November 3, 2021 1:01 PM

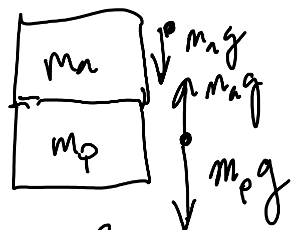
Buoyancy Frequency

Book - An Introduction to atmospheric gravity waves

$$\vec{F}_b = -g(m_p - m_a)\hat{z}$$

m_a → displaced air

m_p → test parcel of air



$\uparrow \delta z$

$$m_p \frac{d^2(\delta z)}{dt^2} = -g(m_p - m_a)$$

...

$$\frac{d^2(\delta z)}{dt^2} = -g\frac{\rho_p - \rho_a}{\rho_p}$$

... get wave eq

Natural frequency of gravity waves

Brunt-Väisälä frequency

=

Buoyancy freq

Gravity Wave Induced Brunt-

On Blowing ...
Vaisala Oscillations

Bomb explosion from far away can cause long-period gravity wave together with a short-period "acoustic frequency wave"

↓

Tuan 1975

Short period - natural from atmosphere

long period from source

$$N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho(z)}{\partial z}}$$

↳ density of parcel

Using the International Monitoring System infrasound network to study gravity waves

IMS detects gravity waves at low freq

Typical
T = few minutes - 24 hrs
$f = 0.017 - 0.00028$ Hz
infrasound

lower than initial

PSD shows 12 and 24 h period

Buoyancy frequency estimate for each station

Acoustic Ray-Tracing in the atmosphere: with gravitational effect and attenuation considered

Related to attenuation, but usually ignored

Waves > Acoustic Cut-off
↳ acoustic Waves

Waves < Buoyancy freq
↳ gravi waves

$N = 2\pi f$ (angular freq)

Global morphology of Infrasound

Dr. Pierre

Douglas,

>0.01 Hz no buoyancy affects

under ray approximation

using classical ray approximation

plane-parallel acoustic wave front in a locally linear medium

Potential Temperature

$$\theta = T\left(\frac{P_0}{P}\right)^{R/c_p}$$

$\frac{R}{C_p} = 0.286$ for air

Temperature if brought adiabatically to P_0

$$N = \sqrt{\frac{-g}{\theta}\frac{\partial\theta}{\partial z}}$$

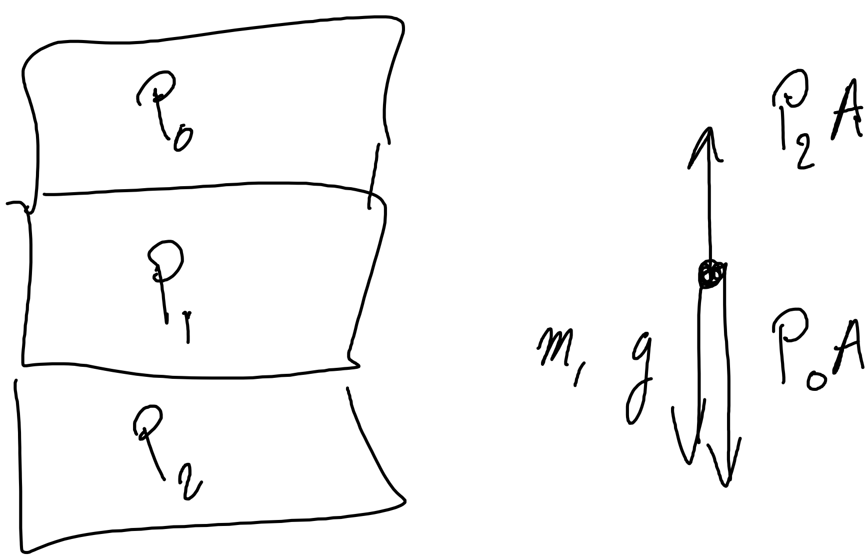
$$P = P_0 e^{-Hz}$$

$H = \frac{g_0 M}{RT_0}$

$$\theta = T e^{Hz\frac{R}{c_p}}$$

Beyond Ray Tracing for Internal Waves I Small amplitude waves accelerate

long periods (High energies) buoyancy effects are significant



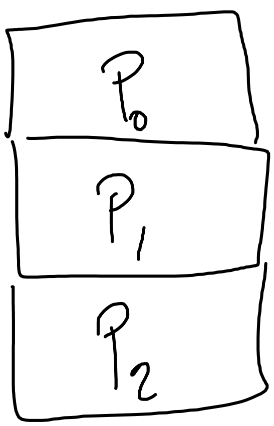
$$F = 0 = (P_2 - P_0)A - mg$$

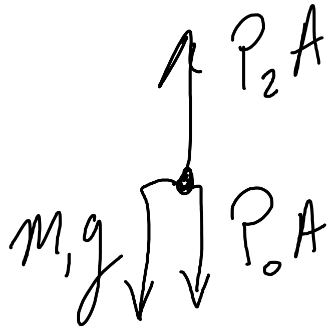
$$mg = -dP \cdot A$$

$$\rho V g = -dPA$$

$$\rho A\, dz\, g = -dPA$$

$$-\frac{dP}{dz} = \rho g$$





$$m_1 a = (P_2 - P_0)A - m_1 g$$

$$m_1 \frac{d^2(\delta z)}{dt^2} = -dP A - m_1 g$$

$$\rho_1 V \frac{d^2(\delta z)}{dt^2} = \rho_0 g \cancel{\delta z A} - \rho_1 V g$$

$$\frac{d^2(\delta z)}{dt^2} = \left(\frac{\rho_0 - \rho_1}{\rho_1}\right) g$$

$$\frac{d^2(\delta z)}{dt^2} = \frac{d\rho}{\rho_1} g$$

$$\frac{d^2(\delta z)}{dt^2} = \frac{g}{\rho_1}\frac{d\rho}{dz}\delta z$$

$$\frac{d^2(\delta z)}{dt^2} + -\frac{g}{\rho_1}\frac{d\rho}{dz}\delta z = 0$$

$$\omega^2 = N^2$$

$$N = \sqrt{-\frac{g}{\rho_0}\frac{d\rho}{dz}}$$