Optical and Quantum Electronics 31: 583–598, 1999.

Understanding and controlling laser intensity noise

T. C. RALPH, E. H. HUNTINGTON, C. C. HARB, B. C. BUCHLER, P. K. LAM, D. E. MCCLELLAND AND H.-A. BACHOR
Department of Physics, Faculty of Science, The Australian National University, ACT 0200, Australia

Received 13 February 1998; accepted 14 August 1998

Abstract. We present unified theoretical expressions for laser intensity noise in the presence of injection locking and feedback. We discuss optimum control strategies for different configurations and frequency regions. We illustrate the various effects with experimental results from Nd : YAG non-planar ring oscillator lasers.

Key words: feedback, injection locking, lasers, quantum optics

1. Introduction

Low noise light sources are important for applications in communications and high precision interferometry. In particular ultra-high sensitivity laser-based metrology requires the laser's intensity to be as high as possible whilst its intensity noise should be as low as possible. To achieve this a clear understanding of the various noise sources of lasers and how they may be controlled is required.

Two fruitful techniques for controlling laser noise are injection locking (Seigman 1986) and opto-electronic control (Koechner 1996). Laser injection locking is a well established technique in which light from a well controlled laser (the master) is directed into the cavity of a more powerful laser (the slave) such that the slave laser becomes synchronized or phase locked to the master laser. In this way desirable properties of the master laser such as single mode operation and frequency stability can be transferred to the slave (Nabors *et al.* 1989). The slave laser also takes on some of the intensity noise characteristics of the master (Freitag and Welling 1994; Harb *et al.* 1996; Barillet *et al.* 1996). Opto-electronic control includes a range of devices in which part of the light from a laser is detected and the resultant photocurrent is used to feedback or forward to various control elements either in the beam or to the laser itself. Noise can be suppressed by such devices if the detected fluctuations are fed-back π out of phase, so-called negative feedback (Kane 1990).

The performance of these systems is ultimately limited by quantum mechanical effects. Optimum performance conditions when the noise levels of the light are well above the standard quantum noise limit (QNL) can be quite different from those required close to the QNL. This is due to the presence of quantum noise sources. For example negative feedback can actually add noise rather than remove it when close to the QNL (Shapiro *et al.* 1987; Masalov *et al.* 1994; Wiseman and Milburn 1994). The laser output is also affected by quantum noise. The laser dynamics can amplify quantum noise such that it is the dominant noise source, even far above the QNL (Harb *et al.* 1997). Hence a quantum theory is essential in understanding these systems. Classical control problems are traditionally couched in terms of transfer functions between noise inputs and outputs. Ideally we would like a theory that is expressible in terms of 'quantum' transfer functions.

Recently we have developed such a theory and tested it experimentally. In this paper we present this theory in a unified form and use it to examine and contrast the noise properties of lasers when they are free-running, have electro-optic feed-back, are injection locked and have combinations of feedback and injection locking. We particularly focus on solid-state lasers, using our experimental results from Nd : YAG non-planar ring oscillator (NPRO) (Kane and Byer 1985) as examples. In sections 2 and 3 we discuss the basic principles and present a simplified, but rigorous, quantum transfer function theory. In sections 4 and 5 we discuss control strategies and compare theory and experiment for various cases. In section 5 we present experimental results demonstrating optimum use of feedback and injection locking together.

2. Theory

In this section we outline the theory of, and present results for, an injection locked slave laser following, and generalizing, the approach of Ralph *et al.* (1996) and Buchler *et al.* (1998). The basic approach used is to solve the quantum Langevin equation for the active atoms and optical cavity mode of the laser in the presence of coupling to various external quantum mechanical reservoirs (Haken 1970; Gardiner and Collett 1985). The external reservoirs produce dissipation and introduce noise into the laser system. The pump and master laser inputs are also modeled quantum mechanically. By linearizing the quantum Langevin equation about its steady-state solution (Reynavd *et al.* 1989; Ralph *et al.* 1996) an expression for the intensity noise spectrum of the laser output can be obtained. The solution is in the form of transfer functions between the various quantum and classical noise sources and the laser output.

The laser output field $\hat{A}$ may be written as

$$\hat{A} = (\bar{A} + \delta\hat{A})e^{i\phi(t)}$$

where $\bar{A}$ is the semiclassical field amplitude, $\delta\hat{A}$ is the operator for the field fluctuations about $\bar{A}$ and ϕ is the semiclassical phase which varies with time due to phase diffusion of the laser. The photon number $\hat{n}$ is then evaluated as

$$\hat{n} = \hat{A}^{\dagger}\hat{A} \cong \bar{A}^2 + \bar{A}\delta\hat{X}_A \tag{1}$$

where second order terms have been discarded and the amplitude quadrature fluctuation operator is defined as

$$\delta\hat{X}_A = \delta\hat{A} + \delta\hat{A}^{\dagger}$$

Equation 1 therefore shows that the intensity fluctuations in the laser output are proportional to $\delta\hat{X}_A$. The dependence of $\delta\hat{X}_A$ on the noise inputs to the laser can be obtained from the quantum Langevin equation and is straightforward to solve in frequency space (Ralph *et al.* 1996). A general expression for this solution is

$$\begin{aligned}\delta X_{\mathrm{A}} = {} & F_{\mathrm{m}}(\omega)\delta X_{\mathrm{m}} + F_{\mathrm{p}}(\omega)\delta X_{\mathrm{p}} + F_{\mathrm{spont}}(\omega)\delta X_{\mathrm{spont}} + F_{\mathrm{dipole}}(\omega)\delta X_{\mathrm{dipole}} \\ & + F_{\mathrm{losses}}(\omega)\delta X_{\mathrm{losses}}\end{aligned} \tag{2}$$

where the δX's are the amplitude fluctuation operators, in Fourier space, for the various noise inputs and the $F(\omega)$'s are transfer functions describing their effect on the output noise. Noise inputs are from the master laser field entering at the output coupler ($|\delta X_{\mathrm{m}}|^2 = V_{\mathrm{m}}$); absorbed pump source intensity noise ($|\delta X_{\mathrm{p}}|^2 = V_{\mathrm{p}}$); spontaneous emission noise ($|\delta X_{\mathrm{spont}}|^2 = V_{\mathrm{spont}} = 1$); dipole fluctuation noise ($|\delta X_{\mathrm{dipole}}|^2 = V_{\mathrm{dipole}} = 1$); and noise introduced from intra-cavity losses ($|\delta X_{\mathrm{losses}}|^2 = V_{\mathrm{losses}} = 1$). The intensity noise spectrum of the laser (normalized to the QNL) is given by the absolute square of the Fourier transformed output amplitude fluctuations; i.e. $V(\omega) = |\delta X_{\mathrm{A}}|^2$. From Equation 2 this gives

$$\begin{aligned}V(\omega) &= |\delta X_{\mathrm{A}}|^2 \\ &= |F_{\mathrm{m}}(\omega)|^2|\delta X_{\mathrm{m}}|^2 + |F_{\mathrm{p}}(\omega)|^2|\delta X_{\mathrm{p}}|^2 + |F_{\mathrm{spont}}(\omega)|^2|\delta X_{\mathrm{spont}}|^2 \\ &\quad + |F_{\mathrm{dipole}}(\omega)|^2|\delta X_{\mathrm{dipole}}|^2 + |F_{\mathrm{losses}}(\omega)|^2|\delta X_{\mathrm{losses}}|^2\end{aligned} \tag{3}$$

The active atoms are modeled by a three level system with lasing occurring between the upper two levels, see Fig. 1. Pumping occurs at a rate Γ, spontaneous emission from the upper laser level occurs at a rate γ_t and from the lower lasing level at a rate γ. G describes the coupling of the lasing transition to the laser mode and is proportional to the stimulated emission cross-section of the transition. The total cavity decay rate is $2\kappa = 2(\kappa_{\mathrm{m}} + \kappa_{\mathrm{l}})$ comprising a

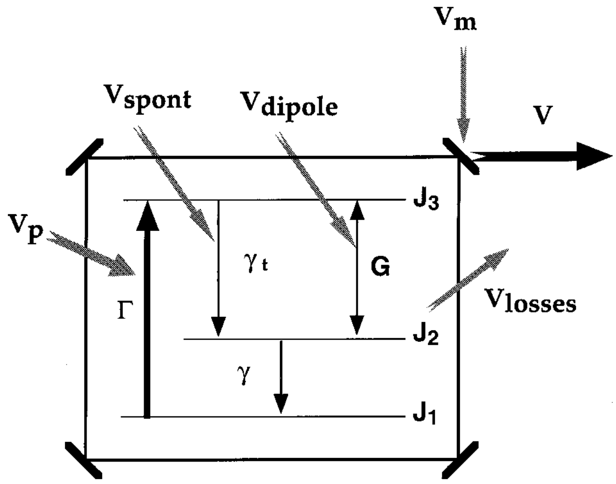

Fig. 1. Schematic of laser model showing atomic energy level scheme and the various noise sources.

component due to output coupling κ_{m} and intra-cavity losses κ_{l}. We assume that the decay rate from the lower lasing level is very rapid. Under these conditions the spontaneous emission noise from this decay is negligible. Substituting expressions for the transfer functions, as calculated by Ralph *et al.* (1996), into Equation 3 we obtain the following intensity noise spectrum for an injection locked laser:

$$\begin{aligned} V(\omega) = &\left| \frac{2\kappa_{\mathrm{m}}(i\omega + \gamma_{\mathrm{l}})}{(\omega_r^2 + \Delta\gamma_{\mathrm{l}} - \omega^2) + i\omega(\gamma_{\mathrm{l}} + \Delta)} - 1 \right|^2 V_{\mathrm{m}} \\ &+ \left| \frac{\sqrt{2\kappa_{\mathrm{m}} G^2 \alpha^2 \Gamma}}{(\omega_r^2 + \Delta\gamma_{\mathrm{l}} - \omega^2) + i\omega(\gamma_{\mathrm{l}} + \Delta)} \right|^2 V_{\mathrm{p}} \\ &+ \left| \frac{\sqrt{2\kappa_{\mathrm{m}} G^2 \alpha^2 \gamma_t J_3}}{(\omega_r^2 + \Delta\gamma_{\mathrm{l}} - \omega^2) + i\omega(\gamma_{\mathrm{l}} + \Delta)} \right|^2 V_{\mathrm{spont}} \\ &+ \left| \frac{\sqrt{2\kappa_{\mathrm{m}} G J_3}((\gamma_t + \Gamma) + i\omega)}{(\omega_r^2 + \Delta\gamma_{\mathrm{l}} - \omega^2) + i\omega(\gamma_{\mathrm{l}} + \Delta)} \right|^2 V_{\mathrm{dipole}} \\ &+ \left| \frac{\sqrt{4\kappa_{\mathrm{m}} \kappa_{\mathrm{l}}}(\gamma_{\mathrm{l}} + i\omega)}{(\omega_r^2 + \Delta\gamma_{\mathrm{l}} - \omega^2) + i\omega(\gamma_{\mathrm{l}} + \Delta)} \right|^2 V_{\mathrm{losses}} \end{aligned} \tag{4}$$

where Δ is given by

$$\Delta = \sqrt{2\kappa_{\mathrm{m}} \frac{A_{\mathrm{in}}^2}{\alpha^2}} \tag{5}$$

and A_{in}^2 is the master field photon flux. The intra-cavity photon number when injection locked is approximately given by

$$\alpha^2 = \left(\alpha_{\mathrm{f}} + \frac{A_{\mathrm{in}}}{\sqrt{8\kappa}}\right)^2 \tag{6}$$

provided $A_{\mathrm{in}} \ll \sqrt{8\kappa}\alpha_{\mathrm{f}}$ and α_{f}^2 is the free running (i.e. with $A_{\mathrm{in}} = 0$) intra-cavity photon number of the lasing mode, given by:

$$2\kappa\alpha_{\mathrm{f}}^2 = \Gamma - \gamma_t J_{3,0} \tag{7}$$

All the photon numbers (fluxes) are scaled by the number of active atoms in the slave laser. The occupation probability of the upper lasing level when free running, $J_{3,0}$, is given by:

$$J_{3,0} = 2\frac{\kappa}{G} \tag{8}$$

The occupation probability of the upper lasing level when injection locked, J_3, is approximately given by

$$J_3 = 2\frac{(\kappa - \Delta)}{G} \tag{9}$$

The denominators in the expression for the spectrum have the form expected for a damped oscillation. The frequency of the oscillation, ω_r, is given by:

$$\omega_r = \sqrt{G^2\alpha^2 J_3} \tag{10}$$

and the damping rate of the oscillation, γ_1, is given by:

$$\gamma_1 = G\alpha^2 + \gamma_t + \Gamma \tag{11}$$

The intensity noise spectrum, V, is normalized to the QNL. That is, $V = 1$ indicates QNL operation. The quantum noise sources are all at this level, i.e. $V_{\mathrm{spont}} = V_{\mathrm{dipole}} = V_{\mathrm{losses}} = 1$. However the pump noise spectrum, V_{p}, and the master laser spectrum, V_{m}, are arbitrary. The locking range (i.e. the maximum master/slave detuning for which they remain phase locked) of the slave laser is given by:

$$\Delta_l = \sqrt{2\kappa_{\mathrm{m}}\frac{A_{\mathrm{in}}^2}{\alpha_{\mathrm{f}}^2}} \tag{12}$$

which is approximately equal to Δ for small injected fields.

3. Including feedback

We now consider the effect of feeding back electronically to the drive current of the diode pump laser and/or to an electro-optic modulator in the master field (see Fig. 2). The master field and/or pump field input fluctuations will then become a function of the output fluctuations and the transfer function of the feedback electronics (W). Using the approach of Ref. (Buchler *et al.* 1998) we obtain the following general expression for the out-of-loop amplitude fluctuations;

$$\delta X_{Af} = \frac{\sqrt{1-\varepsilon}(Z + F_{\rm m}(\omega)\delta X_{\rm m} + F_{\rm p}(\omega)\delta X_{\rm p})}{1 + \varepsilon F_{\rm m}(\omega)W_{\rm m}(\omega) + \varepsilon F_{\rm p}(\omega)W_{\rm p}(\omega)} + \frac{\sqrt{\varepsilon}(1 - F_{\rm m}(\omega)W_{\rm m}(\omega) - F_{\rm p}(\omega)W_{\rm p}(\omega))\delta X_{\rm vac}}{1 + \varepsilon F_{\rm m}(\omega)W_{\rm m}(\omega) + \varepsilon F_{\rm p}(\omega)W_{\rm p}(\omega)} \tag{13}$$

where we have assumed unit in-loop detection efficiency and $\delta X_{\rm vac}$ are the vacuum fluctuations that enter at the feedback beamsplitter. This beamsplitter reflects a proportion ε of the beam to the in-loop detector. The internal quantum noise sources have been lumped together in

$$Z = F_{\rm spont}(\omega)\delta X_{\rm spont} + F_{\rm dipole}(\omega)\delta X_{\rm dipole} + F_{\rm losses}(\omega)\delta X_{\rm losses} \tag{14}$$

The laser transfer functions (F's) remain as defined by Equations 3 and 4. The out-of-loop spectrum, $V_{\rm f}$, can be obtained as before by taking the absolute square of δX_{Af}. The no feedback case can be retrieved by setting the electronic transfer to zero, $W_{\rm m,p} = 0$ and/or making the beamsplitter completely transmitting ($\varepsilon = 0$). Stable noise suppression can be achieved by making $F_{\rm m}(\omega)W_{\rm m}(\omega)$ and/or $F_{\rm p}(\omega)W_{\rm p}(\omega)$ large and positive. By convention this is referred to as negative feedback. A detailed discussion of the stability of feedback loops can be found in (Dorf and Bishop 1995). The important things to note from Equation 13 are: (i) The noise suppression at a particular

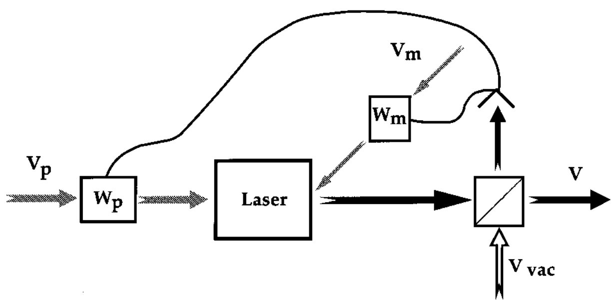

Fig. 2. Schematic of Feedback arrangement.

frequency depends on both the transfer function of the electronics and the relevant laser transfer function at that frequency. For example if $F_p(\omega)$ is very small at a particular frequency then $W_p(\omega)$ will have to be very large to produce significant noise suppression via feedback to the pump. (ii) The presence of $F_{m,p}W_{m,p}$ in the numerator means there is a limit to the noise suppression, namely $V_f \rightarrow 1/\varepsilon$ as $F_{m,p}W_{m,p} \rightarrow \infty$. This is the quantum limit for electro-optic feedback and is due to the amplification by the feedback loop of anti-correlated vacuum noise entering at the beamsplitter. Depending on the beamsplitter ratio, the limiting noise suppression can be significantly bigger than the QNL (Shapiro *et al.* 1987; Taubman *et al.* 1995).

4. The free running laser

We now look at the consequences of the preceding theory for controlling laser intensity noise. Let us consider first the behaviour of the free-running slave, i.e. with $A_{in} = \Delta = 0$ and only vacuum noise entering the output mirror ($V_m = 1$). In Fig. 3 we plot the laser's intensity noise relative to the QNL with parameters typical of NPROs (see Table 1). 0 dB indicates that the noise level is equal to QNL (i.e. $V = 1$). In this diagram we show the intensity noise level of the laser when being pumped by a QNL pump source, i.e. $V_p = 1$. We show the full intensity noise spectrum, curve (a), as well as the contributions of the different noise sources to the spectrum; curve (b) is vacuum noise entering at the output port (first term in Equation 4); (c) is the pump noise (second term

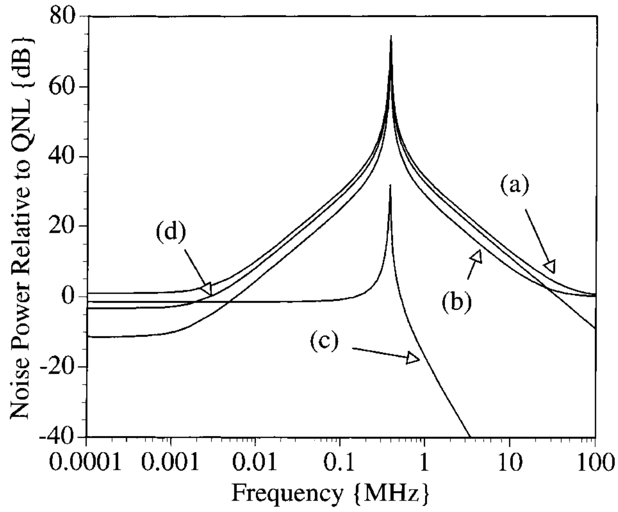

Fig. 3. Frequency dependence of the various noise sources contributing to free-running laser output noise: (a) Total laser noise, (b) contribution from vacuum entering at output mirror V_m, (c) contribution from pump noise V_p, and (d) contribution from internal quantum noise sources V_{spont}, V_{dipole} and V_{losses}.

Table 1. Parameters of the Nd : YAG laser

Parameter	Value
κ_m	$7.5 \times 10^7\ s^{-1}$
κ_l	$4.7 \times 10^7\ s^{-1}$
γ_t	$4.3 \times 10^3\ s^{-1}$
γ	$3.3 \times 10^7\ s^{-1}$
G	$6.6 \times 10^{11}\ s^{-1}$

in Equation 4); and (d) are the other quantum noise sources (final three terms in Equation 4). There is an under-damped oscillation normally referred to as the resonant relaxation oscillation (RRO) dominating the spectrum which is excited by all the noise sources.

In discussing these spectra it is useful to consider two forms of control: passive control, in which the noise of one of the sources is simply suppressed; and active control, in which information from the output field is fed-back to one of the noise sources. For the free-running laser there is only one noise source that can be controlled, i.e. the pump. Passive control, by suppressing the pump noise, is only of benefit at very low frequencies where pump noise dominates. At frequencies around the RRO the vacuum noise sources are greatly amplified, completely dominating the spectrum. At very high frequencies the spectrum is dominated by vacuum noise reflected off the output mirror. These effects are illustrated by the experimental results shown in Fig. 4. In trace (i) the NPRO is pumped by a noisy diode laser array producing over 40 dB of noise above the QNL at low frequencies. In trace (iii) the NPRO is pumped with a much quieter single element diode laser. There is

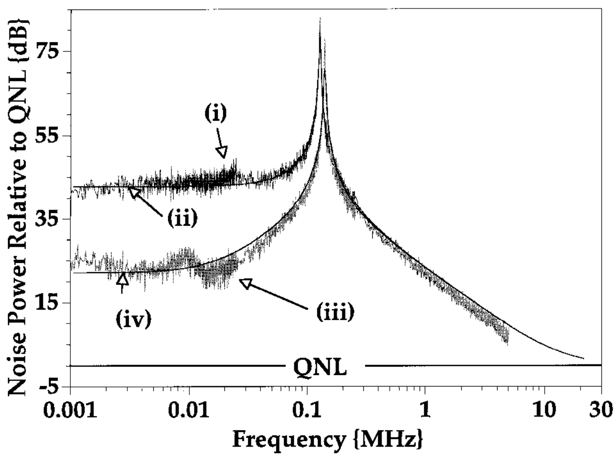

Fig. 4. Intensity noise spectra of the NPRO for a diode array pump, (i), and a quieter single element diode pump, (ii). Theoretical traces also shown with $\Gamma = 3.6\,s^{-1}$, $V_p = 100,000$ for trace (ii) and $\Gamma = 3.8\,s^{-1}$, $V_p = 1,000$ for trace (iv). Laser output power was 10.3 mW and 1.86 mW was detected.

almost 20 dB of noise reduction at low frequencies due to the quieter pump. However, there is only a minor reduction of the RRO peak and virtually no change at higher frequencies due to the dominance of the amplified quantum noise. Theoretical traces are also shown ((ii) and (iv)) using Equation 4. In principle the noise at low frequencies will be as low as that of the pump and can even be sub-QNL (squeezed) if the pump is also sub-QNL (limited by the internal losses). This effect has been demonstrated in diode lasers with pump current noise suppression (Yamamoto and Machida 1987).

Active control, by negative electronic feedback to the pump laser, can be effective over a wider range of frequencies as there is good transfer of pump fluctuations up to frequencies just above the RRO. At higher frequencies there is a rapid roll off of the pump transfer. This is illustrated experimentally in Fig. 5. Trace (i) is for no feedback whilst traces (ii) and (iii) show the effect of feedback with different levels of electronic gain. Theoretical traces are also shown using Equation 13. The feedback loop reduces noise at frequencies up to and beyond ω_r, completely removing the RRO. Feedback is clearly superior to passive control. However, as noted in section 3, the maximum noise suppression possible with a feedback loop is limited. Generalizing to non-unit detection efficiency (Taubman *et al.* 1995) the high gain limit becomes $V = 1 + P_{\text{out}}/P_{\text{in}}$ where P_{out} is the power detected out-of-loop and P_{in} is the power detected in-loop (see Fig. 2). For example, with equal detected powers in-loop and out-of-loop the theoretical limit is 3 dB above QNL. Hence, in principle, better noise reduction can be achieved at low frequencies by passive control.

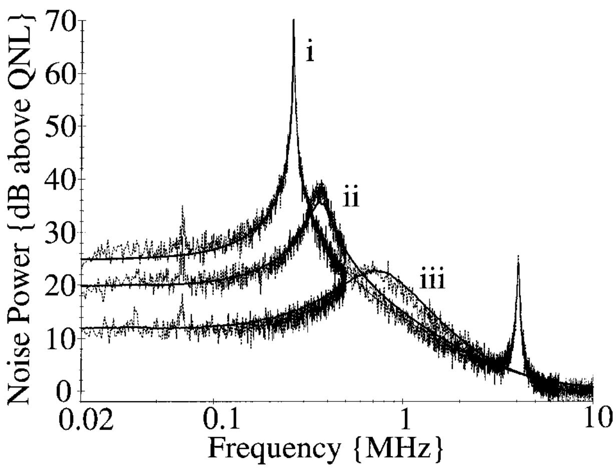

Fig. 5. Intensity noise spectra for NPRO with varying amounts of feedback. (i) Free running – no feedback; (ii) feedback operational with 20 dB of electronic attenuation in the feedback loop; (iii) feedback with no attenuation in-loop, i.e. best performance of loop. Laser output power was 100 mW of which 1 mW was detected both in and out-of-loop. Other parameters were: $\epsilon = 0.5$, $\Gamma = 5.7\,\text{s}^{-1}$ and $V_{\text{p}} = 40{,}000$.

The resonance in the pump transfer function at the RRO leads to a rapid phase flip between the input and output pump fluctuations. This changes negative feedback to positive feedback as we cross the RRO. This must be compensated for in the feedback electronics. The lack of perfect compensation results in the small amount of excess noise seen at frequencies above the RRO.

5. The injection locked laser

We now examine how the control problem changes for the injection locked slave. In Fig. 6 we plot the intensity noise spectrum when there is an injected field. Both the master and pump noises are set at the QNL ($V_p = V_m = 1$). Once again we show the total spectrum, (a), and the individual contributions from the master field entering at the output port, (b), the pump fluctuations, (c), and the internal quantum noise sources, (d). The spectrum is very different to the free running case. The effective damping of the oscillation has been increased from γ_l to $\gamma_l + \Delta$. This means the oscillation is now overdamped and hence no RRO appears. There is still amplification of the quantum noise at mid-range frequencies, but it is far less than in the free running case.

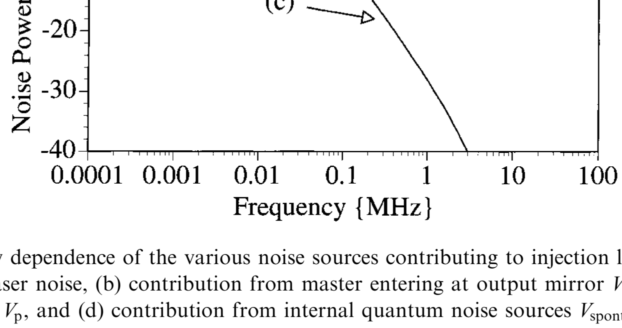

Fig. 6. Frequency dependence of the various noise sources contributing to injection locked laser output noise: (a) Total laser noise, (b) contribution from master entering at output mirror V_m, (c) contribution from pump noise V_p, and (d) contribution from internal quantum noise sources V_{spont}, V_{dipole} and V_{losses}. Power amplification factor between the master and slave is $H = 50$.

There are now two noise inputs, the master and the pump, that can be manipulated in order to control the output noise. Passive control of the pump noise can reduce noise at low frequencies where the pump is still the dominant noise source. This has been demonstrated in diode laser squeezing experiments in which pump noise suppression produces sub-QNL operation in the presence of an injected field (Wang *et al.* 1993). The pump transfer rolls off very rapidly with increasing frequency so reduction in pump noise has no effect at higher frequencies. There is amplification of the master noise at mid-range frequencies of the same magnitude as the power amplification ratio between the master and the slave ($H \cong 2\kappa a^2/A_{\mathrm{in}}^2$). In fact in this region the output behaves just like that from a linear optical amplifier. Reduction of master noise will reduce slave noise in this region but cannot reduce it below $V \approx H$ due to the presence of the amplified quantum noise. At high frequencies, outside the cavity linewidth of the slave, the output spectrum is dominated by unamplified master noise, such that

$$V = V_{\mathrm{m}} \tag{15}$$

In this region passive control of the master noise is clearly very effective. We illustrate this in Fig. 7 which plots the master spectral variance against the slave spectral variance at 73 MHz. This plot was obtained by placing am-

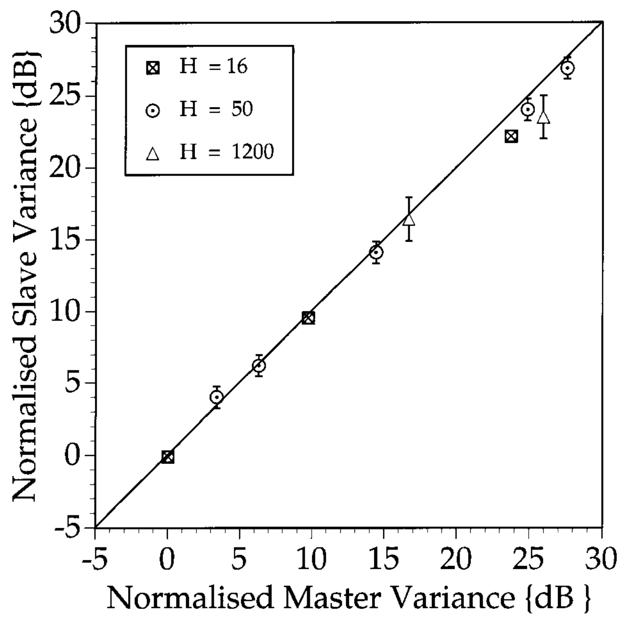

Fig. 7. The normalised spectral variance of an injection locked NPRO as a function of the normalised variance of the master laser at a detection frequency of 73 MHz. H is the power amplification factor between master and slave.

plitude signals of various sizes on a QNL master and measuring their size (relative to the QNL) on the slave output. The results have been corrected for detection losses. The linearity with slope one of Fig. 7 confirms Equation 15. In fact linearity right down to the QNL implies that if the master could be made sub-QNL at these frequencies then the slave would also be sub-QNL.

The pump transfer rolls off rapidly with increasing frequency and the master transfer does not roll on till mid-range frequencies, therefore active control across a large range of frequencies requires feedback to both the pump and the master fields. Injection locking does confer some advantages for feedback: (i) the absence of the RRO means there is less noise to suppress; and (ii) there is no longer a resonance in the pump transfer so there is no longer a sharp phase flip. This greatly simplifies the electronics required in the pump feedback loop. The master transfer has a phase flip across the amplification region, but this is also quite slow and easier to control than in the free running case. In the next section we present experimental results from a combined feedback and injection locking experiment with NPROs.

6. Injection locking and feedback

A diagram of the experiment is shown schematically in Fig. 8. More details about the experimental set-up can be found in Huntington *et al.* (1998). The output of the master laser was split by a beamsplitter (PBS, Fig. 8a) into in-loop and out-of-loop fields. The in-loop field was used as the error signal to electronically suppress the RRO of this laser ('LOOP 1' in Fig. 8a) and the out-of-loop field was used to injection lock the slave laser (shown in Fig. 8b). The output of the injection locked laser was also split (at BS1 Fig. 8b) into in-loop and out-of-loop fields. The in-loop field was used as the error signal for both the feedback loop to the pump diode lasers of the slave laser, 'PUMP LOOP', and the feedback loop to the amplitude modulator (AM) in the master beam, 'MASTER LOOP'.

The measured out-of-loop intensity noise traces for the slave laser under four different operating conditions are shown in Fig. 9. These traces were all measured under the conditions that the output power of the slave laser was 160 mW and, when injection locked, the amplification ratio was $H = 40$. The optical detection efficiency $\eta = (P_{\mathrm{measured}}/P_{\mathrm{actual}})$ for these traces was $\eta = 0.014$ as was the detection efficiency for the in-loop field. The noise powers shown in Fig. 9 are given relative to the measured QNL. We note that as the detected powers in-loop and out-of-loop are equal the electronic noise suppression is limited to 3 dB above the QNL (see Section 4).

Figure 9a shows the intensity noise of the free-running slave laser (trace (i)), the slave laser injection locked to the master laser with its RRO suppressed (trace (ii)) and the injection locked laser with the 'master loop'

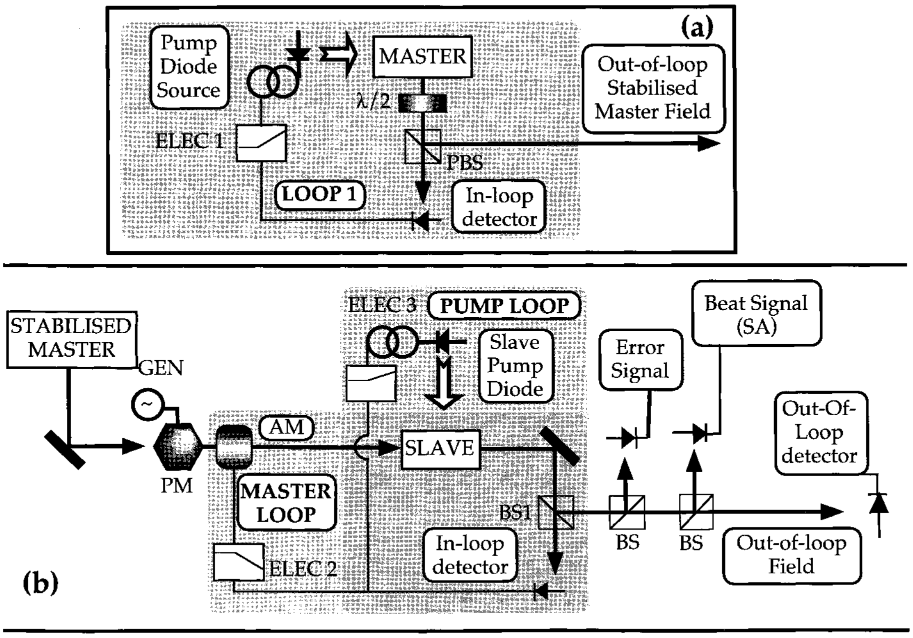

Fig. 8. The schematic diagrams for the injection locking and electronic feedback. Part (a) shows the system used to stabilize the intensity noise of the master laser and part (b) shows the injection locking experiment and the feedback loops therein. The abbreviations are: $\lambda/2$ 1064 nm half wave plate; PBS polarizing beamsplitter; BS beamsplitter (50/50); AM electro-optic amplitude modulator; PM electro-optic phase modulator; GEN sinusoidal signal generator – signal at 13 MHz; ELEC amplifying and filtering electronics; LOOP feedback loop.

operating (trace (iii)). Figure 9b presents a more detailed view of the low frequency region shown in Fig. 9a with the inclusion of the feedback loop to the slave laser pump source running (trace (iv)). A calibration signal on the slave laser pump source at approximately 100 kHz is also shown. The first thing to note from Fig. 9a is that, as a technique for suppressing the intensity fluctuations of the slave laser due to its pump source, injection locking is highly successful. Not only is the RRO that appears when free running not apparent when injection locked, but the magnitude of the calibration signal at 100 kHz is suppressed by approximately 30 dB. The suppression of this signal is due to the greater damping of the pump transfer function that occurs when the laser is injection locked. For RF frequencies greater than approximately 100 kHz the intensity noise of the injection locked laser is measured to be almost identical to that of the master laser. We note here that the master laser intensity fluctuations are measured for a certain photocurrent, I_p. These fluctuations are amplified by the slave laser (near the RRO) and, if we measure the same I_p for the injection locked laser fluctuations, are attenuated by the same amount. We therefore expect to measure identical

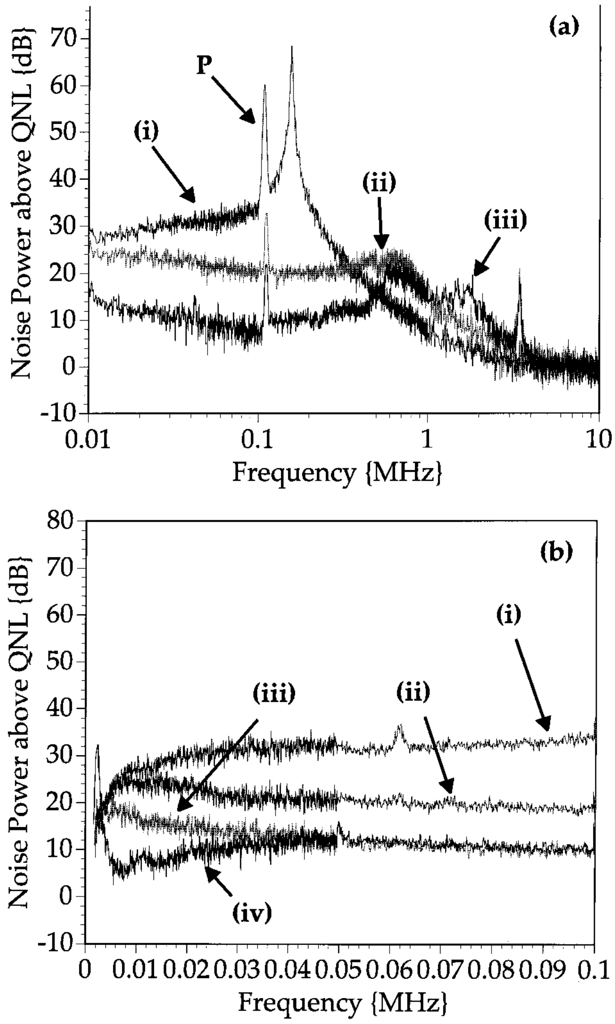

Fig. 9. The measured intensity noise of the slave laser when (i) free-running; (ii) injection locked; (iii) injection locked and with the feedback loop to the AM in the master beam running; (iv) injection locked with the feedback loops to the master beam and the slave pump source running. All traces are taken with 1.12 mW of detected optical power and calibrated to the QNL for that power. The total output power of the injection locked laser is 160 mW. Traces (ii)–(iv) are taken with $H \approx 40$. P is the calibration peak.

fluctuations in this case. This result is in agreement with previous observations (Freitag and Welling 1994; Harb *et al.* 1996).

The limitations of injection locking as an intensity noise suppressor are also apparent from Fig. 9. We observe that the intensity noise of the injection locked laser (trace (ii)) rolls up to that of the free-running slave laser at low frequencies. Below 10 kHz the intensity noise is unaltered by injection locking. This agrees with our earlier observation that at low frequency pump noise dominates for both free-running and injection locked lasers.

When we turn on the 'master loop' we observe from Fig. 9, trace (iii) that, compared to the simple injection locked case, trace (ii), the intensity noise is uniformly decreased by approximately 12 dB from 20 to 300 kHz. We observe that the calibration signal is also decreased by 12 dB by this control loop indicating that, as we would expect, the fluctuations are suppressed irrespective of their origin. The noise suppression rolls off at high frequencies because of the electronic low pass filter used to ensure that the feedback loop was stable. At 2 MHz there is a slight increase in intensity noise above that of the injection locked laser without the feedback loop to the master laser. This increase is due to the non-perfect compensation of the master transfer function by the electronics. At low frequencies, the noise suppression decreases due to the roll-off of the master transfer function and the roll-on of the transfer function of the slave pump noise.

When the additional 'pump loop' is turned on (trace (iv) in Fig. 9b), we obtain noise suppression of 14 dB at 5 kHz decreasing to 0 dB at 40 kHz. The minimum intensity noise measured here is 4 dB above the QNL, approaching to within 1 dB the theoretical limit for this system. This degree of suppression was only possible because of the greater damping of the slave pump source transfer function after injection locking. Without the need for large phase advance at 100's of kHz, the DC gain available was considerably greater (in our case $\approx$20 dB) than for a free-running laser. Also, this degree of noise suppression was only achieved with both feedback loops operating.

7. Conclusion

We have used general expressions for the intensity noise spectrum of an injection locked laser with electro-optic feedback to discuss the source and control of laser noise. We examined and contrasted passive control (in which input noise is simply suppressed) and active control (in which feedback is used). We found active techniques are generally superior across the spectrum, however, for specific frequency regions, passive techniques can, in principle, achieve greater noise suppression. Experimental examples of each of the techniques were presented.

Acknowledgments

We thank I. Freitag and M. Gray for contributions to this work. This work was supported by the Australian Research Council. T. C. Ralph is an ARC postdoctoral fellow.

References

Siegman, A.E. *Lasers*, University Science, Mill Valley, Calif., 1986.
Koechner, W. *Solid-State Laser Engineering*, 5th Ed., Springer, Berlin, 1996.
Nabors, C.D., A.D. Farinas, T. Day, S.D. Yang, E.K. Gustafson and R.L. Byer. *Opt. Lett.* **14** 1189, 1989.
Freitag I. and H. Welling, *Appl. Phys.* B**58** 537, 1994.
Harb, C.C., T.C. Ralph, Ingo Frietag, E.H. Huntington and H.-A. Bachor. *Phys. Rev.* **A54** 4370, 1996.
Barillet, R., A. Brillet, R. Chiche, F. Cleva, L. Latrach and C.N. Man. *Meas. Sci. and Tech.* **7** 162, 1996.
Kane, T.J. *IEEE Phot. Tech. Lett.* **2** 244, 1990.
Shapiro, J.H., G. Saplakoglu, S.-T. Ho, P. Kumar, B.E.A. Saleh and M.C. Teich. *JOSA B* **4** 1604, 1987.
Masalov, A.V., A.A. Putilin and M.V. Vasilyev. *J. Mod. Opt.* **41** 1941, 1994.
Wiseman H.M. and G.J. Milburn. *Phys. Rev.* **A49** 1350, 1994.
Harb, C.C., T.C. Ralph, E.H. Huntington, D.E. McClelland and H.-A. Bachor. *JOSA B* **14** 2936, 1997.
Kane, T.J. and R.L. Byer, *Opt. Lett.* **10** 65, 1985.
Ralph, T.C., C.C. Harb and H.-A. Bachor. *Phys. Rev.* **A54** 4359, 1996.
Buchler, B.C., E.H. Huntington, C.C. Harb and T.C. Ralph. *Phys. Rev.* **A57** 1286, 1998.
Haken, H. Light and Matter lc. In *Handbuch der Physik*, Springer, Berlin, 1970.
Gardiner, C.W. and M.J. Collett, *Phys. Rev.* **A31** 3761, 1985.
Reynaud, S., C. Fabre, E. Giacobino and A. Heidmann. *Phys. Rev.* **A40** 1440, 1989.
Dorf, R.C. and R.H. Bishop, *Modern control systems*, 7th Ed., Addison Wesley, 1995.
Taubman, M.S., H. Wiseman, D.E. McClelland and H.-A. Bachor. *JOSA B* **12** 1792, 1995.
Yamamoto, Y. and S. Machida. *Phys. Rev.* **A35** 5114, 1987.
Wang, H., M.J. Freeman and D.G. Steel, *Phys. Rev. Lett.* **71** 3951, 1993.
Huntington, E.H., B.C. Buchler, C.C. Harb, T.C. Ralph, D.E. McClelland and H.-A. Bachor. *Opt. Comm.* **145** 359, 1998.