

V-Mobile Mathematical Formulation

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1 Sets

$i := 1, \dots, 3$ carriers
 $j := 1, \dots, 5$ destinations
 $k := 1, \dots, 3$ price intervals
 $t := 1, 2$ time periods

2 Parameters

$p_{i,j,k,t}$ = price per call minute at carrier i to destination j in price interval k in month t
 $pen_{i,j,t}$ = penalty per call minute at carrier i to destination j in month t
 $d_{j,t}$ = forecasted volume to destination j in month t
 $LT_{i,k}$ = lower threshold for carrier i in price interval k
 $UT_{i,k}$ = upper threshold for carrier i in price interval k
 $LB_{i,t}$ = lower bound on number of call minutes at carrier i in month t
 $UB_{i,t}$ = upper bound on number of call minutes at carrier i in month t

3 Decision Variables

$X_{i,j,k,t}$ = Number of call minutes at carrier i to destination j in price interval k in month t
 $bin_{i,k} = \begin{cases} 1 & \text{if } \sum_j \sum_t X_{i,j,k,t} \text{ falls into range of price interval } k \\ 0 & \text{otherwise} \end{cases}$

4 Objective Function

minimize Cost: Number of minutes * (Cost + Penalty)

$$\sum_t \sum_k \sum_i \sum_j [X_{i,j,k,t} * (p_{i,j,k,t} + pen_{i,j,t})]$$

5 Constraints

5.1 Capacity Limits

$$\sum_k \sum_j X_{i,j,k,t} \leq UB_{i,t} \quad \forall i, t$$

$$\sum_k \sum_j X_{i,j,k,t} \geq LB_{i,t} \quad \forall i, t$$

5.2 Price Intervals

$$bin_{i,k} * LT_{i,k} \leq \sum_j \sum_t X_{i,j,k,t} \quad \forall i, k$$

$$bin_{i,k} * UT_{i,k} \geq \sum_j \sum_t X_{i,j,k,t} \quad \forall i, k$$

$$\sum_k bin_{i,k} \leq 1 \quad \forall i$$

5.3 Forecasted Volume

$$\sum_k \sum_i X_{i,j,k,t} \geq d_{j,t} \quad \forall j, t$$