# ALTERNATE EXPLANATIONS OF URBAN RANK–SIZE RELATIONSHIPS[1]

BRIAN J. L. BERRY AND WILLIAM L. GARRISON

*University of Washington*

PICK any large area. It will likely contain many small cities, a lesser number of medium-size cities, and but few large cities. This pattern of city sizes has been observed to be quite regular from one area to another. That is, when the frequency of occurrence of city sizes in any area is compared with the frequency of occurrence of sizes in another area, the two frequencies are very much alike. An example is furnished by a comparison of city sizes and ranks in the Republic of Korea and the state of Washington (Figure 1). Frequencies for the two areas are quite similar. Such empirical regularities of city size have been noted many times and have long posed a challenge to those who would explain or interpret them.[2]

Several explanatory schemes directly or indirectly related to the problem of repeated regularities of patterns of occurrence of city sizes have been proposed. The present discussion brings these schemes together for comparison, namely, the schemes or theories of G. K. Zipf, W. Christaller, N. Rashevsky, and H. A. Simon.[3] In the ensuing discussion it will be noted that city size regularities associated with Zipf have been explained by Simon using very simple probability notions. Too, it will be noted that the city size rule of Zipf is consistent in special cases with the theories of Rashevsky and Christaller. Since Rashevsky's scheme is a contribution to the general theory of urbanization and economic opportunity, and since Christaller's theory is the generic base of theories of urban size, function, and arrangement (subsequently generalized in several respects by August Lösch[4]), city size relations are consistent with more general theory. The alternatives and issues involved in the researchers' choice between the simple

[1] This article is entirely expository. It elaborates diverse ideas that the authors have found difficult to resolve and it is hoped that it will stimulate more detailed explanations. For such explanations we are already indebted to Messrs. Vir Bhatia, Harvard University, Richard Quandt, Princeton University, and Duane Marble, University of Washington.

[2] Edgar M. Hoover has pointed out the need for an adequate explanation in "The Concept of a System of Cities: A Comment on Rutledge Vining's Paper," *Economic Development and Cultural Change*, Vol. 3 (1955), pp. 196–98.

[3] George K. Zipf, *Human Behavior and the Principle of Least Effort* (Cambridge: Addison-Wesley Press, Inc., 1949); W. Christaller, *Die zentralen Orte in Süddeutschland* (Jena: Gustav Fischer, 1933, translated in 1954 at the Bureau of Population and Urban Research, University of Virginia, by C. Baskin); N. Rashevsky, *Mathematical Theory of Human Relations*, Mathematical Biophysics Monograph Series, No. 2 (Bloomington: The Principia Press, Inc., 1947); Herbert A. Simon, "On a Class of Skew Distribution Functions," *Biometrika*, Vol. 42 (1955), pp. 425–40, reprinted as Chapter 9 of *Models of Man* (New York: Wiley, 1957). Note that when practicable in our ensuing discussion of these studies the terminology of original authors is maintained.

[4] August Lösch, *Die räumliche Ordnung der Wirtschaft* (Jena: Gustav Fischer, 1939), translated by W. H. Woglom and W. F. Stolper as *The Economics of Location* (New Haven: Yale University Press, 1954).

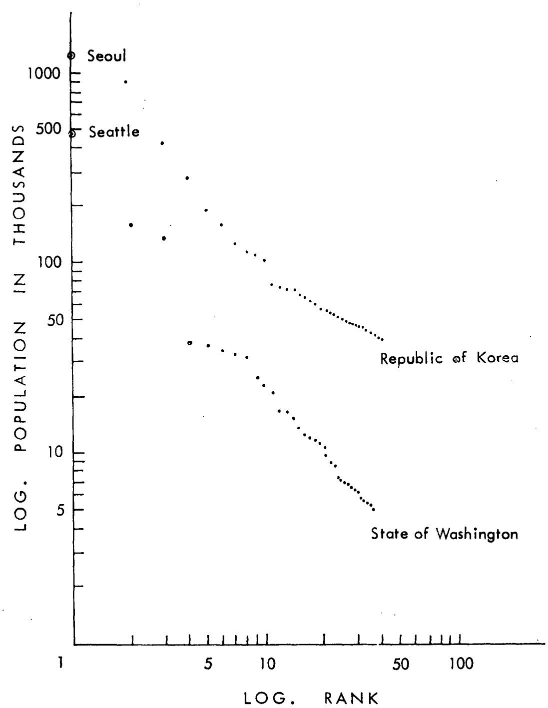


FIG. 1. City rank–size relationships: Republic of Korea and State of Washington. Sources: *Census of the Republic of Korea, Section 1, 1955* (Seoul, 1956), and *Population of Towns and Cities in the State of Washington* (Seattle: Washington State Census Board, 1954).

explanation offered by Simon and the less simple alternates of others pose the concluding problem of the discussion.

## CITY SIZES

The problem before us is posed by a comparison of city populations and ranks. In any area, say a nation, cities may be ranked from the largest to the smallest according to population. The largest city would rank number 1, the second largest number 2, etc. When these ranks are then plotted against city population size a regular relationship emerges. There are few large cities and many small ones and there is an apparent empirical relationship between the rank of a city and its population. The size relationship takes the form $r_i\ (p_i^q) = K$, when $q$ and $K$ are constants, $r_i$ is the rank of the $i$th city and $p_i$ is the population of that city. Observed distributions are concave upward and are linear or nearly so when plotted using logarithmic axes.

These city size relations, termed the rank–size rule, have been noted by a number of persons.[5] Valuable empirical studies are due to Zipf who has compiled and published data for several countries.[6] As W. Isard has pointed out, however, a number of persons had noted the empirical relationships earlier.[7] The empirical evidence is formidable and has recently been reviewed by Vining[8] and Isard,[9] and augmented by G. R. Allen[10] and C. H. Madden.[11] For these reasons it will not be reviewed here.

Any empirical regularity poses problems for theory. Observations of city rank–size relationships have served both to initiate and to verify theory, the two classic functions of empirical work. Zipf has attempted to erect a scheme to explain rank regularities and Rashevsky has turned to Zipf's observations for partial verification of his theoretical scheme. Like Zipf, Simon has attempted a theoretical scheme to explain empirical regularities, and as with Rashevsky, observations of rank–size relationships are consistent with the scheme of Christaller. Empirical regularities, then, are a common element in a variety of undertakings and they serve as a link among divergent approaches to a common problem.

In connection with rank–size relationships the question has been asked, Is there in fact a regularity?[12] A scatter of dots formed by plotting the sizes of cities versus their rank never presents a perfect alignment of dots along a mean line. One dot will fall above the line, another below, and so on. Some misfit is expected, of course, due to different definitions of the area of the city and other data liabilities.[13] The problem is whether regularity occurs, leaving these sources of error in the data aside.

It is sufficient for the present problem that regularities may exist. One point about the problem of empirical validation should be made, though. There is never perfect agreement of data with theory. Thus a number of statistical tools have been developed to test hypotheses of agreement. Paralleling these

[5] Well-known references to city size relationships in the geographical literature are those of Mark Jefferson, e.g., "The Law of the Primate City," *Geographical Review*, Vol. 29 (1939), pp. 226–32, and J. Q. Stewart, "Empirical Mathematical Rules Concerning the Distribution and Equilibrium of Population," *Geographical Review*, Vol. 37 (1947), pp. 461–85, especially pp. 462–67. A recent reference to expected city size relationships is in P. E. James and Speridiav Faissol, "The Problem of Brazil's Capital City," *Geographical Review*, Vol. 46 (1956), pp. 301–17. One of the very early studies was that by Felix Auerbach, "Das Gesetz der Bevolkungskonzentration," *Petermann's Geographische Mitteilungen*, Vol. 59 (1913).

[6] *Op. cit.*, and *National Unity and Disunity* (Bloomington: The Principia Press, 1941).

[7] Walter Isard, *Location and Space Economy* (New York: John Wiley and Sons, Inc., 1956), pp. 55–60.

[8] "A Description of Certain Spatial Aspects of an Economic System," *Economic Development and Cultural Change*, Vol. 3 (1955), pp. 147–95.

[9] *Op. cit.*

[10] "The 'Courbe des Populations', A Further Analysis," *Bulletin of the Oxford University Institute of Statistics*, Vol. 16 (1954), pp. 179–89. Allen fitted the Pareto formula to 44 countries and finds: "The main conclusion is, therefore, that the Pareto Law can be used with much success to summarize the relationships between size of towns and number of towns above a specified size" (p. 184). The Pareto formula is similar to what Zipf has termed the rank–size rule. Explanations of it are to be found in R. G. D. Allen, *Mathematical Analysis for Economists* (London: MacMillan, 1950), pp. 407–08, and D. G. Champernowne, "A Model of Income Distribution," *Economic Journal*, Vol. 63 (1953), pp. 318 *et seq.* The earlier empirical application of Pareto's law to city sizes was by H. W. Singer, "The 'Courbe des Populations': A Parallel to Pareto's Law," *Economic Journal*, Vol. 46 (1936), pp. 254–63.

[11] "The Growth of Cities in the United States: An Aspect of the Development of an Economic System" (unpublished Ph.D. thesis, University of Virginia, 1954).

[12] Isard, *op. cit.*, p. 57.

[13] These problems are noted by G. R. Allen, *op. cit.*, and Allen also provides a measure of error or lack of fit.

tests but apart from the statistical problems are problems of interpreting degrees of agreement. Plausibility plays a part here. To the extent that regularity is plausible it is easy to see regularity. If no plausible basis for regularity is known, regularity is difficult to see.

The present discussion reviews plausible bases for empirical regularities between city sizes and ranks, and to the extent that plausibility is provided, the present discussion serves to verify asserted regularities.

### ZIPF AND THE RANK–SIZE RULE

Perhaps the best-known elaboration of rank–size regularities was developed by G. K. Zipf.[14] Zipf's discussion was set within a broad context—a general theory of human behavior —in which rank–frequency relationships are noted for many expressions of human behavior (e.g., number of times words were used in a novel). The general theory was presented to explain these regularities.

The discussion as a whole has been reviewed elsewhere.[15] Too, the portion of the discussion treating the sizes of cities may be extracted from the larger work without distorting Zipf's analysis. Thus, the general works will not be reviewed at the present time. Zipf presented evidence of strong rank–size relationships. As a case in point, a study of the 100 largest metropolitan districts in the United States in 1940 yielded the best-fitting equation $r = (P^{-1})$ 10,000,000. [16] This equation, Zipf noted, indicated that $K = 10{,}000{,}000$ and was approximately equal to the population of the largest metropolitan district. Since the exponent $q$ equals 1, it indicated the equality of the forces of diversification and unification.[17] The latter requires elaboration.

*Diversification versus unification.* Zipf's explanation of rank–size relationships is in broad outline quite simple. On the one hand, it is postulated that there is a tendency for the population to be split into many small autarchic communities. This process, labeled the force of diversification, is that of economical location relative to raw materials. Persons are located to minimize the transfer cost of obtaining raw materials. A society using many scattered raw materials would be highly disaggregated in location; a society using few strongly localized raw materials would be highly aggregated in location.

On the other hand, there is a reverse tendency, the force of unification. Diversification tends to minimize the difficulty of moving raw materials to the places where they are to be processed; unification tends to minimize the difficulty of moving processed materials to the ultimate consuming populace. If all persons in the society were located at the same point, then maximum unification would be achieved. When both the forces of diversification and unification are at work a distribution of population is presumed to occur that is at optimum with reference to both forces.

*The domains of goods.* An additional concept of Zipf's theory is the notion of the domain in which goods may be economically traded. Zipf notes that different goods have different cost relationships. Some are marketed in many small domains, others in large domains, and so on. Each good is made from a unique mix of raw materials, but groups of goods may be produced in the same community.

*Formulation of the rank–size rule.* Thus far the Zipf scheme is simple and straightforward. However, the manner by which one moves from these simple notions to the rank–size rule is less clear, namely,

> Since the Force of Diversification makes for a larger $n$ number of small $P$ communities, whereas the Force of Unification makes for a smaller $n$ number of larger $P$ communities, then, if we interpret the relationship as a best straight line on doubly logarithmic co-ordinates, the result will be that the $n$ number of different communities, when ranked $r$, in the order of their decreasing $P$ size will follow the equation (approximately):
>
> $r = P^{-q}K$ . . . [18]

*Contributions of Zipf.* It is certainly not clear just what are the logical links between the scheme proposed by Zipf to explain rank–size regularity and observed rank–size regularities. Thus, it would not be proper to credit Zipf with an articulated empirical and theoretical analysis of the rank–size problem. On the other hand, in several important ways Zipf deserves great credit.

Zipf's works have called emphatic attention to the rank–size problem. The number of students who have been attracted to the problem as stated by Zipf is, of course, unknown. But two of the contributions to the rank–size prob-

[14] *Human Behavior . . . , op. cit.*

[15] See, for example, the review by Read Bain in *Social Forces*, Vol. 28 (1950), pp. 340–41.

[16] Zipf, *Human Behavior . . . , op. cit.*, p. 375.

[17] *Ibid.*, p. 376.

[18] *Ibid.*, p. 359.

lem (both to be reviewed), those of Rashevsky and Simon, give direct credit to Zipf for calling attention to the problem. Both of these give credit to the empirical substantiation of rank–size regularities, rather than to Zipf's theoretical formulation of the problem.

Too, Zipf has proposed, or at least restated, components of the emerging general theory of location. His discussion of unification and diversification is related to Weberian localization schemes and the notion of the range of a good is similar to that of Christaller.[19] In this latter sense, Zipf's statements are not original, but they do have the virtue of consistency with other literature.

## CHRISTALLER AND SIZE CLASSES OF CITIES

A well-known alternate scheme to that of Zipf, both regarding the size of cities and the processes causing size regularities, is that of W. Christaller.[20] It is quite surprising that the scheme of Christaller has never (to the writers' knowledge) been compared with the scheme or the empirical observations of Zipf in terms of city sizes. The two schemes are quite similar. Both utilize notions of the domains of cities (domains of goods) for the performance of various economic activities.[21] Too, the rules of behavior leading to the spatial system of central places and associated arrangements of city sizes (diversification and unification) are quite similar.[22] In both schemes as the population of cities increases, the number of centers of this population diminishes.

It should be recalled that Christaller's work was somewhat broader than that of Zipf; Christaller was concerned with the spatial arrangement, function, and size of urban centers. Too, it should be recalled that Christaller did not provide a general theory in formal terms.[23] So far as general theory is concerned, there is no explicit way the scheme of Christaller may be compared with that of Zipf despite the fact that the notions of the two schemes are, as already noted, quite similar.

*Special cases.* Turning to special cases, however, it is practicable directly to compare the scheme of Christaller with that of Zipf. Christaller did provide a formal statement of one network of cities in homogeneous space. This is the well-known $k=3$ network, with one "primate city" of population $K$, and $r=1$; three cities of population $K/3$, and $r=2$; nine cities of population $K/9$ and $r=3$; twenty-seven of population $K/27$; etc.[24] This arrangement of city sizes may be compared directly with the rank–size rule provided by Zipf.

In order to make the comparison the steps of the hierarchy of centers in the $k=3$ scheme are taken as ranks, $r=1, 2, 3$, etc., and $P_i = K, K/3, K/9$, etc. A rank–size distribution in the manner of Zipf is formed if the exponent

$$q = \frac{\log\ (K/r)}{\log\ (K/3^{r-1})}.$$

Thus, where $r=2$

$$q = \frac{\log\ (K/2)}{\log\ (K/3^{2-1})}$$

and if $q \cdot \log P = \log\ (K/r)$

then $$\log\ P = \log\ (K/2) \cdot \frac{\log\ (K/3)}{\log\ (K/2)}$$

and $P = K/3$ as required by Christaller's theory.

*Implications.* In fact no great difference exists between Zipf and Christaller; on the level of the intuitive statement of basic notions the two schemes seem very much alike. Even when one turns to special cases generated by the schemes certain relationships are evident.[25] By the proper choice of relationships, the class hierarchical scheme of Christaller may take on the rank–size character of Zipf's

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[19] Zipf was cognizant of these works and used them as references.

[20] Christaller, *op. cit.*, especially pp. 63–4.

[21] *Ibid.*, pp. 20, 31, 54. Both Hoover and Vining link the schemes of Zipf and Christaller through the notion of domain.

[22] *Ibid.*, pp. 63–74.

[23] Christaller addressed the general problem of city size and arrangement on an informal and intuitive level. His discussion is formal only with his example case of the $k=3$ network. It is not clear why Christaller has been criticized for lack of generality. See the comments upon this common criticism in Brian J. L. Berry, "Geographic Aspects of the Size and Arrangement of Urban Centers" (unpublished Master's Thesis, University of Washington, 1956), pp. 7–30. Our attention has been called to C. W. Baskin's, "A Critique and Translation of Walter Christaller's *Die zentralen Orte in Süddeutschland*" (unpublished Ph. D. dissertation, Department of Economics, University of Virginia, 1957), but we have not had the opportunity of examining this study.

[24] A discussion of this network as well as other networks is available in August Lösch, *op. cit.*, p. 131.

[25] It is to be noted, of course, that our Christaller formulation refers to a rank–size regularity of hierarchical classes of city sizes.

observations. The Christaller scheme is consistent with Zipf's empirical observations, but consistency requires a rigorous choice of relationships.

### RASHEVSKY, URBANIZATION AND ECONOMIC OPPORTUNITY

As was true of both Christaller and Zipf, N. Rashevsky's deductions bearing on city size were set within a larger context.[26] Rashevsky set out a general theory of human relations in mathematical form. In this general theory the spatial distribution of individuals and the size of cities were core topics.[27] Observed empirical regularities in the distribution of city sizes were approached in a different manner from the two topics just mentioned.[28]

*Rural versus urban opportunities.* Rashevsky's approach to the urbanization problem was via an evaluation of alternate economic opportunities.[29] The rural–urban division of population was seen directly related to the level of opportunities in each area. There are $N_r$ rural persons and $N_u$ urban ($N_r + N_u = N$) and $p_r$ and $p_u$ indicate the corresponding production per person. In general,

$$p_u = f_u(N_u, N_r); \quad p_r = f_r(N_u, N_r)$$

and the condition $p_u = p_r$ is assumed. This is an extremely interesting system that leads to some interesting results bearing on relations between urbanization and population changes.[30]

While the simple system just mentioned offers a first approximation to the urbanization problem it throws no light on the city size problem, which is approached by enlarging the scheme to identify the number and sizes of cities. If the productivity, $p_i$, per person in cities of population size $n_i$ is

$$p_i = f(n_i, N_i)$$

(where $N_i$ is the total number of persons in all cities of size $n_i$) and $p_1 = p_2 = , \ldots , = p_n$ (where $i = 1, 2, \ldots , n$),

then: $$p = f(n_i, N_i)$$

and all city sizes are presumed determined.

Now, like many schemes, once presented, these notions of Rashevsky's seem utterly elementary and obvious and this is to Rashevsky's credit. An equilibrium in city size is presumed to be reached when the production per person in each city is the same. There is no allowance for lag and it is not completely explicit how equilibrium processes work. But the system seems a reasonable first approximation of reality. Is it consistent with observed rank–size regularities of the present problem? The answer to this is a qualified No; this production opportunity view of equilibrium is not one which generates rank–size regularities directly.[31]

*Types of activities.* On the other hand, the evidence offered by Zipf's rank–size observations was so strong that Rashevsky attempted to reformulate his city size system in a manner that would tailor directly to rank–size observations. To do this the distribution function of the gradation of numbers of persons performing different types of activities in urban centers was considered. (For example, one could consider the distribution in urban centers of groups of persons associated with government activities). After considerable manipulation of this idea, it was shown that city size as a function of the distribution of activities can in a special case approximate the observed rank–size distributions.

*Implications.* As was the case with Christaller's scheme, observed rank–size regularities would seem neither to contradict nor to support the scheme of Rashevsky, which will produce rank–size relationships as a special case. As with Christaller, the scheme of Rashevsky's is consistent with Zipf's empirical observations, but consistency requires a set choice of relationships.

### SIMON AND A PROBABILITY EXPLANATION

Like Rashevsky, H. A. Simon attempted an explanation of observed rank–size regularities.[32] Simon's explanation formed an integral part of an approach to a general systems theory based on broad analogies between the

[26] N. Rashevsky, *op. cit.*

[27] *Ibid.*, chaps. 10 and 11.

[28] *Ibid.*, chap. 12.

[29] Rashevsky's discussion is entirely in economic terms, i.e., he speaks of the opportunities for the production of economic goods. As he points out (*ibid.*, p. ix), however, the discussion could have been cast around satisfaction functions. This practicability removes the argument that Rashevsky's discussion is overly restricted.

[30] *Ibid.*, pp. 85–87, and Rashevsky, "Contribution to the Theory of Human Relations: Outline of a Mathematical Theory of the Size of Cities, VII," *Psychometrica*, Vol. 8 (1943), pp. 87–90.

[31] This conclusion is reached by Rashevsky, *ibid.*, pp. 94–5. For some interesting empirical observations relating to population distribution and city sizes see A. H. Hawley, *The Changing Shape of Metropolitan America* (Glencoe, Illinois: The Free Press, 1956), pp. 34 *et seq.*

[32] *Op. cit.*

frequency distribution of a wide variety of biological, social, and economic phenomena observed by Zipf and others.[33] Such phenomena include distributions of words in prose samples by frequency of occurrence, distributions of numbers of scientists by numbers of papers published, distributions of incomes by size (Pareto distributions), and distributions of biological genera by numbers of species, in addition to the distributions of city sizes. Simon was unwilling to suppose initially that there was any connection between these phenomena other than that known probability mechanisms might provide satisfactory abstract models of each, and provide the bases of analogies.

*The distributions.* Simon argued that the distribution of city sizes was one of a family of distributions which have the following general characteristics in common:

(a) They are *J*-shaped, or at least highly skewed, with very long upper tails which can be approximated closely by a function of the form

$$f(i) = (a/i^k)b^i$$

where $a$, $b$ and $k$ are constants, and the convergence factor $b$ is so close to 1 that it often may be disregarded. Thus, for example, the number of cities that have a population $i$ is approximately $a/i^k$.

(b) The exponent $k$ is of the form $1<k<2$.

(c) The function describes the distribution not merely in the tail, but also for small values of $i$. In this case it may be shown for example, that $f(2)/f(1) = 1/3$ and $f(2)/f(n) = 1/2$, where $n = \sum_{i=1}^{\infty} f(i)$.

These three properties just identified define the class of functions which Simon terms the Yule distribution. The term is used because G. Udny Yule used the distribution some years ago to explain the distribution of biological genera.[34] Simon reconstructed Yule's probability model using only a weak set of assumptions and modern theory,[35] and made the initial steps in applying it to city sizes.

*Underlying mechanism.* Stated in terms of the city size problem, the distribution derived by Simon is evolved under roughly the following notions. Consider a total population $k$ distributed in cities, with a city considered to be an aggregate of population larger than some threshold size. The probability that the $(k+1)$st person being found in cities of size $i$ is assumed to be proportional to $i[f(i,k)]$. It is also assumed that there is a constant probability $\alpha$ that the $(k+1)$st person will be in cities not previously of threshold size when the total population was $k$.[36]

It must be emphasized that these are extremely weak assumptions. They would hold roughly, for example, if population change were simply proportional to present population.

*The Model.* It is not practicable to reproduce Simon's derivation of the probability model here.[37] The derived set of equations from which expected distributions of cities may be calculated is as follows:

(a) $\alpha = n_k/k$

(b) $f(1) = n_k/2 - \alpha$

(c) $f(i)/f(i-1) = (1-\alpha)\cdot(i-1)/1+(1-\alpha)i$

where $k$ is the total urban population in the $n_k$ cities of greater than threshold size, and $f(i)$ is the number of cities of population $i$. From equation (a) and equation (b), and by successive application of equation (c) the expected distribution of city sizes can be con-

[33] For a development of this notion of a general systems theory see H. A. Simon and A. Newell, "Models: Their Uses and Limitations," in L. D. White, ed., *The State of the Social Sciences* (The University of Chicago Press, 1956), pp. 78–9.

[34] "A Mathematical Theory of Evolution Based on the Conclusions of Dr. J. C. Willis, F. R. S.," *Philosophical Transactions*, Vol. 213 (1924), pp. 21–84. Vining, *op. cit*, recognized the contribution of Yule.

[35] But in several alternate formulations Simon was able to show that a variety of different assumptions did not materially alter the shape of the derived distribution. This he took to indicate the value of the probabilistic explanation.

[36] Whereas the second assumption is very weak, Simon states that the first assumption will be satisfied if the growth rates of cities are stable, not for each city, but for aggregates of cities in each population band. There seems ample evidence of this stability in th pattern of urban growth. See, for example, C. H. Madden, "On Some Indications of Stability in the Growth of Cities in the United States," *Economic Development and Cultural Change*, Vol. 4 (1956), pp. 236–53.

[37] Simon, "On a Class of . . . ," *op. cit.* The use of the derived equations does not follow the precise form prescribed by Simon for city sizes, in which $f(i)$ is the number of cities equal to or greater than size $i$, but the form outlined in Brian J. L. Berry and William L. Garrison, "A Probability Model of the Distribution of City Sizes" (mimeographed manuscript, Department of Geography, University of Washington, 1957).

structed. Since the situation will scarcely, if ever, be found in which $\alpha > \varepsilon$ (where $\varepsilon$ is an extremely small number), we may write this system in the more simplified form:

$(b^*)$ $f(1) = n_k/2$

$(c^*)$ $f(i)/f(i-1) = (i-1)/(1+i)$

From $(b^*)$ and by successive application of $(c^*)$ the expected distribution of city sizes may then be constructed.

*An example.* The accuracy with which the stochastic model developed by Simon represents the distribution of city sizes may be illustrated by data from the state of Washington. Five thousand urban residents were taken to constitute the threshold at which a nucleated settlement became a city (this figure is entirely arbitrary). Hence $f(1)$ was interpreted as the number of cities with populations of 5,000–10,000; $f(2)$ the number of cities with between 10,000 and 15,000 population; $f(3)$, 15,000–20,000, etc.

The number of cities with populations of 5,000 or greater, $n_k$, was 36. Hence, as expected in equation $(a)$, $\alpha$ was extremely small, for $k$ was, relatively speaking, very large. From the simplified form of the model, therefore:

$$f(1) = 36/2 = 18 \qquad (b^*)$$
$$f(i)/f(i-1) = f(2)/f(1) \qquad i = 2$$
$$f(2)/18 = (2-1)/(1+2) \qquad (c^*)$$
$$f(2) = 18 \cdot 1/3 = 6$$

By successive application of equation $(c^*)$ where $i = 3$, $i = 4$, etc., the *a priori* distribution of cities in the various size classes has been calculated for the state in Table 1. Note the close resemblance of actual and *a priori* distributions.

The distribution of $f(i)$, the number of cities of size $i$, may be readily converted into the rank–size distribution, where $r_i$ is the number of cities of size equal to or greater than size $i$, by the use of the following transformation

$$r_i = n_k - f(i_j)$$

where $r_i$ is the number of centers of population equal to or greater than $i$, $n_k$ is as before, and $f(i_j)$ is the total number of centers of population less than $i$. Again, the close resemblance between expected and observed frequencies of city sizes by ranks may be seen in Table 1.

TABLE 1.—CITY SIZES IN THE STATE OF WASHINGTON

| Population of cities in 1,000's | Number of cities of this population $f(i)$ | | Number of cities equal to or greater than this population $(r_i)$ | |
|---|---|---|---|---|
| | Expected | Observed | Expected | Observed |
| 5–10 | 18 | 16 | 36 | 36 |
| 10–15 | 6 | 6 | 18 | 20 |
| 15–20 | 3 | 3 | 12 | 14 |
| 20–25 | 2 | 2 | 9 | 11 |
| 25–30 | 1 | 1 | 7 | 9 |
| 30–35 | 1 | 2 | 6 | 8 |
| 35+ | 5 | 6 | 5 | 6 |

STATUS OF THE PROBLEM

There seems to be no doubt that the empirical regularity with which we are concerned exists. The weight of empirical evidence for regularities has been given substance by Simon's derivation of accurate expected distributions utilizing stochastic processes and probability concepts. Plausibility does not require real understanding of causal processes, however, and it is not immediately clear where one should look for causal explanations.

*The simplest alternate.* Walter Isard has noted that "Zipf . . . has intuitively associated city size with the market area complex . . . although the logic connecting his statistical findings on the one hand and his Forces of Unification and Diversification and the principle of least effort on the other hand is not at all clear."[38] The notions of Christaller and Rashevsky are tenuously linked to the problem of rank–size, relating only in special cases. These theories may be suggested to explain city size regularities, but they seem both divergent and generally inadequate.

If we were to use the dictum of the simplest of alternate hypotheses, the selection of explanations would be clear. Instead of the troublesome theories of Zipf, Rashevsky, and Christaller, one would rely for explanation of city–size regularities on the implications of the work of Simon. Simon's scheme is simple and it works (in Washington state, for example, the expected frequency was very much like the observed distribution of city sizes). The alternatives are to derive explanations of the observed distributions of city sizes from underlying distributions of occupations (Rashevsky), or of agglomeration and dispersion tendencies (Zipf and Christaller). These require very specific and not very plausible assumptions about the distribution of occupa-

[38] Isard *op. cit.*, p. 60.

tions and dispersion and agglomeration tendencies, and such assumptions simply serve to transfer the mystery from the frequency distribution of city sizes to other frequency distributions.[39]

But on the other hand, the city size problem seems to be a case where the available simple explanation is unsatisfactory. For one thing, a probabilistic explanation in some sense refers to the presence of an infinite number of causes and the ability to predict in these terms is not enough; we wish explanations viable in explicit ways within a broad theoretical context. As John Somerville has pointed out, ". . . , no problem is worth working on that does not involve a deliberately formulated hypothesis which has scientific implications beyond the original problem."[40] Some would argue that the probabilistic explanation is meaningful in a theoretical context and has scientific implications, as noted in the discussion of average conditions in paragraphs below. The theory still leaves a central question unanswered, namely: Why is the arrangement of city sizes the outcome of simple probabilistic processes?

*What is needed.* What are the implications from noting our present level of knowledge regarding city sizes? One answer to this question seems clear. Present knowledge of processes of urbanization is skimpy. From the standpoint of numbers of available studies, there has been little concern with a general theory of city size, function, and arrangement. What is more amazing, this is true in spite of our great concern with studies of urban areas. It is obvious that here is a place where there is great need for *articulated* empirical and theoretical research. We also need a point of view; it has to be decided what is important to explain.

*Average conditions.* To elaborate the point of the preceding sentence, one may note that work may be done without explicitly stating the causes of rank–size distributions of city sizes. Madden has noted that the rank–size formulation exhibits stability over time.[41] Others have noted stability from nation to nation, and Simon gives us a simple explanation for this stability. This, then, is an average condition, both from empirical and theoretical points of view. We have already mentioned the notion of a general systems theory.[42] One point emphasized in this theory is that living systems tend to maintain steady states of many variables which keep all subsystems in order of balance both with one another and with their environments. These steady states are described in terms of entropy, in accordance with the second law of thermodynamics, in which entropy is a state of randomly distributed energy, and essentially a "normal or average state" of equilibrium. That the rank–size distribution is a random state is borne out by Simon. As such it is a condition of entropy or equilibrium and is a proper subject of systems theory. Thus city size problems may be treated as average conditions, *and* in the more general context of the development of systems theory.

*Variations.* On the other hand, one may argue that it is variations from and between average conditions which pose the problems for theory.[43] A region containing cities all approximately the same size would pose many interesting questions. A region with a rank–size arrangement of cities merely represents the occurrence of average conditions. Too, many interesting problems would result from comparisons of city-size frequencies in different regions.[44] Absence of significant dif-

[39] These sentences follow remarks made by Simon in a similar context: "Productivity among American Psychologists. An Explanation," *American Psychologist*, Vol. 9 (1954), p. 805.

[40] John Somerville, "Umbrellaology, or Methodology in Social Science," *Philosophy of Science*, Vol. 8 (1941), p. 564.

[41] Madden, *op. cit.*

[42] A lucid development of this idea has been provided by J. G. Miller, "Toward a General Theory for the Behavioral Sciences," *American Psychologist*, Vol. 10 (1955), pp. 513–32.

[43] James and Faissol, *op. cit.*, provide an example of this point of view. It should also be remembered that Mark Jefferson thought in terms of a "law" of the primate city. *Op. cit.*

[44] It would be interesting to treat the rank–size distribution as a lognormal distribution, for as Aitchison and Brown have noted, ". . . many of these distributions [of Zipf] may be regarded as lognormal, or truncated lognormal, with more prosaic foundations in normal probability theory." J. Aitchison and J. A. C. Brown, *The Lognormal Distribution* (Cambridge, 1957), p. 102. It would then be possible to test for significant differences from lognormalcy for any one distribution, and for significant differences between lognormal distributions, using the tests outlined by Aitchison and Brown. Krumbein is one of the several geologists who have used lognormal distributions to characterize distributions. W. C. Krumbein, "Application of Statistical Methods to Sedimentary Rocks," *Journal of the American Statistical Association*, Vol. 49 (1954), p. 51.

ferences between frequencies would indicate that similar processes of urbanization were operating in both regions; presence of significant differences would indicate the operation of differential forces of urbanization.

Average conditions can also contain within them important problems. Madden maintains that if the empirical distributions are plotted for a series of years, then it is possible to trace the fortunes of cities or groups of cities within these average conditions and relative both to other cities and to the general tendency for growth in the economy. These variable fortunes may then be taken and explained as the first step in the explanation of the general processes of economic growth in the economy.[45]

What is apparent from the foregoing is that a variety of points of view of the problem are permissive, but that search for important causes can also take one *away* from factors generating average conditions (the normal state, or entropy), even though, as Madden has pointed out, one may still perform valuable work within the context of average conditions. It is clear that, in any case, the available explanation for city size relationships is a base on which to build or to relate city size relationships to other relationships. It is certainly not the answer to all city size problems.

[45] Madden, *op. cit.*, and "Some Spatial Aspects of Urban Growth in the United States," *Economic Development and Cultural Change*, Vol. 4 (1956), pp. 371–87; J. R. P. Friedmann, "Locational Aspects of Economic Development," *Land Economics*, Vol. 32 (1956), pp. 213–27.

## MAP SUPPLEMENT TO THE ANNALS

A "Map Supplement" to the *Annals* was approved by the Council at the national meeting of the Association of American Geographers in Cincinnati on April 7, 1957. The objective is to provide a publication outlet in the Association for the cartographic portrayal of geographic data on a format larger than *Annals* page size. The Council is convinced that maps dealing with particular areas or subjects can be made to stand alone, requiring little or no explanatory text.

To implement the publication of such maps, the Council approved the appointment of Dr. Erwin Raisz as Map Editor for the Supplement. He is to encourage the preparation of maps in appropriate format and with adequate research significance for publication. The Map Editor is authorized to accept or reject maps submitted for publication in the Supplement and will attempt to obtain funds to cover the costs of publication for maps he accepts. Because color reproduction is very costly, emphasis should be placed on black-and-white maps. If an unusually fine map, dealing with a highly significant topic, is submitted for reproduction in color, it will be given consideration. It is suggested, however, that authors of such maps correspond with the Map Editor during the early stages of planning and compilation.

The general specifications for maps intended for publication in the Supplement are similar to those for sheets of the *National Atlas of the United States*. These specifications will be published in a forthcoming issue of the *Professional Geographer*, with suggestions on the mechanics of preparing the maps.

The Council indicated that only the highest quality maps should be given consideration. It is hoped that several such maps will be submitted within the next year to initiate and demonstrate the value of this new channel for the dissemination of geographic knowledge in cartographic form. Members of the Association who have compiled maps representing significant research, and who will draft them according to specifications, are encouraged to submit the manuscript drawings to Dr. Erwin Raisz, 107 Washington Avenue, Cambridge 40, Massachusetts.

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