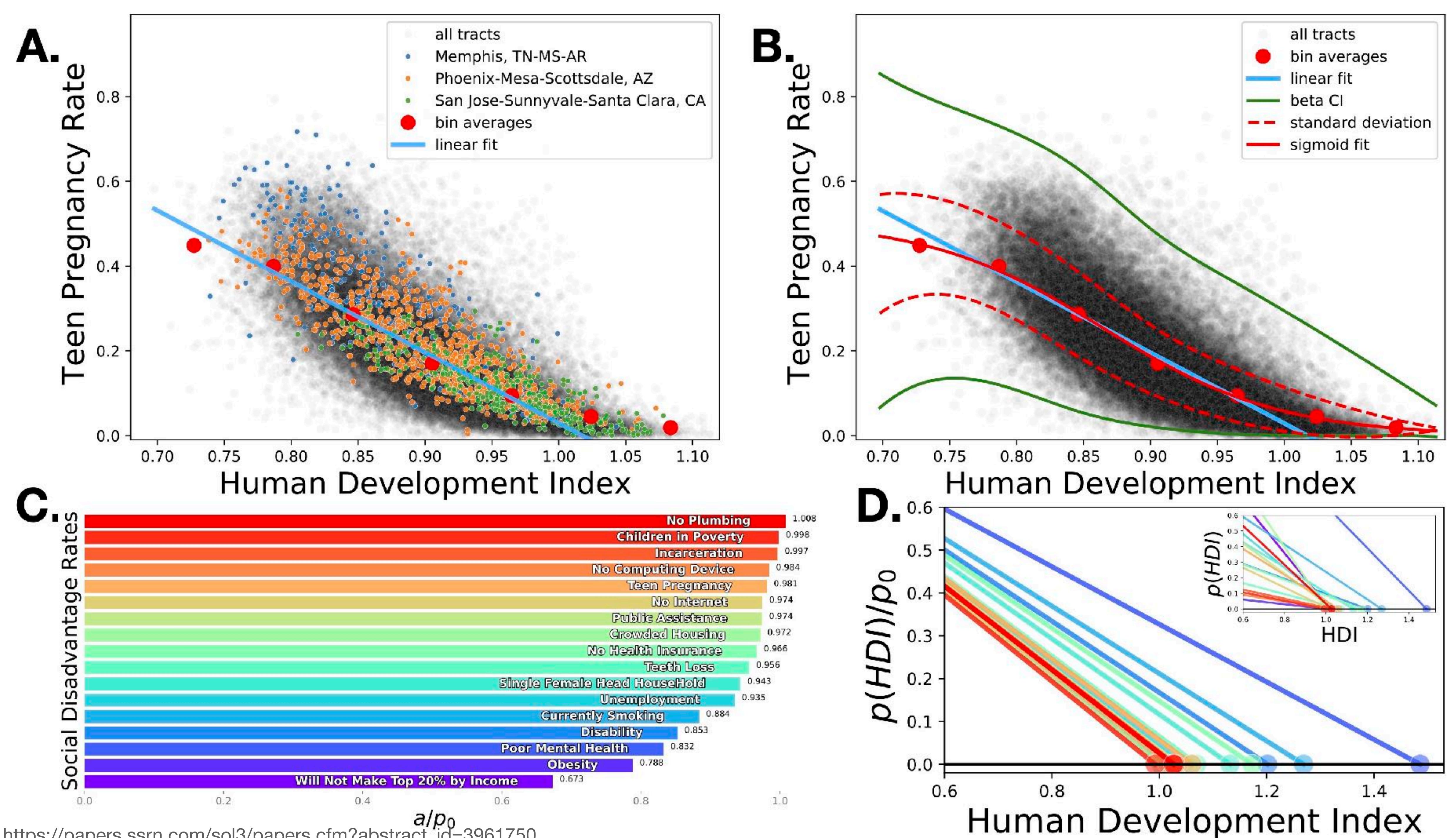






State/Federal District	HDI	Metropolitan Area	HDI
Massachusetts	0.9407	Boulder, CO	0.9624
Connecticut	0.9378	Corvallis, OR	0.9622
Vermont	0.9348	Ann Arbor, MI	0.9570
New Hampshire	0.9345	Iowa City, IA	0.9567
Minnesota	0.9330	San Jose-Sunnyvale-Santa Clara, CA	0.9557
New Jersey	0.9281	Bridgeport-Stamford-Norwalk, CT	0.9553
District of Columbia	0.9273	Ames, IA	0.9532
Maryland	0.9271	Lawrence, KS	0.9496
North Dakota	0.9244	Boston-Cambridge-Newton, MA-NH	0.9491
Colorado	0.9240	San Francisco-Oakland-Hayward, CA	0.9486
Texas	0.8964	Gadsden, AL	0.8647
South Carolina	0.8935	McAllen-Edinburg-Mission, TX	0.8641
Nevada	0.8919	Lake Havasu City-Kingman, AZ	0.8637
Oklahoma	0.8879	Laredo, TX	0.8614
Tennessee	0.8874	Dalton, GA	0.8614
Kentucky	0.8843	Brownsville-Harlingen, TX	0.8605
Alabama	0.8839	Yakima, WA	0.8604
Louisiana	0.8838	Pine Bluff, AR	0.8601
West Virginia	0.8832	Bakersfield, CA	0.8595
Arkansas	0.8798	Visalia-Porterville, CA	0.8579
Mississippi	0.8762	Madera, CA	0.8572





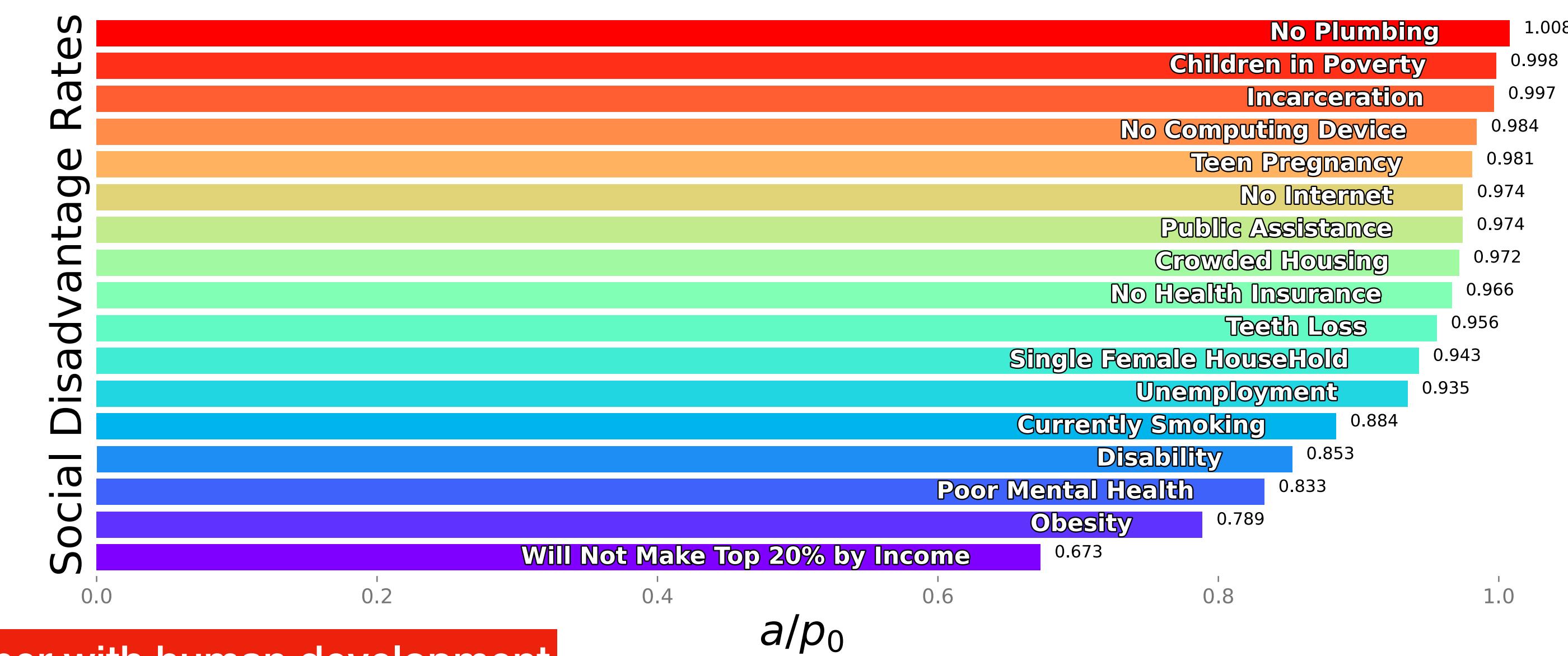
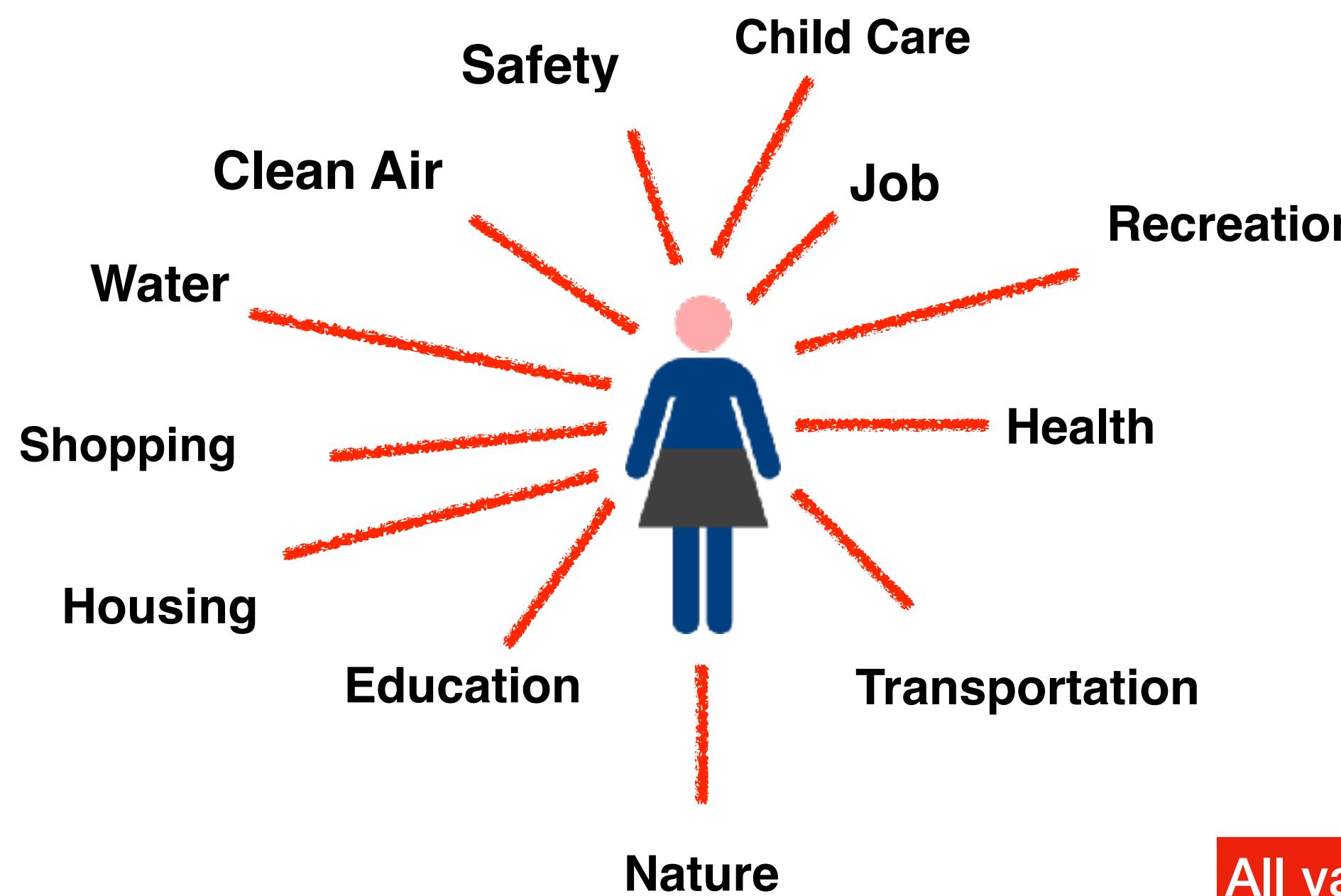
“Anna Karenina Principle” of Human Development

A deficiency in any **one of a number of factors** dooms an endeavor to failure.

Consequently, a successful endeavor (subject to this principle) is one for which every possible deficiency has been avoided.

https://en.wikipedia.org/wiki/Anna_Karenina_principle

**All *high* human development communities are alike (“no problems”);
but each *low* human development community is challenged in its own way**



All vary together with human development

Solution: ensure systemic human capabilities = education and health and decent real income