Prove: That any two eigenvectors, of a symmetric matrix A are orthogonal.
w/ distinct eigenvalues

Proof: Let u and v be two eigenvectors of A, with eigenvalues λ and γ, respectively.

This means: $Au = \lambda u$ & $Av = \gamma v$ & $\lambda \neq \gamma$

We want to show that $u \cdot v = 0$ (or $u^T v = 0$, or $v^T u = 0$).

Let's start with this expression: $\lambda u^T v$, and use some of the above relationships, along with the fact that $A = A^T$, to arrive at $u^T v = 0$.

We know $Au = \lambda u$

$\Rightarrow [Au = \lambda u]^T$

$\Rightarrow u^T A^T = \lambda u^T$

$\Rightarrow \lambda u^T v = (\lambda u^T) v = u^T A^T v$

$\Rightarrow u^T A^T v = u^T A v$ (b/c $A^T = A$)

$\Rightarrow u^T A v = u^T \gamma v = \gamma u^T v$

Okay, so $\lambda u^T v = \gamma u^T v$.

$\Rightarrow \lambda u^T v - \gamma u^T v = 0$

$\Rightarrow (\lambda - \gamma) u^T v = 0$

Because $\lambda - \gamma \neq 0$ (they are distinct), $u^T v = 0$!