Lecture 6

Network Models of Cities

6.1 Scale Invariance and Urban Scaling

IUS 3.1, 3.2

Network Effects

The value of a network is proportional to the number of its connections

Connections grow faster than proportionally to the number of nodes

$$Y \sim N^2$$

Metcalfe's Law

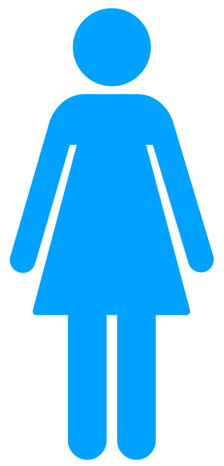
Can this happen for cities?

Cities !!

What does data say about their properties ?

IUS 3.2

10^9



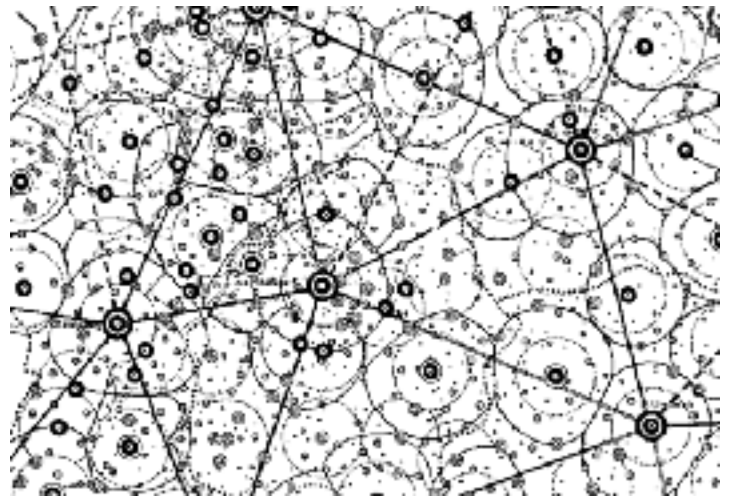
billions

10^6

millions

10^3

thousands



Scale	Integration	Mechanism
Individual		**cognition & agency** **life-course** human development seeds of economic growth
↕	**livability** **safety** **schools**	
Neighborhood		**neighborhood effects** cumulative (dis)advantage
↕	**selection** **segregation** **mixing** **inequality**	
City		**scaling and agglomeration** social + physical networks economy, information, land uses
↕	**migration** **trade**	
Urban Systems		**laws of geography** gravity law, Zipf's law urban hierarchy

Urban Theory

When a city doubles in size

its economic productivity per capita increases by 15%

Sveikauskas 1975

https://academic.oup.com/qje/article/89/3/393/1896685?login=true

When a city doubles in size

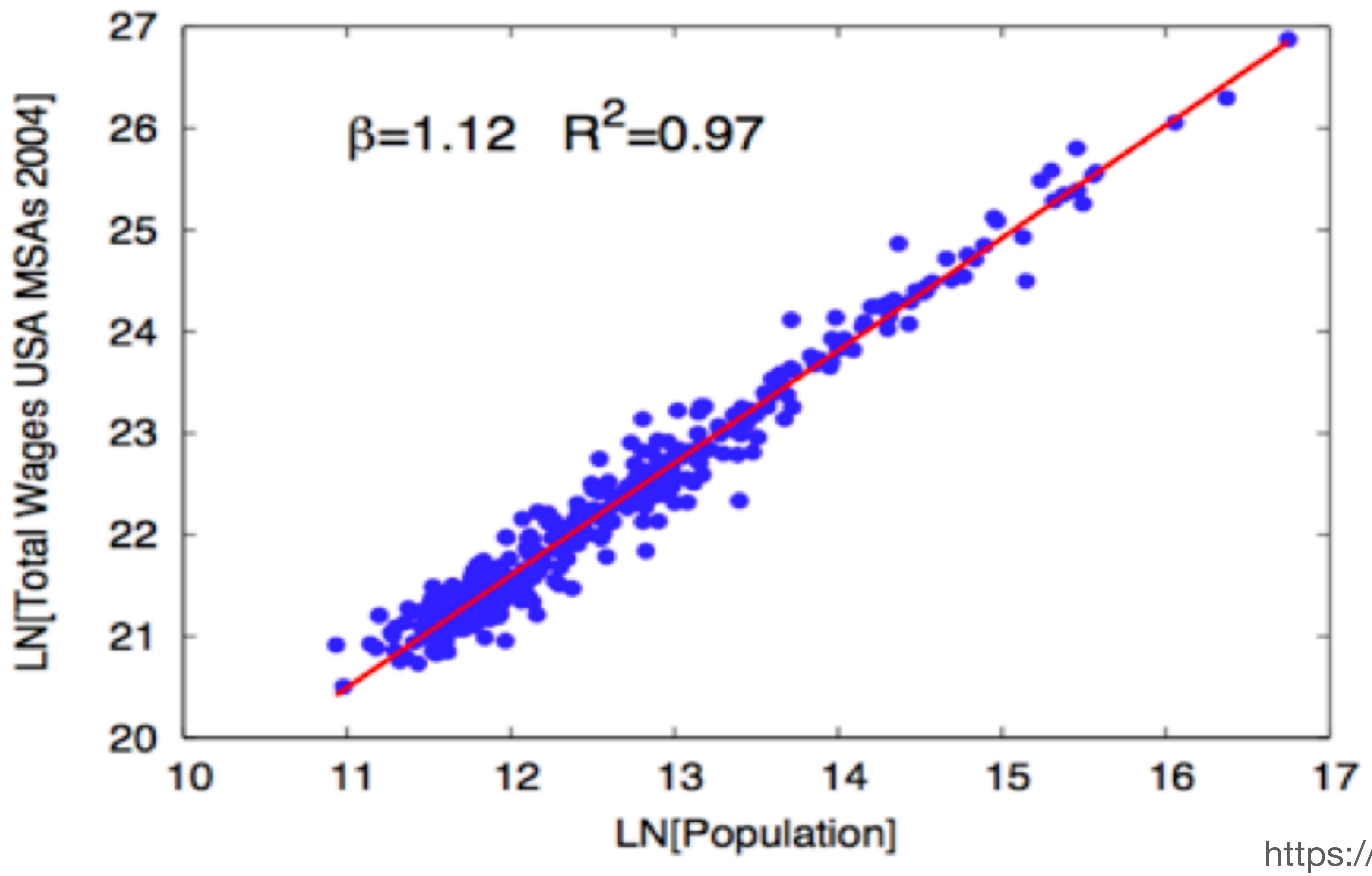
its per capita violent crime increases by 16%

Glaeser & Sacerdote 1999

https://www.journals.uchicago.edu/doi/abs/10.1086/250109

Increasing returns to city population size

USA



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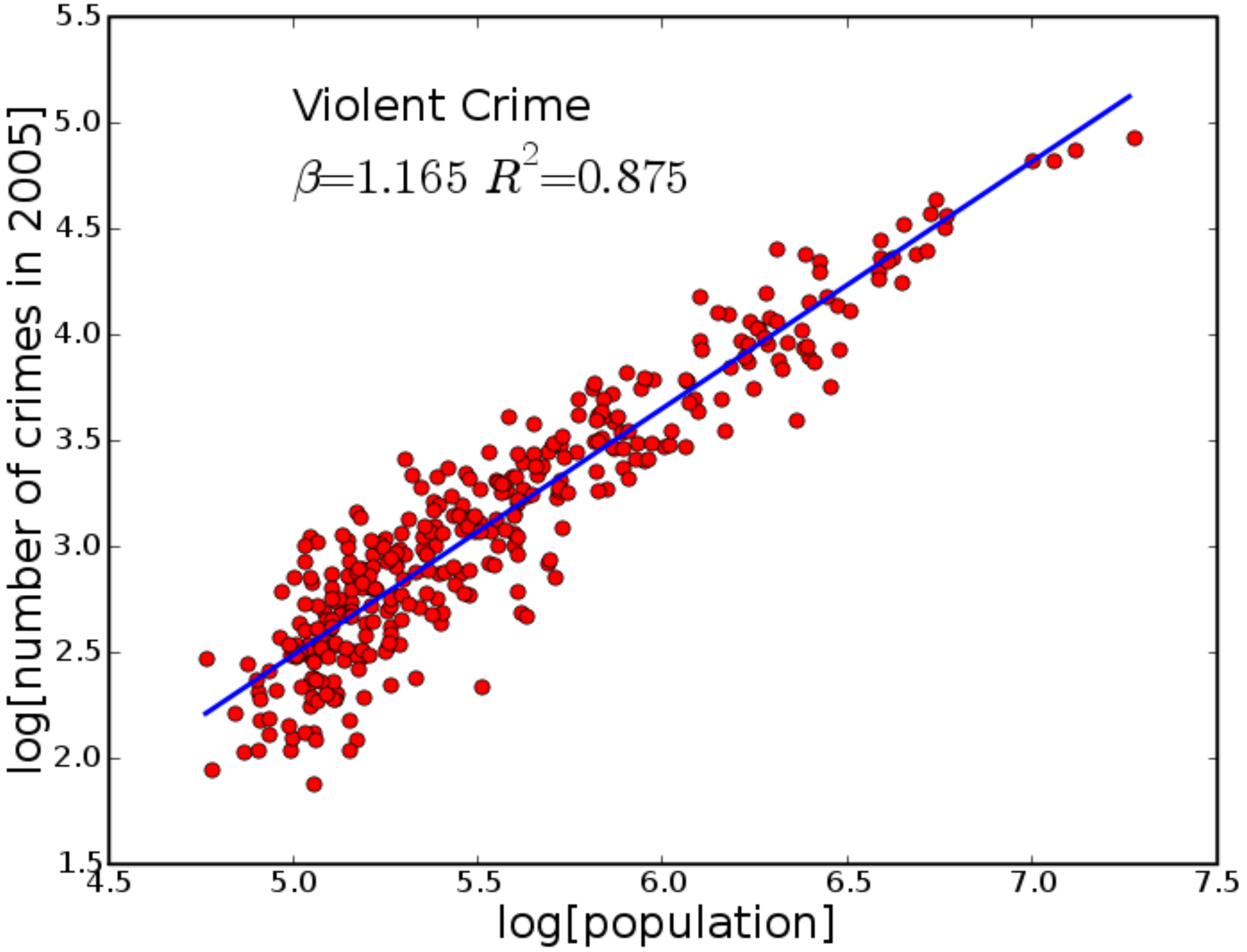
When a city doubles in size its wages increase by 12%

Tokyo GMP US$1.191 trillion (2005) ~Brazil, > India

New York GMP US$0.901 trillion (2005) ~Russia

Violent Crime in US cities

When a city doubles in size its wages increase by 16.5%



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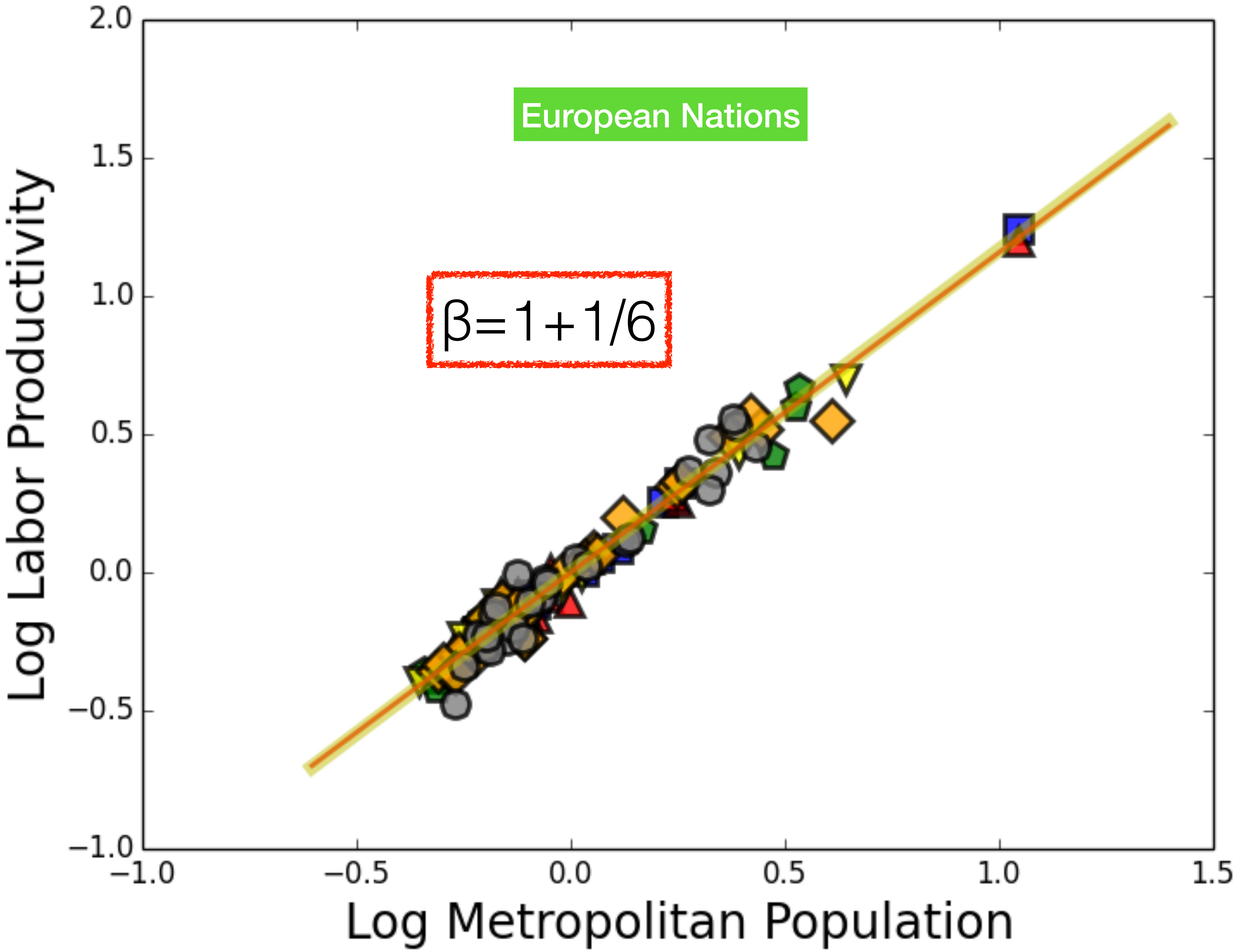
When a city doubles in size its labor productivity per capita increases by 16%



https://royalsocietypublishing.org/doi/10.1098/rsif.2016.0005

Table 1. Scaling exponents for urban indicators vs. city size

Y	β	95% CI	Adj-R^2	Observations	Country–year
New patents	1.27	[1.25,1.29]	0.72	331	U.S. 2001
Inventors	1.25	[1.22,1.27]	0.76	331	U.S. 2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	U.S. 2002
"Supercreative" employment	1.15	[1.11,1.18]	0.89	287	U.S. 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	U.S. 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
Total wages	1.12	[1.09,1.13]	0.96	361	U.S. 2002
Total bank deposits	1.08	[1.03,1.11]	0.91	267	U.S. 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999–2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
Total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
New AIDS cases	1.23	[1.18,1.29]	0.76	93	U.S. 2002–2003
Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003
Total housing	1.00	[0.99,1.01]	0.99	316	U.S. 1990
Total employment	1.01	[0.99,1.02]	0.98	331	U.S. 2001
Household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	U.S. 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	U.S. 2001
Length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
Road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

Data sources are shown in *SI Text*. CI, confidence interval; Adj-R^2, adjusted R^2; GDP, gross domestic product.

Scaling Relation	Exponent	Error	Observations	Region/Nation	Urban Unit	Year	Reference
Land area							
administrative	$\alpha = 0.75$	NR	412	USA	Cities (political)	1940	*(39)*
administrative	$\alpha = 0.75$	$R^2 = 0.87$	157	England and Wales	Cities (political)	1951	*(39)*
urbanized	$\alpha = 0.66$	[0.65,0.67]	1,800	Sweden	Tätort (urban area)	1960	*(18)*
urbanized	$\alpha = 0.65$	[0.64,0.66]	1,800	Sweden	Tätort (urban area)	1965	*(18)*
urbanized	$\alpha = 0.63$	0.62-0.64	329	USA	MSA	1980-2000	*(42)*
developed	$\alpha = 0.57$	0.56-0.59	329	USA	MSA	1992, 2000	*(42)*
light emissions	$\alpha = 0.65$	$R^2 = 0.62$	4,851	USA	Night-light clusters	1992	*(47)*
Average land area	$\alpha = 0.67$	[0.56,0.75]					
built area	$\alpha = 0.78$	NR	89	USA	Cities (political)	1960	*(44)*
built area, radial	$\alpha = 0.88$	NR	368†	Michigan, USA	Cities (political)	1969	*(40)*
built area	$\alpha = 0.96$	[0.89,1.04]	70	Norfolk, UK	Settlements	1981	*(41)*
built area, radial	$\alpha = 0.87$	[0.75,0.99]	70	Norfolk, UK	Settlements	1981	*(41)*
built area	$\alpha = 0.87$	NR	51	Ontario, Canada	Urban Areas	1966	*(46)*
Average land area*	$\alpha = 0.75$	[0.56,1.04]					
Network area (or volume)							
impervious surfaces	$\nu = 0.85$	0.84-0.86	3,629	World	Cities$> 100,000$	2000	*(16)*
impervious surfaces	$\nu = 0.86$	$R^2 = 0.74$	119	EU	Agglomerations$> 200,000$	1990	*(50)*
built area	$\nu = 0.82$	$R^2 = 0.84$	660	China	Urban Areas	2005	*(51)*
area of roads	$\nu = 0.85$	[0.81,0.89]	451	USA	MSA	2006	Fig. 1A
area of roads	$\nu = 0.83$	[0.74,0.92]	29	Germany	LUZ	2002	*(12)*
Average network volume	$\nu = 0.84$	[0.74,0.92]					
Network length							
length of pipes	$\lambda = 0.67$	[0.55,0.78]	12	Japan	MA	2005	Fig. S1
Socioeconomic rates							
GDP	$\beta = 1.13$	[1.11,1.15]	363	USA	MSA	2006	Fig. 1B,*(12)*
GDP	$\beta = 1.22$	[1.11,1.33]	273	China	Prefectural Cities	2005	Fig. S2A
GDP	$\beta = 1.10$	[1.01,1.18]	35	Germany	LUZ	2004	Fig. S2B
income	$\beta = 1.12$	[1.07,1.17]	12	Japan	MA	2005	Fig. S1A
wages	$\beta = 1.12$	[1.07,1.17]	363	USA	MSA	1969-2009	Fig. S3
violent crime	$\beta = 1.16$	[1.11,1.19]	287	USA	MSA	2003	*(12)*
violent crime	$\beta = 1.20$	[1.07,1.33]	12	Japan	MA	2008	*(62)*
violent crime	$\beta = 1.20$	[1.15,1.25]	27; 5,570	Brazil	MA; Municipios	2003-07	*(25)*, *(62)*
new AIDS cases	$\beta = 1.23$	[1.17,1.29]	93	USA	MSA	2002-3	*(12)*
new patents	$\beta = 1.27$	[1.22,1.32]	331	USA	MSA	1980-2001	*(11, 12)*
supercreative jobs	$\beta = 1.15$	[1.13,1.17]	331	USA	MSA	1999-2001	*(11)*
R&D employment	$\beta = 1.19$	[1.12,1.26]	227-278	USA	MSA	1987-2002	*(11)*
Average socioeconomic rates	$\beta = 1.17$	[1.01,1.33]					
Social interactions							
cell phones	$\beta = 1.12$	[1.00,1.25]	415	Portugal	Cities, LUZ, Municipality	2006-7	*(21)*
land lines	$\beta = 1.12$	[1.05,1.17]	24	UK	Cities	2005	*(21)*
Average social interactions	$\beta = 1.12$	[1.00,1.25]					
Power dissipation							
electrical	$\omega = 1.11$	[1.05, 1.17]	380	Germany	Cities	2002	*(12)*
Average land rents							
median house value	$\delta_L = 0.49$	[0.46,0.52]	363	USA	MSA	2006	*(24)*

NR=not reported. Error, in order of availability from the source, is given by: 95% confidence intervals (square brackets), ranges, or R^2 values.
Note: Average quantities are the simple (unweighted) averages across rows. Corresponding error intervals are the union of those from individual studies.
* This estimate of *Average land area* includes all 12 rows above, it mixes explicit measurements of built area with others.
† This estimate was obtained by the author through visual inspection of Fig. 1 in Ref. *(39)*.

Urban Systems:
USA
European Union
Brazil
South Africa
Japan
China
India
…

Settlements:
Aztecs
Inka
Southwest Pueblo
Roman Empire
Medieval Europe
19th Century England

Different in:
- Hunter/Gatherer Camps
- Slums (informal settlements)

https://www.science.org/doi/10.1126/science.1235823

Summary of Exponents

Table 2. Classification of scaling exponents for urban properties and their implications for growth

Scaling exponent	Driving force	Organization	Growth
$\beta < 1$	Optimization, efficiency	Biological	Sigmoidal: long-term population limit
$\beta > 1$	Creation of information, wealth and resources	Sociological	Boom/collapse: finite-time singularity/unbounded growth; accelerating growth rates/discontinuities
$\beta = 1$	Individual maintenance	Individual	Exponential

https://www.pnas.org/content/104/17/7301

Growth, innovation, scaling, and the pace of life in cities

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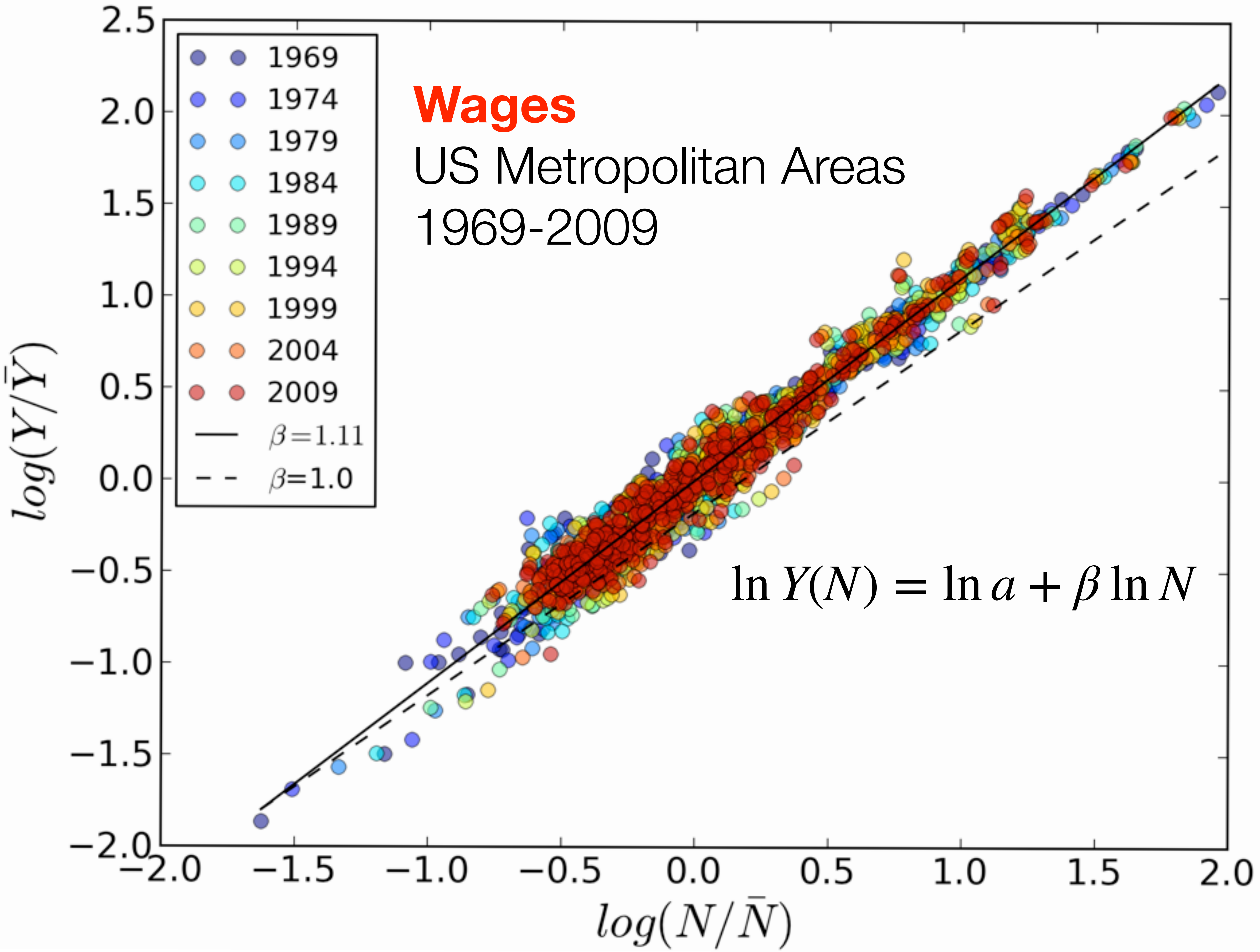
Humanity has just crossed a major landmark in its history with the majority of people now living in cities. Cities have long been known to be society's predominant engine of innovation and wealth creation, yet they are also its main source of crime, pollution, and disease. The inexorable trend toward urbanization worldwide presents an urgent challenge for developing a predictive, quantitative theory of urban organization and sustainable development. Here we present empirical evidence indicating that the processes relating urbanization to economic development and knowledge creation are very general, being shared by all cities belonging to the same urban system and sustained across different nations and times. Many diverse properties of cities from patent production and personal income to electrical cable length are shown to be power law functions of population size with scaling exponents, β, that fall into distinct universality classes. Quantities reflecting wealth creation and innovation have $\beta \approx 1.2 > 1$ (increasing returns), whereas those accounting for infrastructure display $\beta \approx 0.8 < 1$ (economies of scale). We predict that the pace of social life in the city increases with population size, in quantitative agreement with data, and we discuss how cities are similar to, and differ from, biological organisms, for which $\beta < 1$. Finally, we explore possible consequences of these scaling relations by deriving growth equations, which quantify the dramatic difference between growth fueled by innovation versus that driven by economies of scale. This difference suggests that, as population grows,

The increasing concentration of people in cities presents both opportunities and challenges (9) toward future scenarios of sustainable development. On the one hand, cities make possible economies of scale in infrastructure (9) and facilitate the optimized delivery of social services, such as education, health care, and efficient governance. Other impacts, however, arise because of human adaptation to urban living (9, 10–14). They can be direct, resulting from obvious changes in land use (3) [e.g., urban heat island effects (15, 16) and increased green house gas emissions (17)] or indirect, following from changes in consumption (18) and human behavior (10–14), already emphasized in classical work by Simmel and Wirth in urban sociology (11, 12) and by Milgram in psychology (13). An important result of urbanization is also an increased division of labor (10) and the growth of occupations geared toward innovation and wealth creation (19–22). The features common to this set of impacts are that they are open-ended and involve permanent adaptation, whereas their environmental implications are ambivalent, aggravating stresses on natural environments in some cases and creating the conditions for sustainable solutions in others (9).

These unfolding complex demographic and social trends make it clear that the quantitative understanding of human social organization and dynamics in cities (7, 9) is a major piece of the puzzle toward navigating successfully a transition to sustainability. However, despite much historical evidence (19, 20) that cities are the principal engines of innovation and economic growth, a

https://www.pnas.org/content/pnas/104/17/7301.full.pdf

A law in **time** ...



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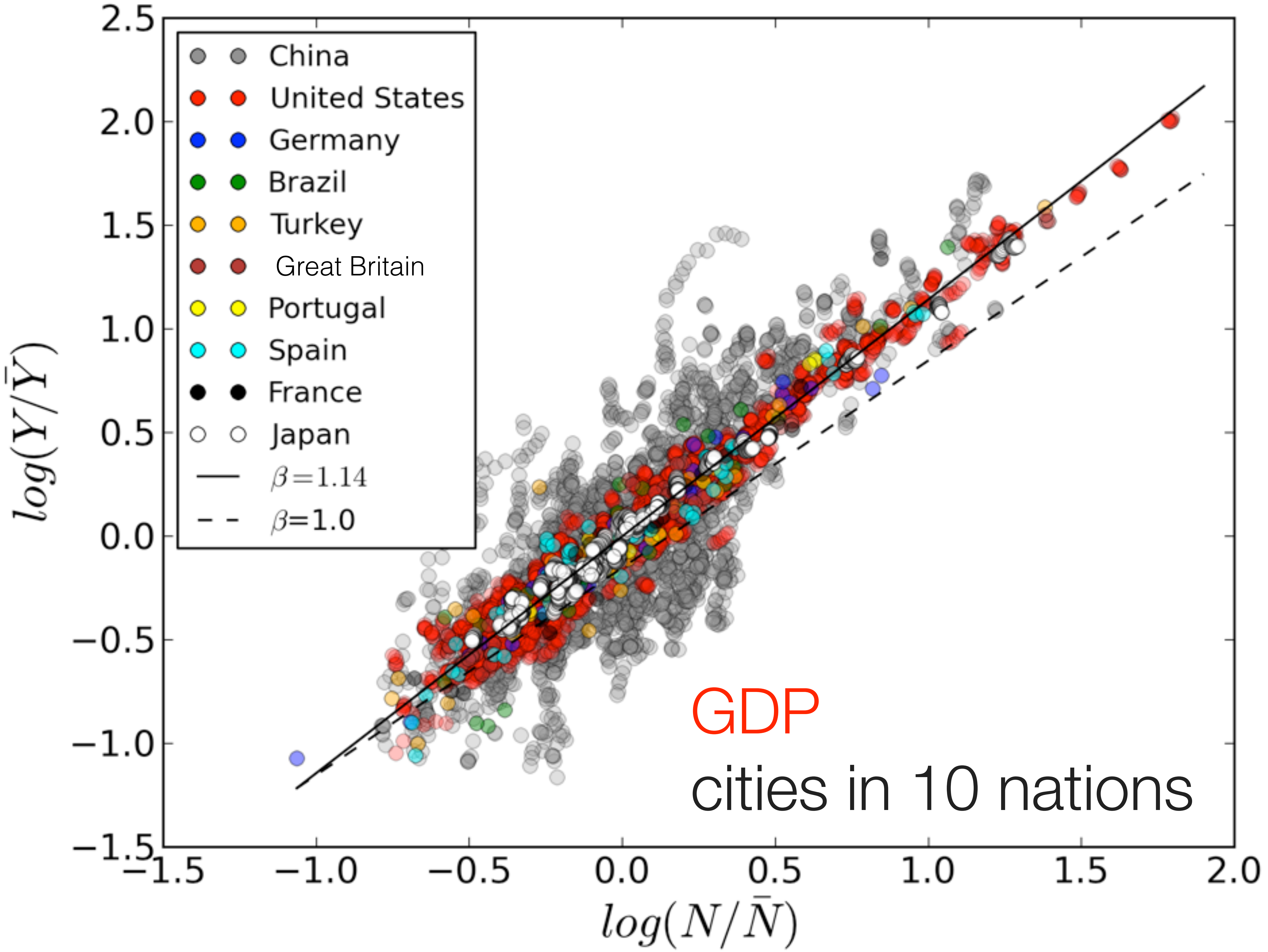
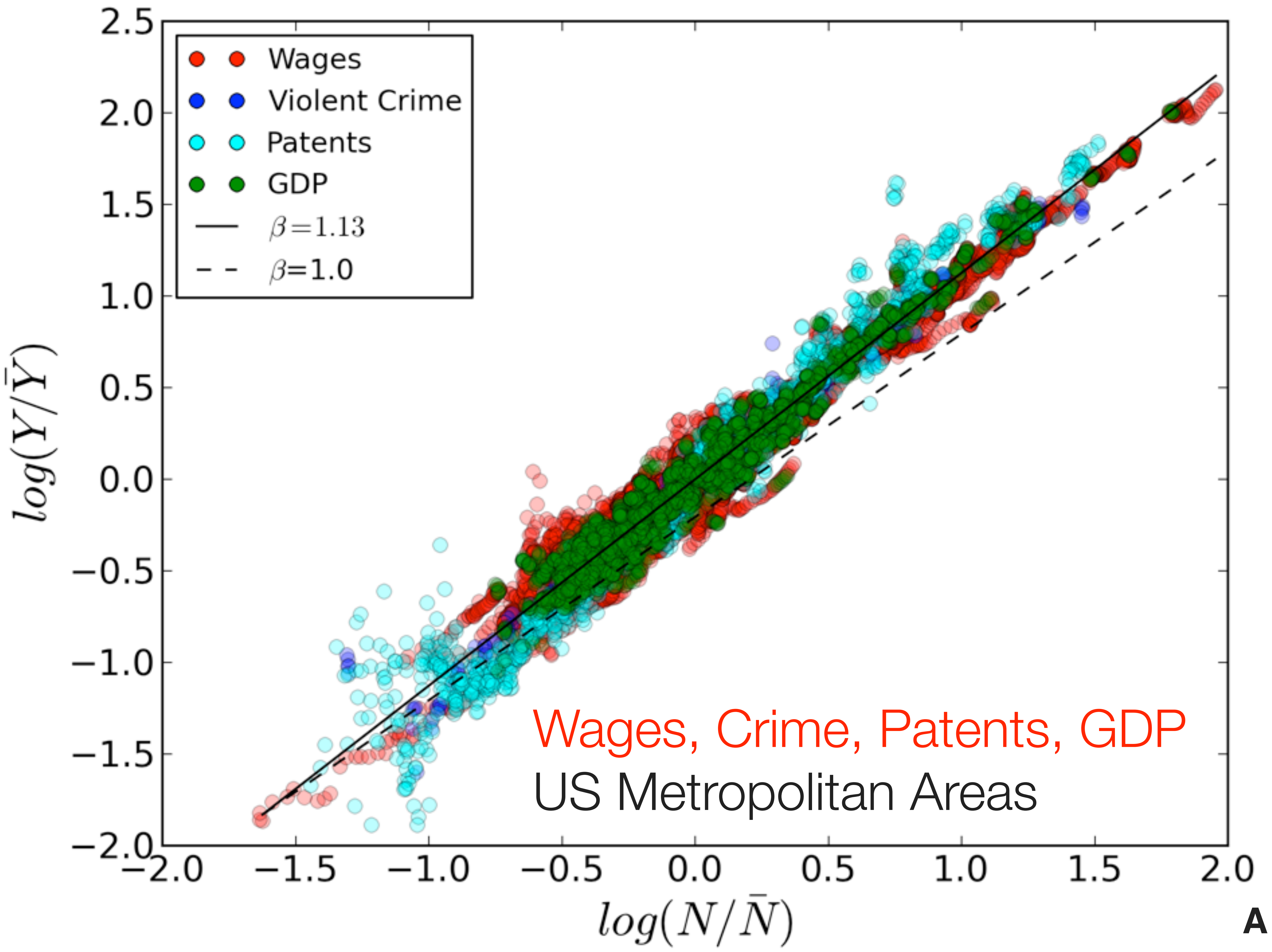
A law in **across nations** ...

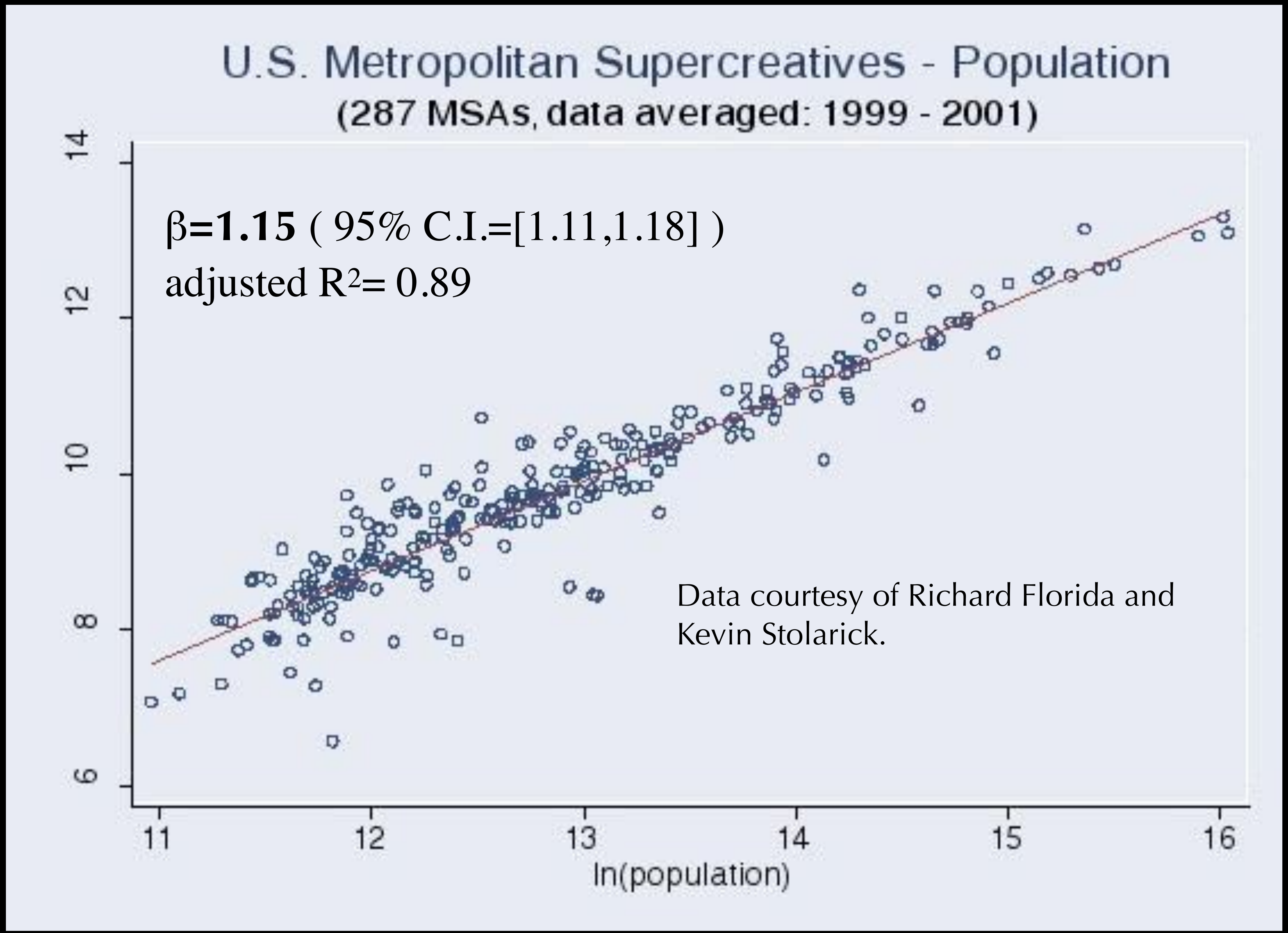


A law in **across quantities** ...



A unified theory of urban living

https://www.nature.com/articles/467912a



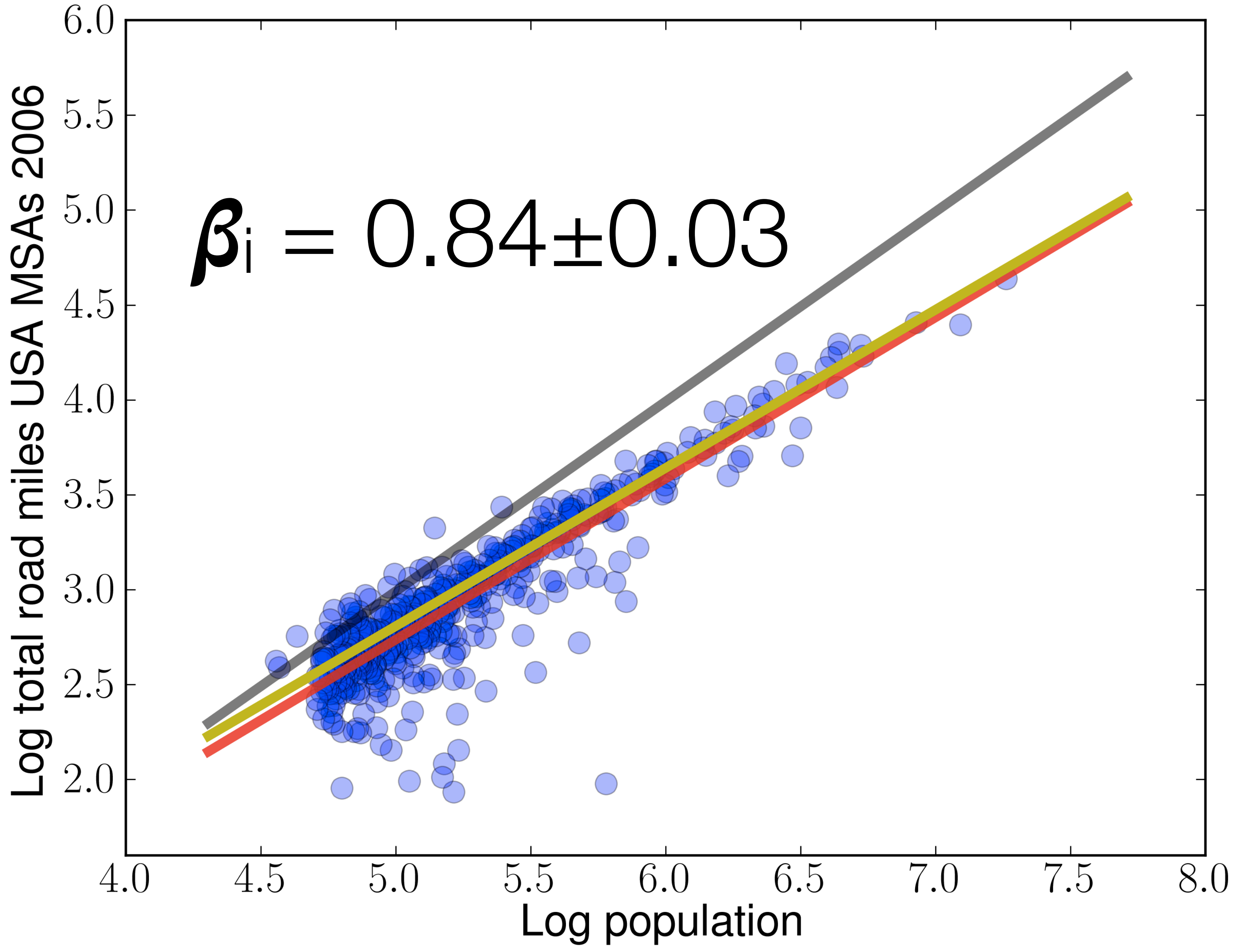
Supercreative Professionals [Florida 2002] are “Computer and Mathematical, Architecture and Engineering, Life Physical and Social Sciences Occupations, Education training and Library, Arts, Design, Entertainment, Sports and Media Occupations”.

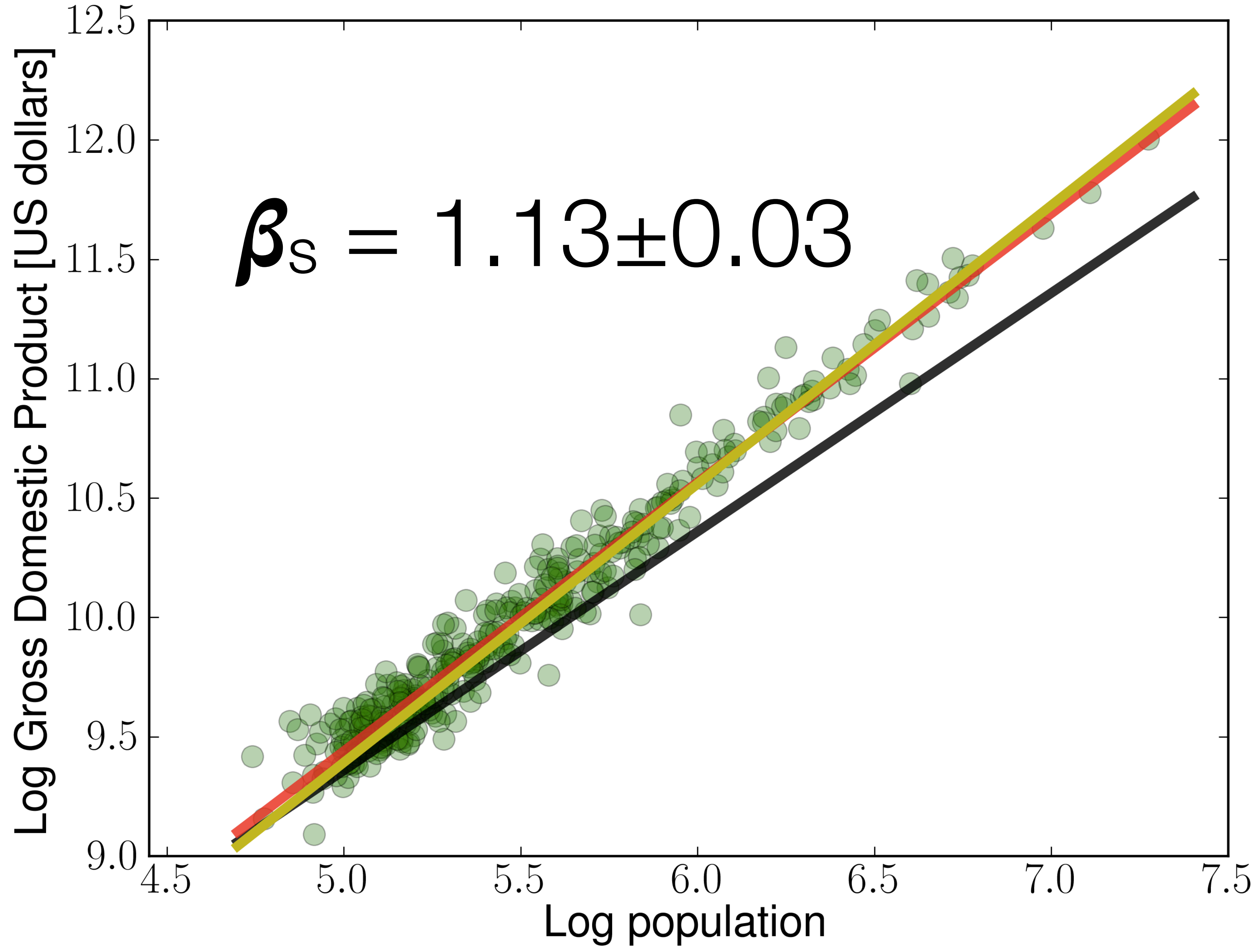
Derived from Standard Occupation Classification System of the U.S. Bureau of Labor Statistics

https://www.sciencedirect.com/science/article/abs/pii/S0048733306001661?via%3Dihub

Infrastructure & socioeconomic rates

United States: Metropolitan Areas





Volume of Infrastructure

$$\sim N^{\beta_i}$$

$$\beta_i = 1 - \delta$$

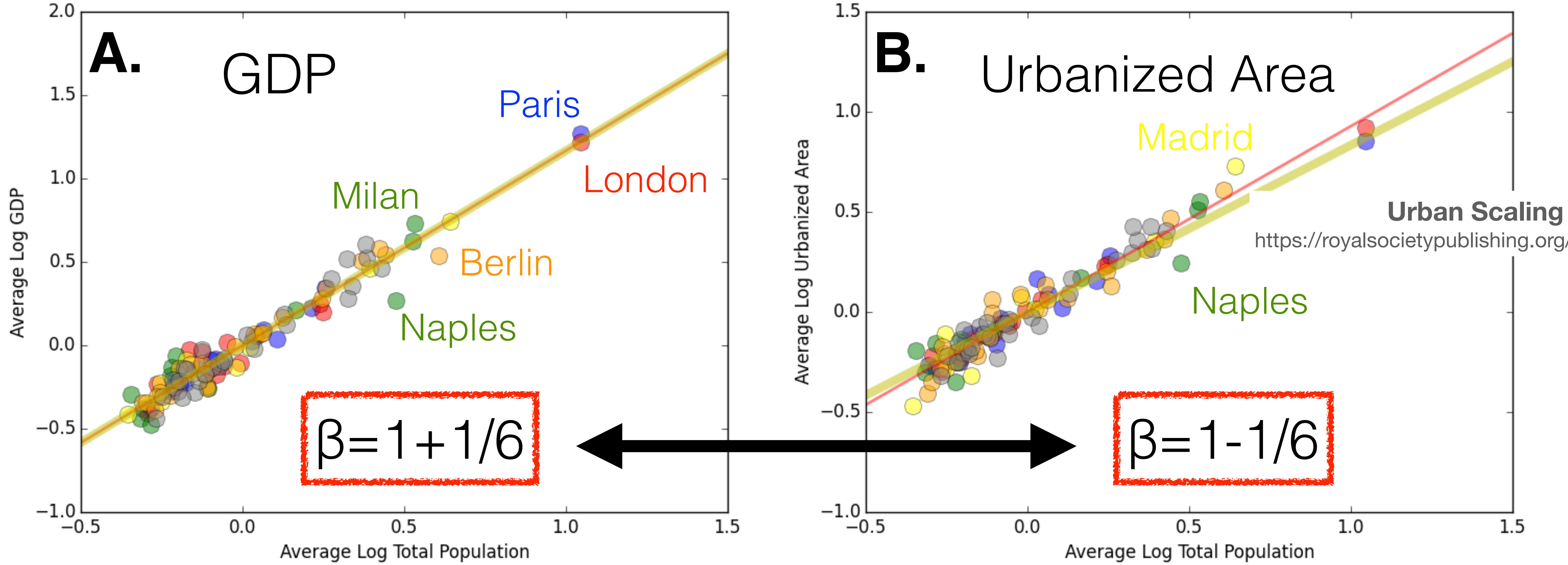
$$\delta \simeq 1/6$$

Social Outputs

$$\sim N^{\beta_s}$$

$$\beta_s = 1 + \delta$$

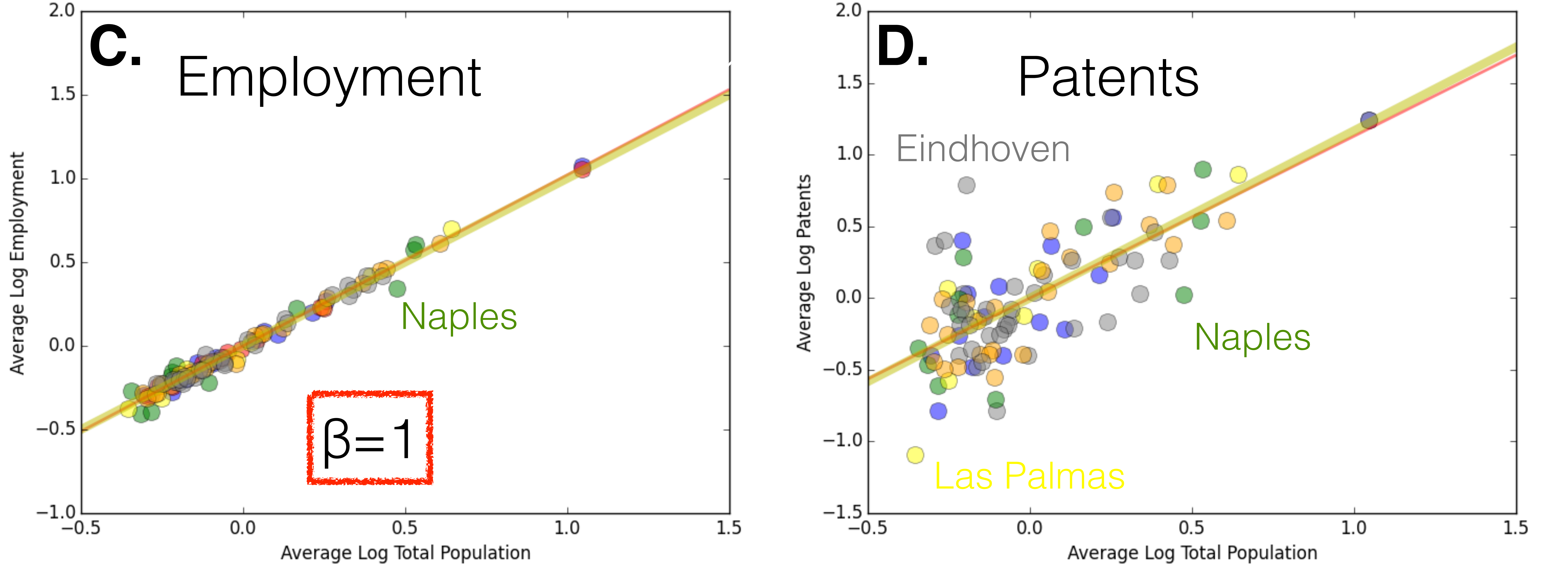
The origins of scaling in cities
https://www.science.org/doi/full/10.1126/science.1235823



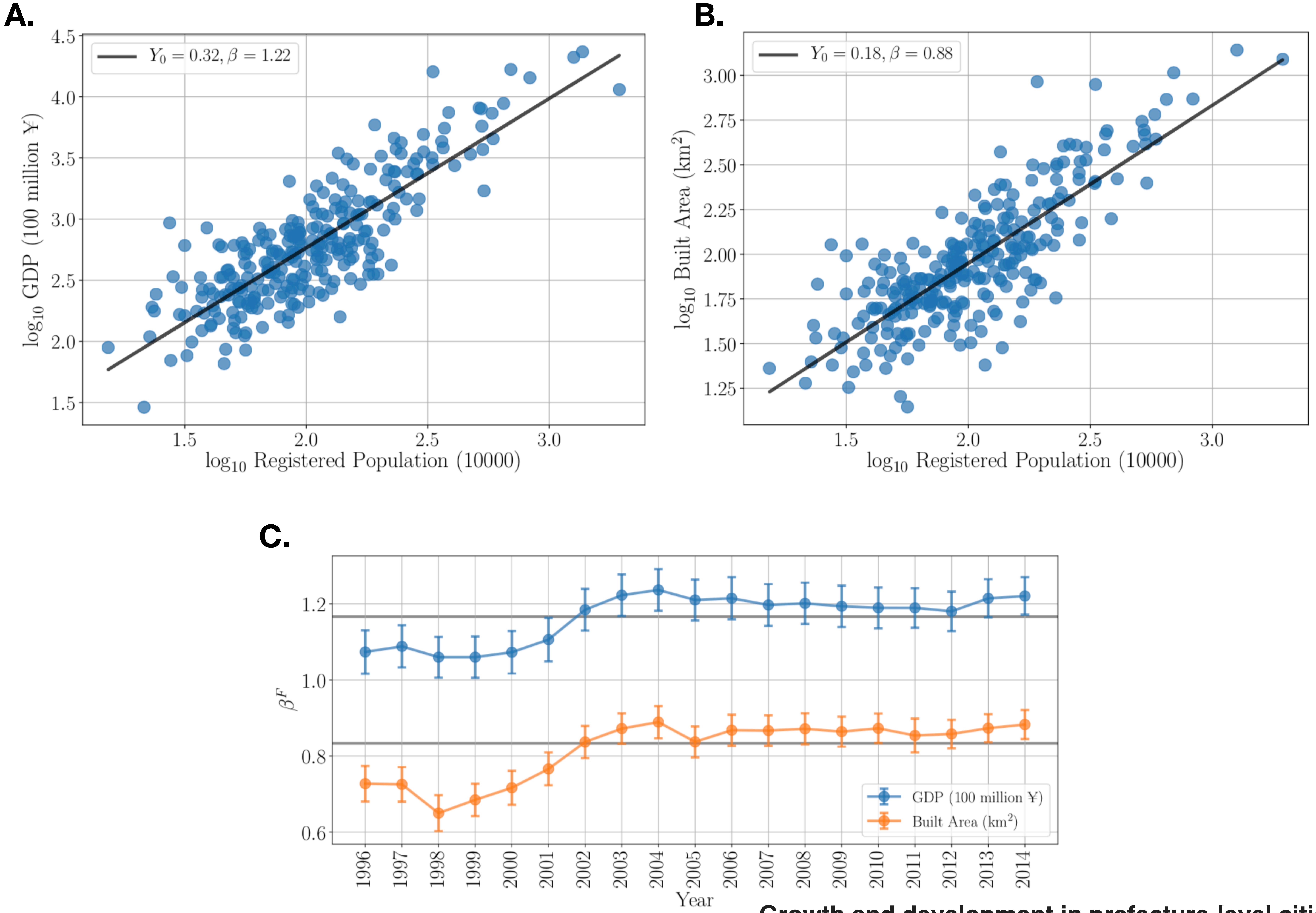
Urban Scaling in Europe

https://royalsocietypublishing.org/doi/10.1098/rsif.2016.0005

Metropolitan Areas in the European Union



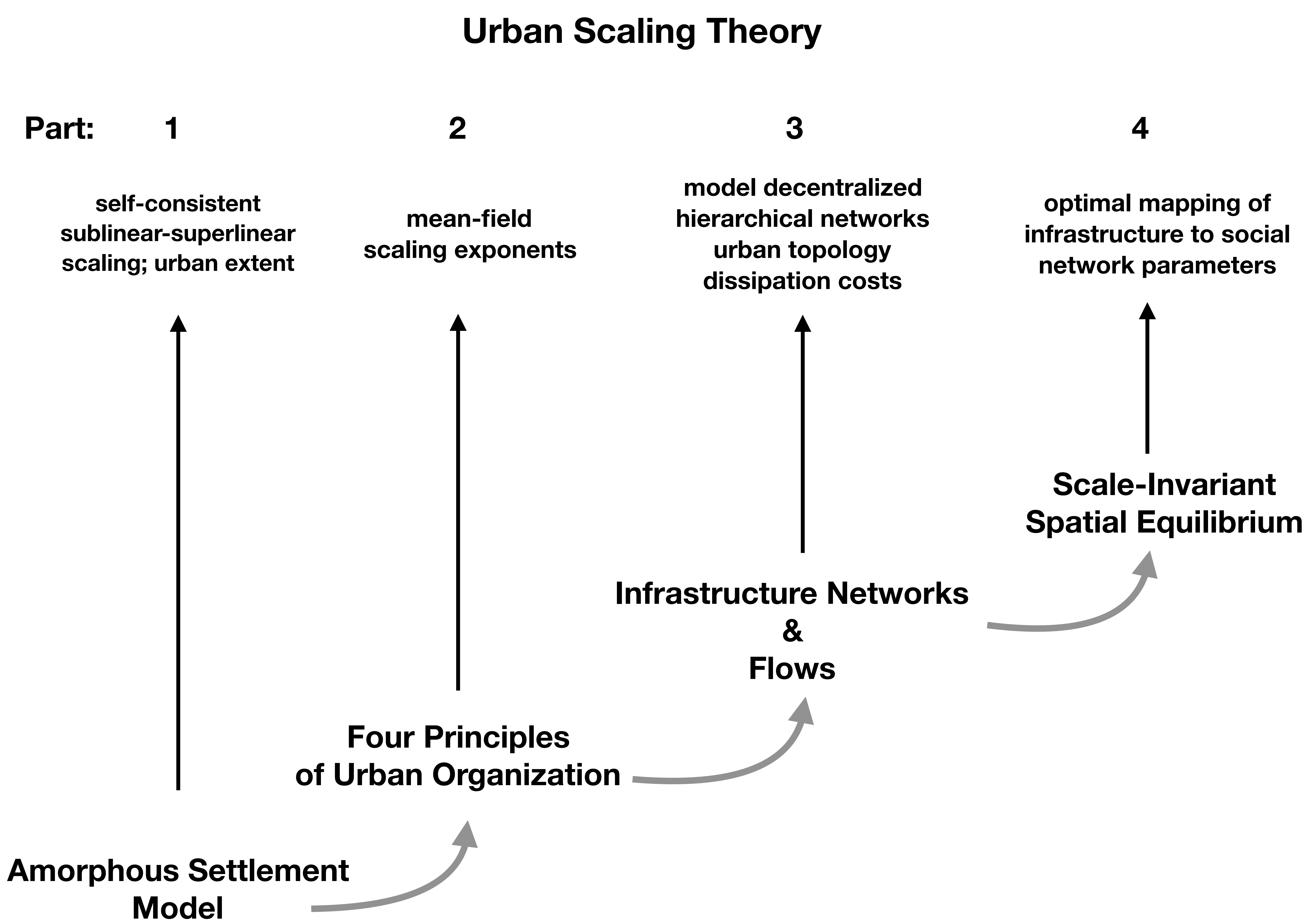
Prefectural Cities in China



Growth and development in prefecture-level cities in China

https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0221017

A roadmap for deriving the scaling properties of cities



IUS Figure 3.5