**Introduction to Urban Science**

**Assignment 8: The Urban System and the Laws of Geography**

**Either:**

**Qualitative Exercise: On the Nature of the Urban System (<500 words)**

Read The Nature of Cities by Harris and Ullman (link Harris\_Ullman.pdf), and relevant parts of IUS, Chapter 8.

1. Consider the ideas of Central Place Theory (CPT): what is the justification for cities (no matter their size) to be equally spaced over the land? [~100 words]
2. Explain why larger cities have more (new) functions than smaller ones: What does this imply for economic exchanges between a larger city and smaller towns nearby?  What kinds of goods and services may be typically exchanged in each direction? [~150 words]
3. Give three reasons, associated with geography, transportation and resource concentration (such as mining or energy) for the urban system to deviate from the regularly spaced hexagonal arrangements of CPT. Give an example of each in terms of cities or regions in the US (or your nation).  [~100 words]
4. Does central place theory imply Zipf’s law for the distribution of population sizes of cities? Why (not)? You may want to Ch8 IUS, and the classic Berry and Garrison’s paper (link Berry\_Garrison.pdf) [~150 words]

**Or:**

**Quantatative Exercise: The 2-city Urban System.**

Consider an urban system with just 2 cities (a “toy model”)! Let’s label them city *1* and city *2*. There’s no foreign migration or rural migration to these cities, so that their demographic change is set by their own births and deaths and the migration flows between them. Then, there are only 6 parameters: the birth rates in each city  ,  (births per person per year), their death rates ,  and their migration currents ,  (number of people migrating from city 1 to city 2 over that year, and vice-versa).

* Write down the difference equations for the population change of the two cities, i.e.

 N\_1 ( t + 1 ) = N\_1 ( t ) + … , N\_2 ( t + 1 ) = N\_2 ( t ) + …

In other words, what are the "…" in each equation?

* Write these equations in matrix form: N ( t + 1 ) = **A** N ( t ) where  is a column vector at each time (Clue: you will need to write , ).
  + What are the 4 entries in the 2x2 matrix ***A***? (show the matrix)
* Solve the eigenproblem **A** N = λ N:
  + Give expressions for the two eigenvalues and eigenvectors in terms of the parameters.
* Compare the magnitude of the two eigenvalues,  . What parameters make the first eigenvalue (, the larger one) be bigger than the second, ? The simplest case is when the vital rates (births minus death rates) for the two cities are the same (). You may want to try that first.
* Discuss if higher versus lower migration rates (the magnitudes of   relative to vital rates) make the urban system emerge more quickly or slowly, in the sense of the dynamics converging to the single larger (dominant) eigenvalue (and eigenvector) faster.  This is the same as asking: how does the magnitude of the 's increase the difference of the eigenvalues?