V-Mobile Mathematical Formulation

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October 2020

1 Sets

$$
\begin{aligned}
c &:= 1, .., 3 \quad \text{carriers} \\
d &:= 1, .., 5 \quad \text{destinations} \\
i &:= 1, .., 3 \quad \text{price intervals} \\
t &:= 1, 2 \quad \text{time periods}
\end{aligned}
$$

2 Parameters

$p_{c,d,i,t}$ = price per call minute at carrier c to destination d in price interval i in month t

$pen_{c,d,t}$ = penalty per call minute at carrier c to destination d in month t

$d_{d,t}$ = forecasted volume to destination d in month t

$LT_{c,i}$ = lower threshold for carrier c in price interval i

$UT_{c,i}$ = upper threshold for carrier c in price interval i

$LB_{c,t}$ = lower bound on number of call minutes at carrier c in month t

$UB_{c,t}$ = upper bound on number of call minutes at carrier c in month t

3 Decision Variables

$X_{c,d,i,t}$ = Number of call minutes at carrier c to destination d in price interval i in month t

$bin_{c,i} = \begin{cases} 1 \text{ if } X_{c,d,i,t} \text{ falls into range of price interval i in month t} \\ 0 \text{ otherwise} \end{cases}$

$z_{c,d,i,t} = bin_{c,i} * X_{c,d,i,t}$

4 Objective Function

minimize Cost: Number of minutes $*$ (Cost + Penalty) $*$ Binary

$$
\sum_{t} \sum_{c} \sum_{d} z_{c,d,it} * (p_{c,d,i,t} + pen_{c,d,t})
$$

5 Constraints

5.1 Capacity Limits

$$\sum_d z_{c,d,i,t} \leq UB_{c,t} \quad \forall c,t$$

$$\sum_d z_{c,d,i,t} \geq LB_{c,t}$$

$\forall c,t$

5.2 Price Intervals

$$\text{bin}_{c,i} * LT_{c,i} \leq \sum_d \sum_t X_{c,d,i,t} \quad \forall c,i$$
$$\text{bin}_{c,i} * UT_{c,i} \geq \sum_d \sum_t X_{c,d,i,t} \quad \forall c,i$$
$$\sum_i bin_{c,i} = 1 \quad \forall c$$

5.3 z-Variable