V-Mobile Mathematical Formulation

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1 Sets

$i := 1, .., 3$	carriers
$j := 1, .., 5$	destinations
$k := 1, .., 3$	price intervals
$t := 1, 2$	time periods

2 Parameters

$p_{i,j,k,t}$	= price per call minute at carrier i to destination j in price interval k in month t
$pen_{i,j,t}$	= penalty per call minute at carrier i to destination j in month t
$d_{j,t}$	= forecasted volume to destination j in month t
$LT_{i,k}$	= lower threshold for carrier i in price interval k
$UT_{i,k}$	= upper threshold for carrier i in price interval k
$LB_{i,t}$	= lower bound on number of call minutes at carrier i in month t
$UB_{i,t}$	= upper bound on number of call minutes at carrier i in month t

3 Decision Variables

$X_{i,j,k,t}$	= Number of call minutes at carrier i to destination j in price interval k in month t
$bin_{i,k}$	$= \begin{cases} 1 \text{ if } X_{i,j,k,t} \text{ falls into range of price interval k in month t} \\ 0 \text{ otherwise} \end{cases}$
$z_{i,j,k,t}$	$= bin_{i,k} * X_{i,j,k,t}$

4 Objective Function

minimize Cost: Number of minutes $*$ (Cost + Penalty) $*$ Binary

$$\sum_t \sum_k \sum_i \sum_j z_{i,j,k,t} * (p_{i,j,k,t} + pen_{i,j,t})$$

$$\sum_t \sum_i \sum_j (\sum_k z_{i,j,k,t} * p_{i,j,k,t}) + (\sum_k z_{i,j,k,t}) * pen_{i,j,t}$$

5 Constraints

5.1 Capacity Limits

$$\sum_k \sum_j z_{i,j,k,t} \leq UB_{i,t} \quad \forall i,t$$

$$\sum_k \sum_j z_{i,j,k,t} \geq LB_{i,t} \quad \forall i,t$$

5.2 Price Intervals

$$bin_{i,k} * LT_{i,k} \leq \sum_j \sum_t X_{i,j,k,t} \quad \forall i,k$$

$$bin_{i,k} * UT_{i,k} \geq \sum_j \sum_t X_{i,j,k,t} \quad \forall i,k$$

$$\sum_k bin_{i,k} = 1 \quad \forall i$$

5.3 z-Variable

$f(x)_{i,t} = X_{i,j,k,t}$
$M_{i,t} = UB_{i,t}$
$m_{i,t} = LB_{i,t}$

$$\sum_j z_{i,j,k,t} \leq bin_{i,k} * UB_{i,t} \quad \forall i,k,t$$

$$\sum_j z_{i,j,k,t} \geq bin_{i,k} * LB_{i,t} \quad \forall i,k,t$$

$$\sum_j z_{i,j,k,t} \leq \sum_j X_{i,j,k,t} - LB_{i,t}(1 - bin_{i,k}) \quad \forall i,k,t$$

$$\sum_j z_{i,j,k,t} \geq \sum_j X_{i,j,k,t} - UB_{i,t}(1 - bin_{i,k}) \quad \forall i,k,t$$

5.4 Forecasted Volume

$$\sum_{k} \sum_{i} X_{i,j,k,t} \geq d_{j,t} \quad \forall j, t$$