Lecture 2:
Fourier Transforms, Filtering, and Convolutions

April 8th, 2015

Lane McIntosh & Kiah Hardcastle

Math Tools for Neuroscience

Today's lecture

When will this be useful?

Filtering in time or space

- What is convolution?
- How much should I filter?

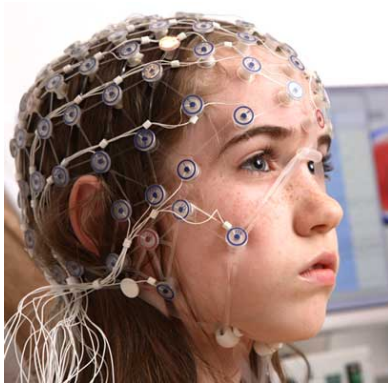
Filtering in frequency

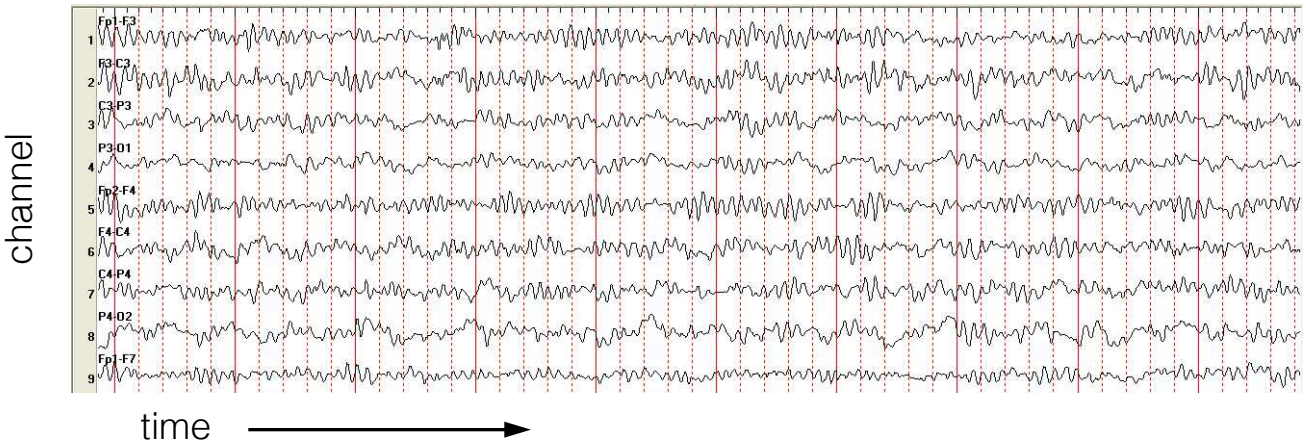
- Fourier series
- What is a Fourier transform?
- Designing filters in frequency domain

Experimental examples of filtering

Experimental examples of filtering

Experimental noise





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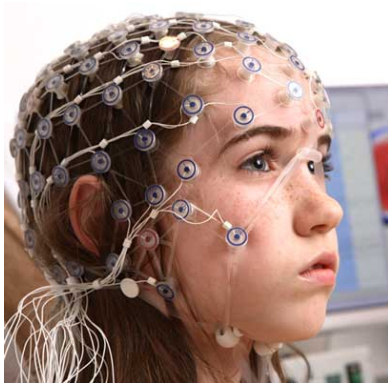
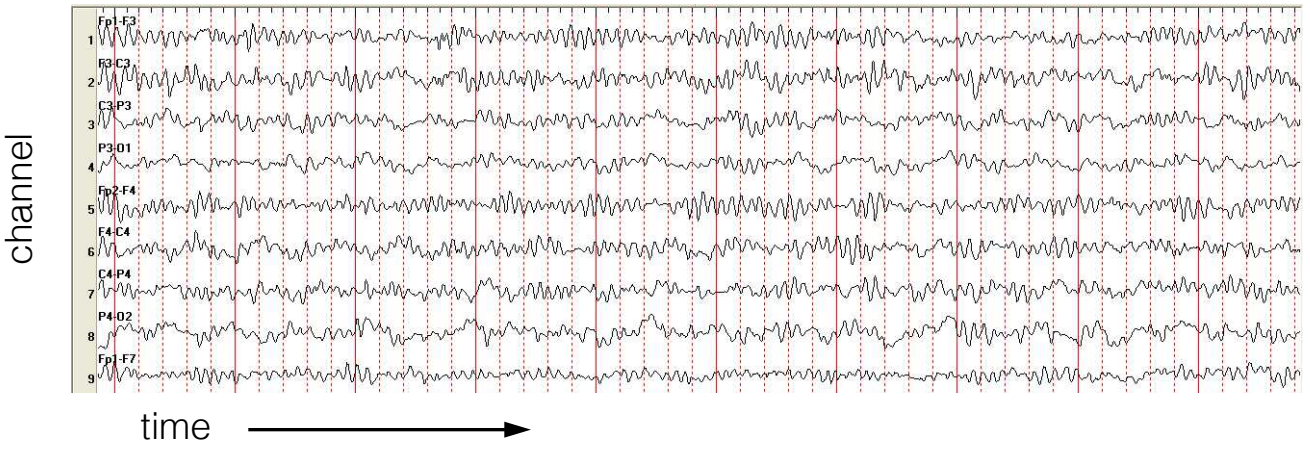
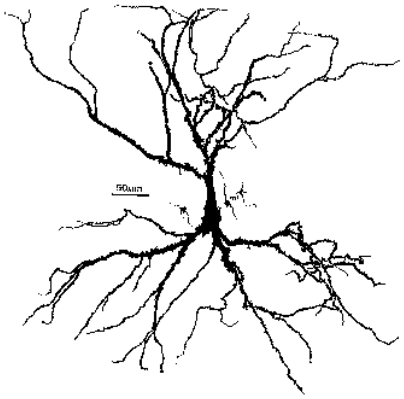
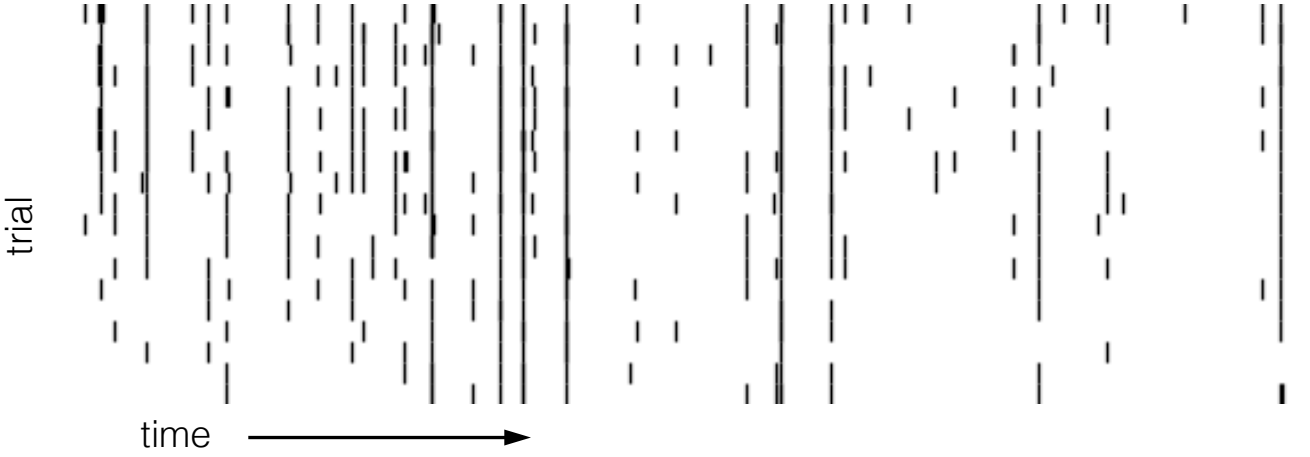




Biological noise





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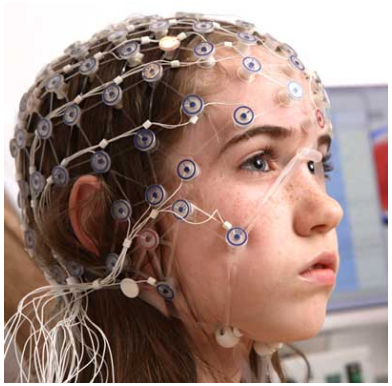

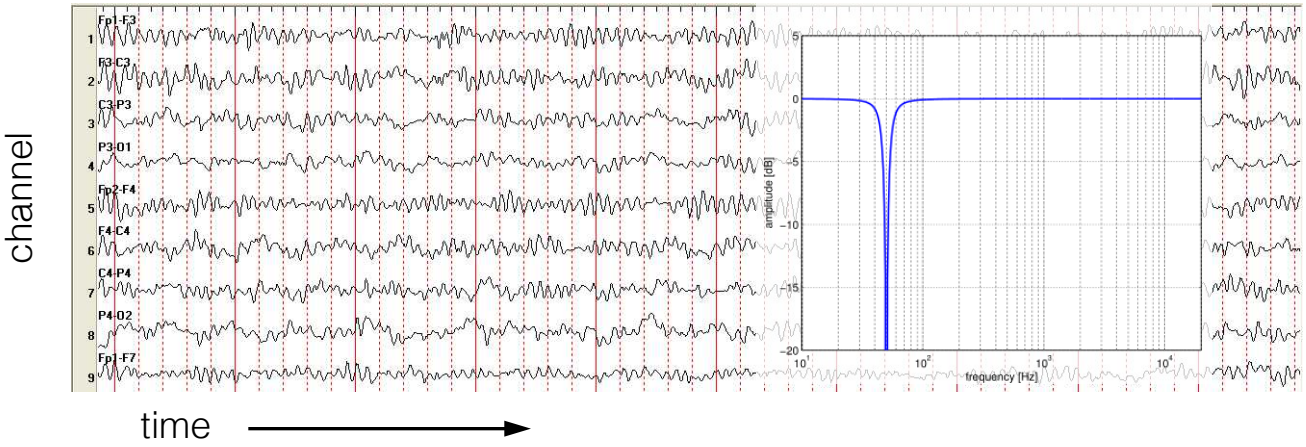

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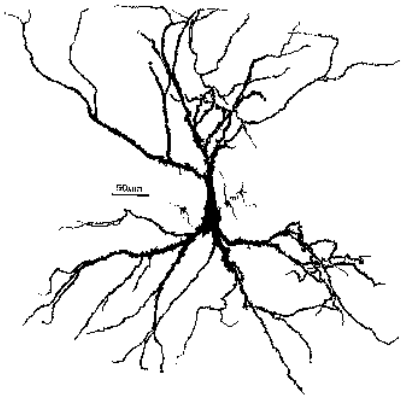

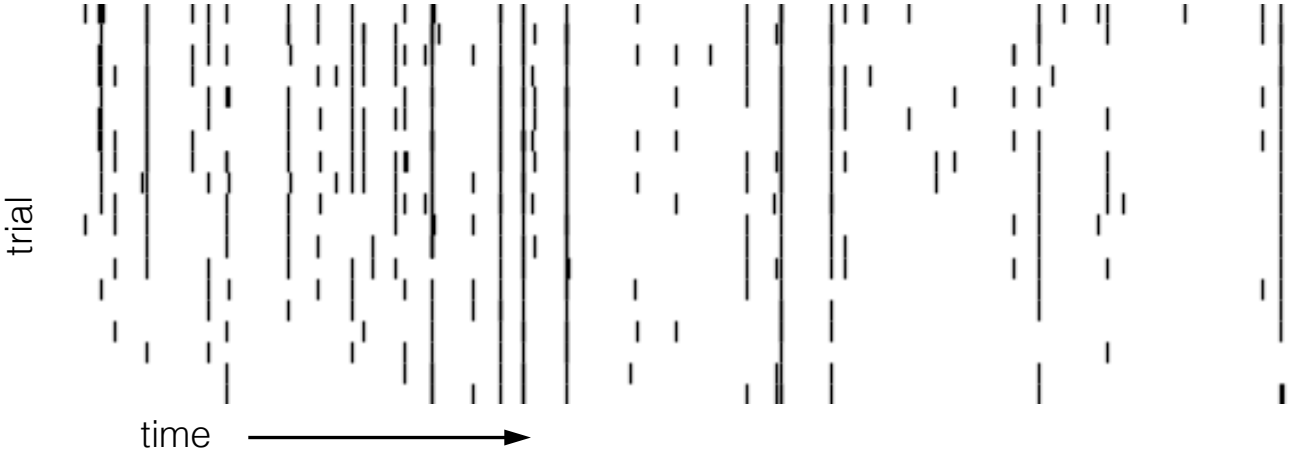

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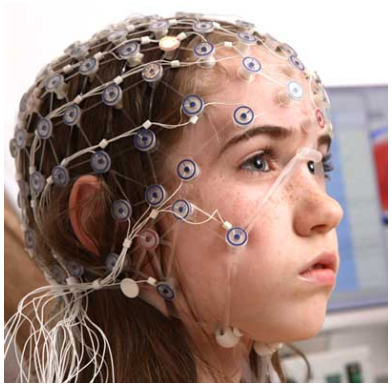

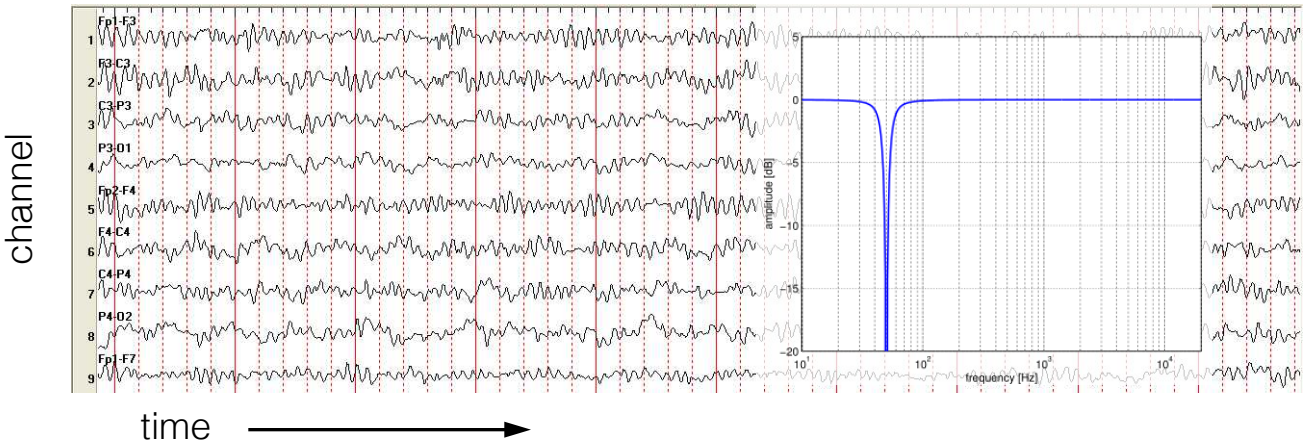

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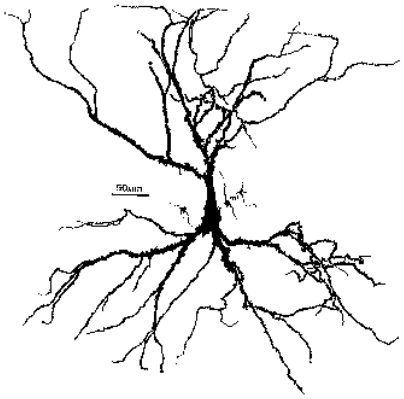

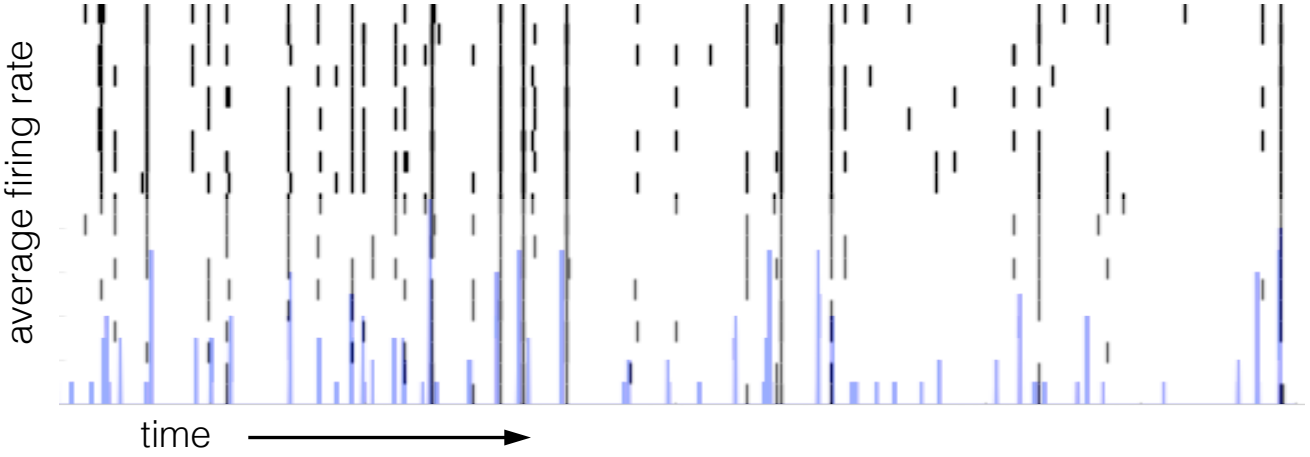

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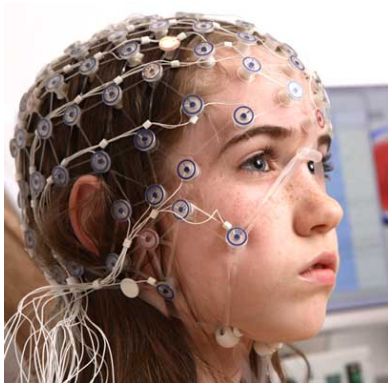

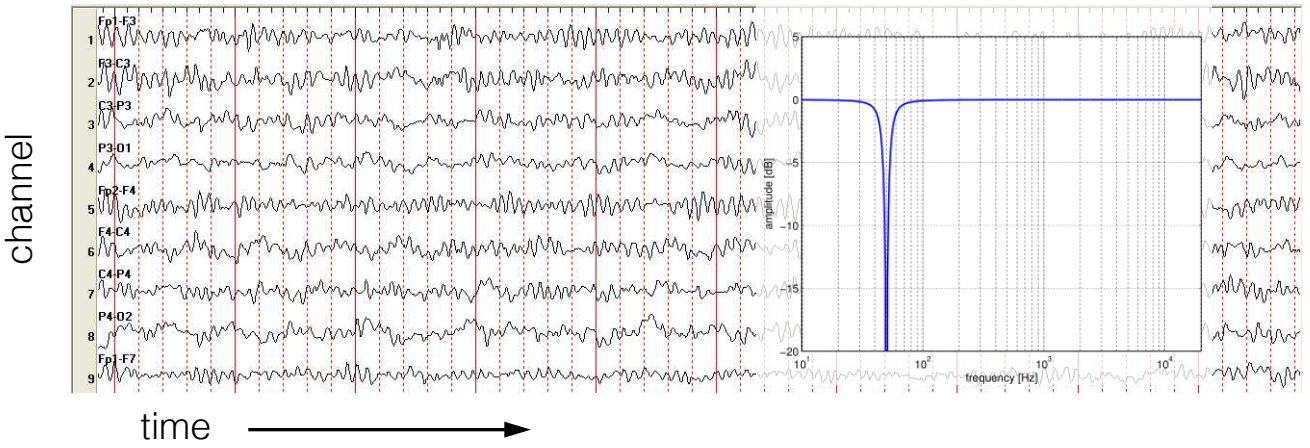

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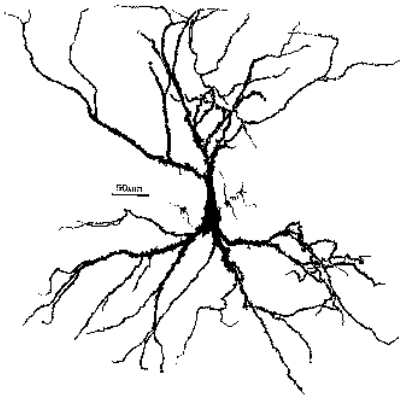

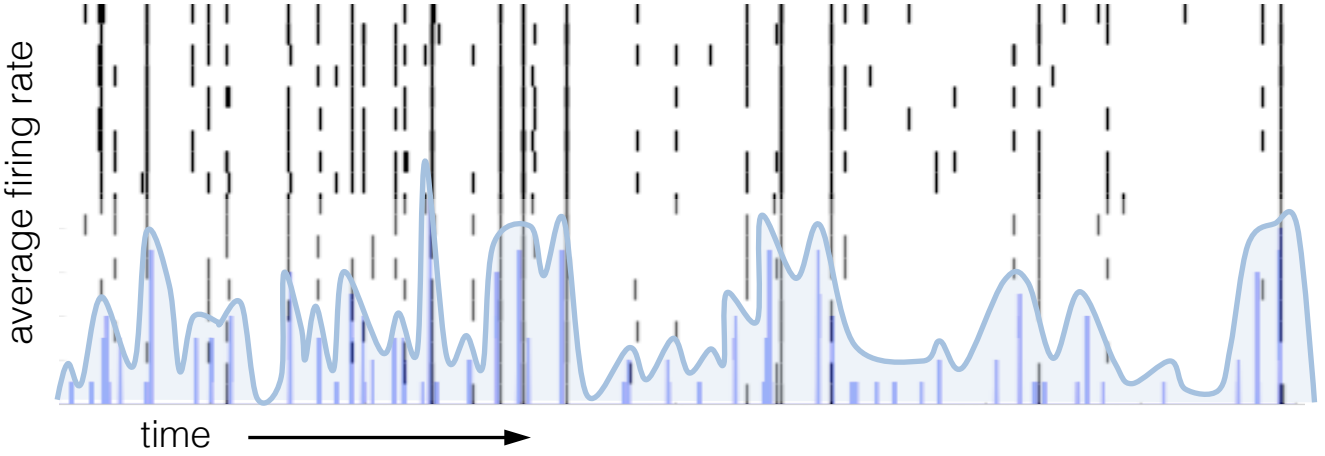

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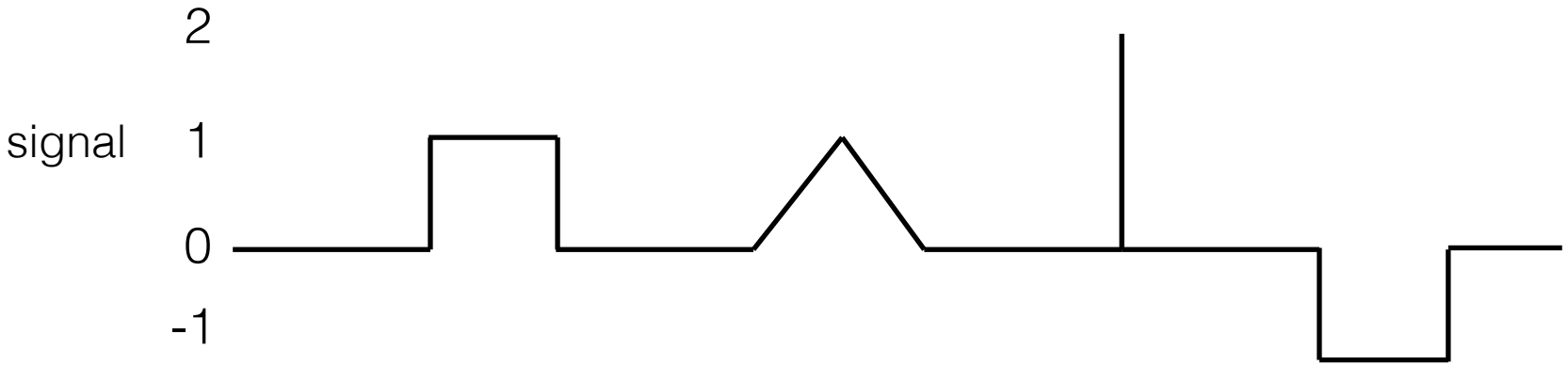

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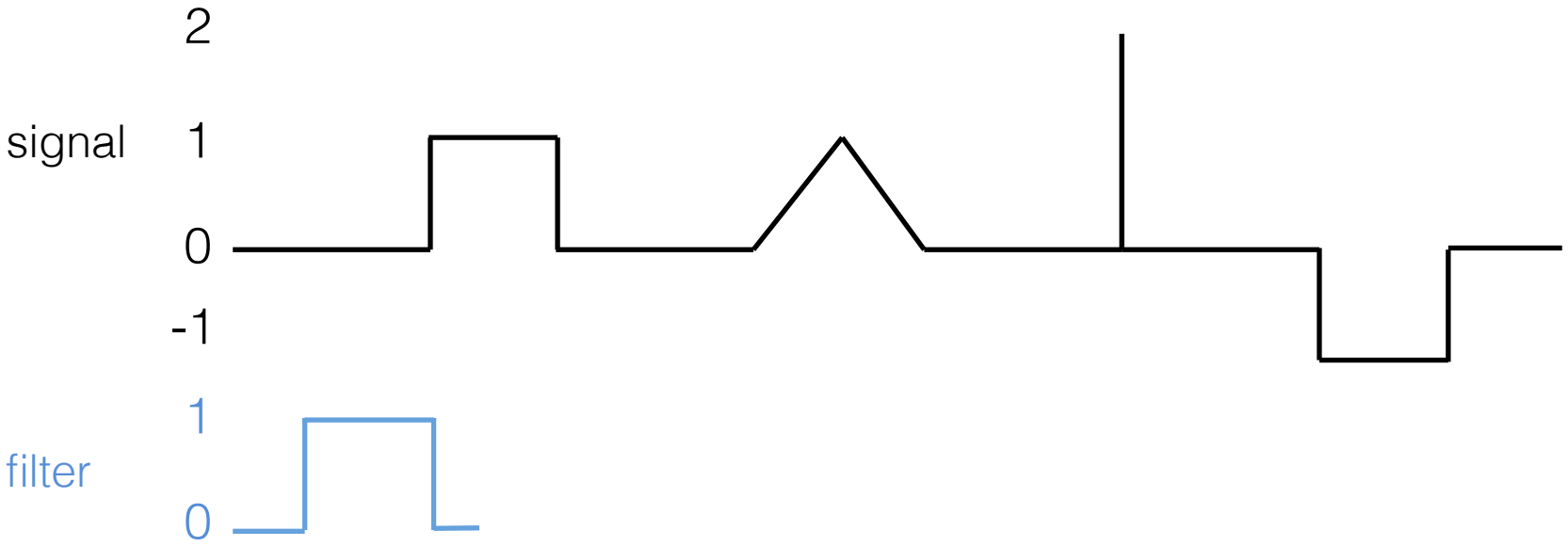

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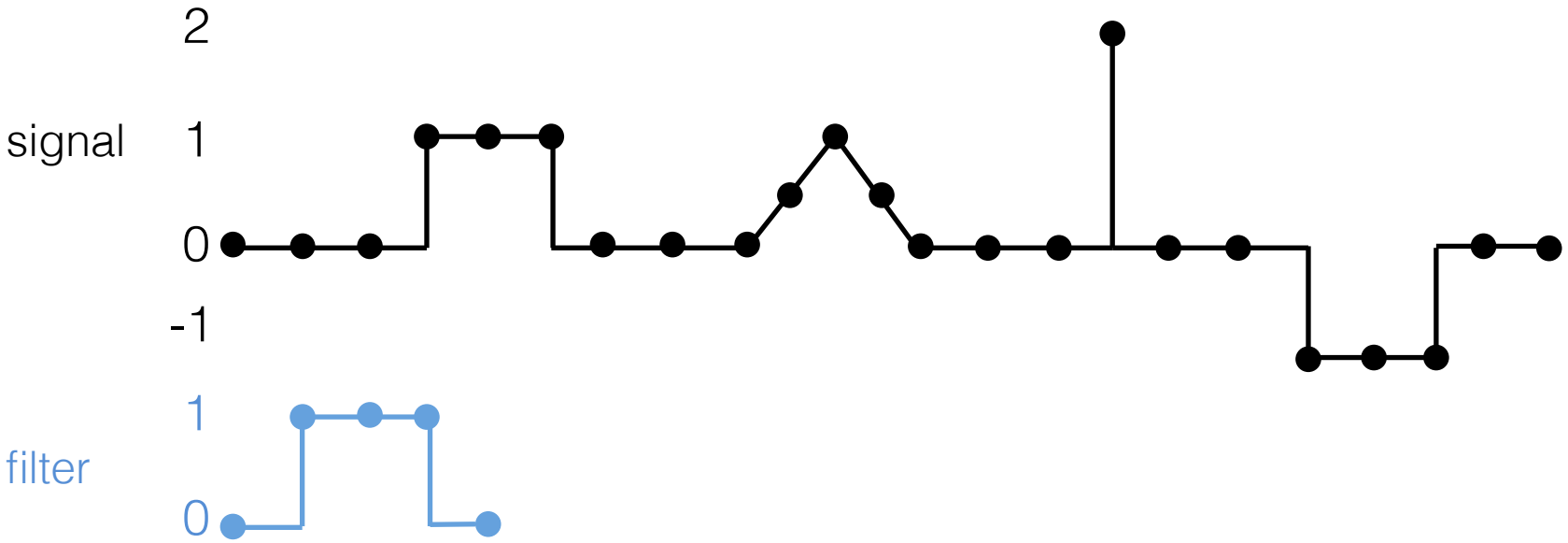

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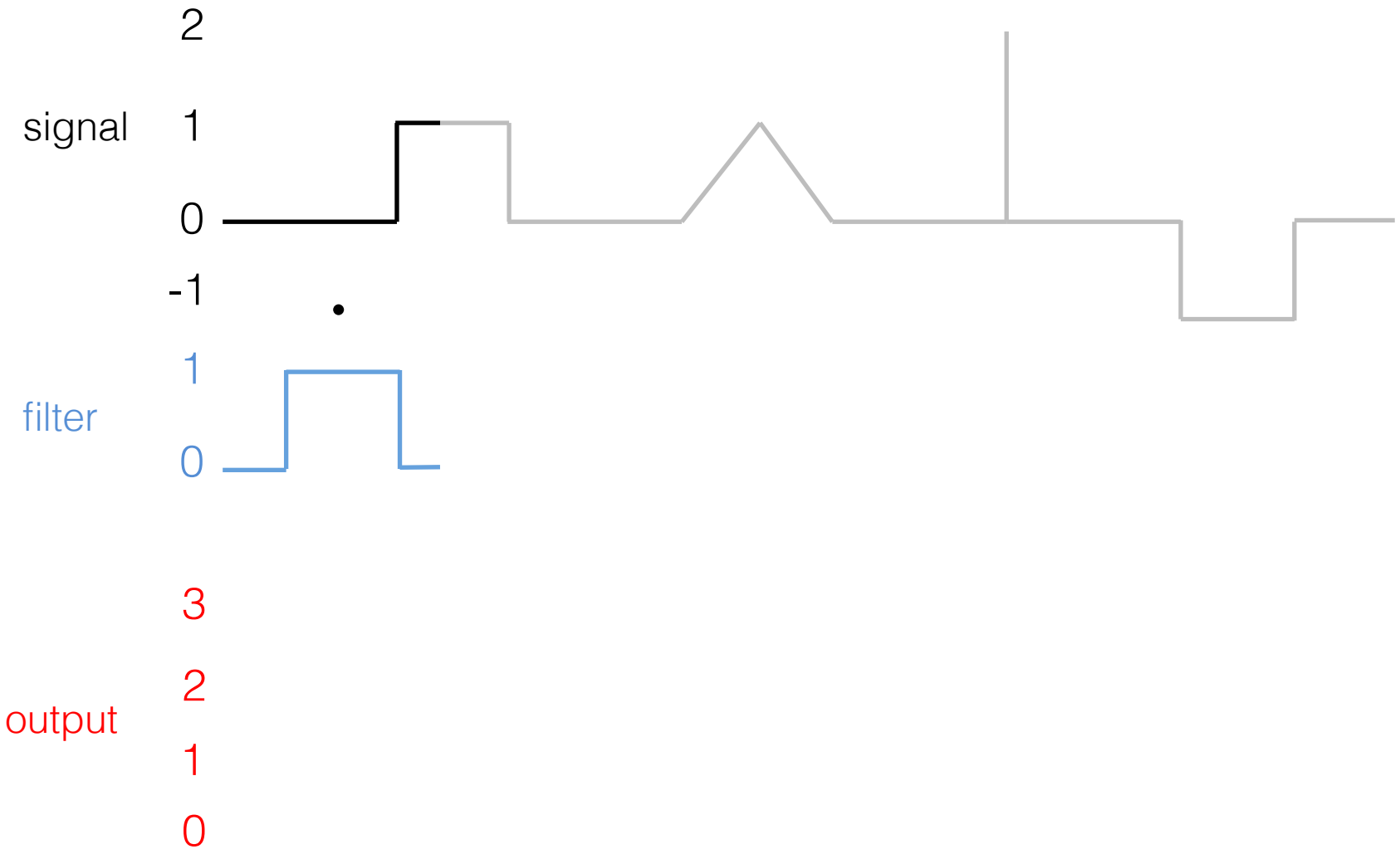

MATLAB: `conv(signal, filter, 'valid')`

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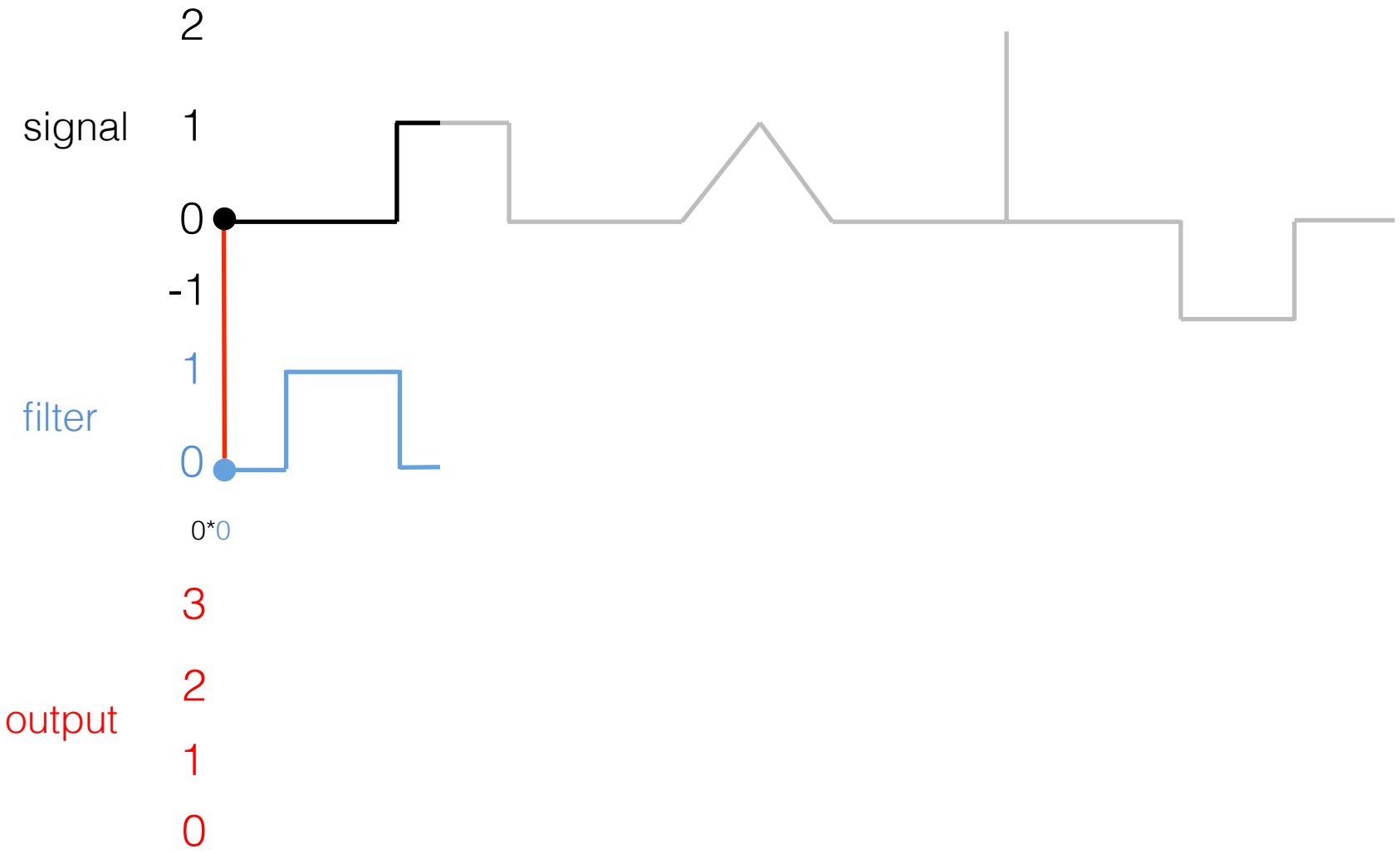

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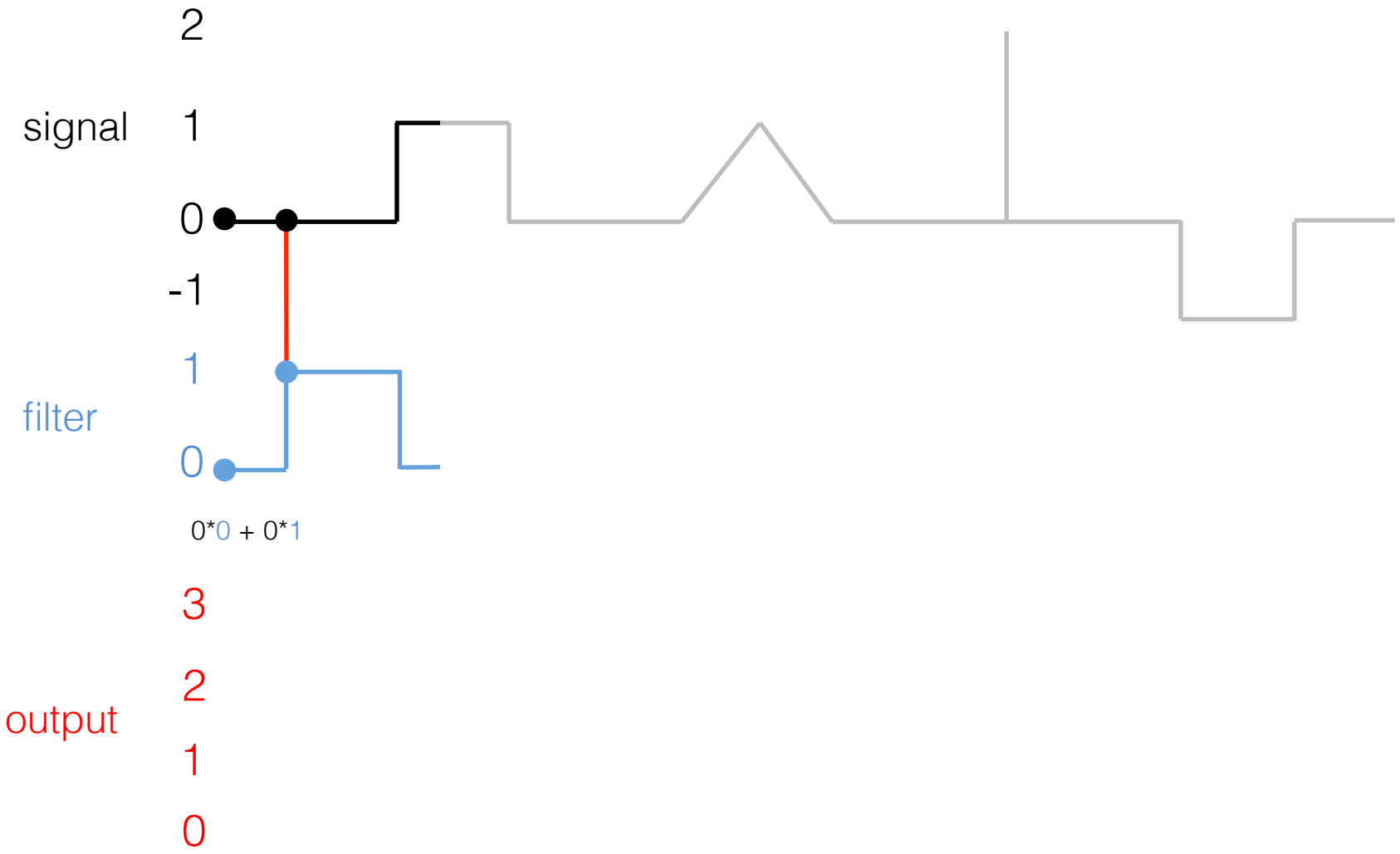

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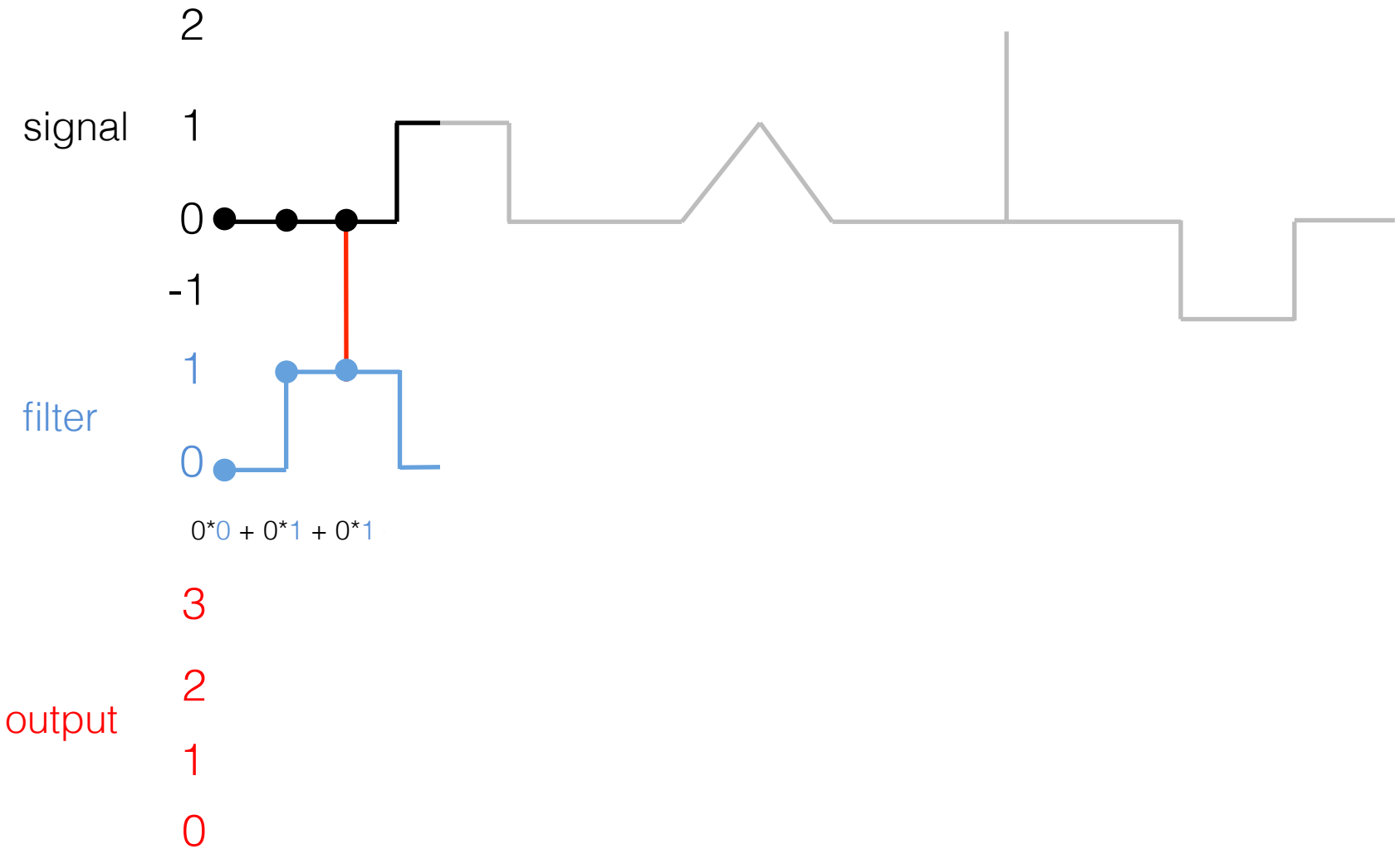

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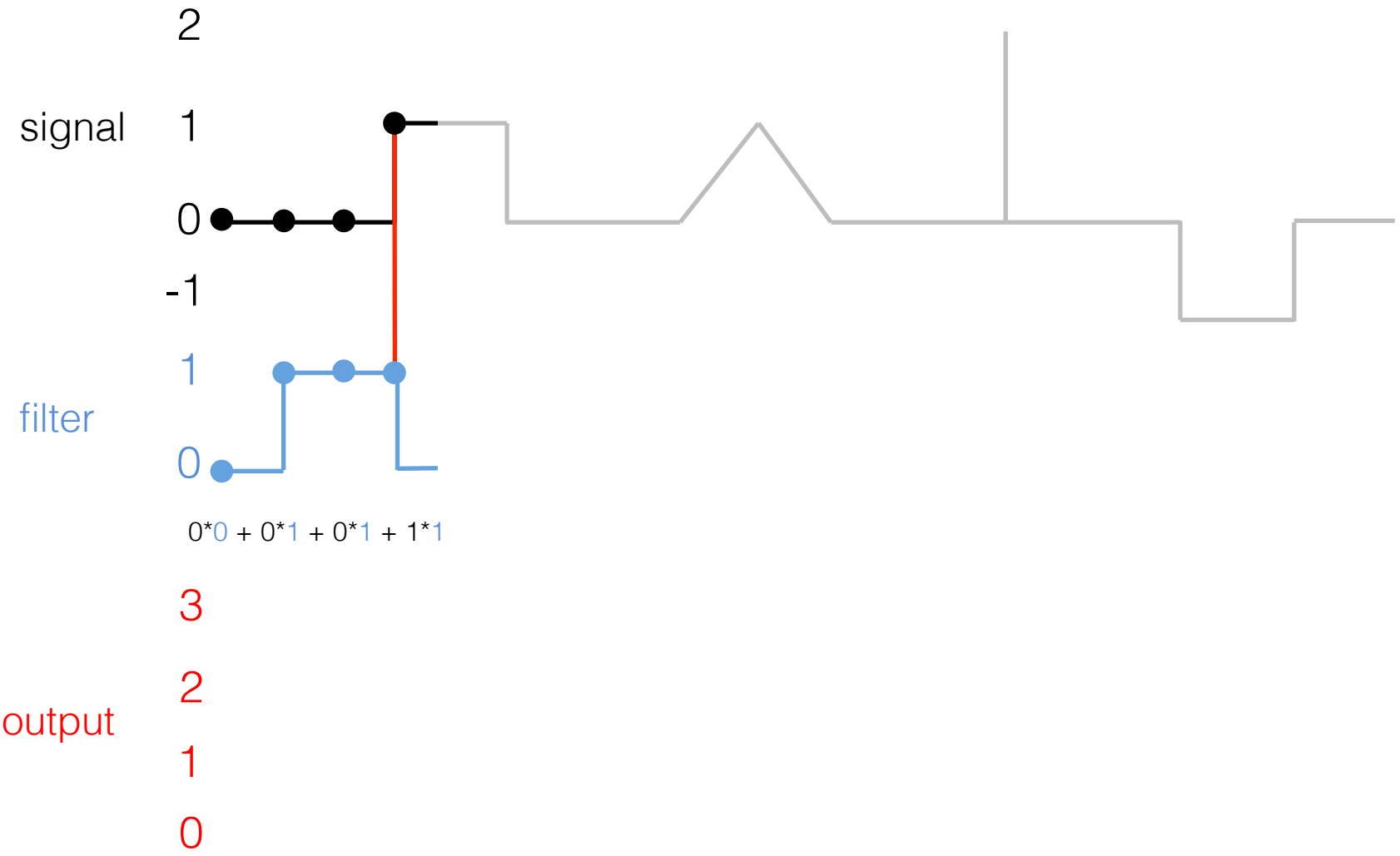

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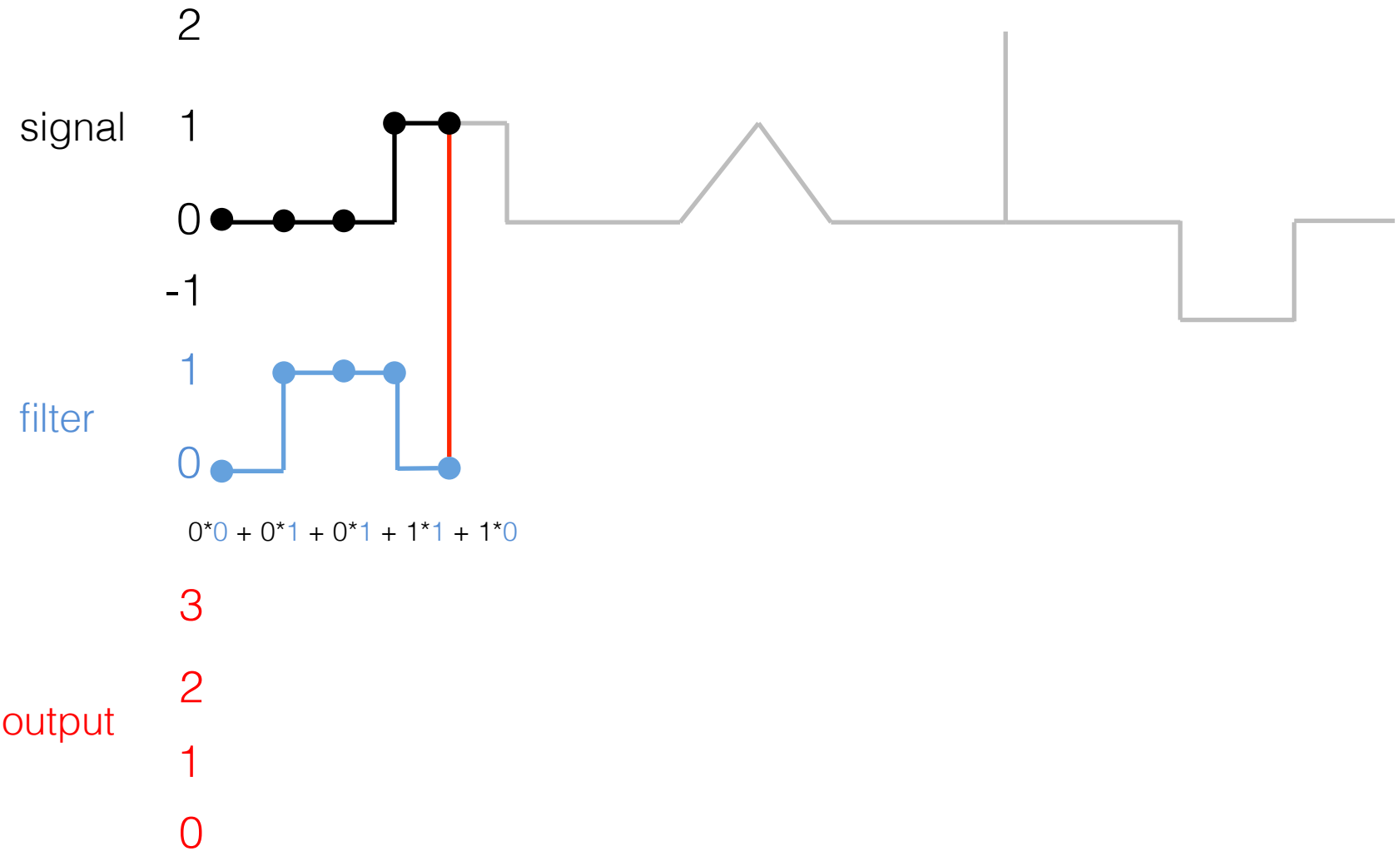

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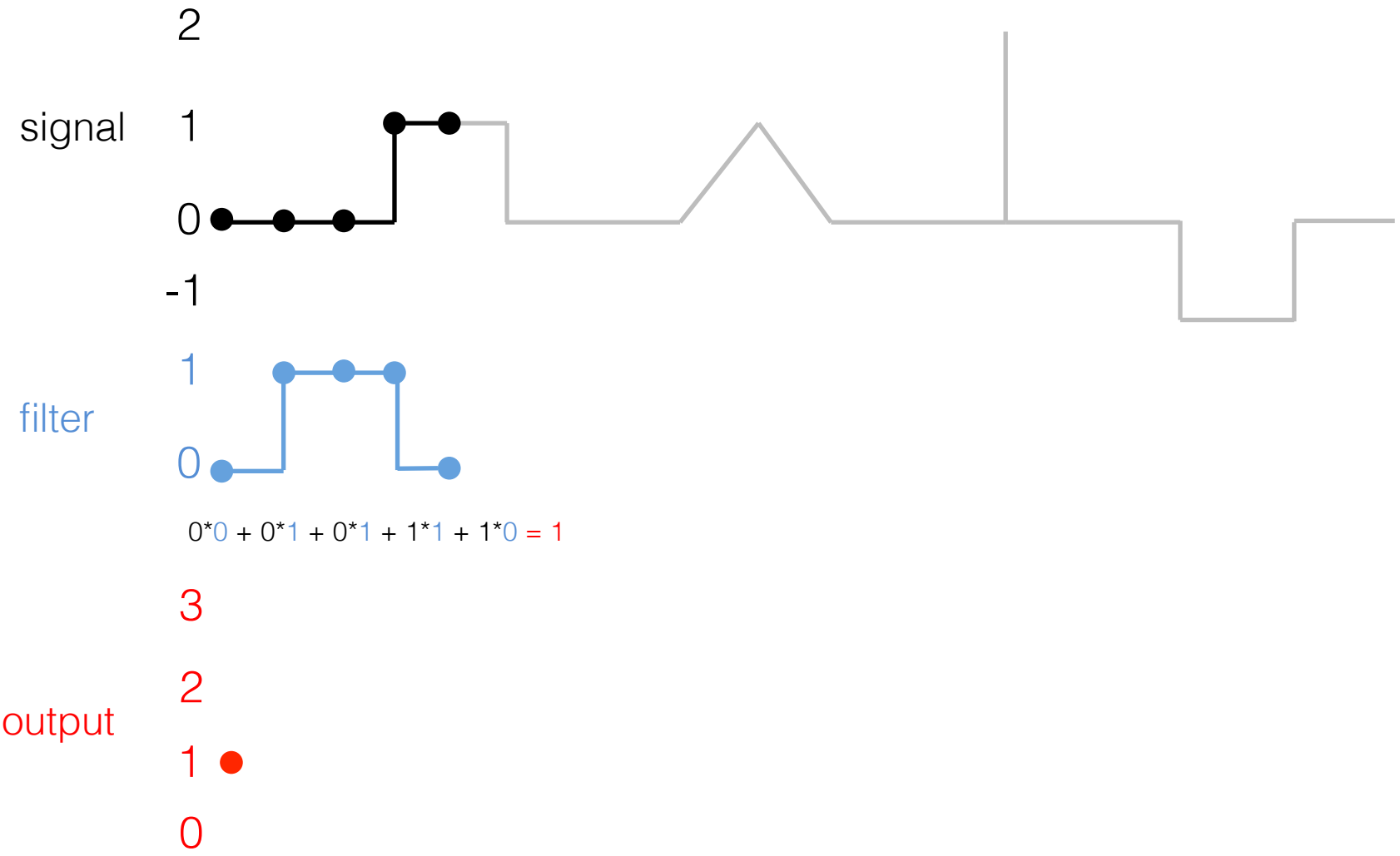

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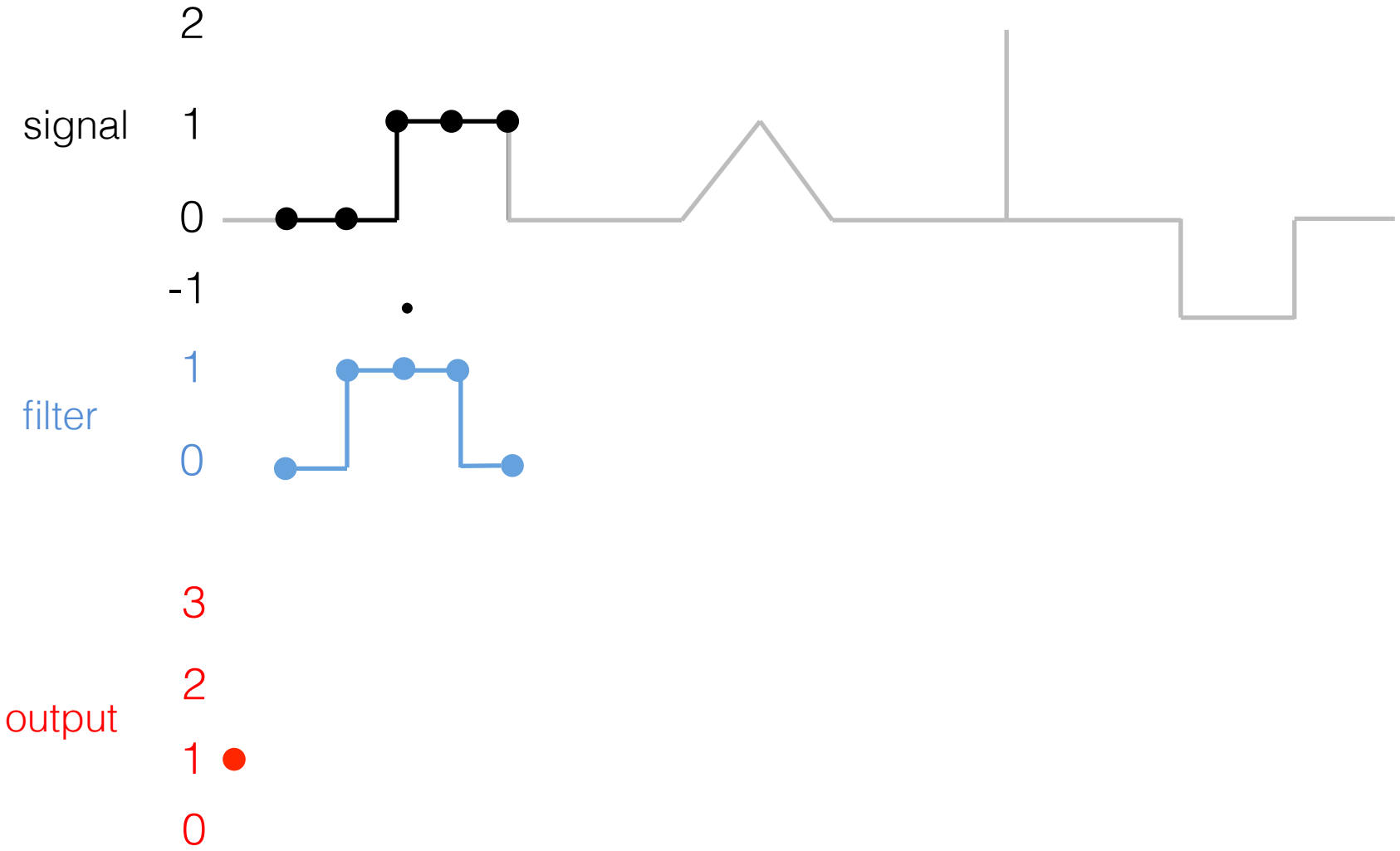

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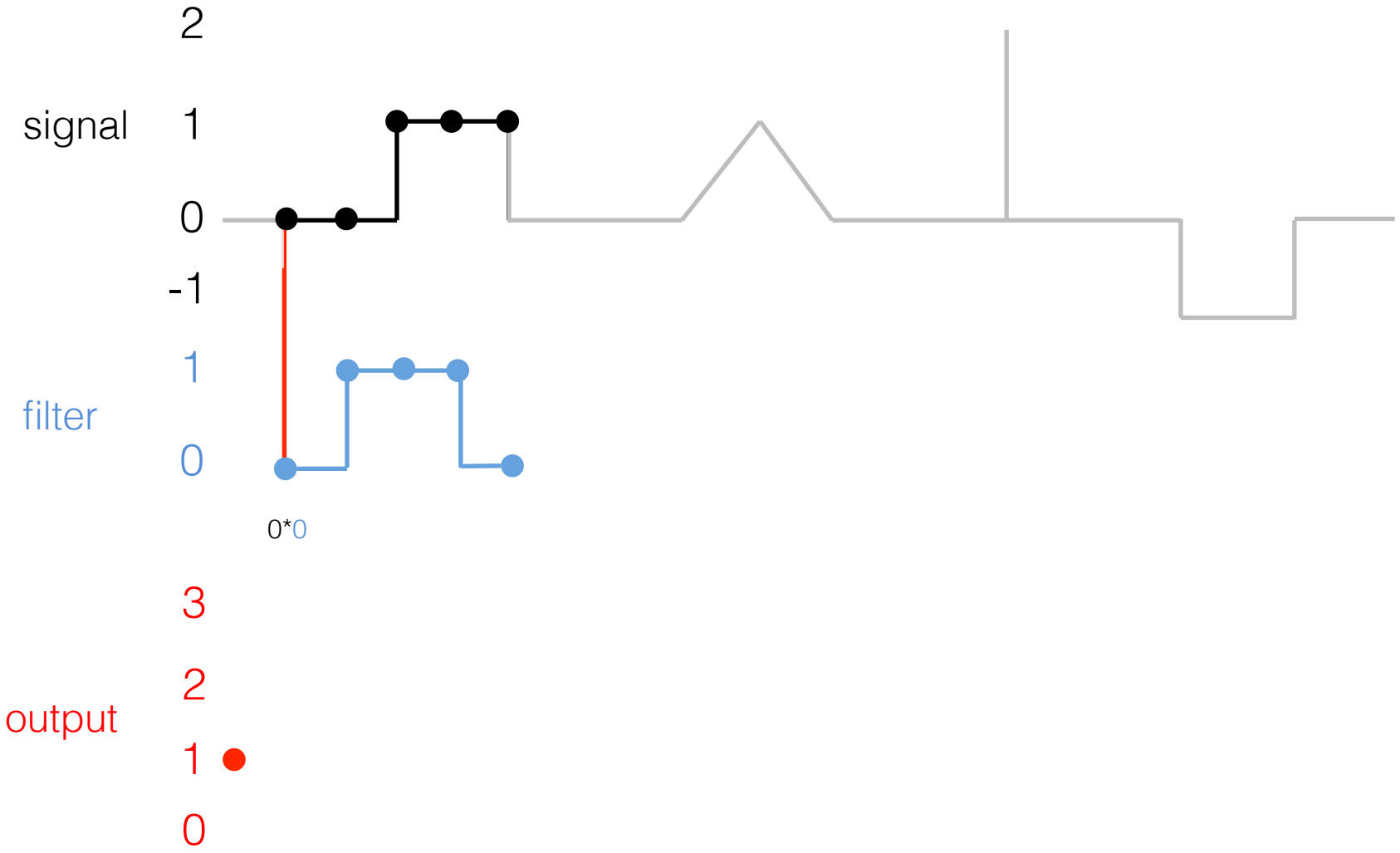

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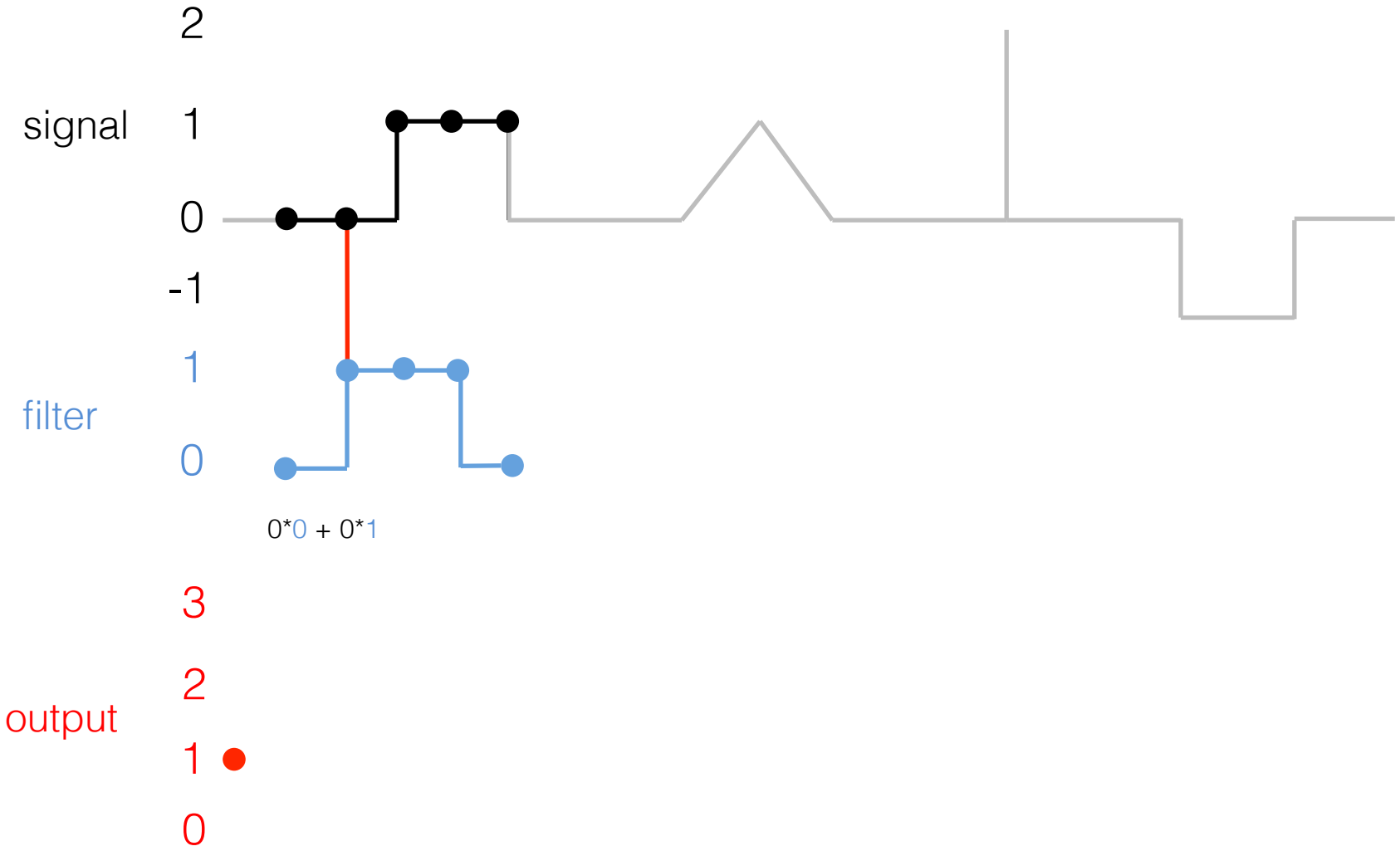

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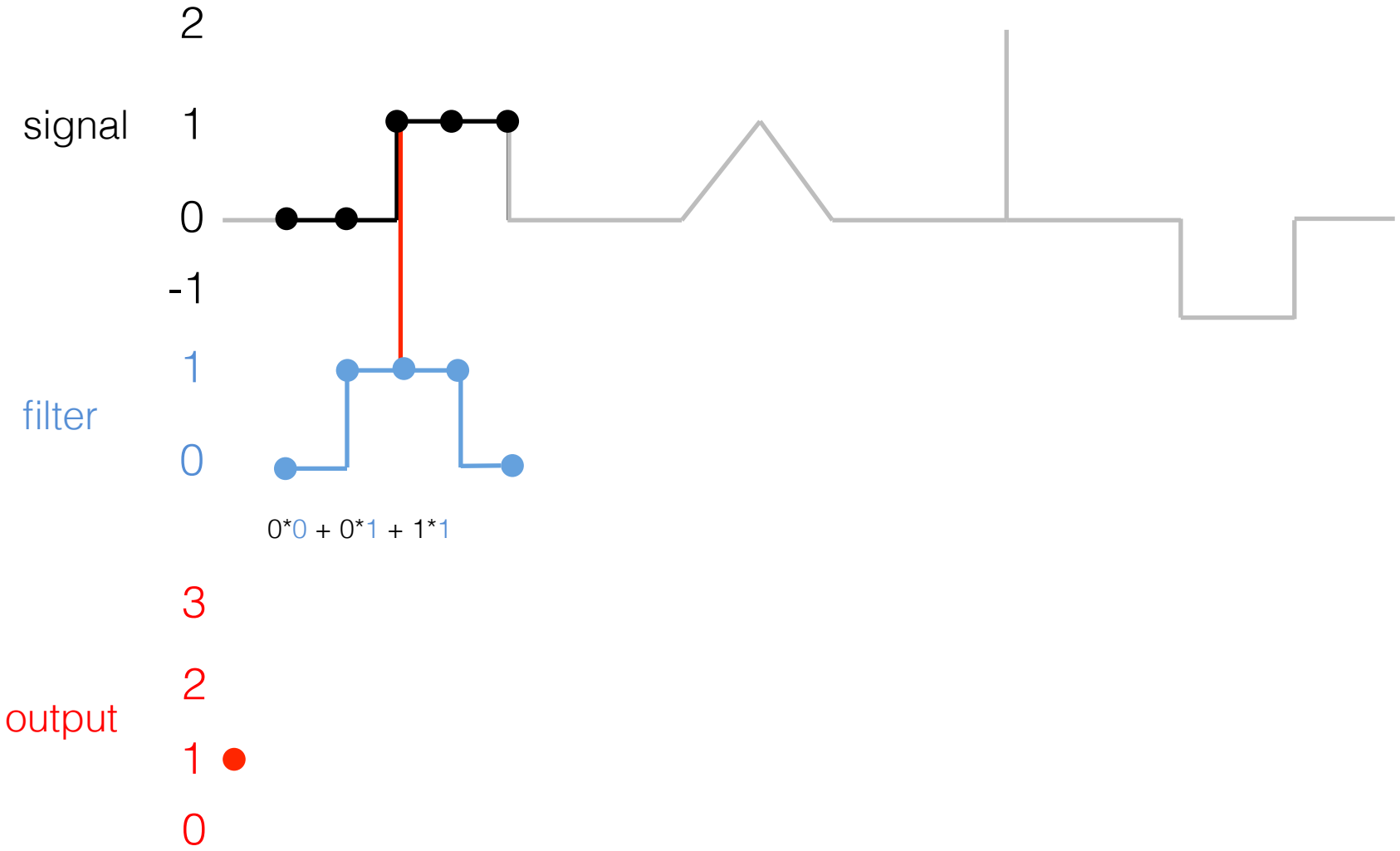

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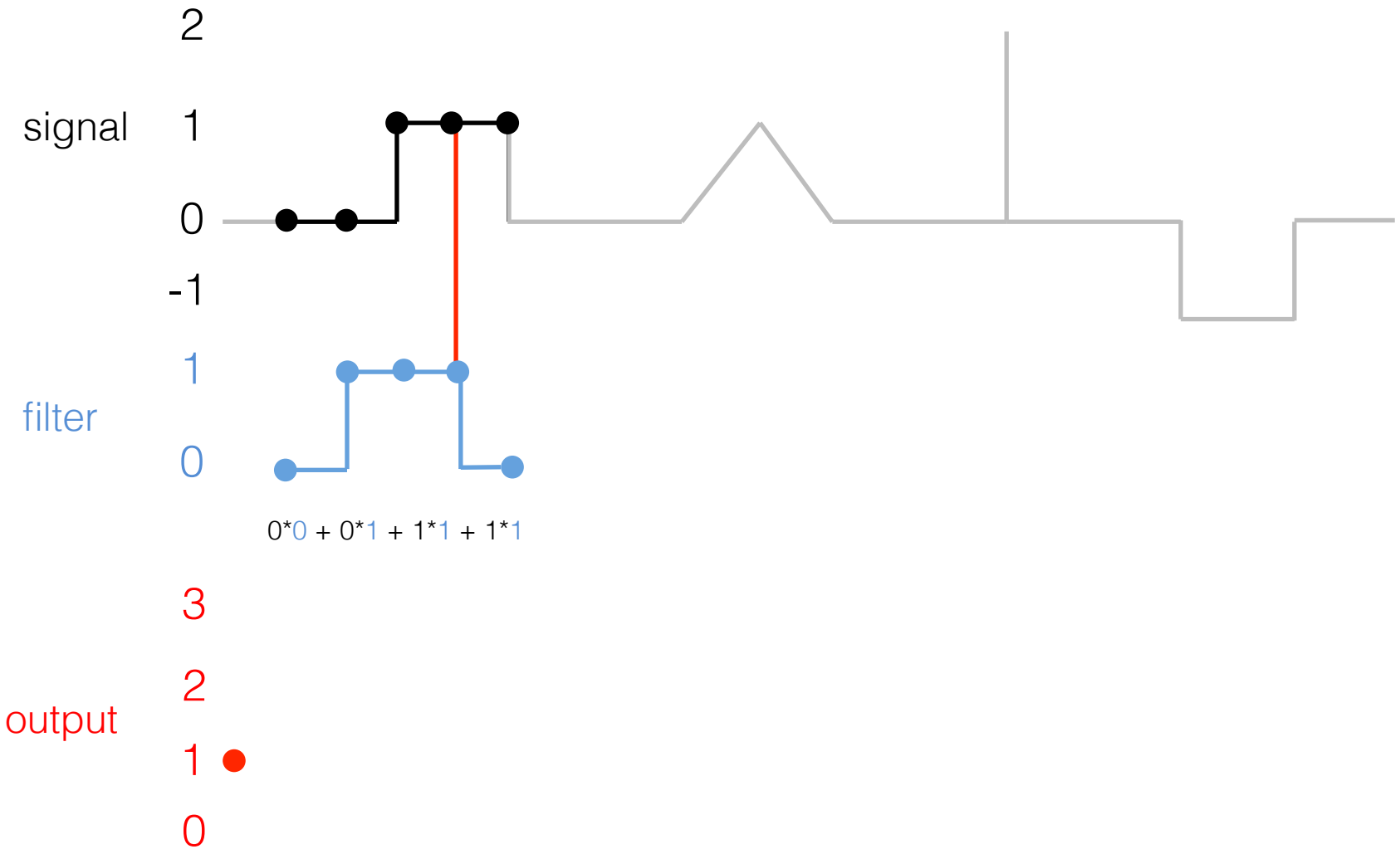

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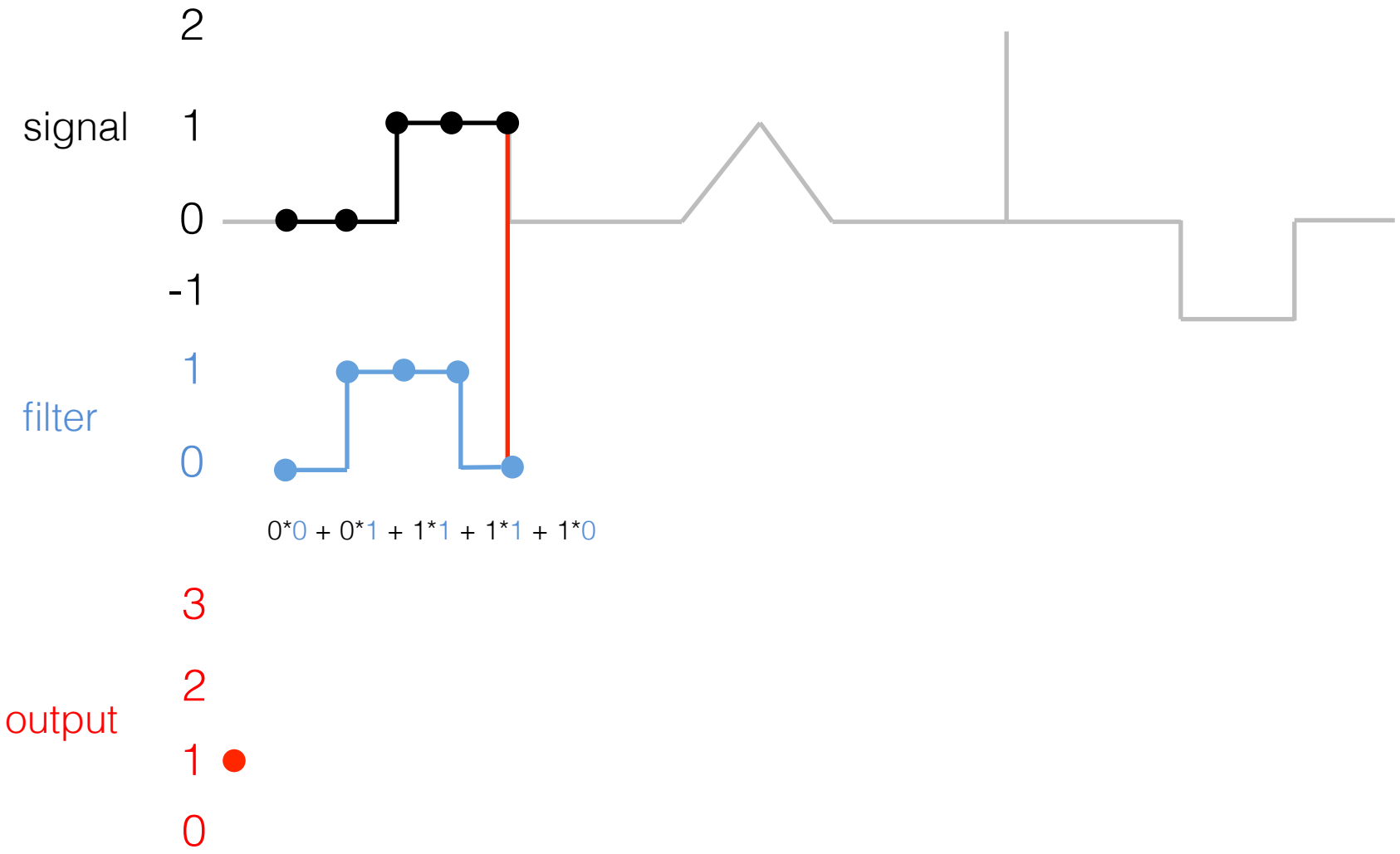

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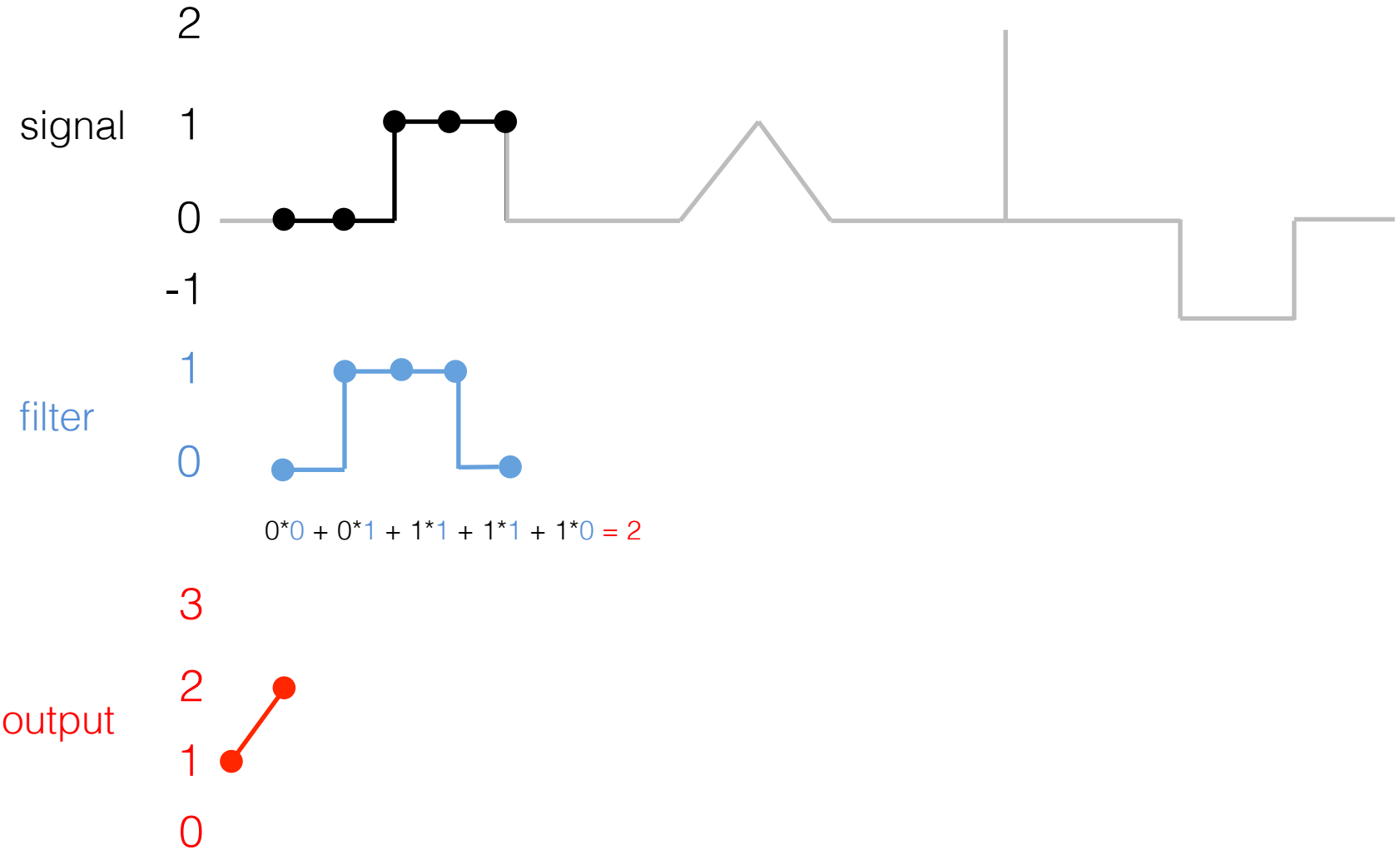

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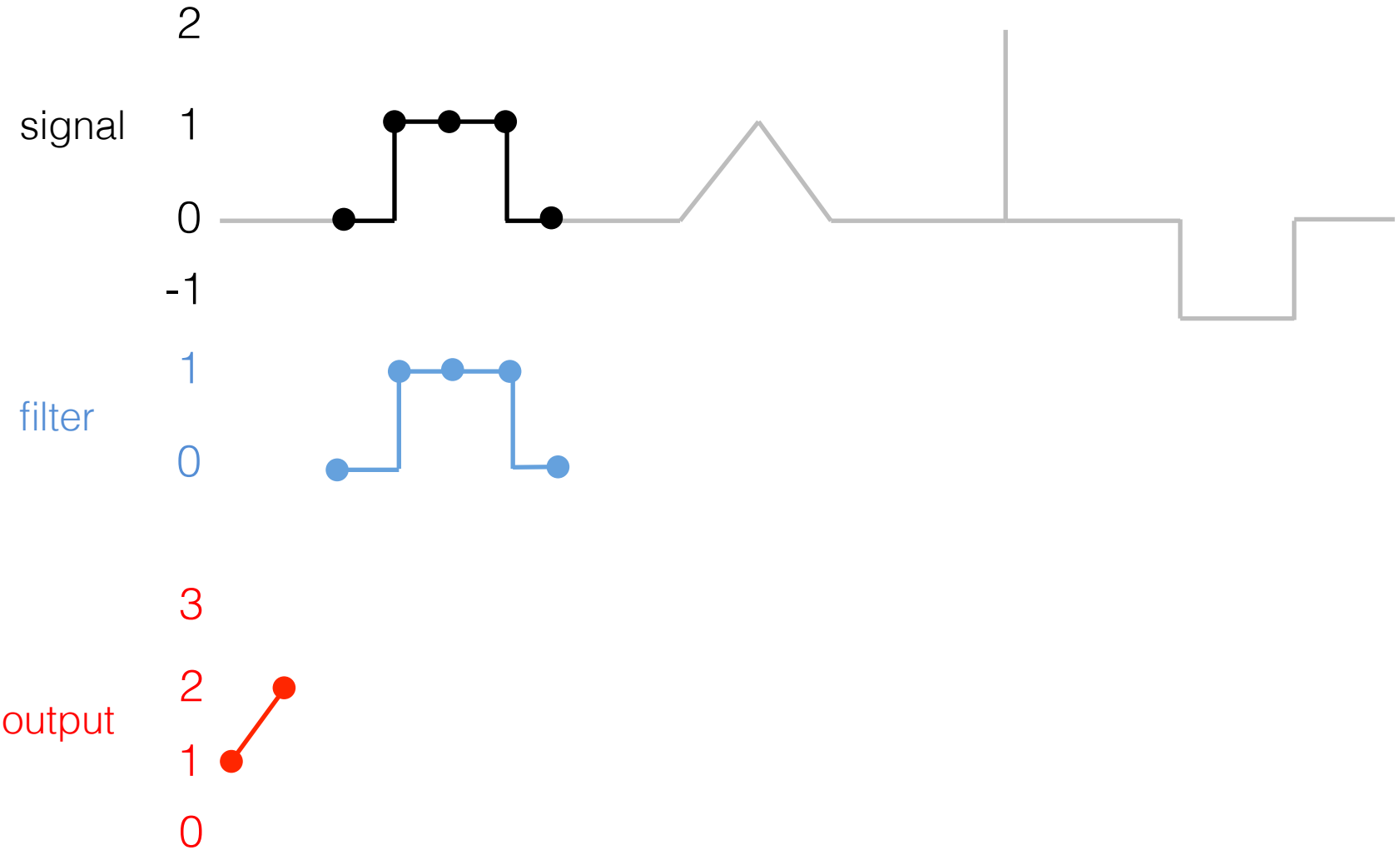

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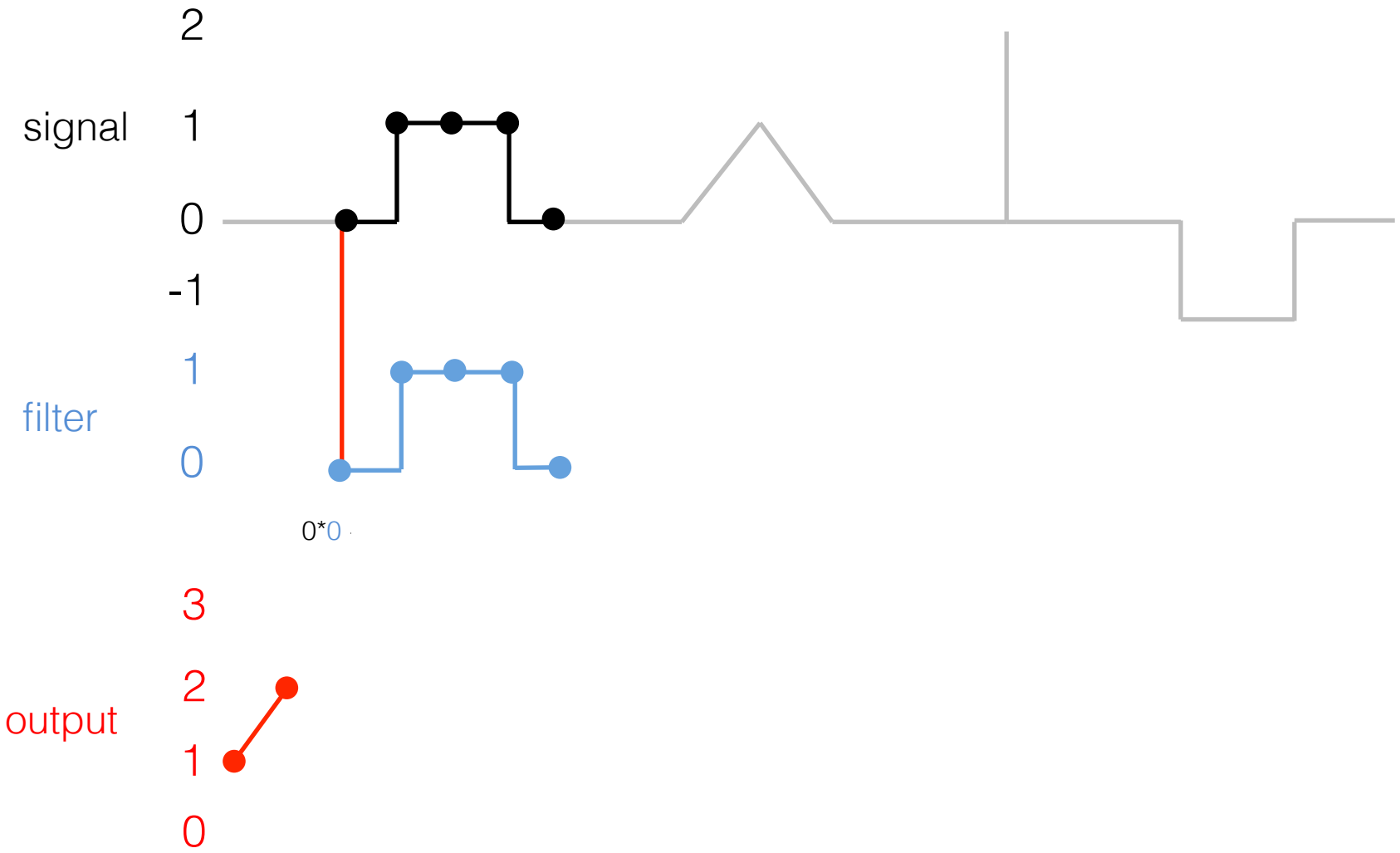

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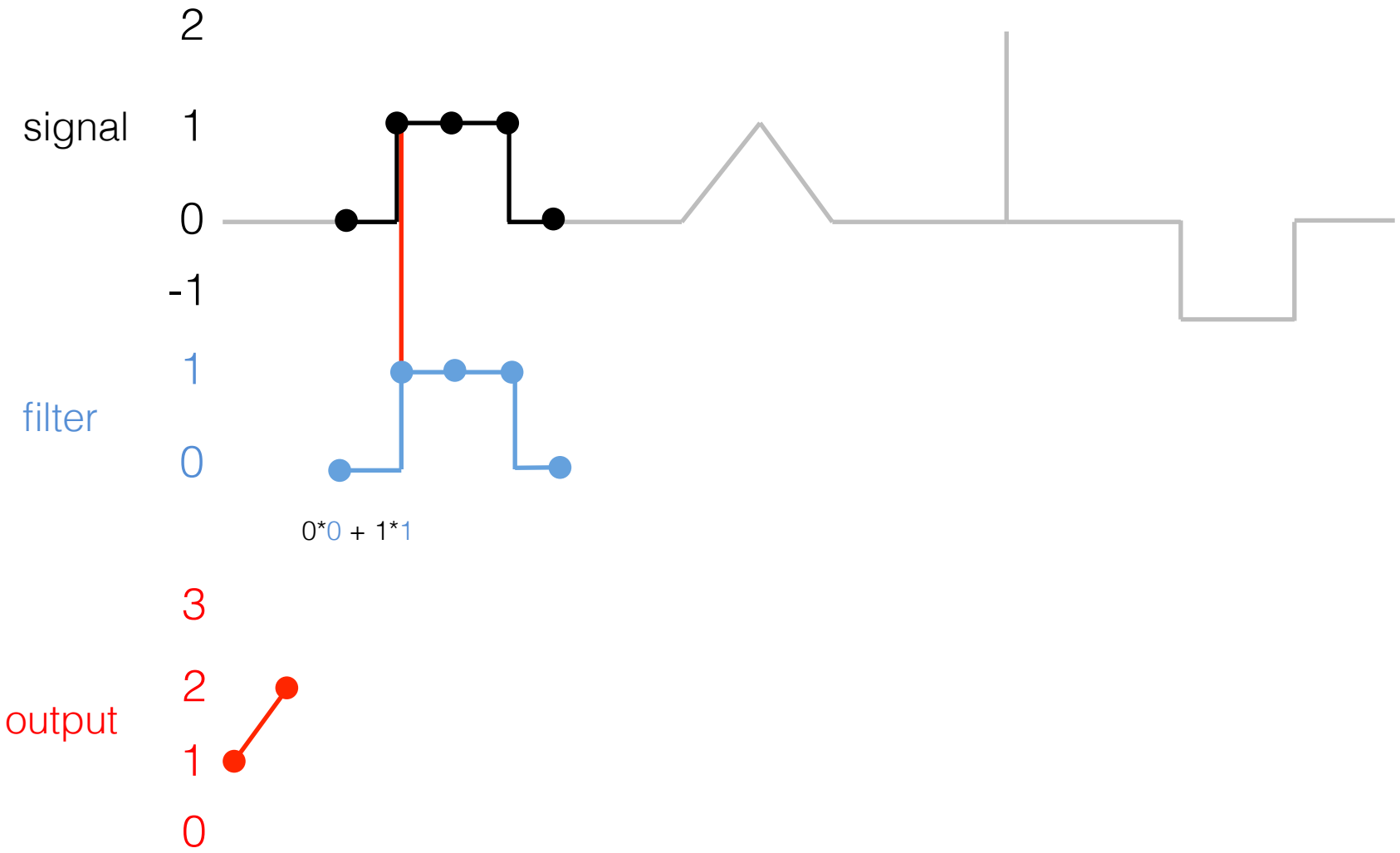

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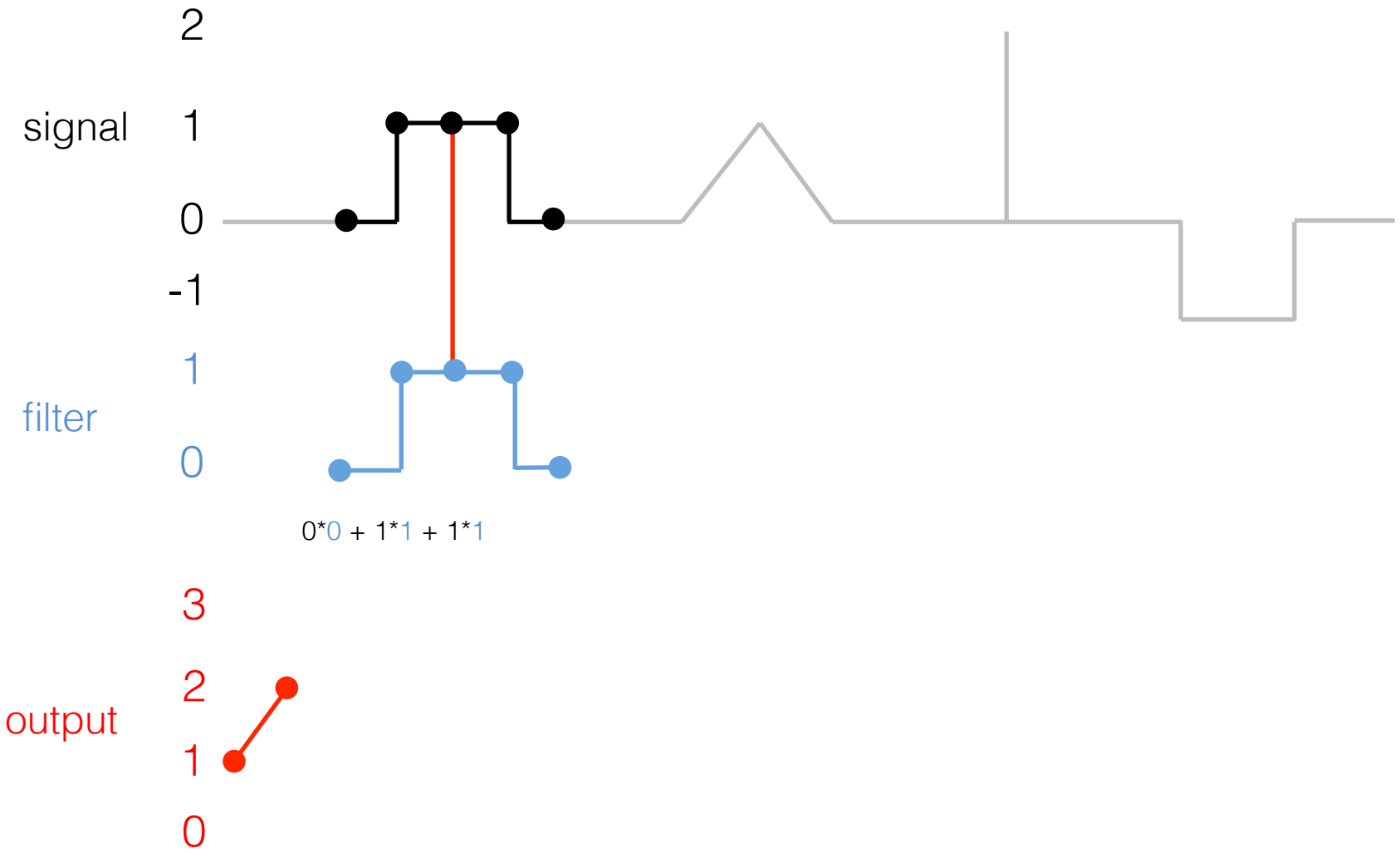

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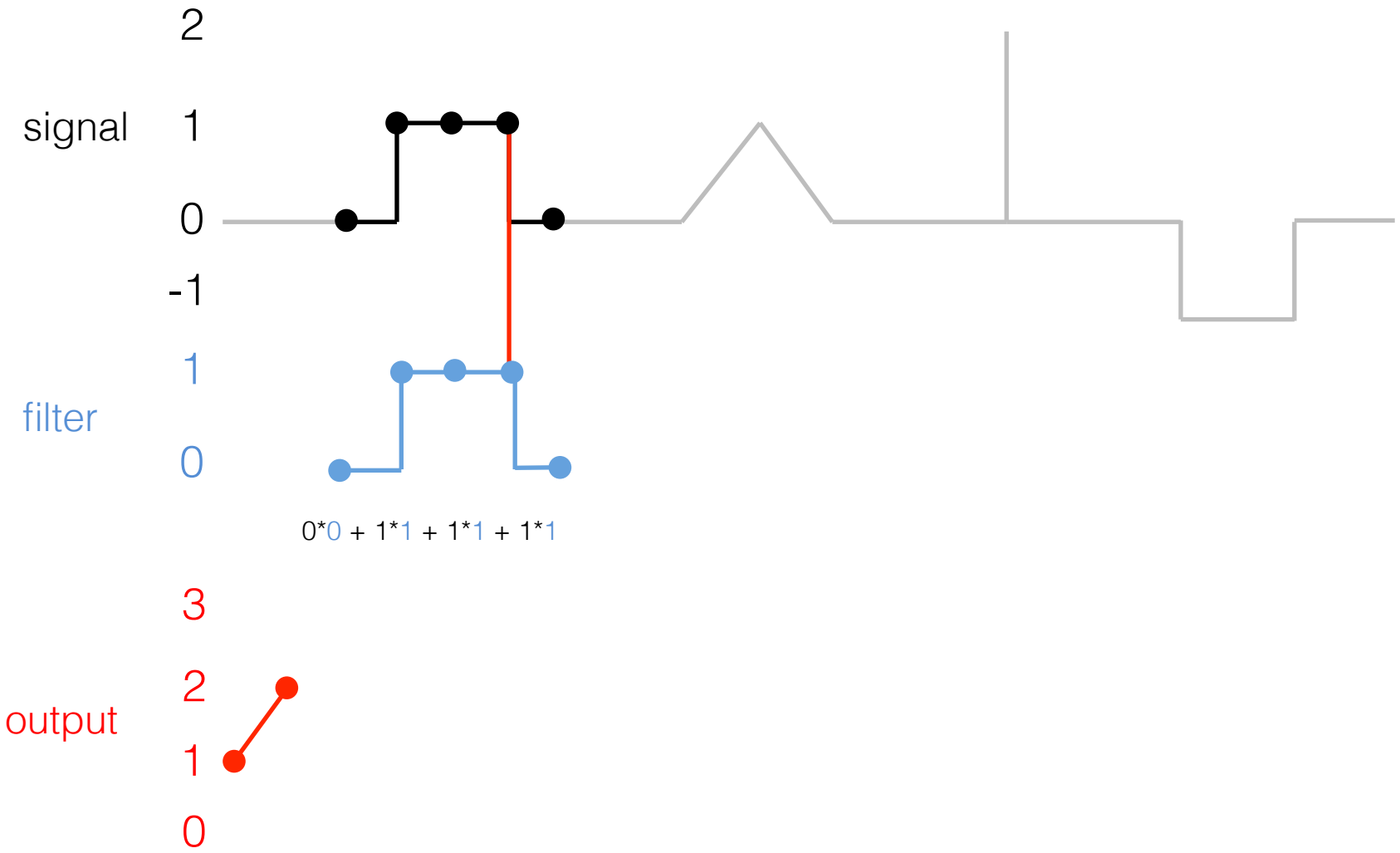

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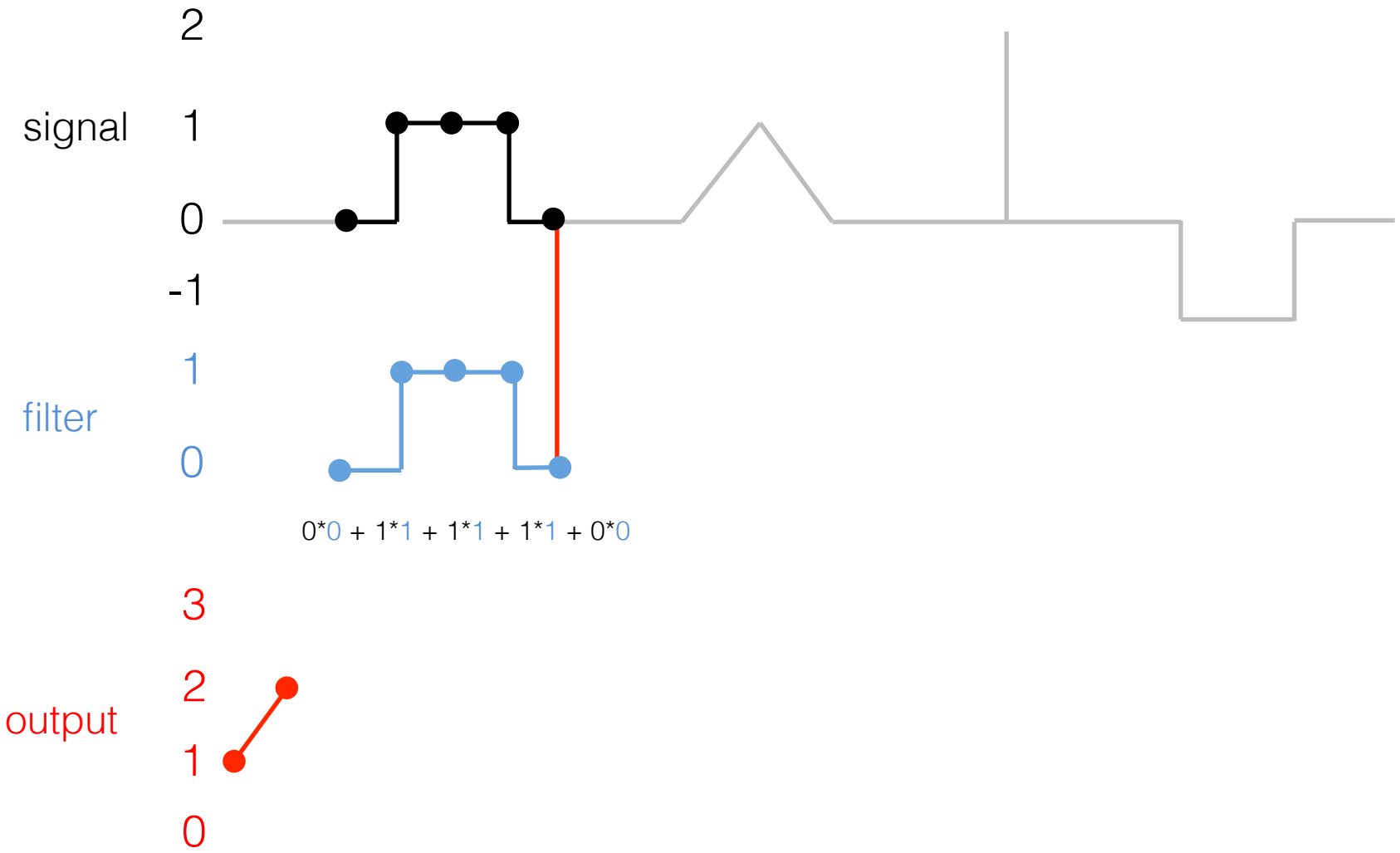

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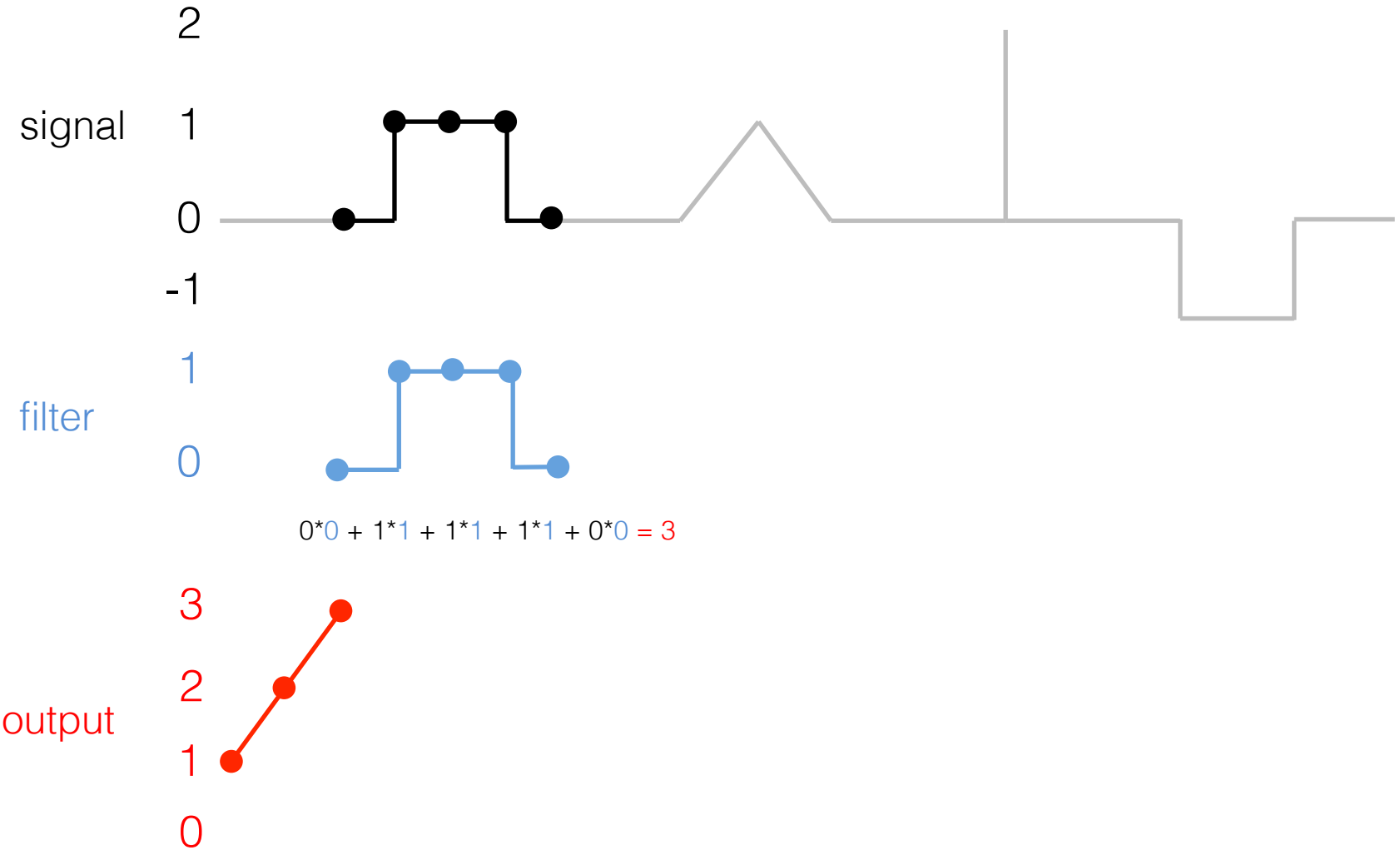

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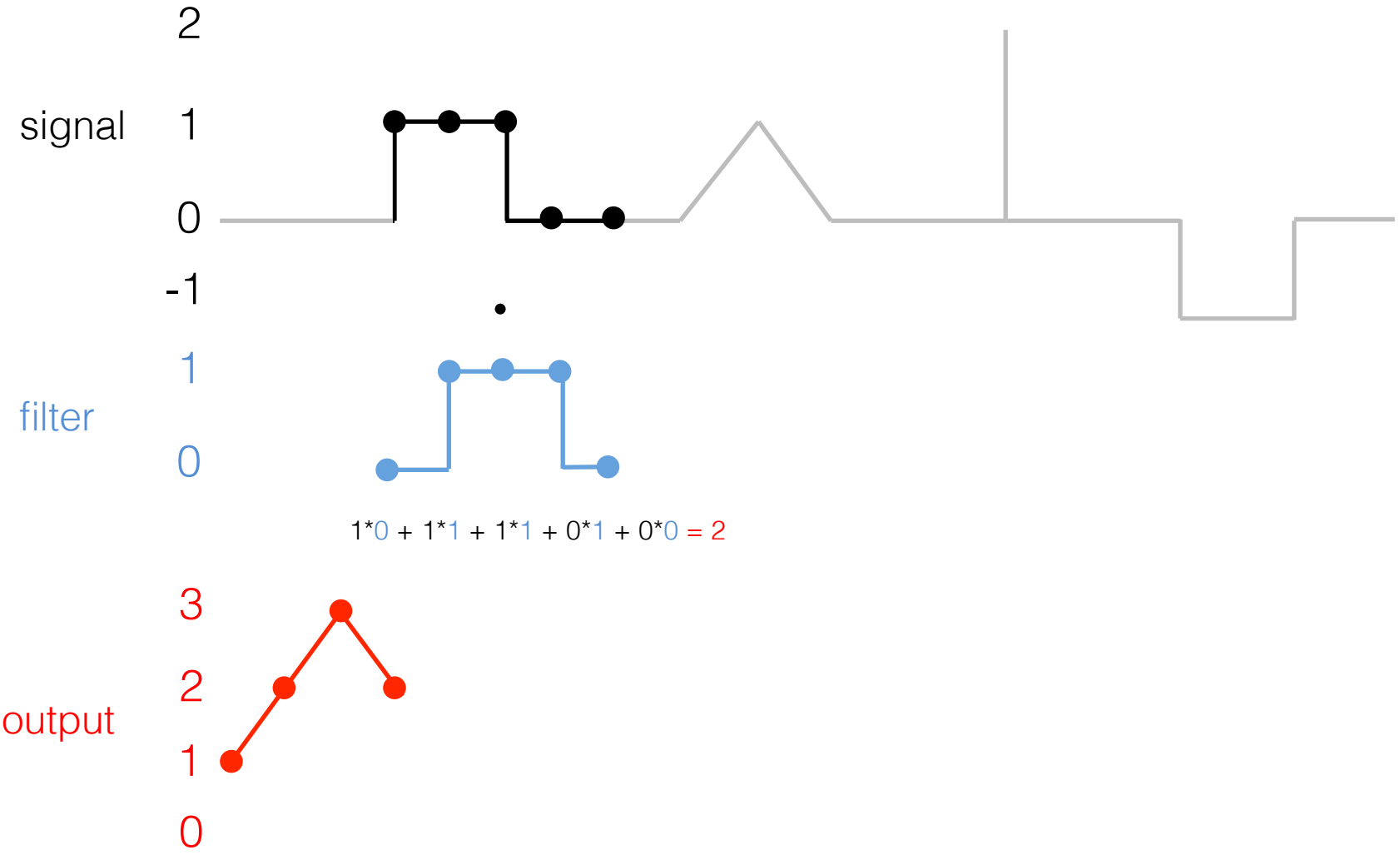

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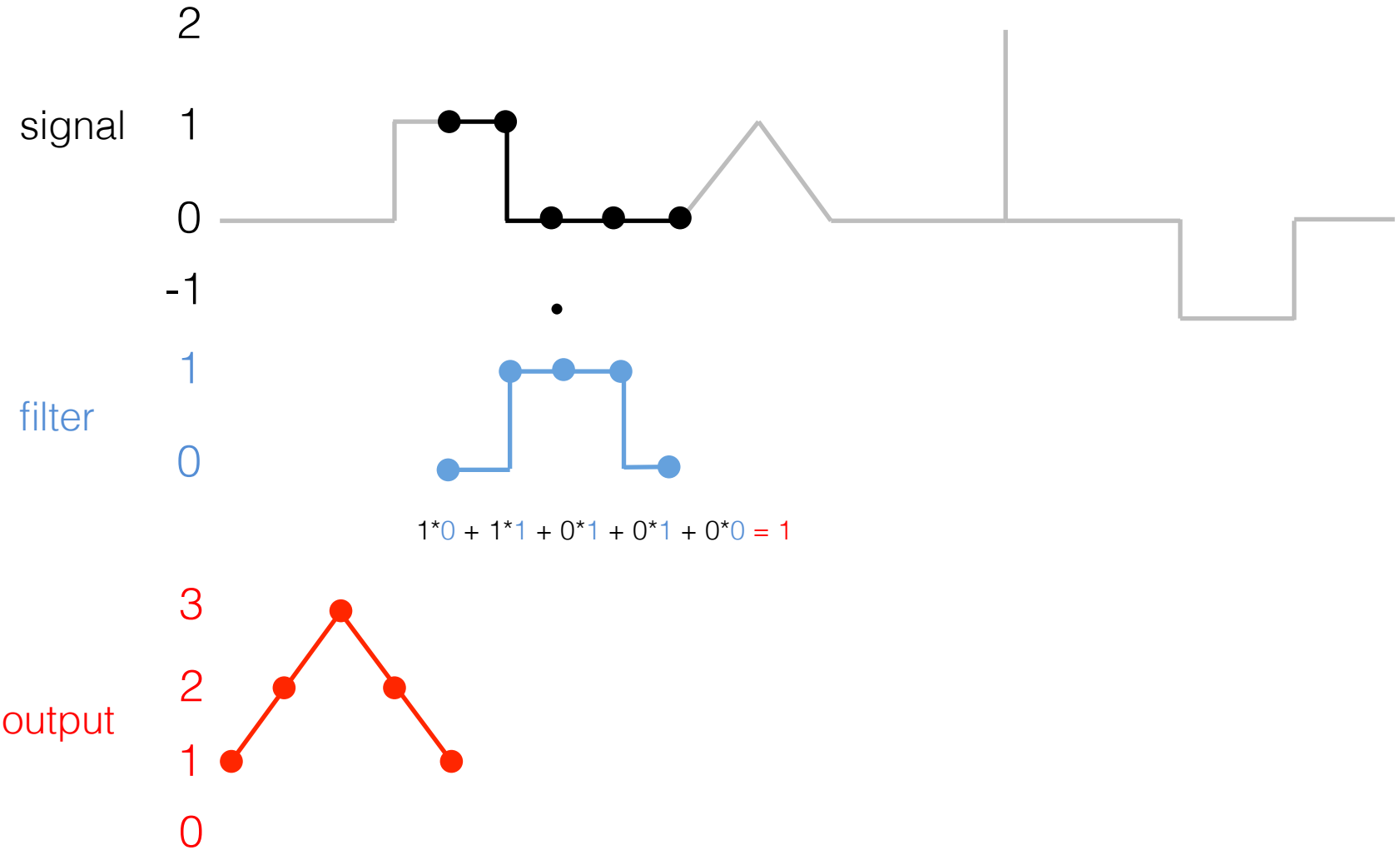

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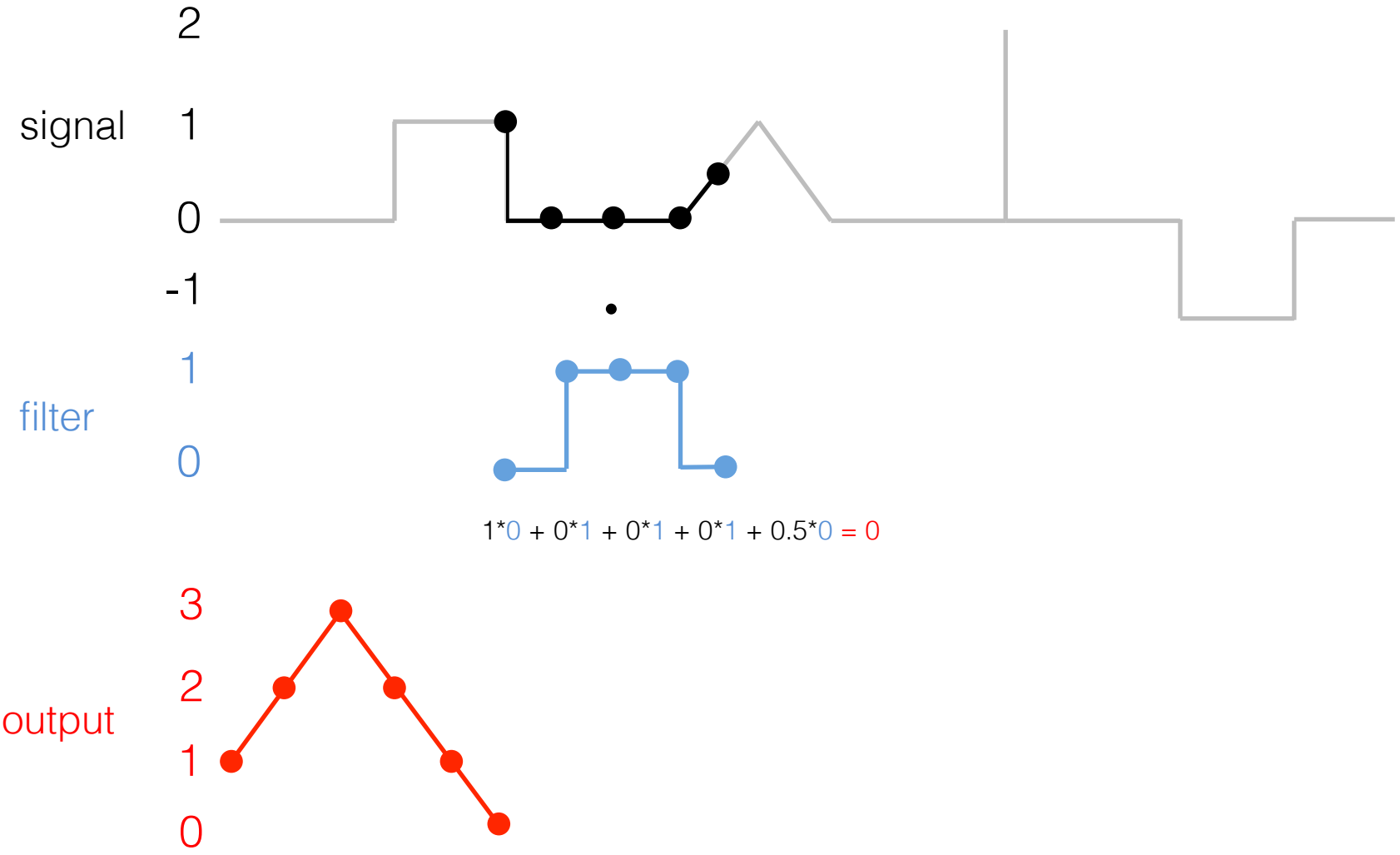

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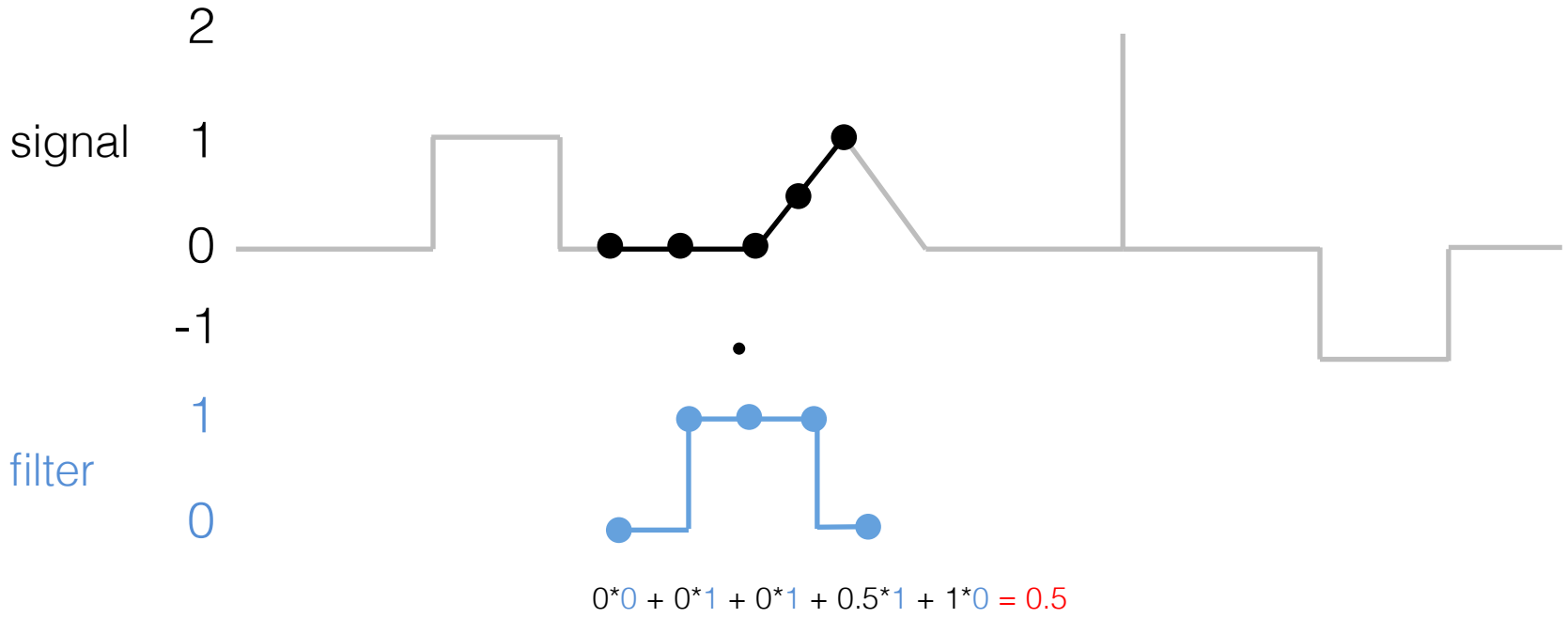
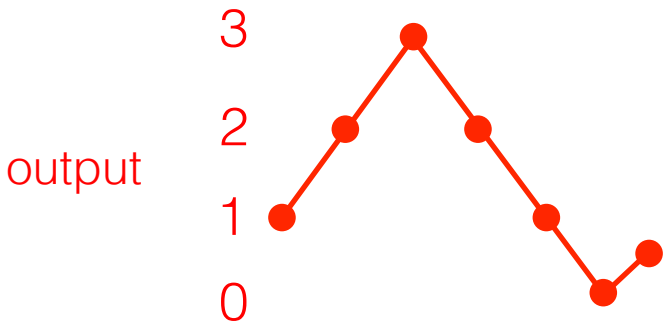


0*0 + 0*1 + 0*1 + 0.5*1 + 1*0 = 0.5



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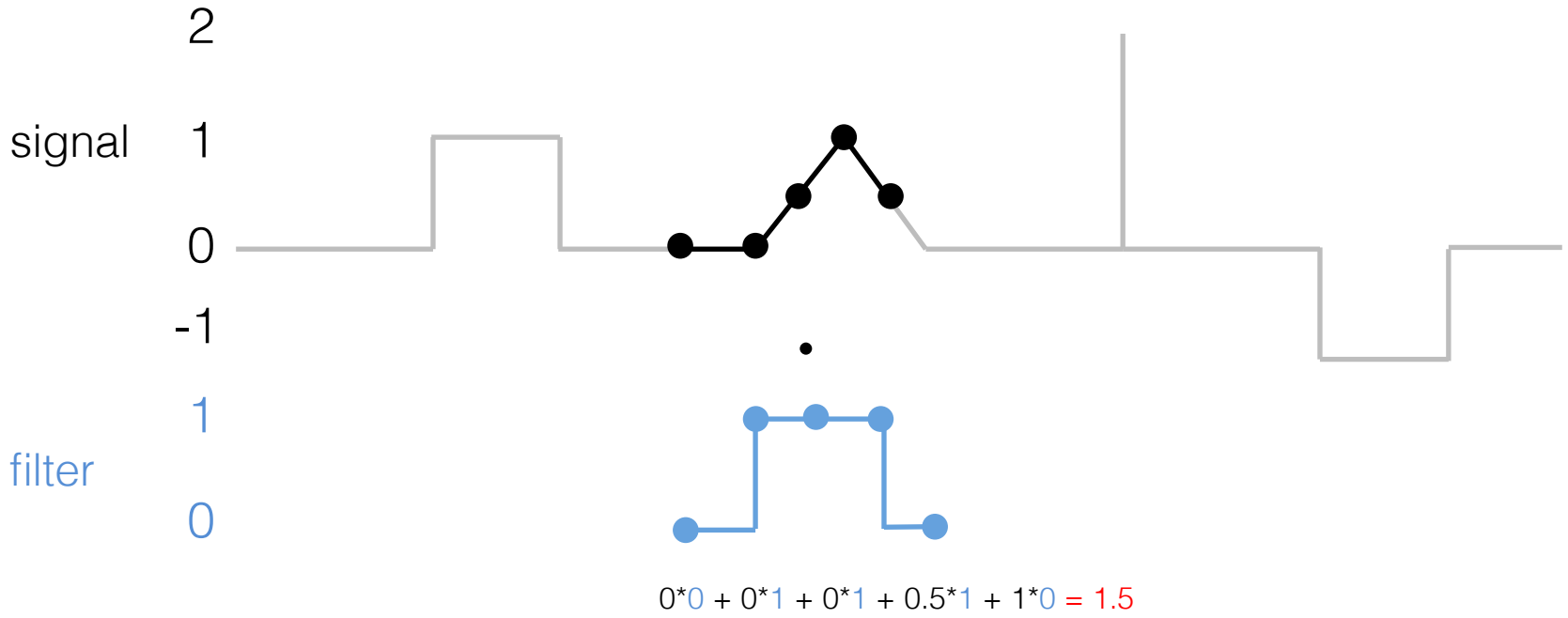
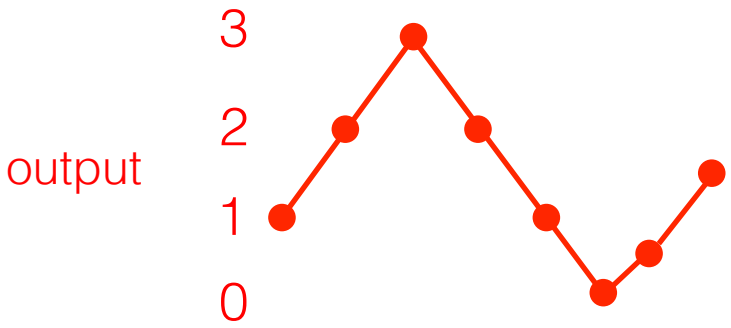


0*0 + 0*1 + 0*1 + 0.5*1 + 1*0 = 1.5



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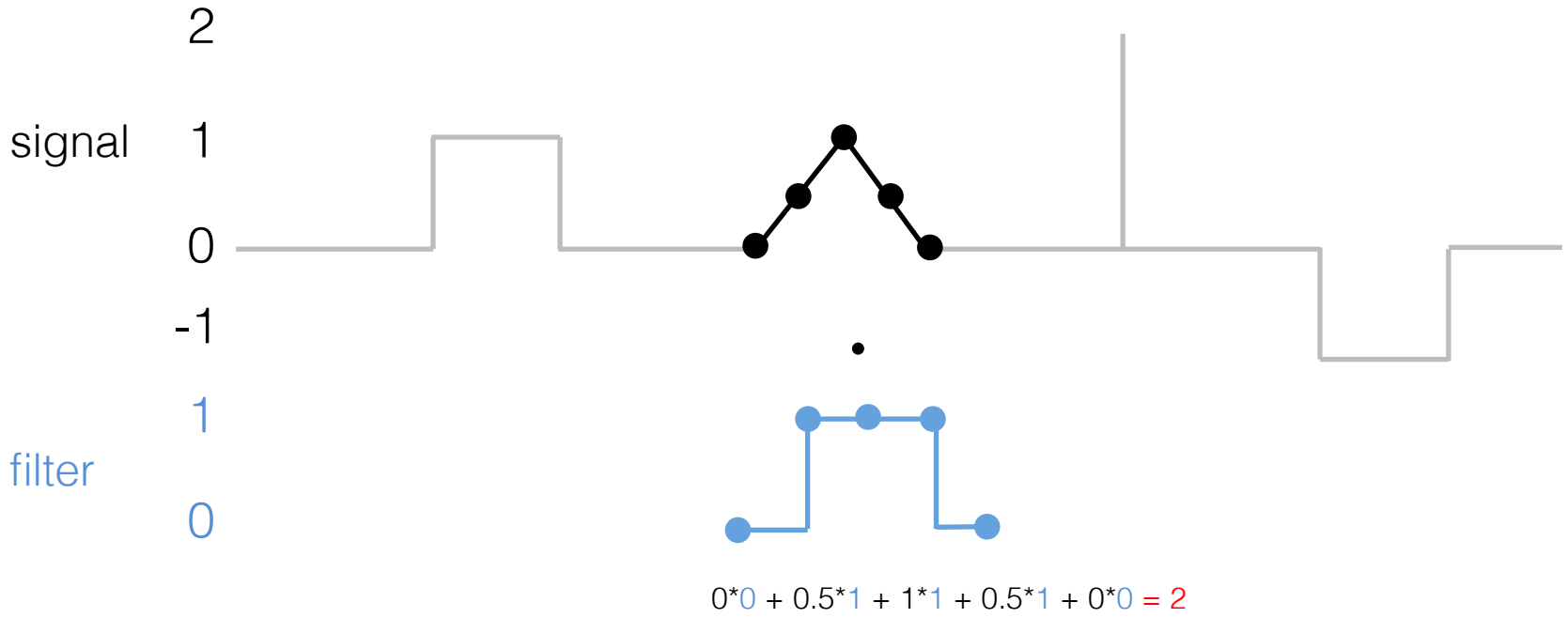
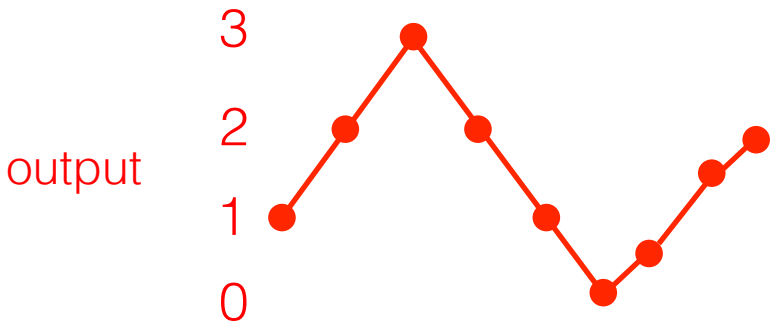


0*0 + 0.5*1 + 1*1 + 0.5*1 + 0*0 = 2



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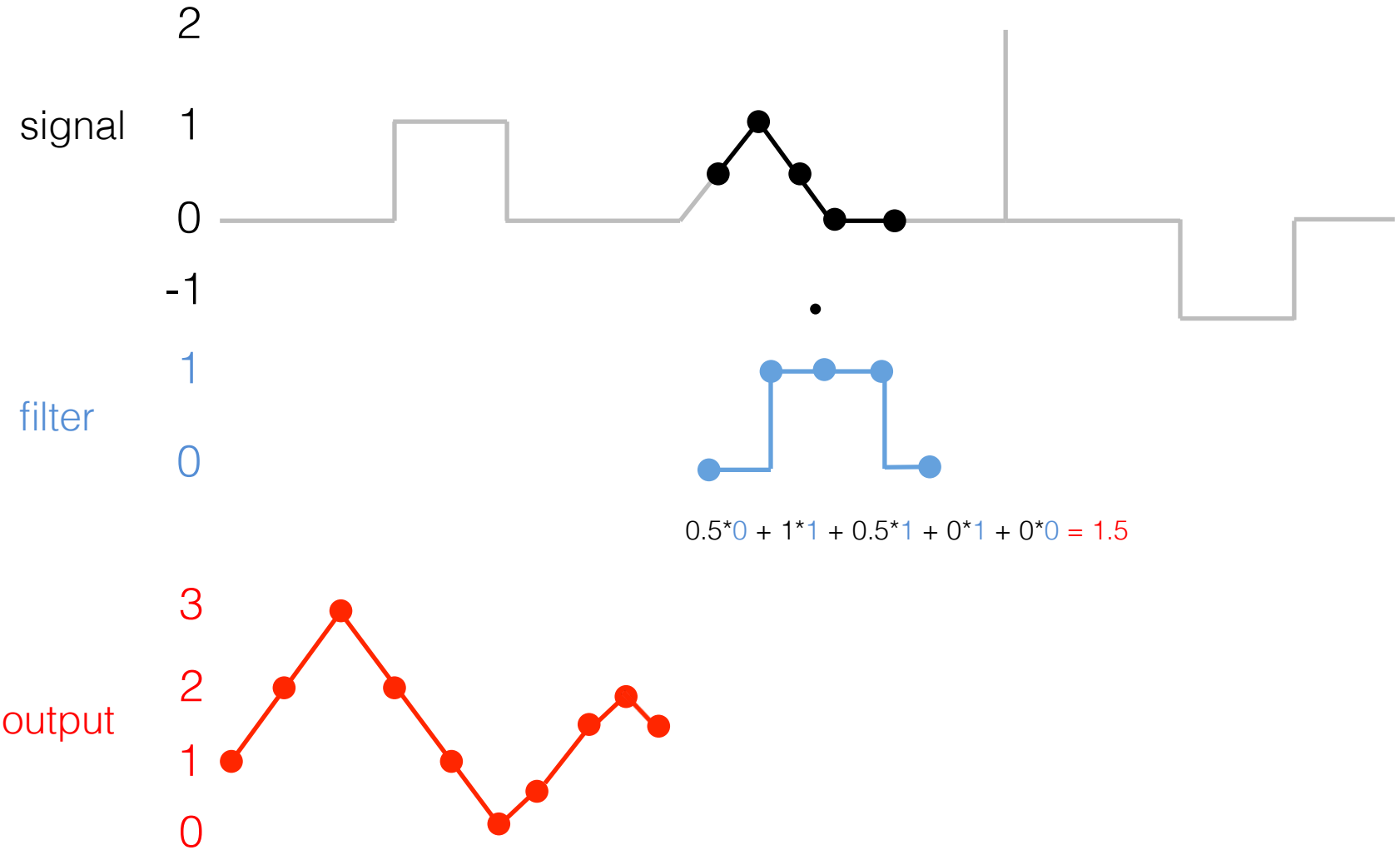

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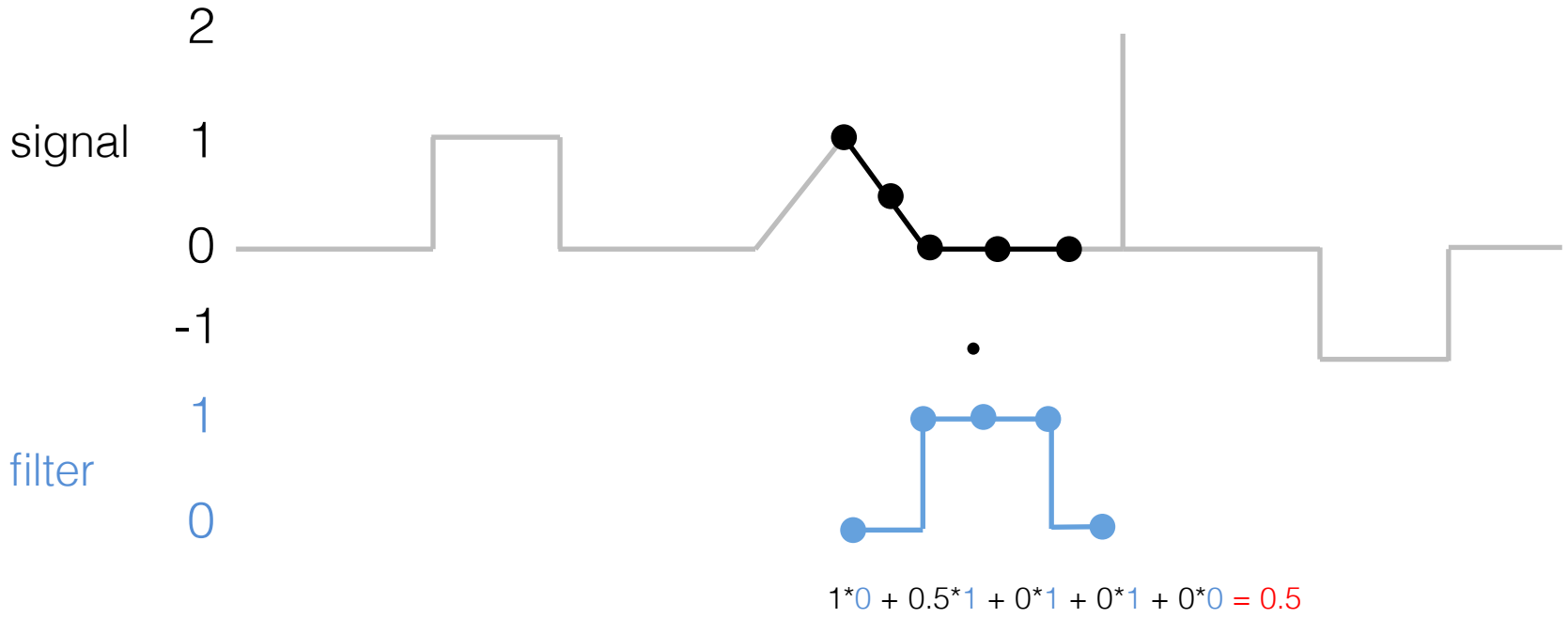

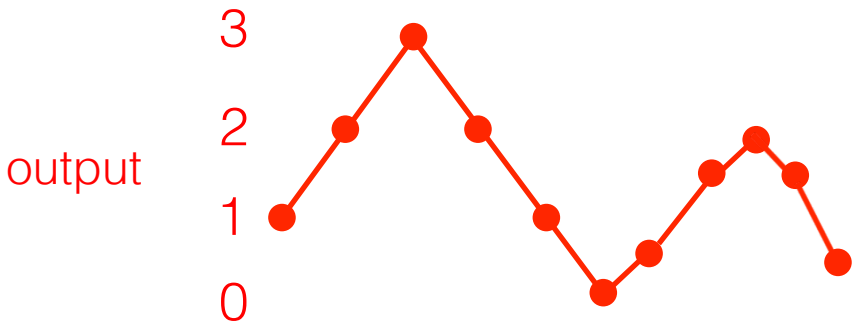

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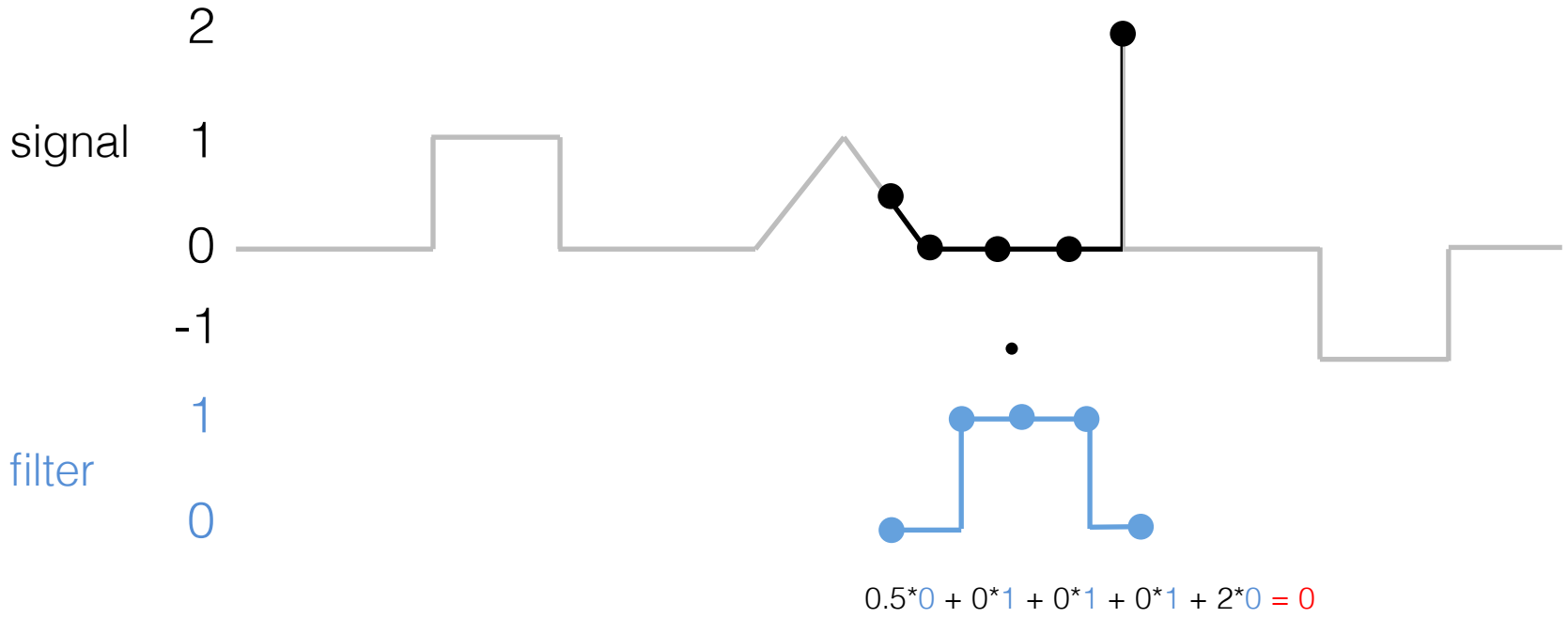
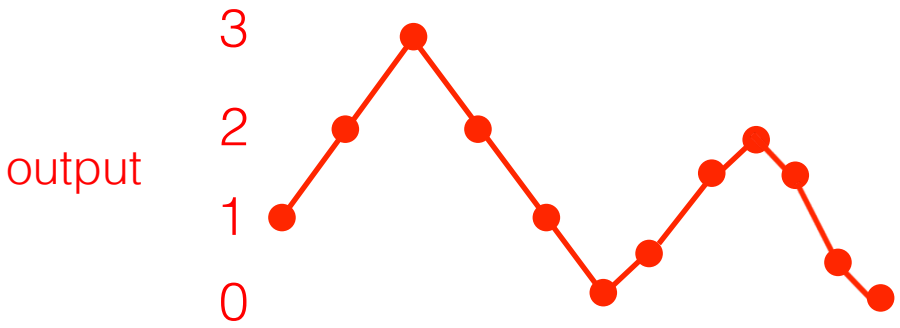
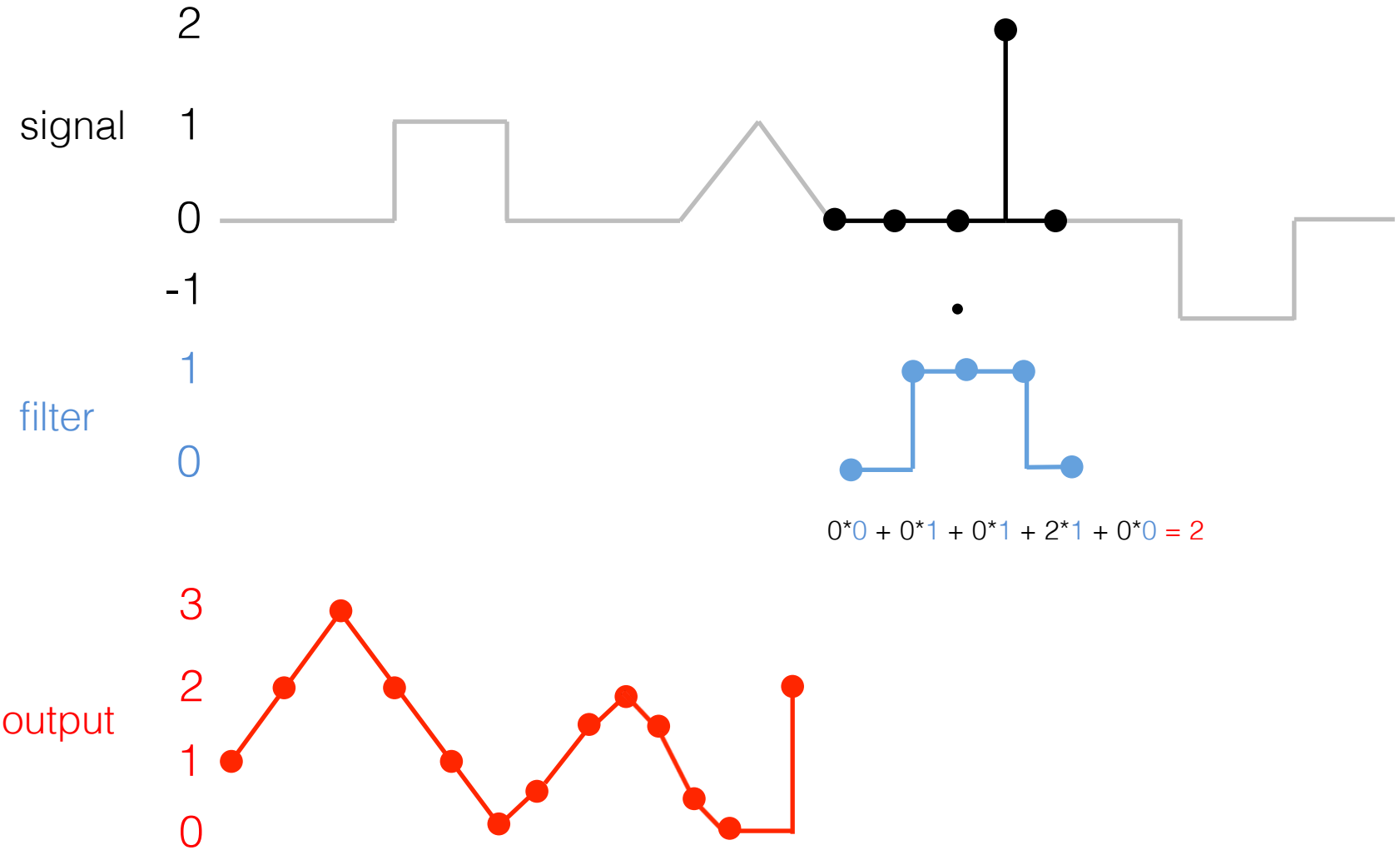




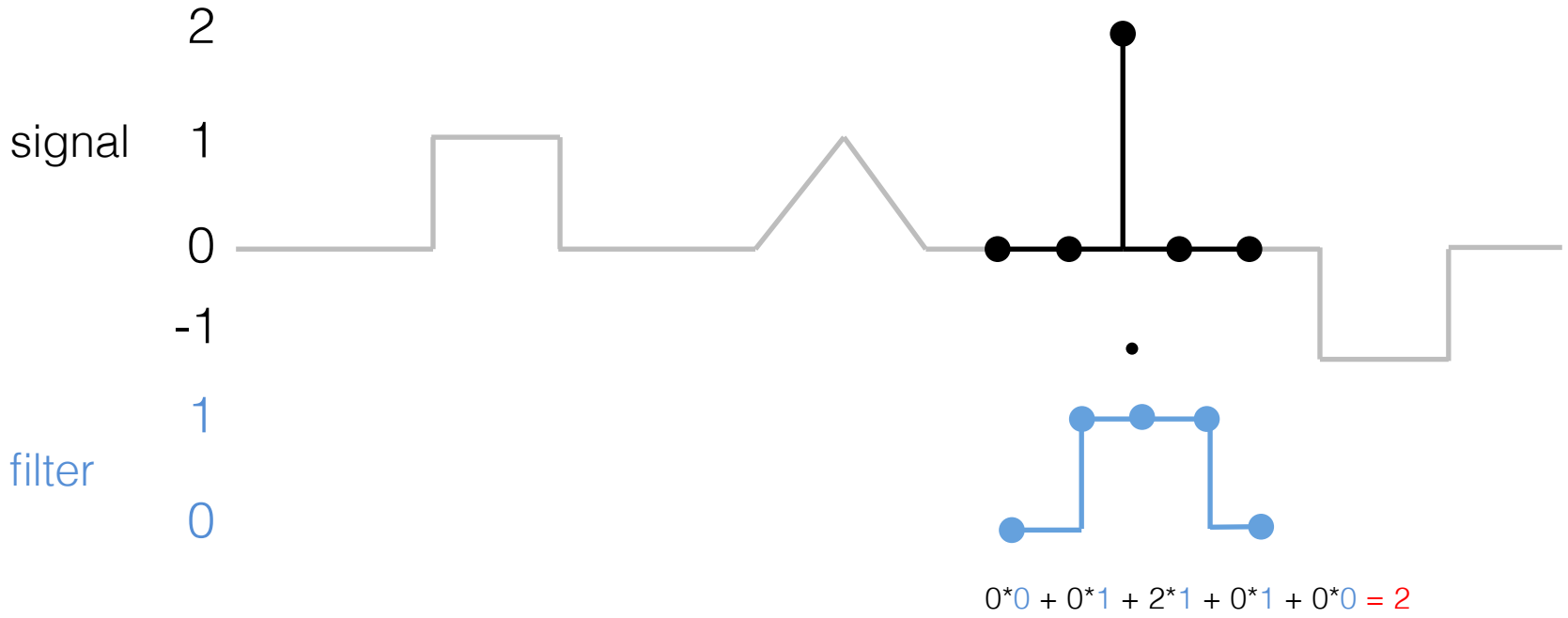
MATLAB: `conv(signal, filter, 'valid')`

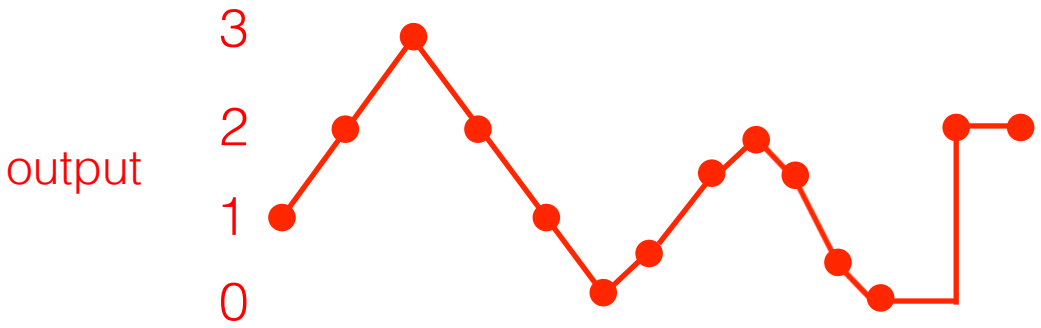
Convolution



MATLAB: `conv(signal, filter, 'valid')`

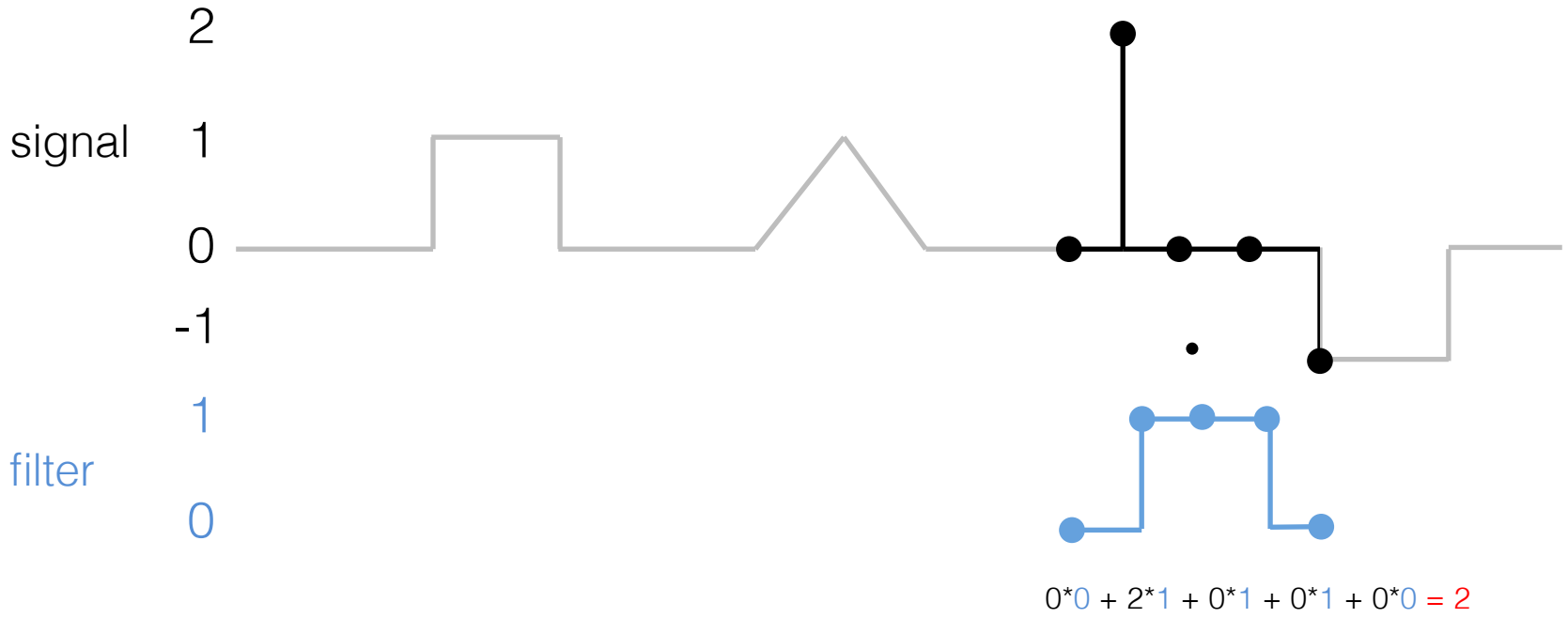
Convolution

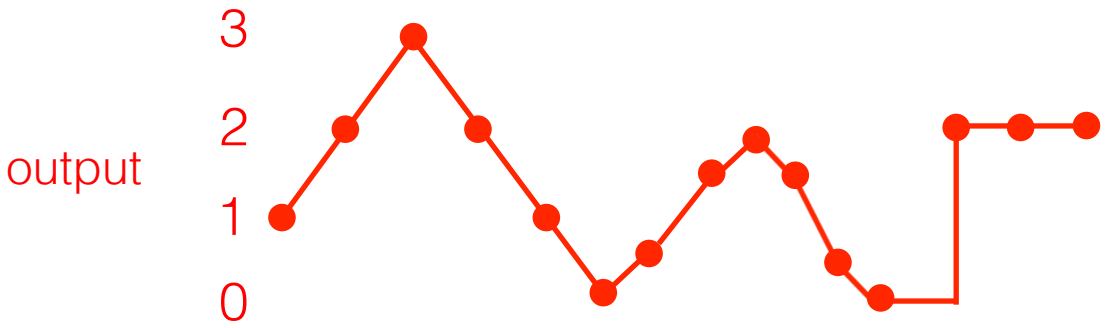




MATLAB: `conv(signal, filter, 'valid')`

Convolution





MATLAB: `conv(signal, filter, 'valid')`

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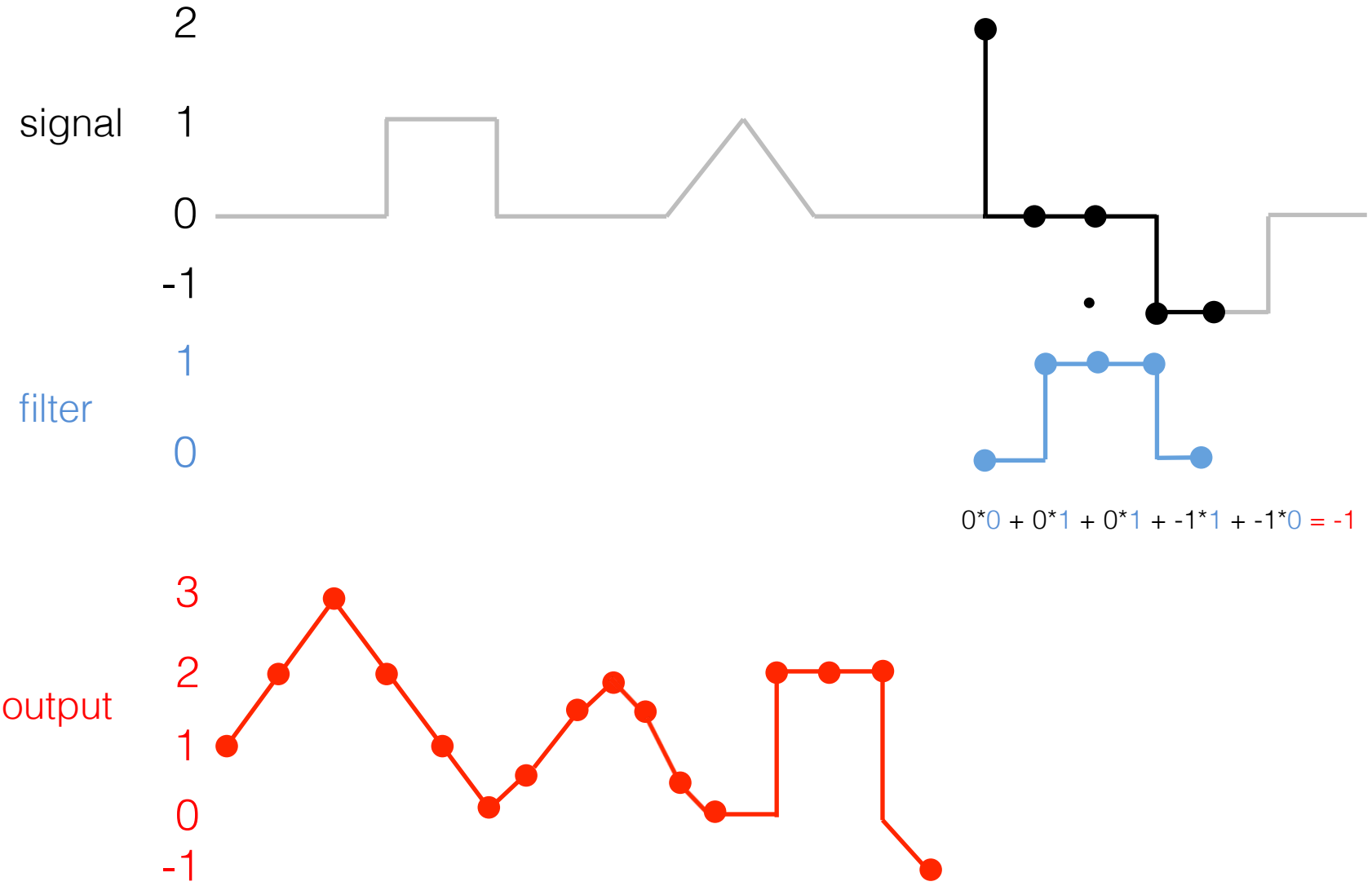
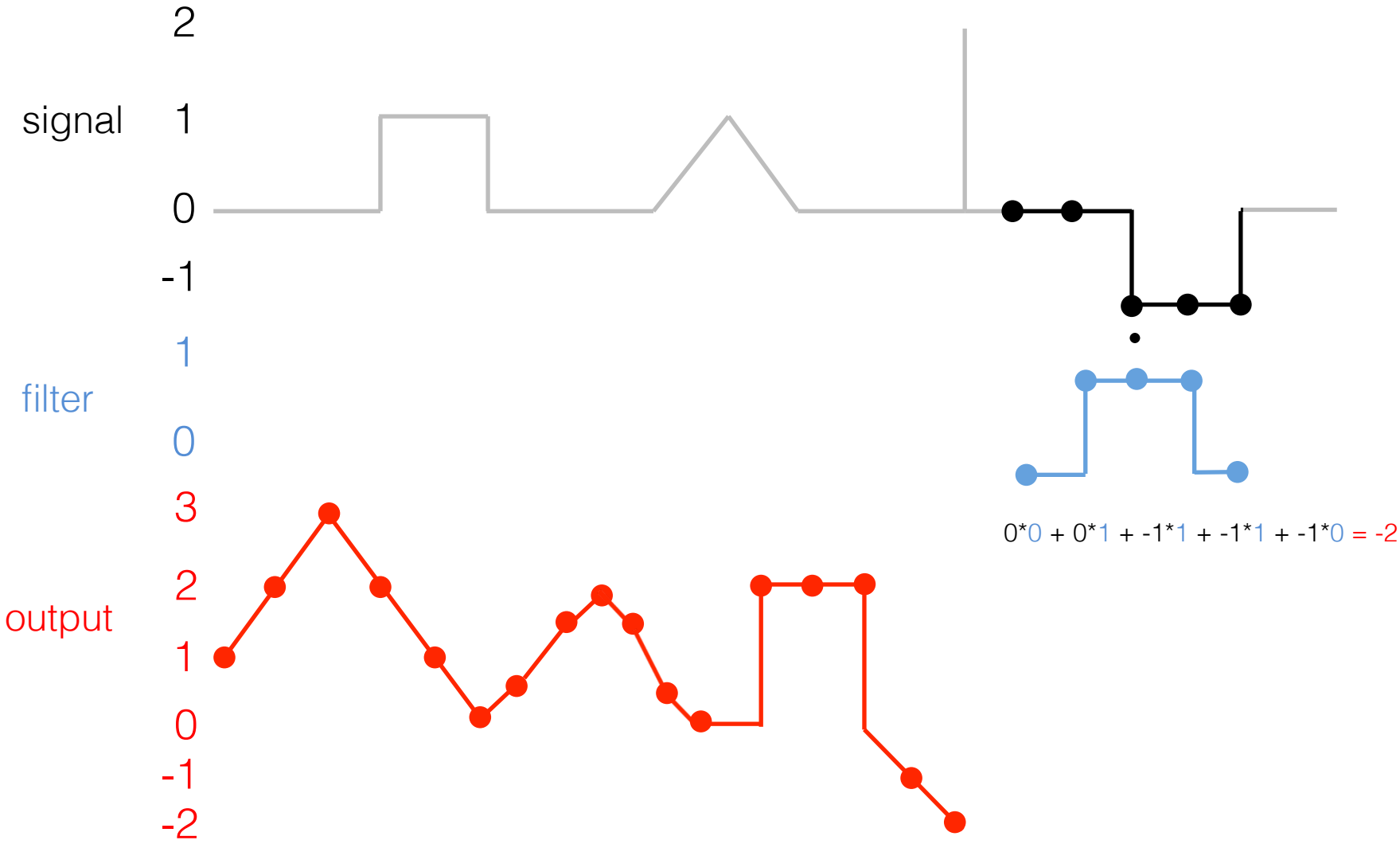
Convolution



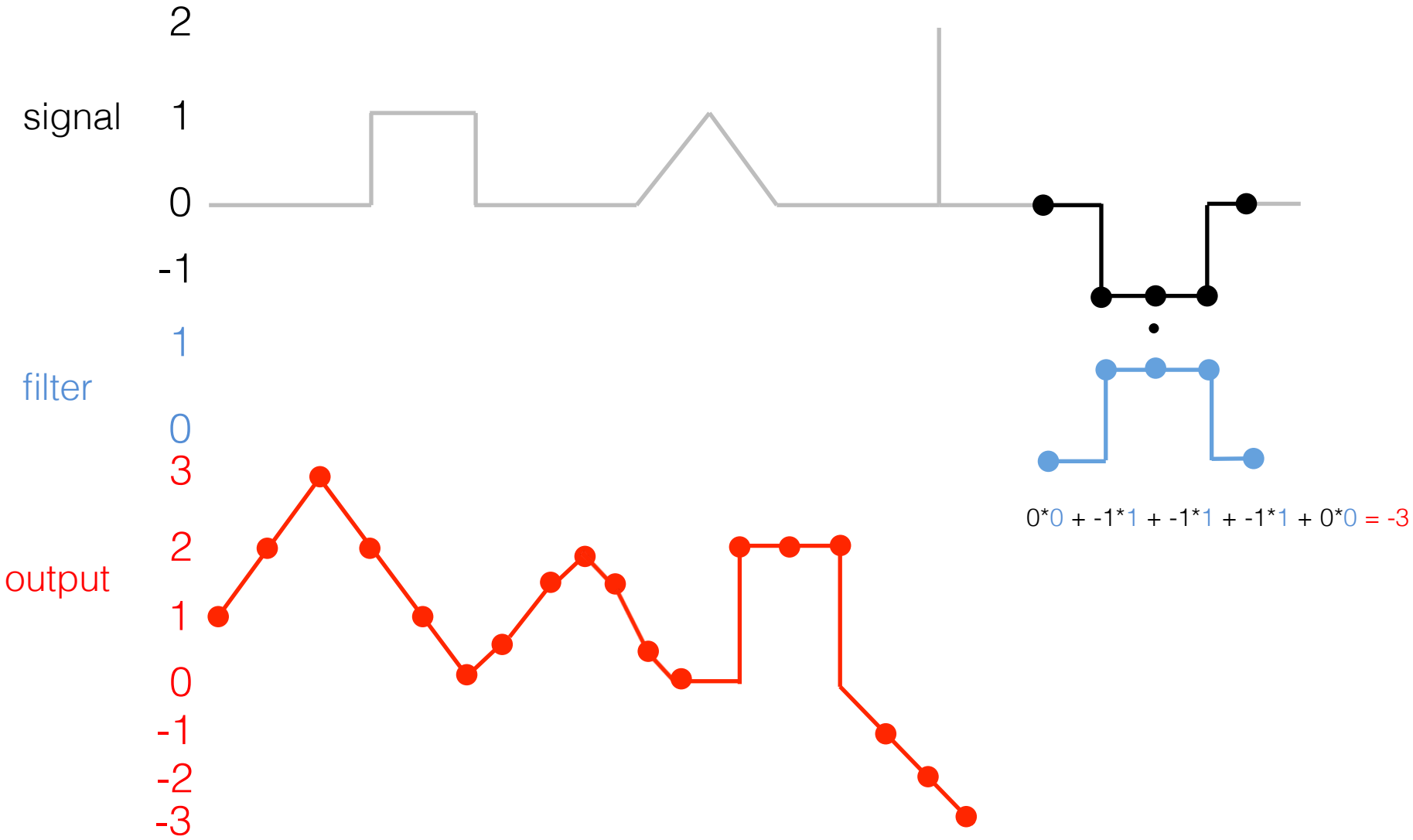
MATLAB: `conv(signal, filter, 'valid')`

Convolution



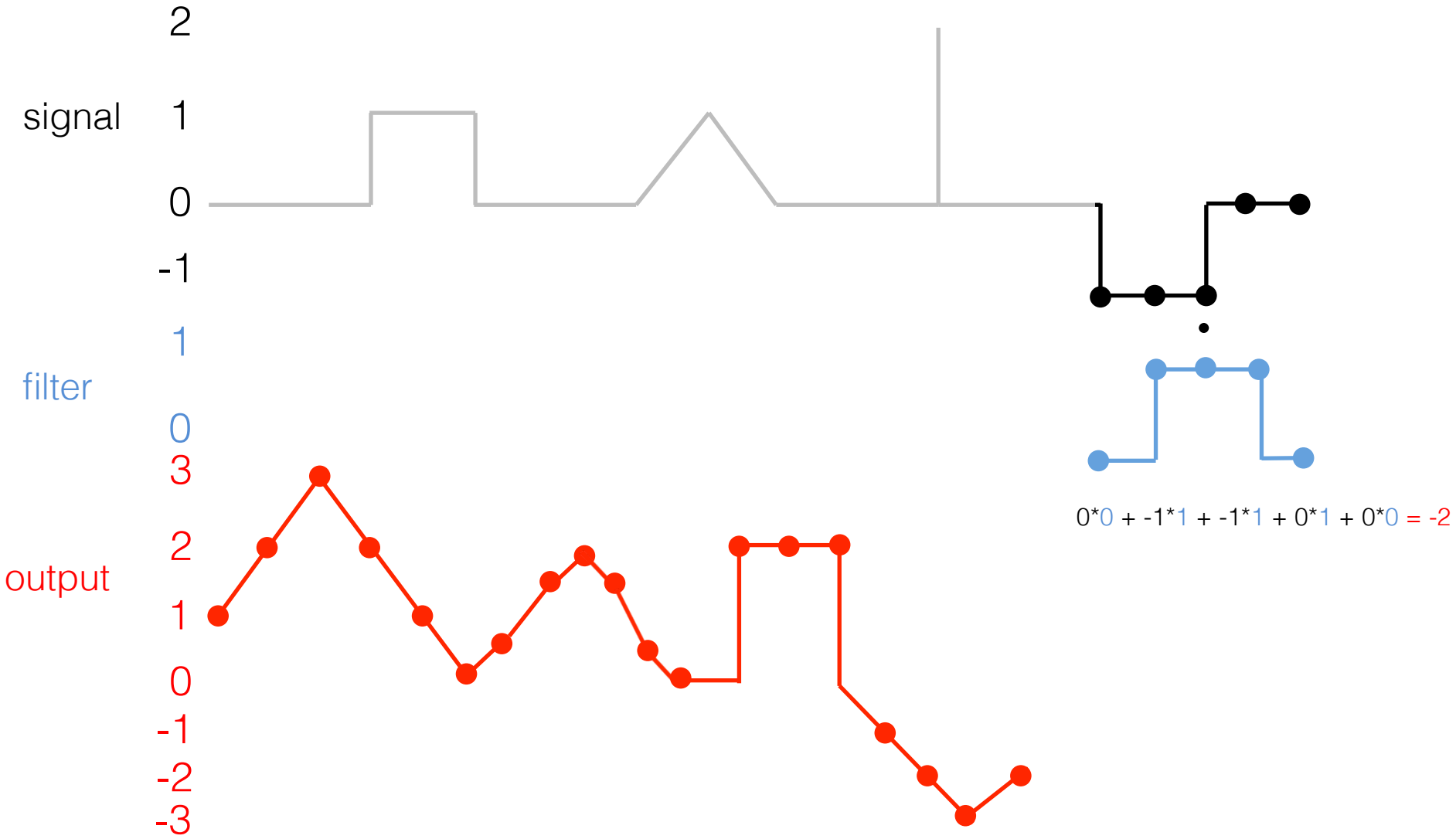
MATLAB: `conv(signal, filter, 'valid')`

Convolution



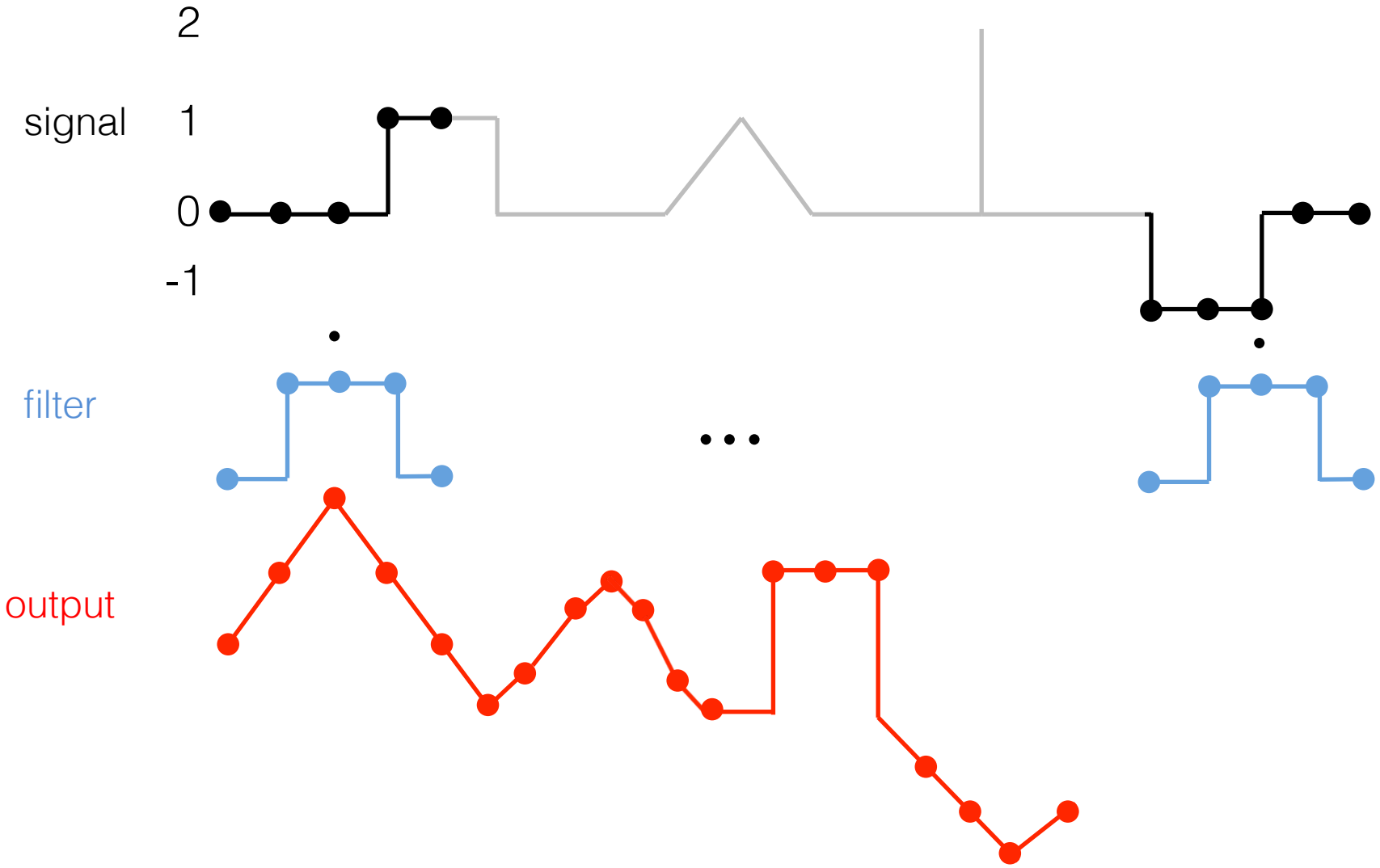
MATLAB: `conv(signal, filter, 'valid')`

Convolution



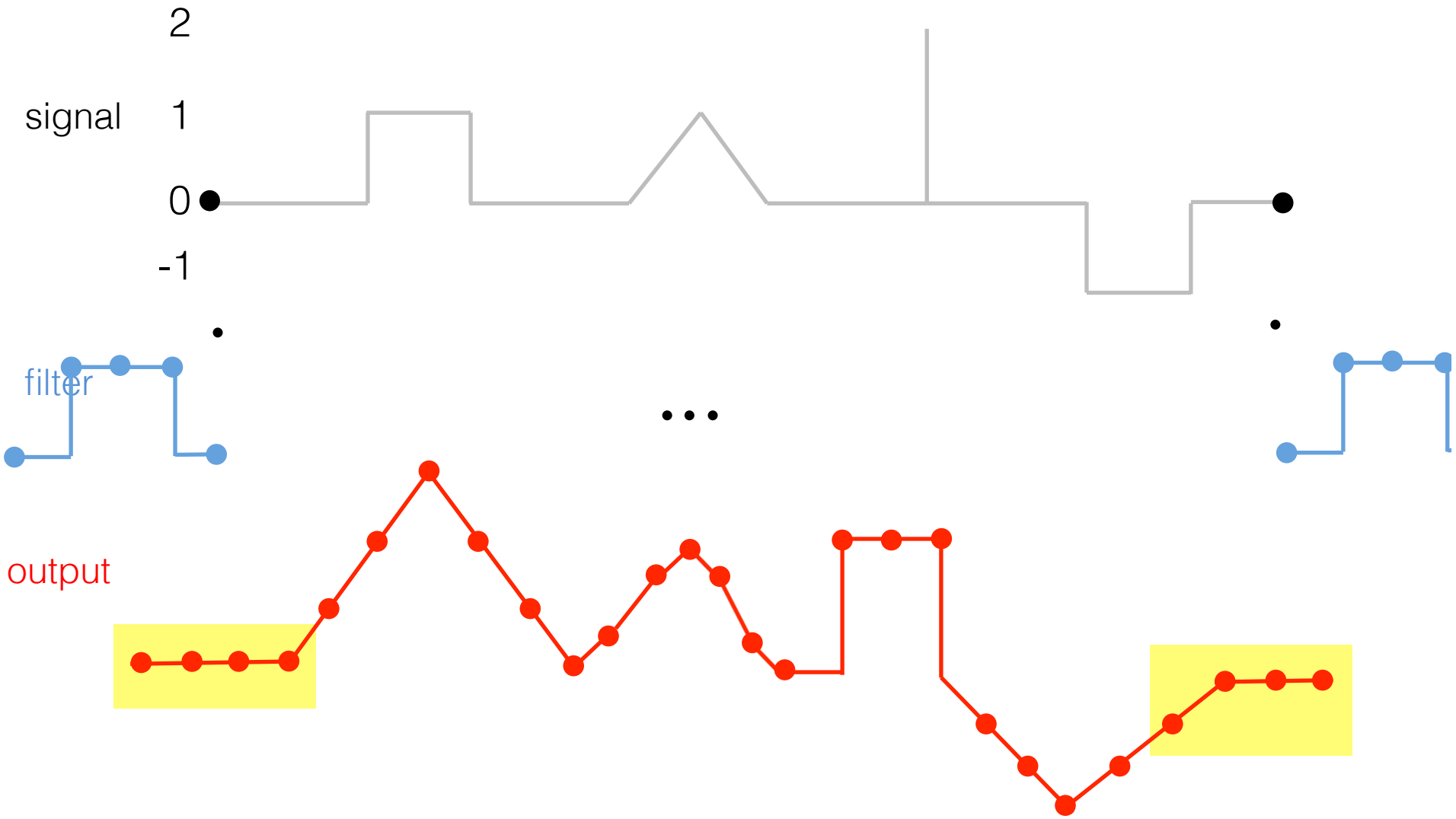
MATLAB: `conv(signal, filter, 'valid')`

Convolution



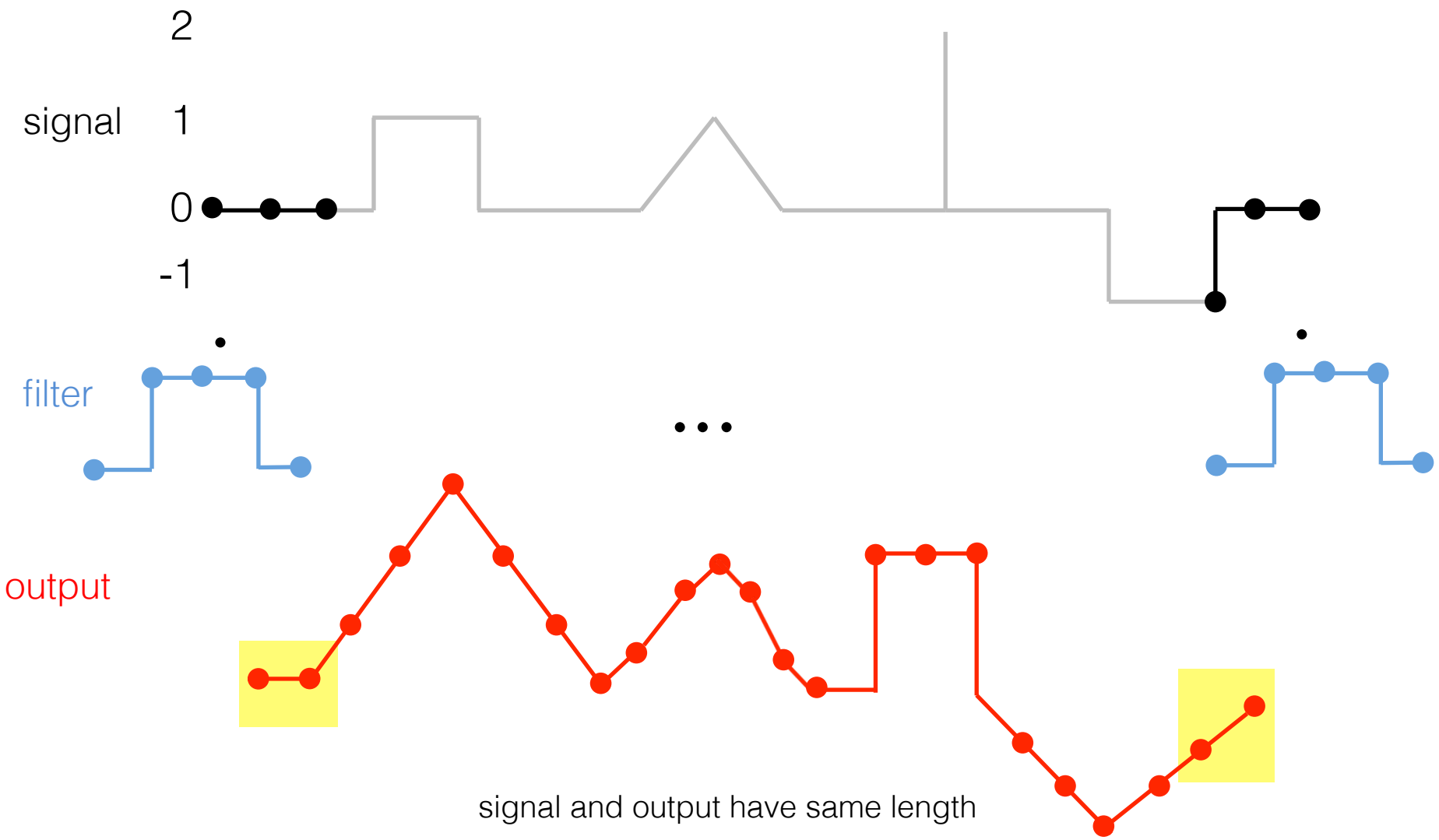
MATLAB: `conv(signal, filter, 'full')`

Convolution



MATLAB: `conv(signal, filter, 'same')`

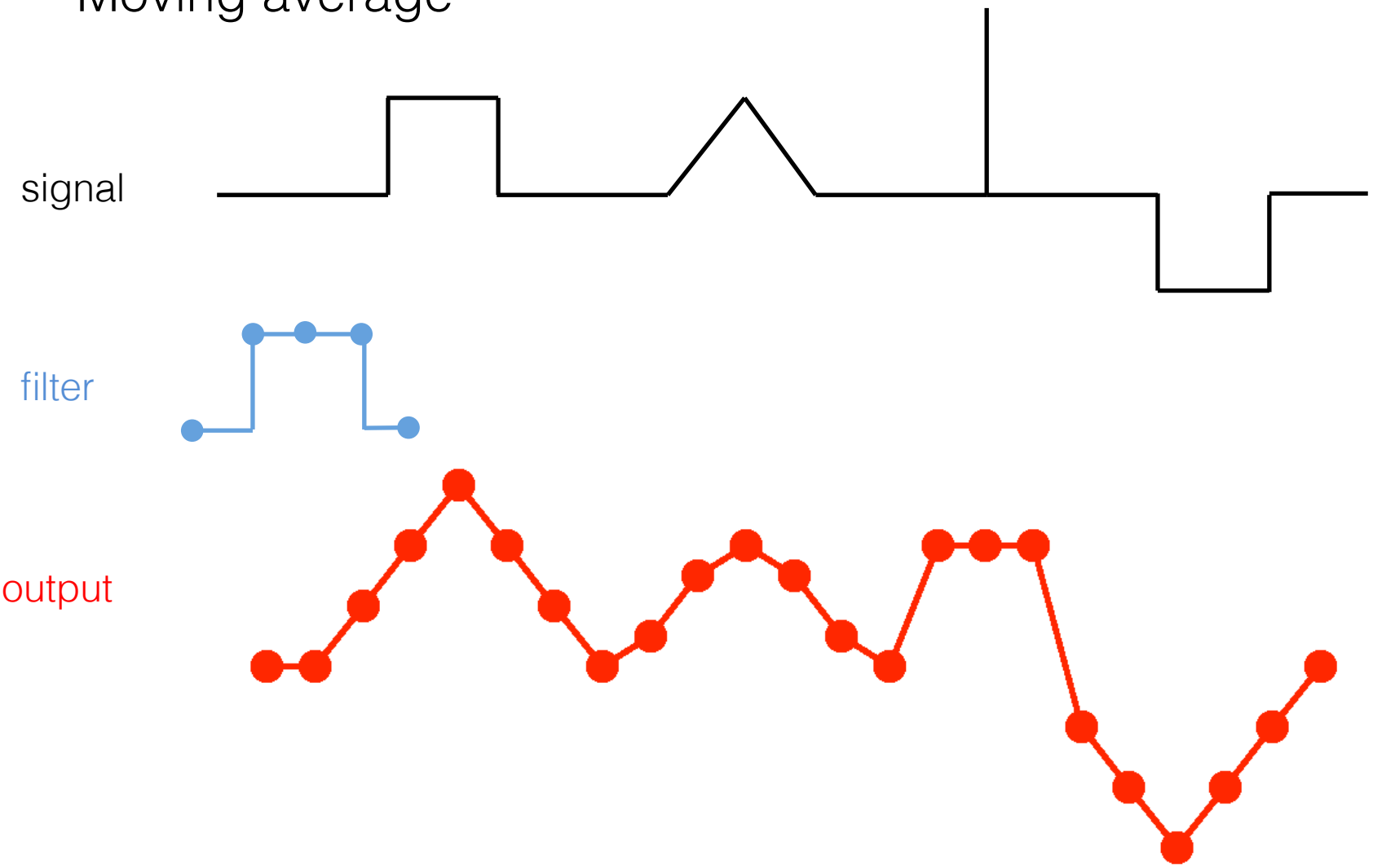
Convolution



MATLAB: `conv(signal, filter, 'same')`

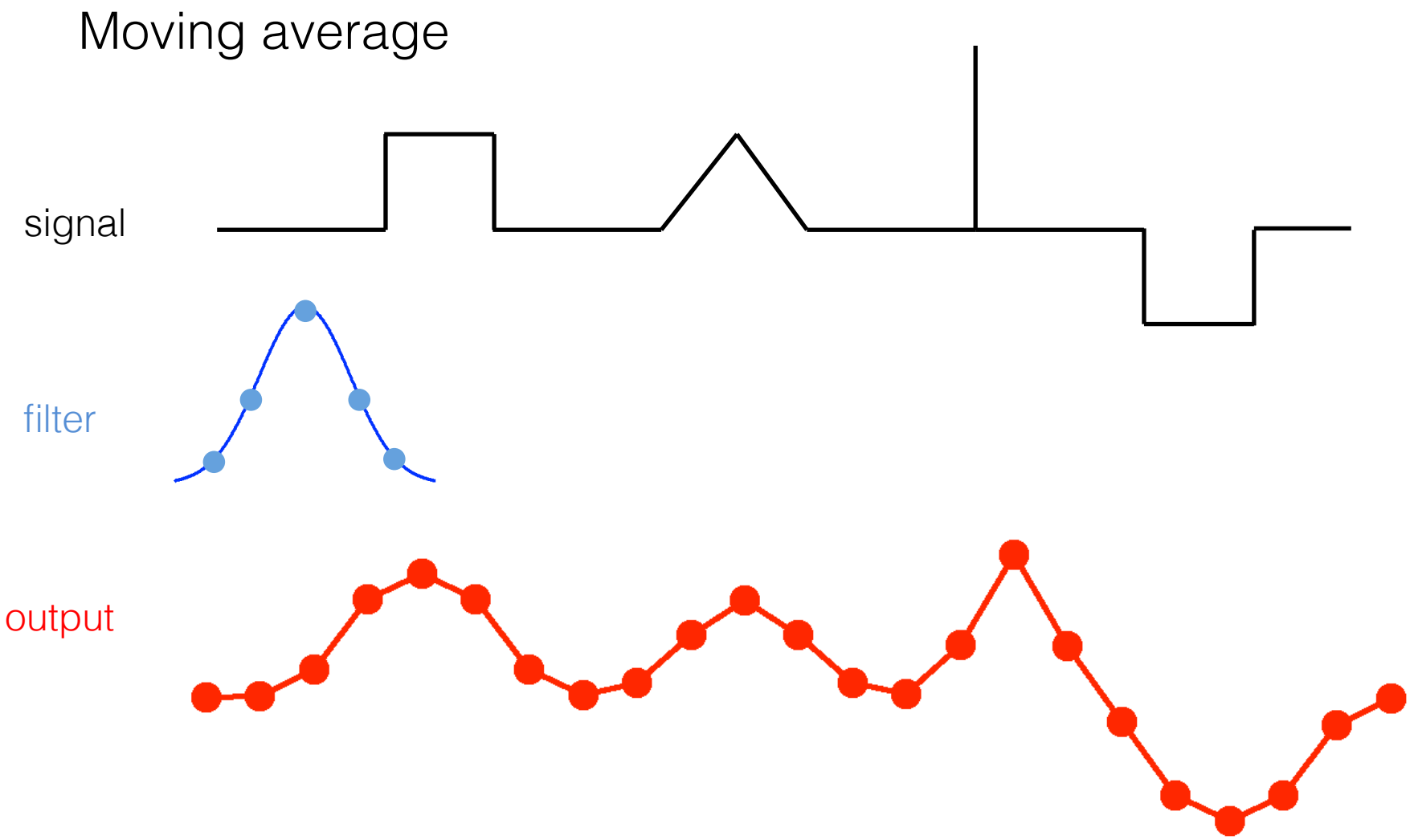
Convolution examples

Moving average



MATLAB: `conv(signal, filter, 'same')`

Convolution examples

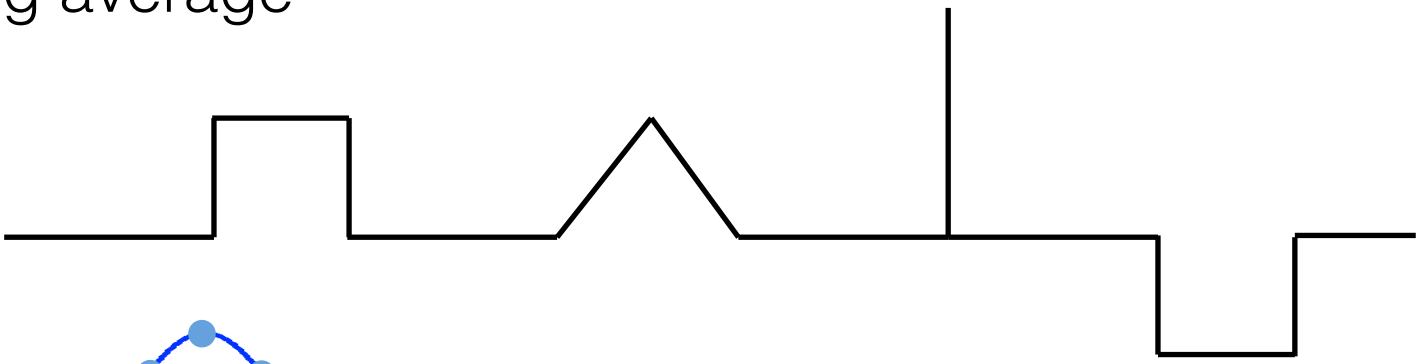


MATLAB: `conv(signal, filter, 'same')`

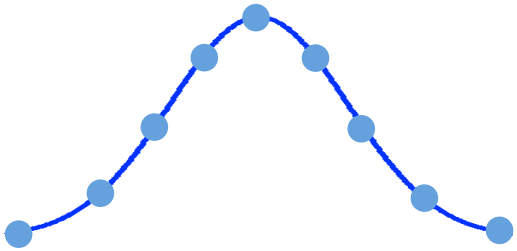
Convolution examples

Moving average

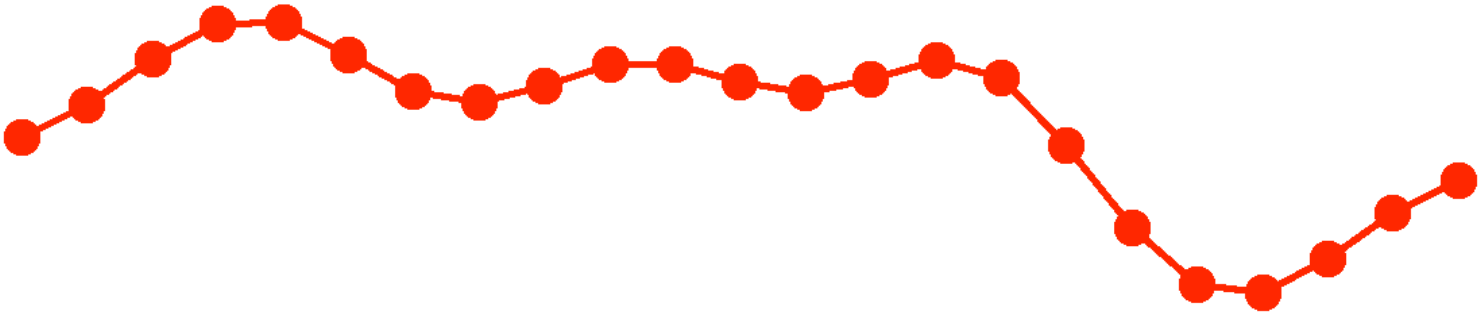
signal



filter

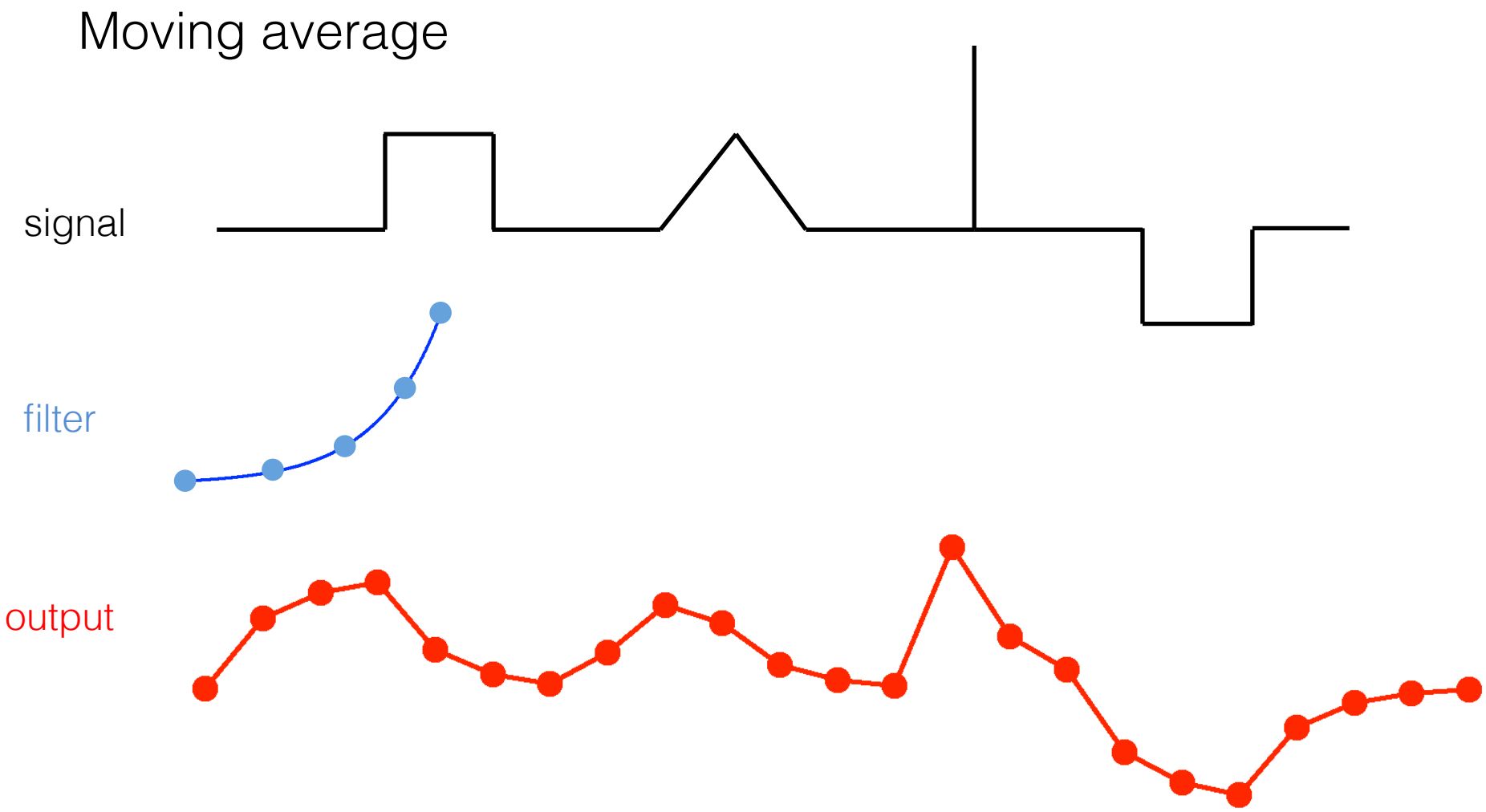


output



MATLAB: `conv(signal, fliplr(filter), 'same')`

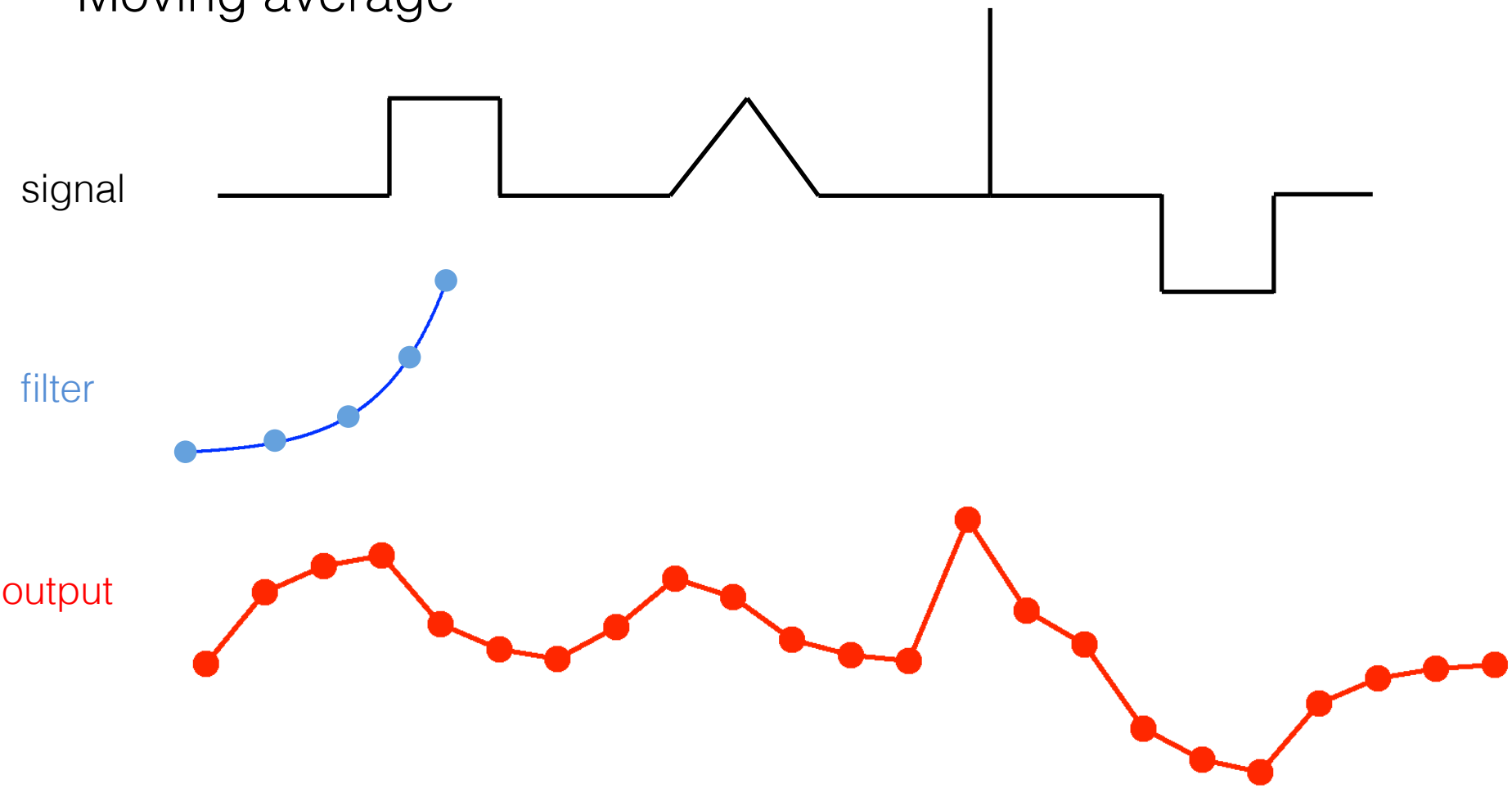
Convolution examples



MATLAB: `conv(signal, fliplr(filter), 'same')`

Convolution examples

Moving average

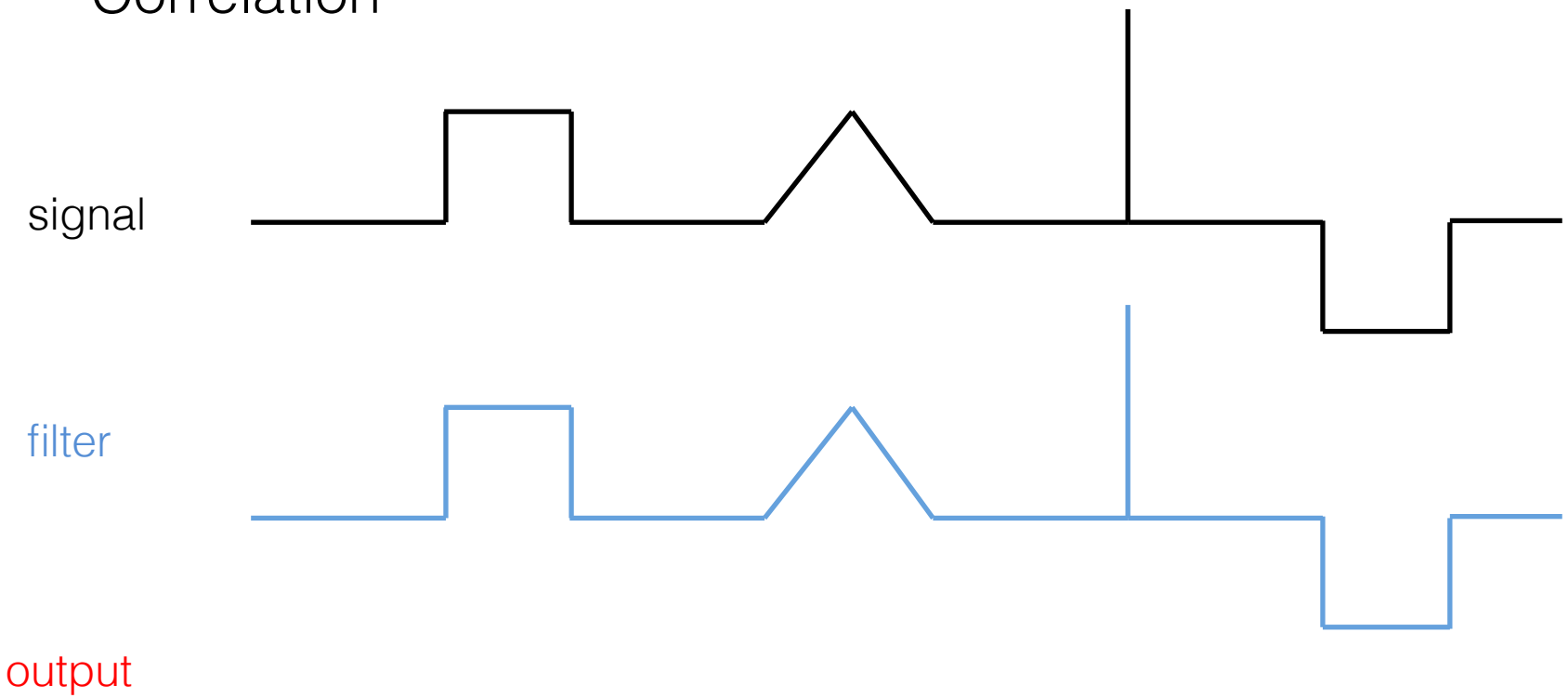


a note about using asymmetric filters in MATLAB

MATLAB: `conv(signal, fliplr(signal), 'same')`

Convolution examples

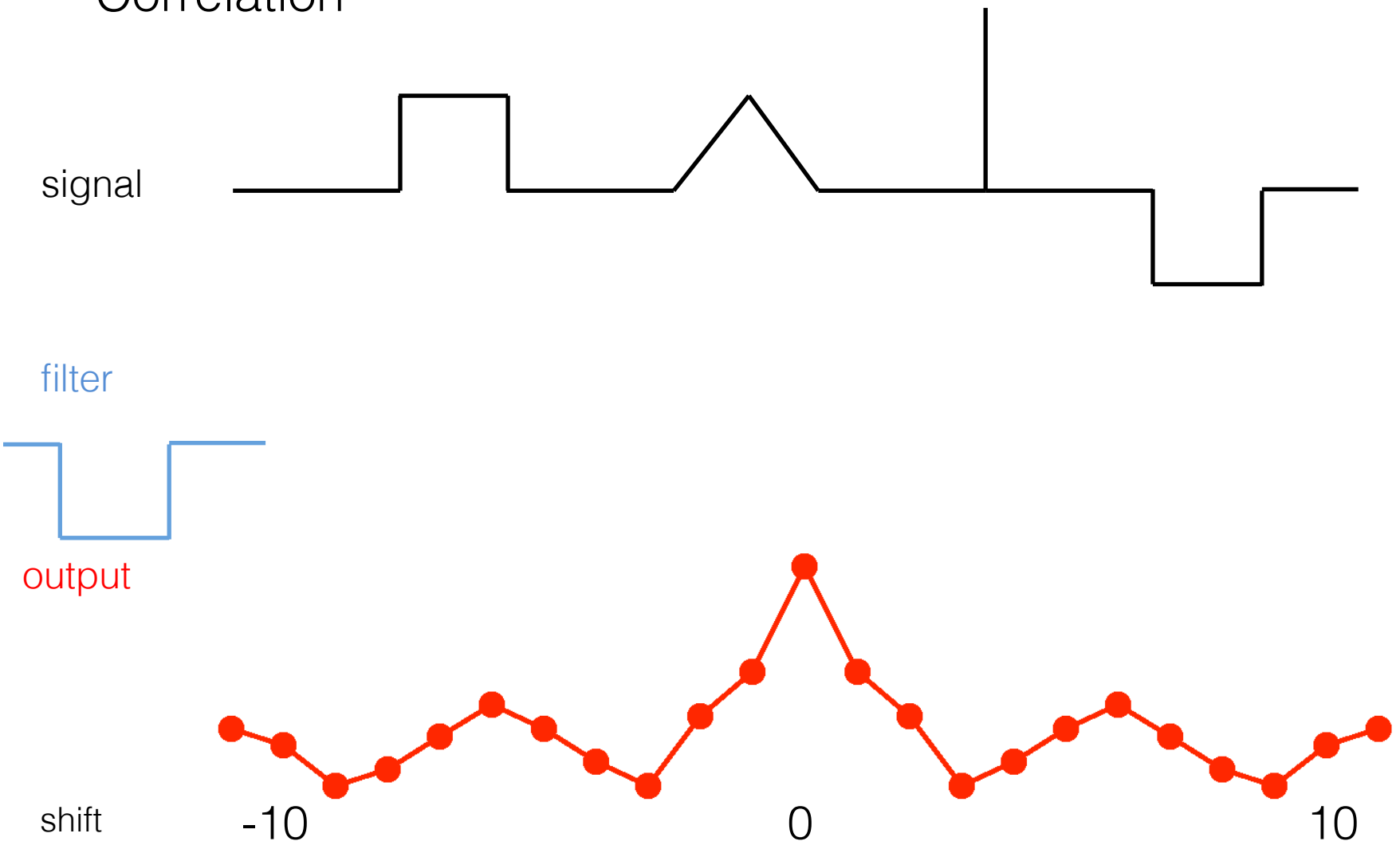
Correlation



MATLAB: `conv(signal, fliplr(signal), 'same')`

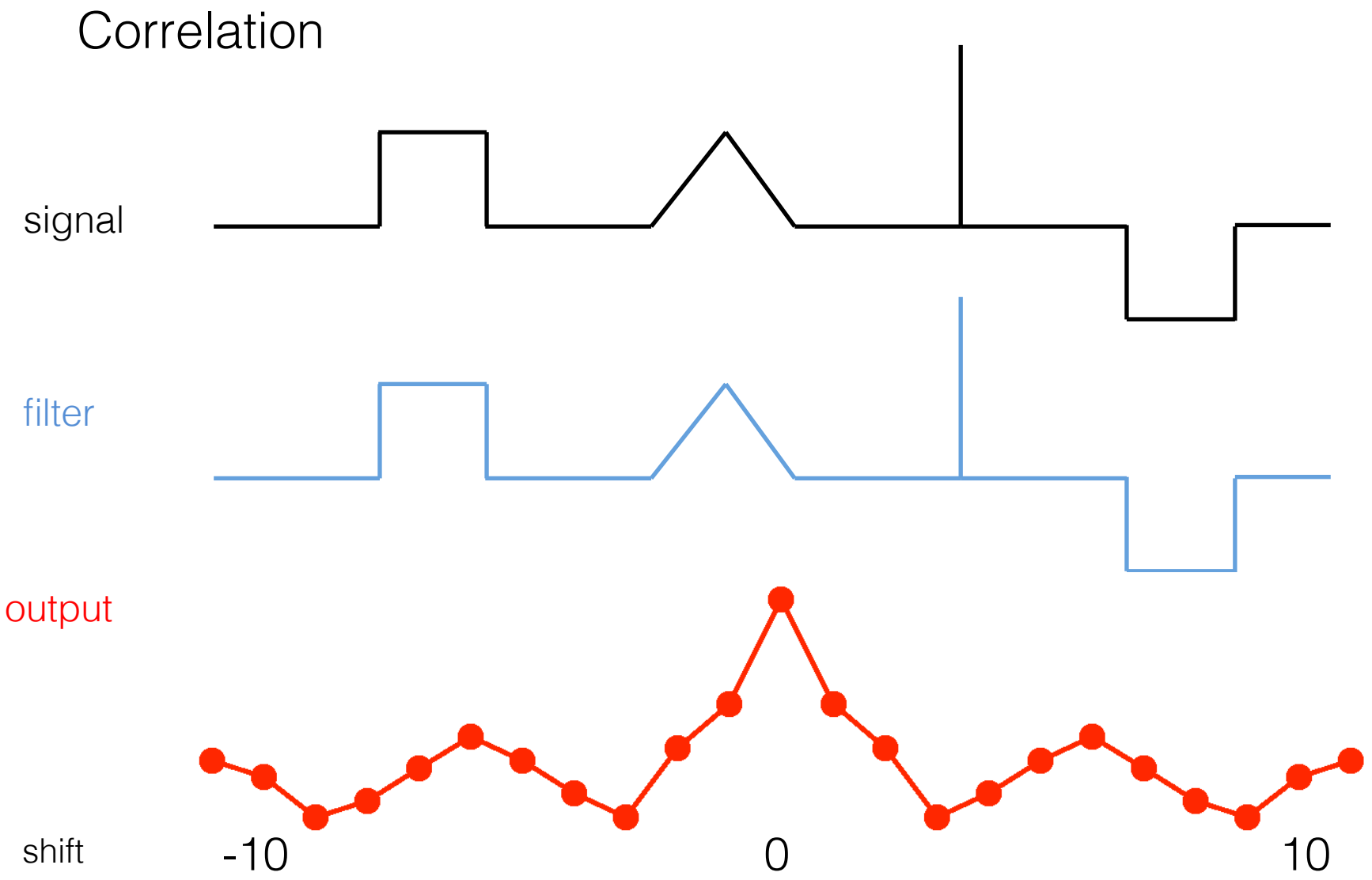
Convolution examples

Correlation



MATLAB: `conv(signal, fliplr(signal), 'same')`

Convolution examples



MATLAB: `conv2(signal, filter, 'same')`

2d Convolution examples

Image processing

signal

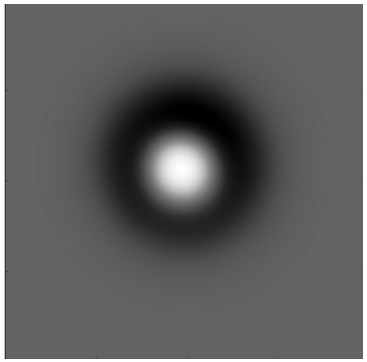
MATLAB: `conv2(signal, filter, 'same')`

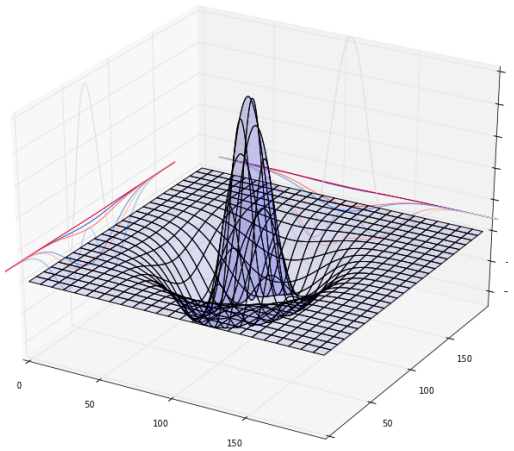
2d Convolution examples

Image processing

signal

filter





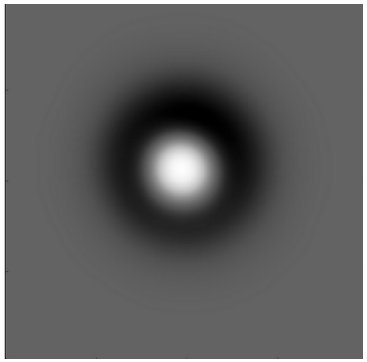
MATLAB: `conv2(signal, filter, 'same')`

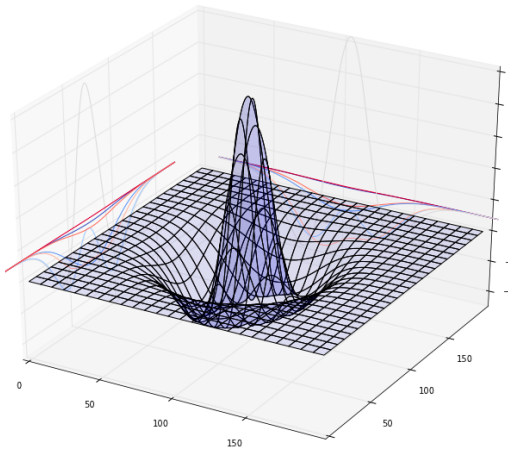
2d Convolution examples

Image processing

signal

filter





output

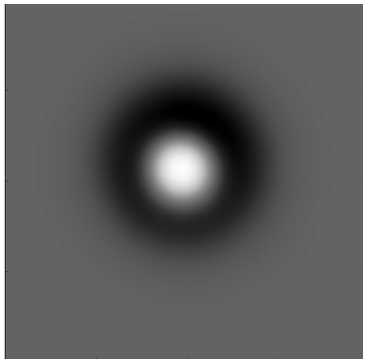
MATLAB: `conv2(signal, filter, 'same')`

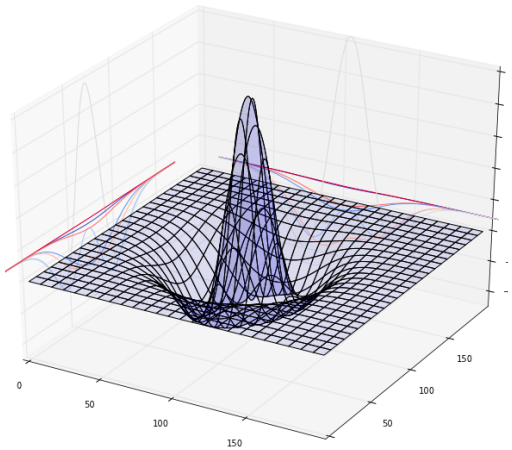
2d Convolution examples

Image processing

signal

filter





output

Today's lecture

When will this be useful?

Filtering in time or space

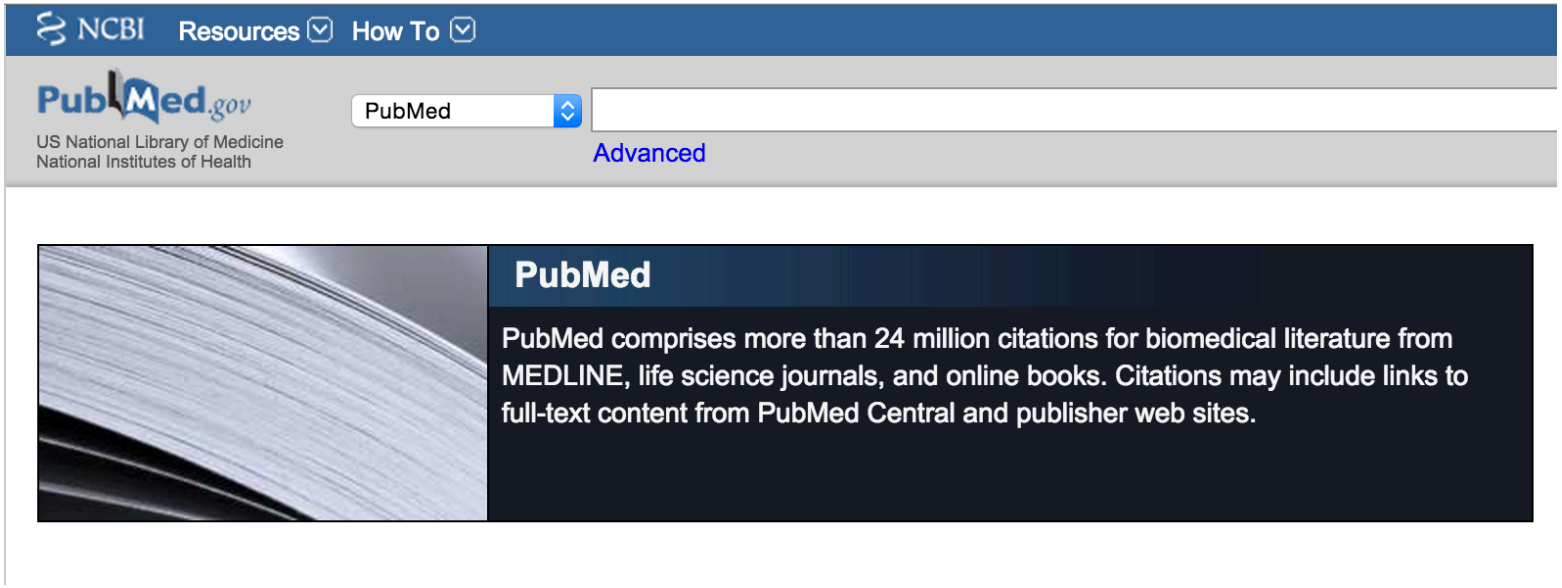
- What is convolution?
- How much should I filter?

Filtering in frequency

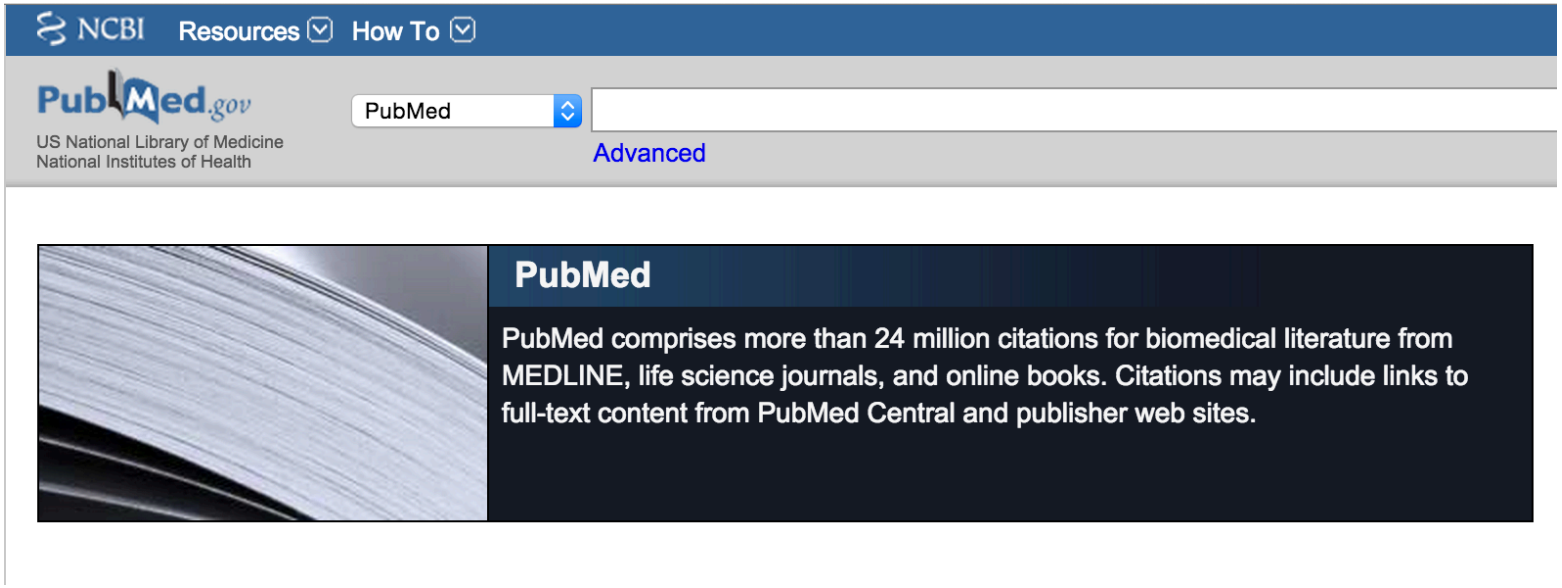
- Fourier series
- What is a Fourier transform?
- Designing filters in frequency domain

Fourier transform

Fourier transform

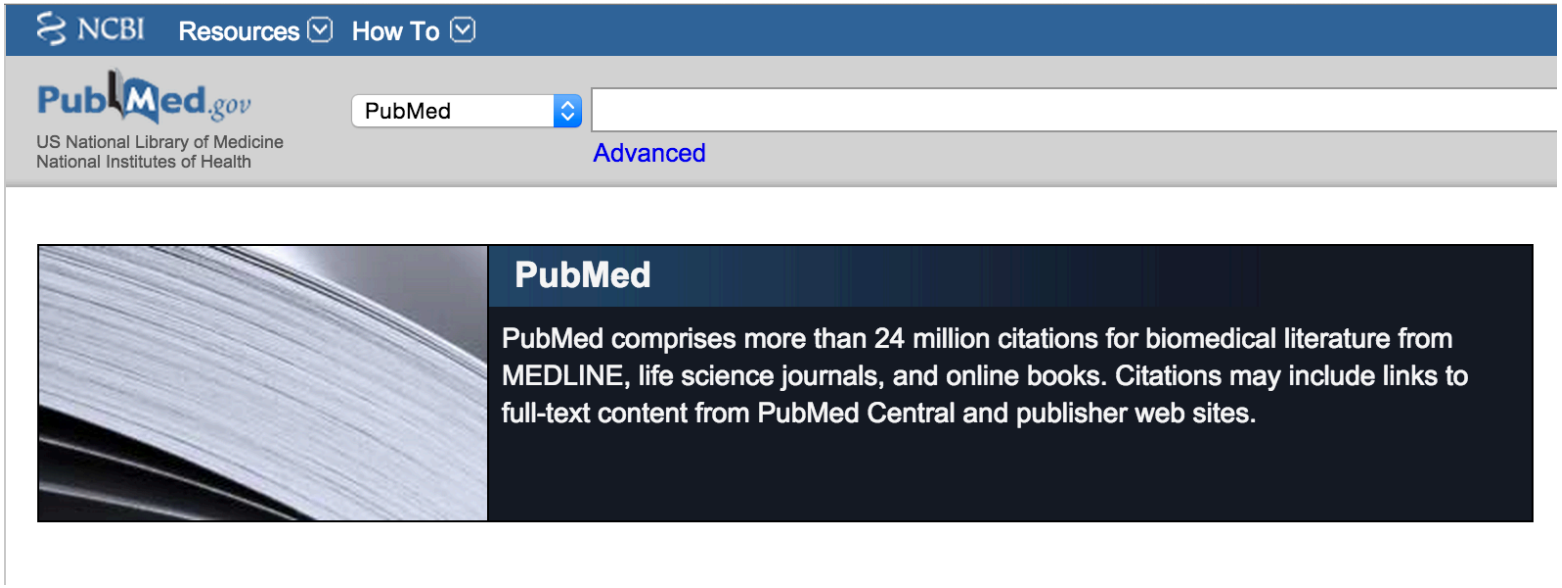


Fourier transform



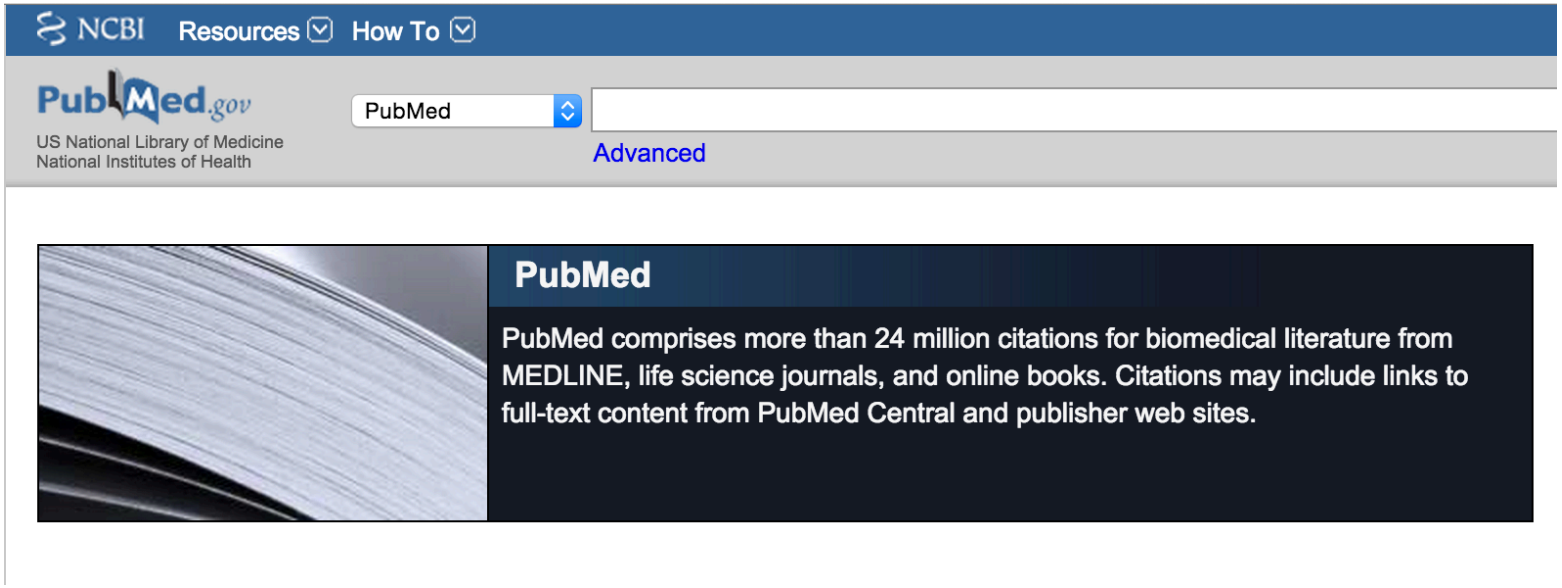
“Fourier” 74,123

Fourier transform



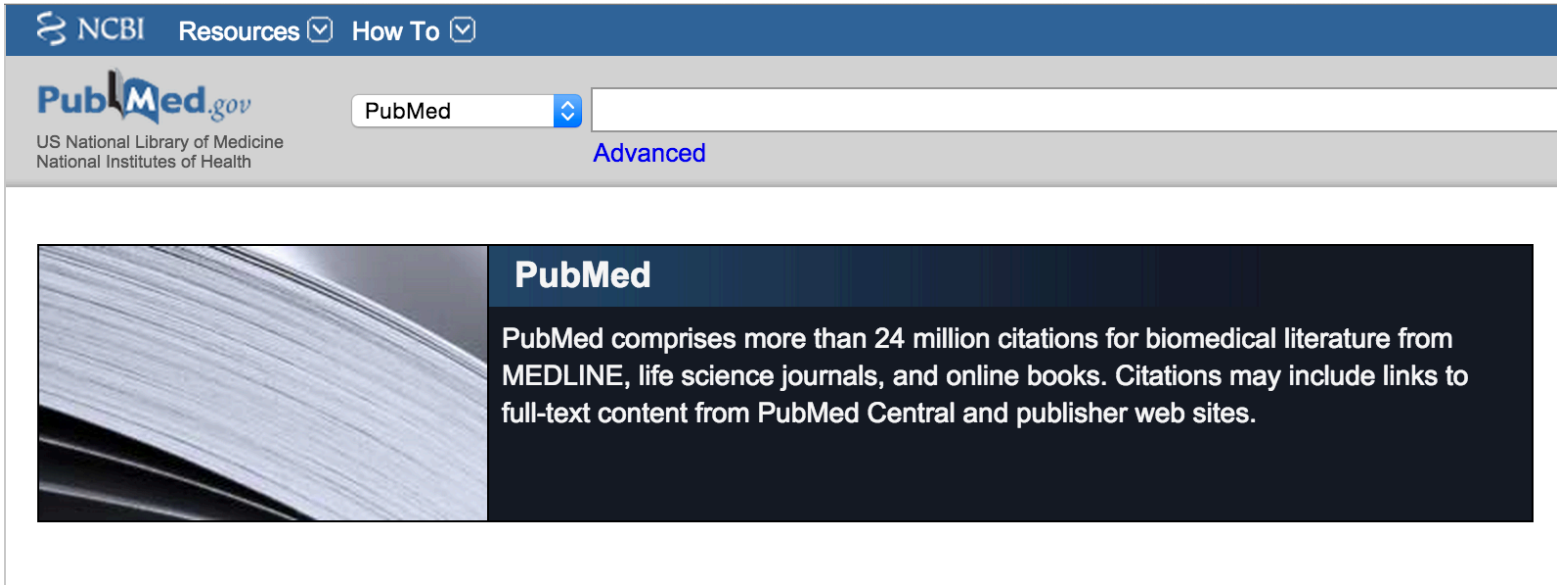
“Fourier”	74,123
“Filter”	55,596

Fourier transform



“Fourier”	74,123
“Filter”	55,596
“Spectral analysis”	54,052

Fourier transform



“Fourier”	74,123	“Drosophila”	89,207
“Filter”	55,596	“Elegans *or* C. elegans *or* Caenorhabditis elegans”	26,069
“Spectral analysis”	54,052		

Other reasons Fourier transform is useful

Ease of computation

Filtering (convolution) is just element-wise multiplication in the frequency domain

Designing filters to remove noise is a lot easier in the frequency domain

Describing things

The cochlea transforms time domain signal into a frequency signal

Many brain regions have oscillations of a particular frequency

Fourier series

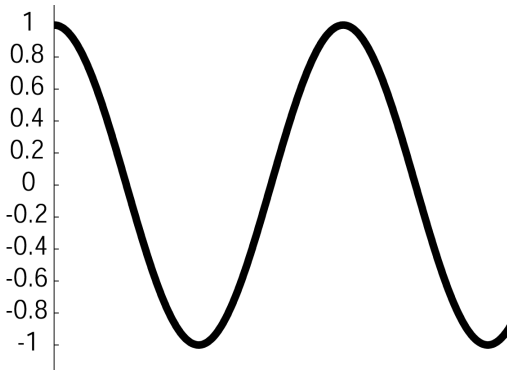
Every signal is equal to a sum of sines and cosines

Fourier series

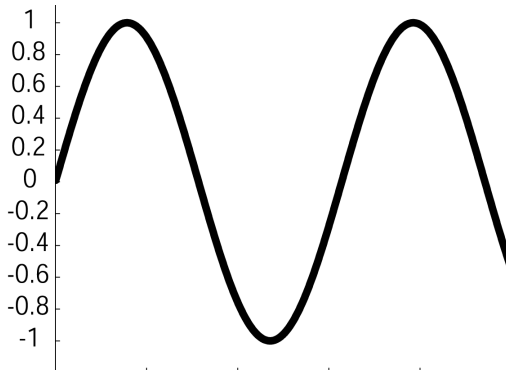
Every signal is equal to a sum of sines and cosines

$$a\cos(2\pi f x) + b\sin(2\pi f x)$$

cosine



sine

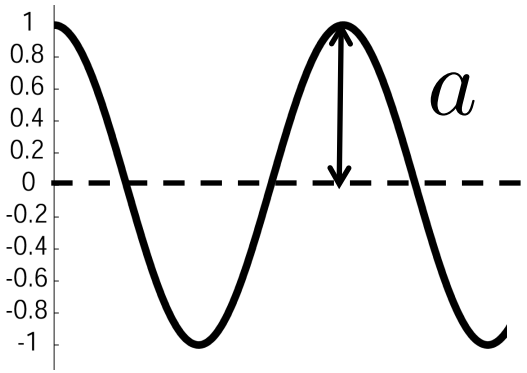


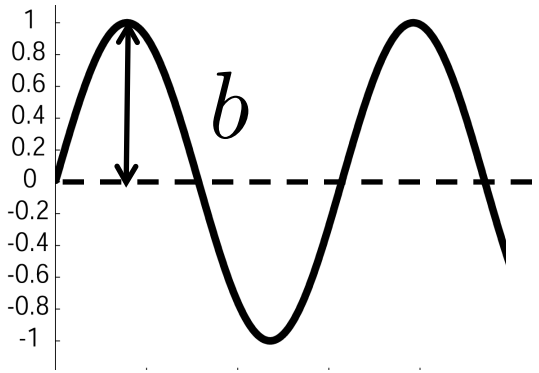
Fourier series

Every signal is equal to a sum of sines and cosines

$$a\cos(2\pi f x) + b\sin(2\pi f x)$$

coefficients (amplitudes)





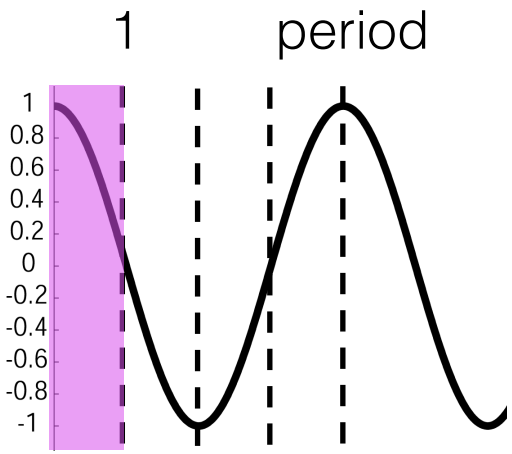
Fourier series

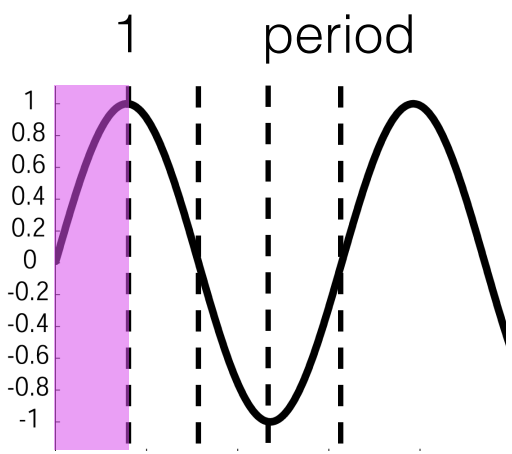
Every signal is equal to a sum of sines and cosines

$$a\cos(2\pi f x) + b\sin(2\pi f x)$$

coefficients (amplitudes)

frequencies





Fourier series

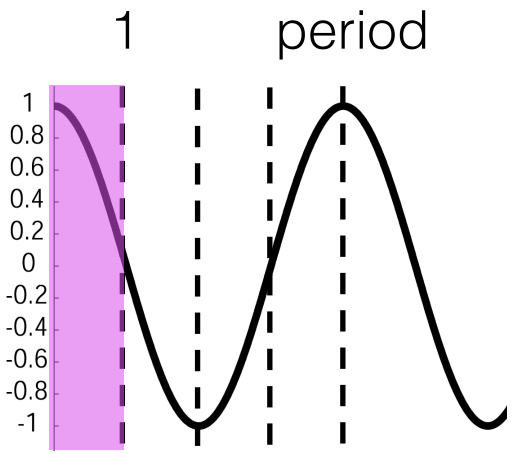
Every signal is equal to a sum of sines and cosines

$$a\cos(2\pi f x) + b\sin(2\pi f x)$$

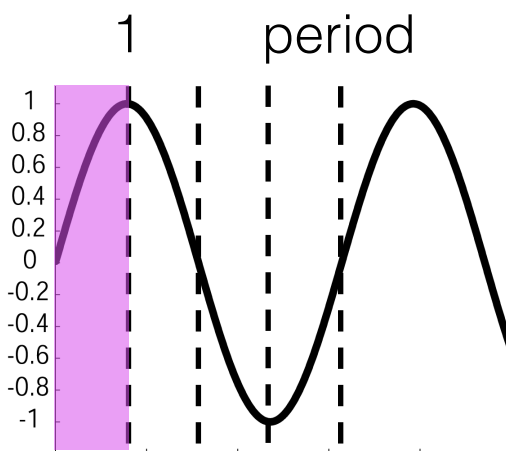
coefficients (amplitudes)

frequencies

$f = 1/4$

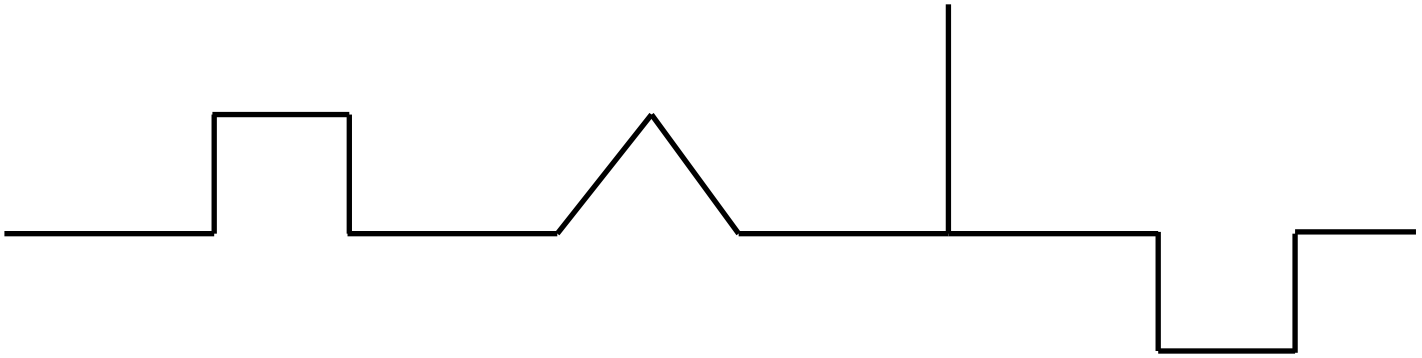


$f = 1/4$

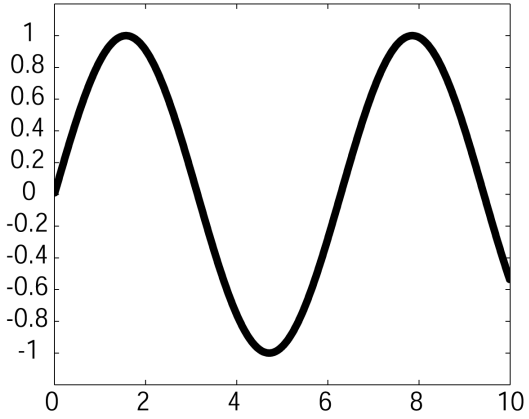


Fourier series

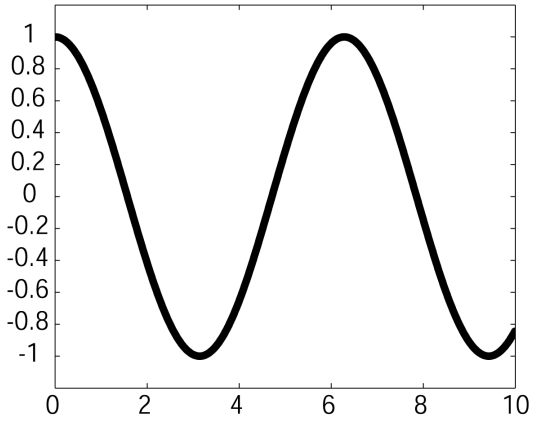
Every signal is equal to a sum of sines and cosines



sine (amplitude 1, period 2pi)



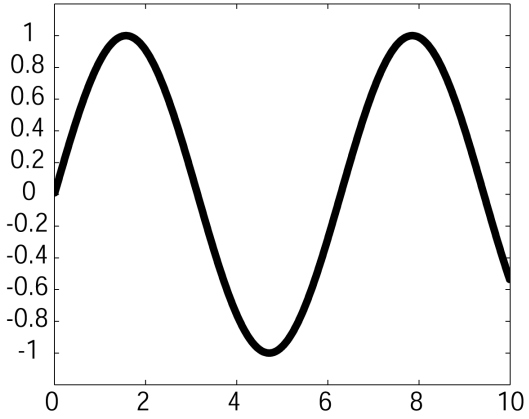
cosine (amplitude 1, period 2pi)



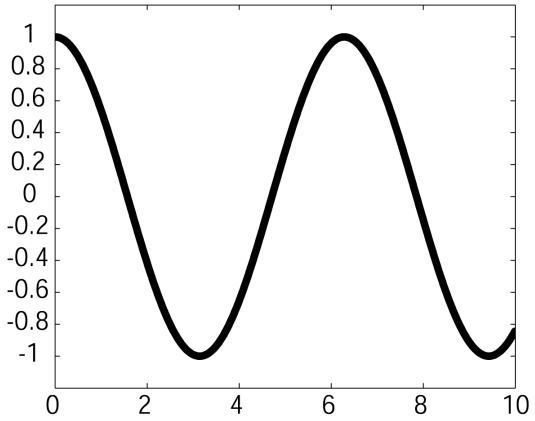
Fourier series

Every signal is equal to a sum of sines and cosines

sine (amplitude 1, period 2pi)



cosine (amplitude 1, period 2pi)



Fourier series

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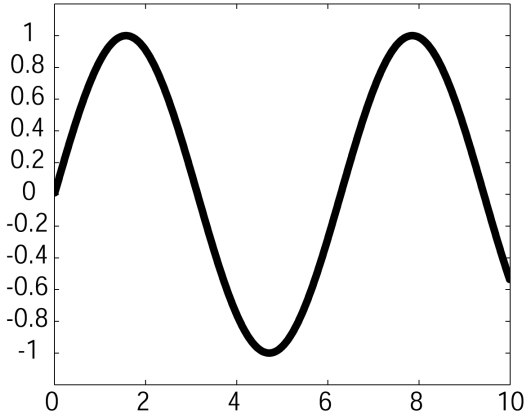
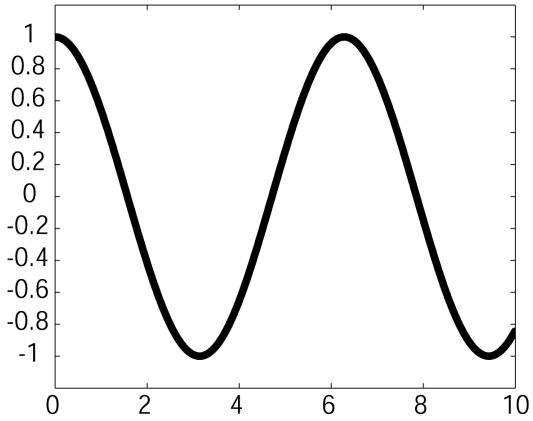
sine (amplitude 1, period 2pi)

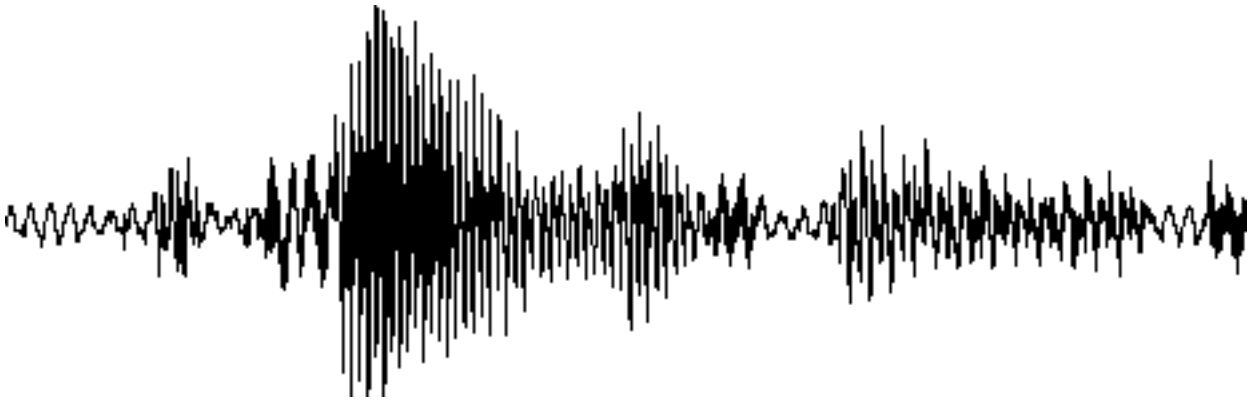


cosine (amplitude 1, period 2pi)

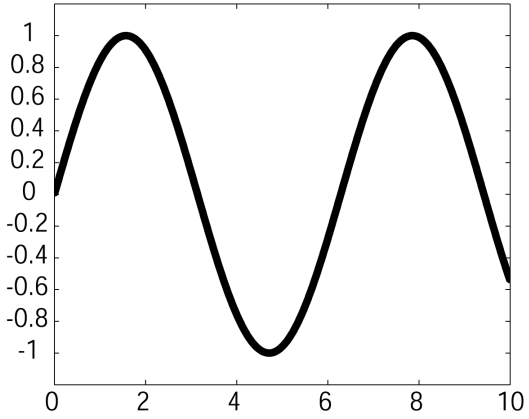


Fourier series

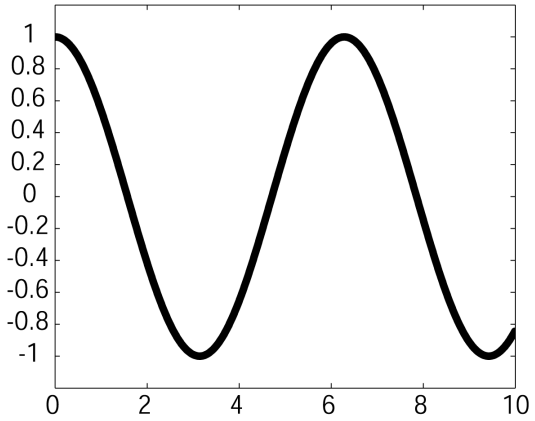
Every signal is equal to a sum of sines and cosines



sine (amplitude 1, period 2pi)



cosine (amplitude 1, period 2pi)



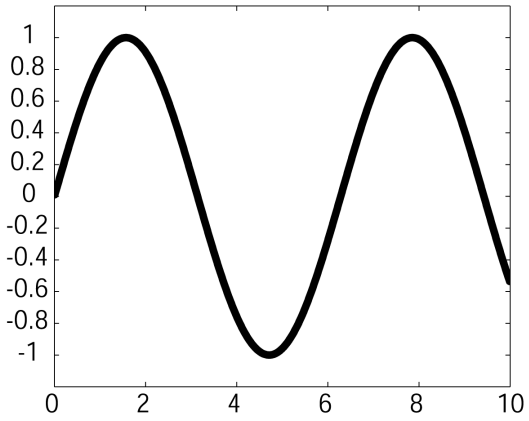
How is this possible?

Let's find the "Fourier series" representation of a simple signal

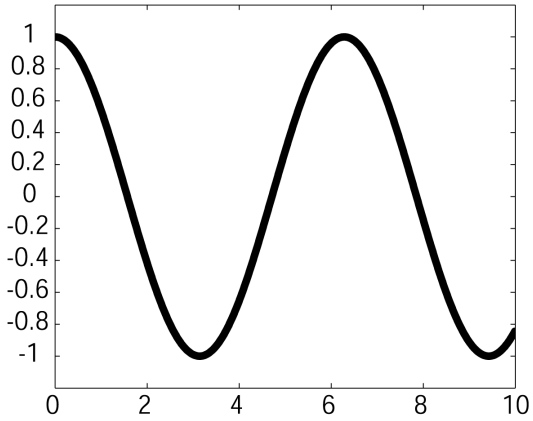
2

-1 ●

sine (amplitude 1, period 2pi)

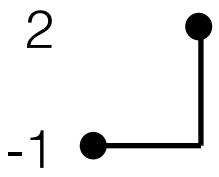


cosine (amplitude 1, period 2pi)

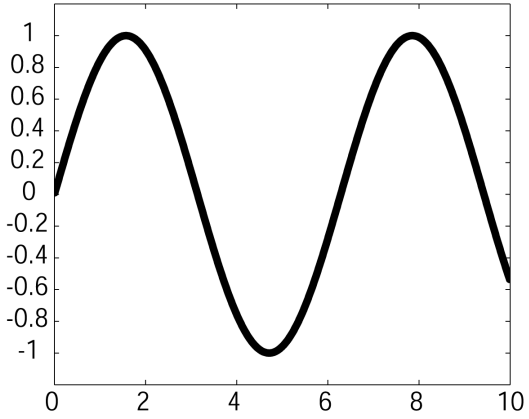


How is this possible?

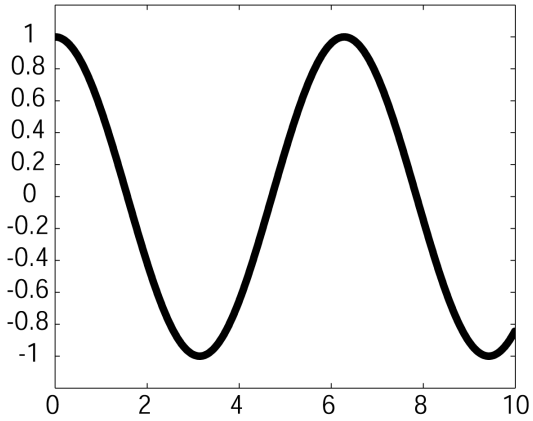
Let's find the "Fourier series" representation of a simple signal



sine (amplitude 1, period 2pi)

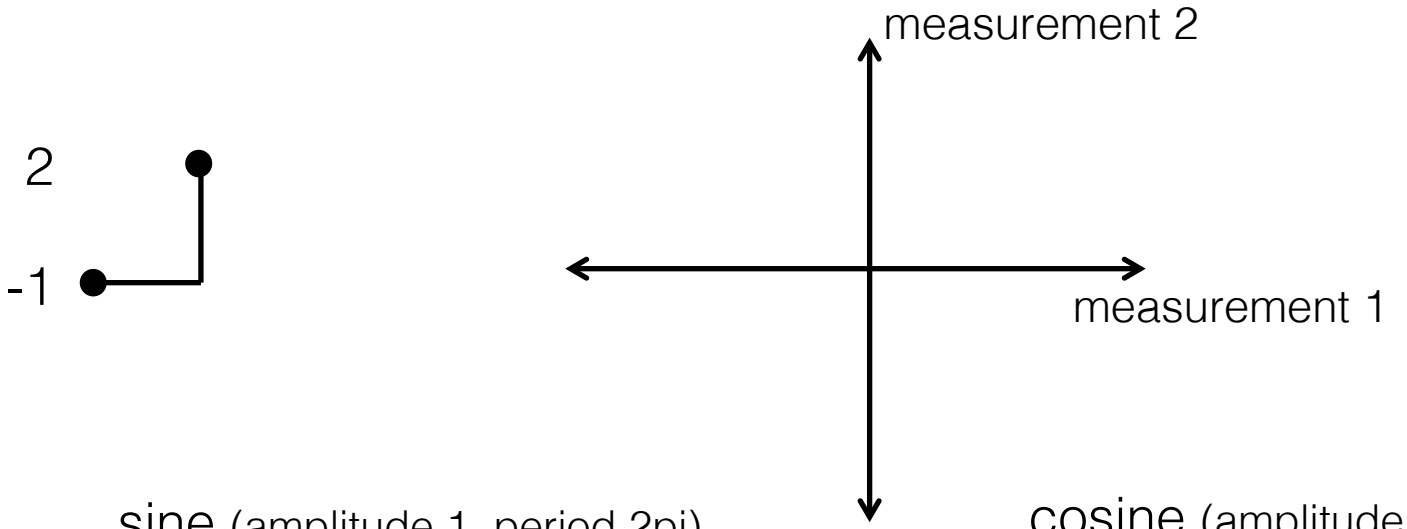


cosine (amplitude 1, period 2pi)

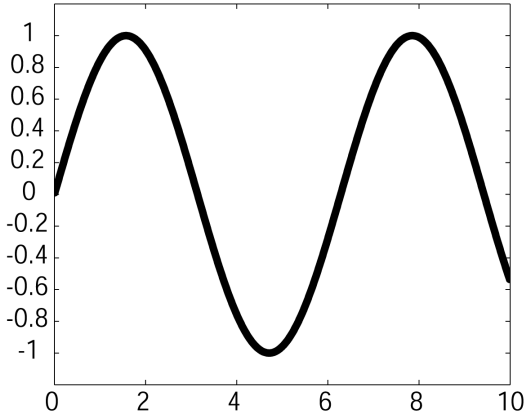


How is this possible?

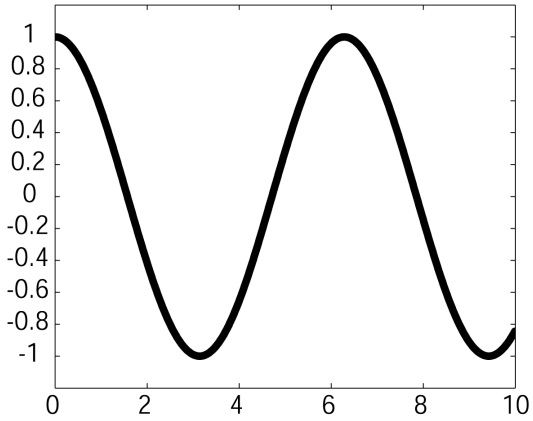
Let's find the "Fourier series" representation of a simple signal



sine (amplitude 1, period 2pi)

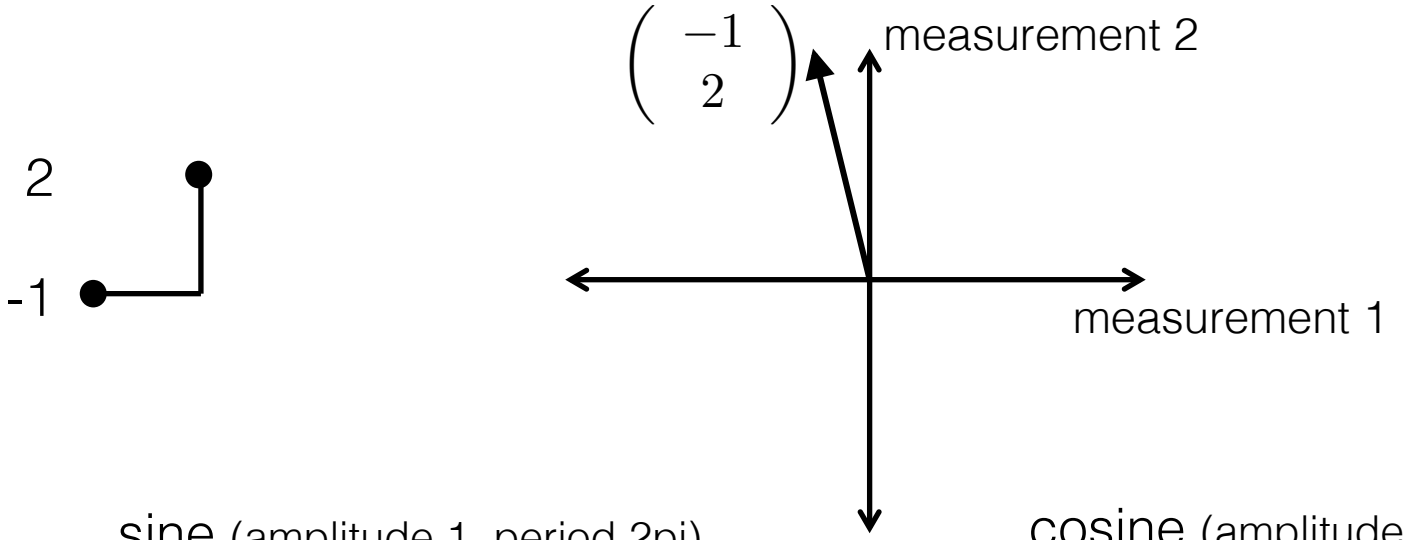


cosine (amplitude 1, period 2pi)

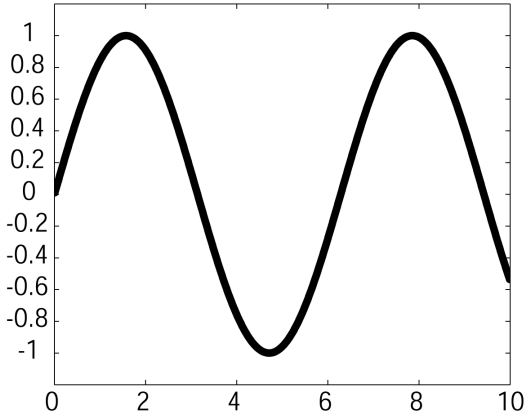


How is this possible?

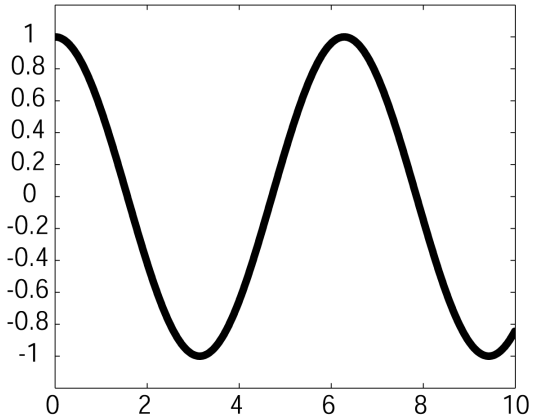
Let's find the "Fourier series" representation of a simple signal



sine (amplitude 1, period 2pi)

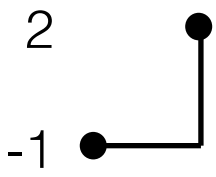


cosine (amplitude 1, period 2pi)

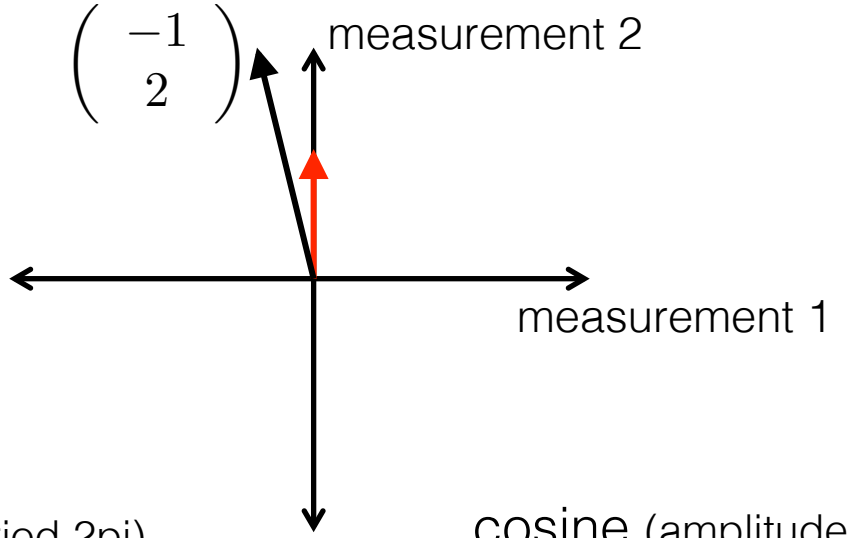


How is this possible?

Let's find the "Fourier series" representation of a simple signal

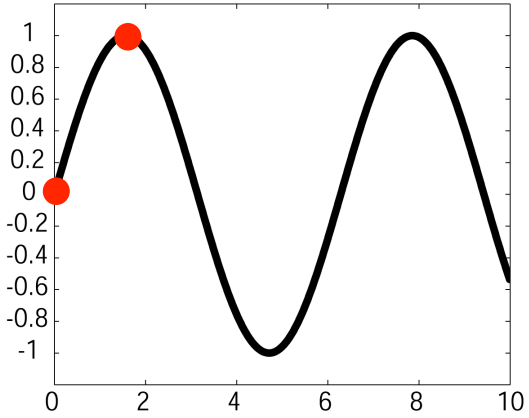


$\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

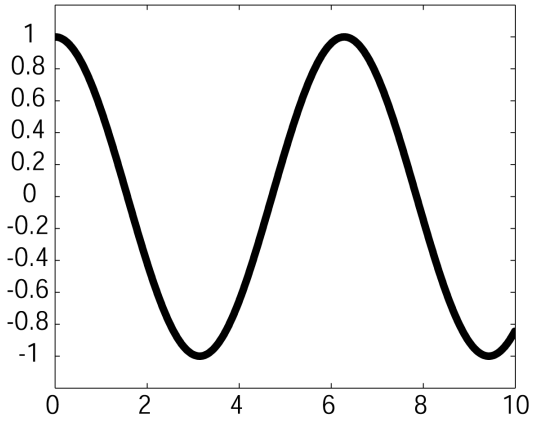


sine (amplitude 1, period 2pi)

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

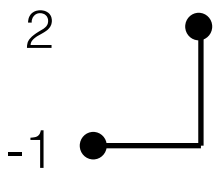


cosine (amplitude 1, period 2pi)

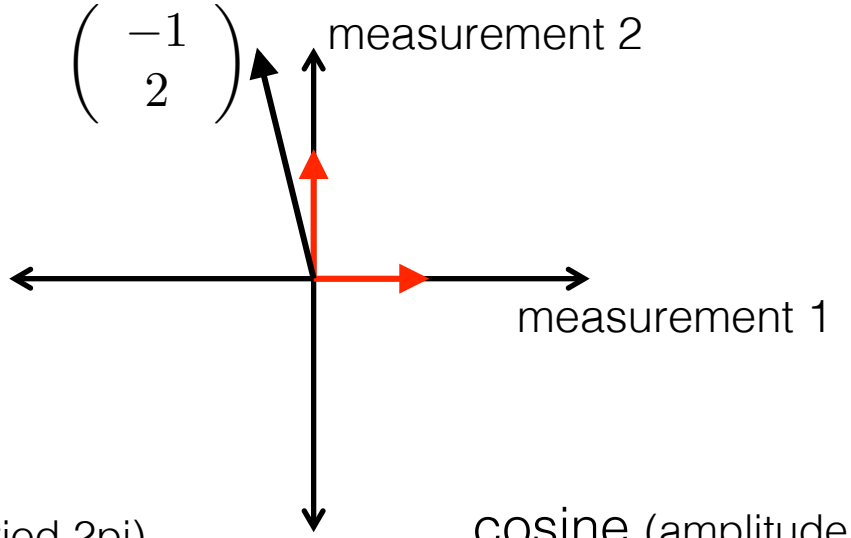


How is this possible?

Let's find the "Fourier series" representation of a simple signal

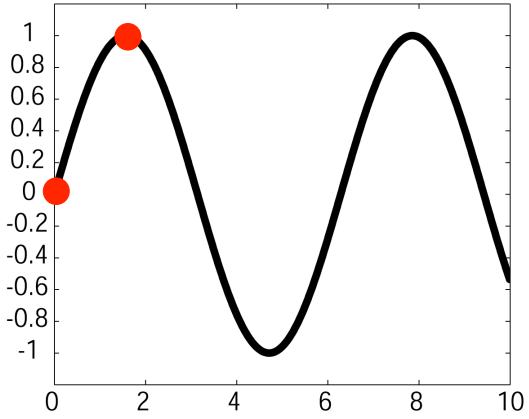


$\begin{pmatrix} -1 \\ 2 \end{pmatrix}$



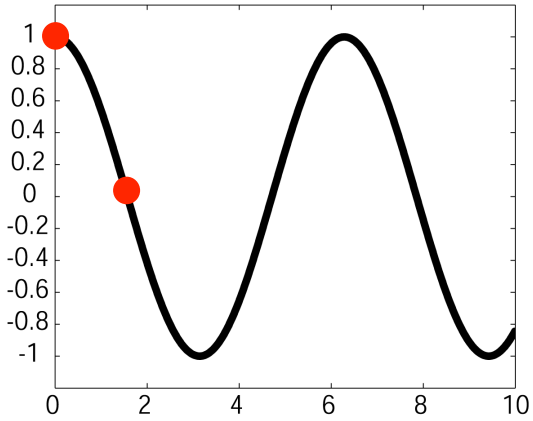
sine (amplitude 1, period 2pi)

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



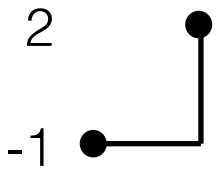
cosine (amplitude 1, period 2pi)

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

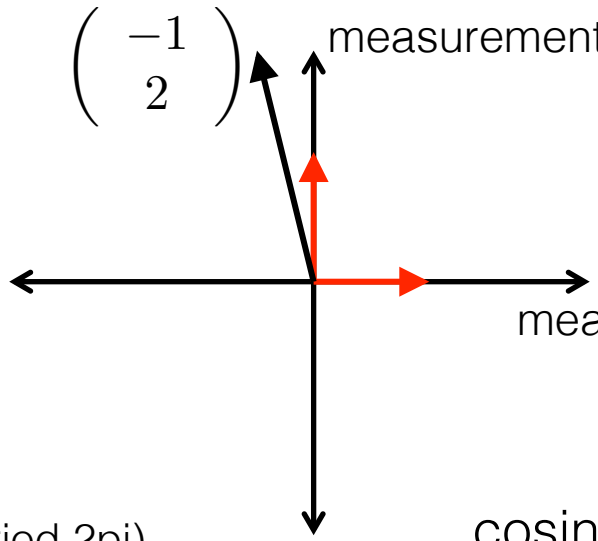


How is this possible?

Let's find the "Fourier series" representation of a simple signal



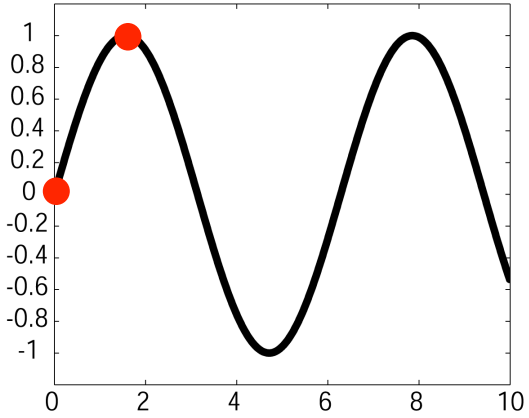
$$2\sin(x) - \cos(x)$$



$$\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

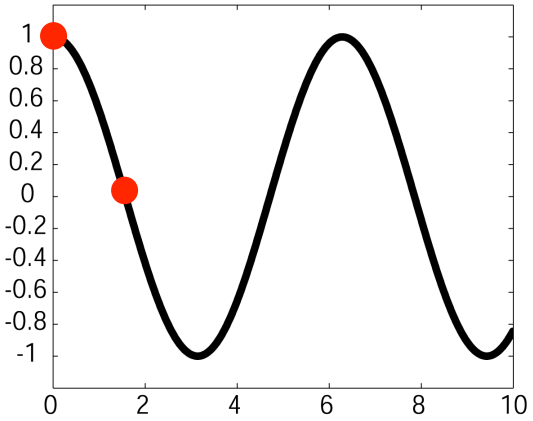
sine (amplitude 1, period 2pi)

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



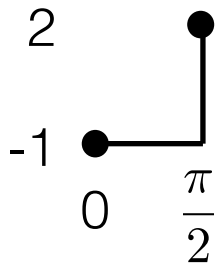
cosine (amplitude 1, period 2pi)

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

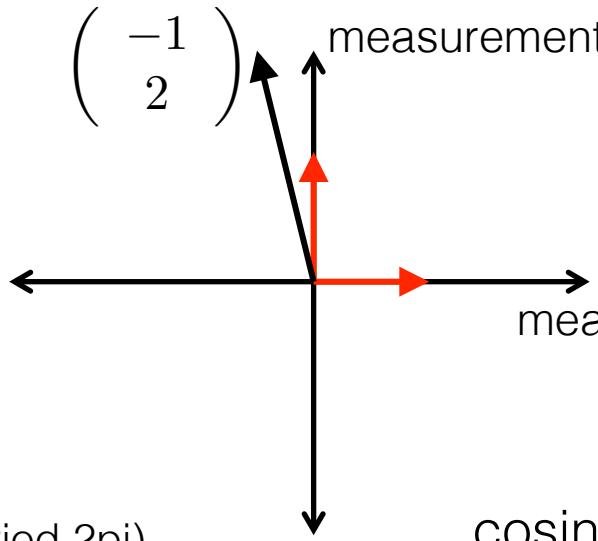


How is this possible?

Let's find the "Fourier series" representation of a simple signal

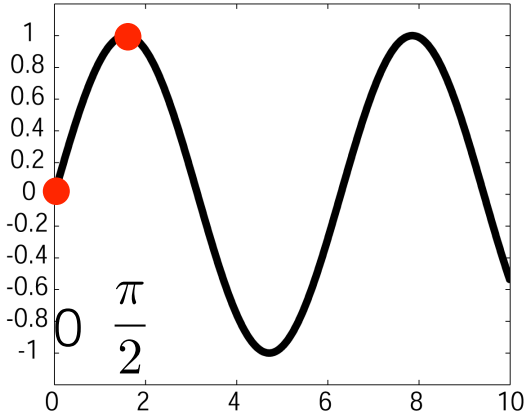


$$2\sin(x) - \cos(x)$$



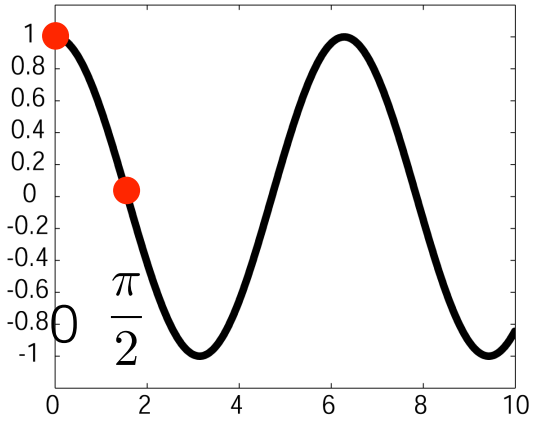
sine (amplitude 1, period 2pi)

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



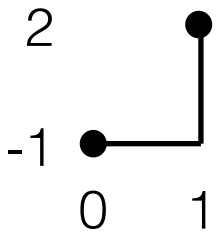
cosine (amplitude 1, period 2pi)

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

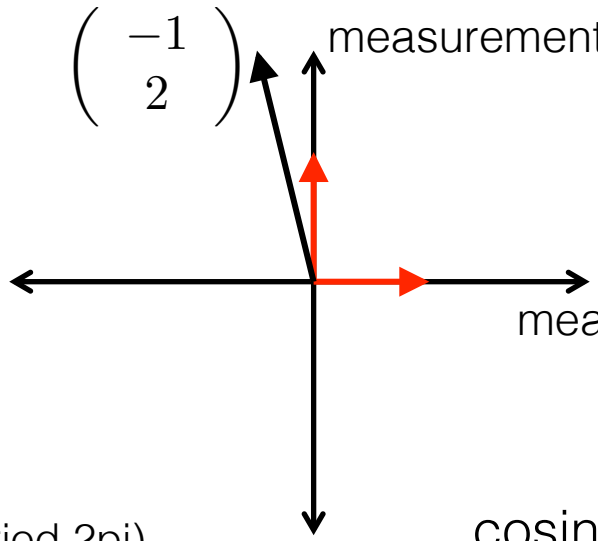


How is this possible?

Let's find the "Fourier series" representation of a simple signal

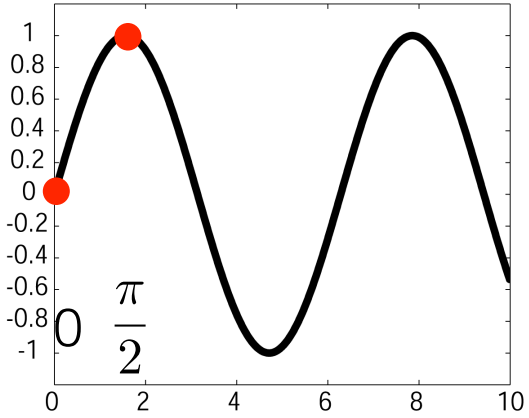


$$2\sin(x) - \cos(x)$$



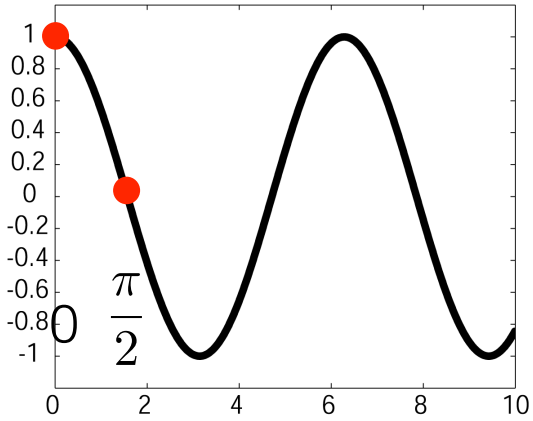
sine (amplitude 1, period 2pi)

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



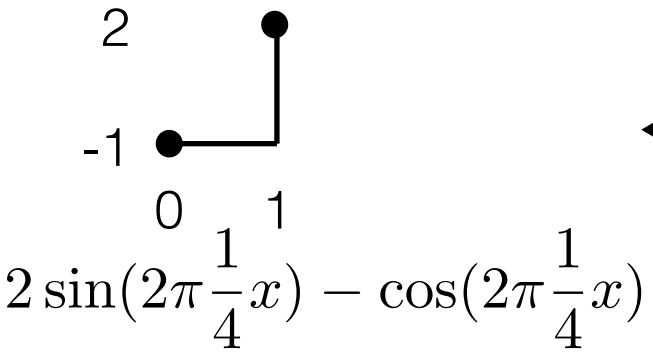
cosine (amplitude 1, period 2pi)

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

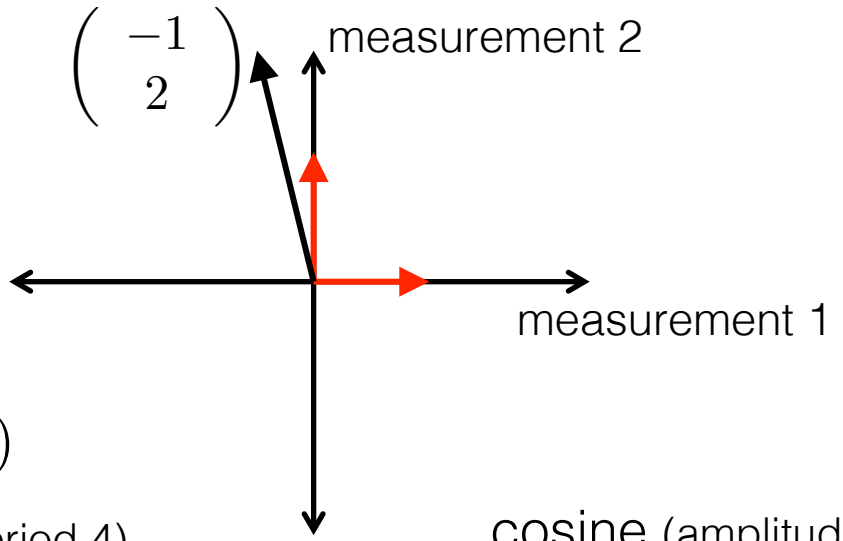


How is this possible?

Let's find the "Fourier series" representation of a simple signal

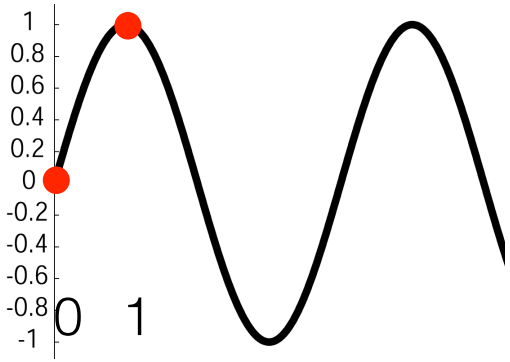


$$2\sin(2\pi\frac{1}{4}x) - \cos(2\pi\frac{1}{4}x)$$



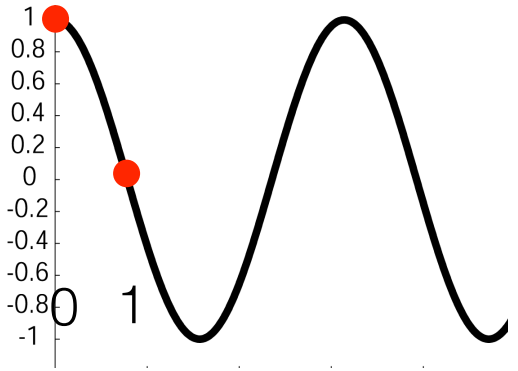
sine (amplitude 1, period 4)

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



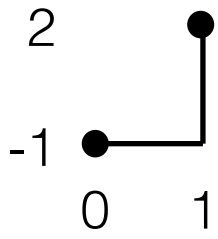
cosine (amplitude 1, period 4)

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

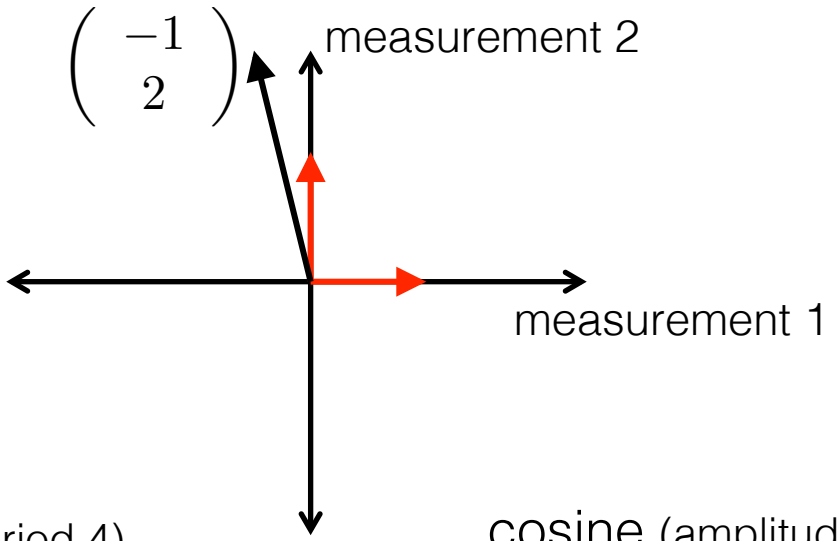


How is this possible?

Let's find the "Fourier series" representation of a simple signal

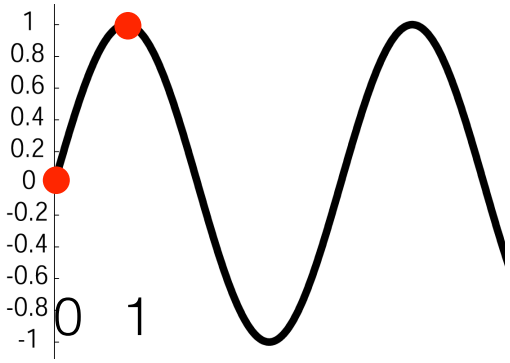


$$2\sin(\frac{\pi}{2}x) - \cos(\frac{\pi}{2}x)$$



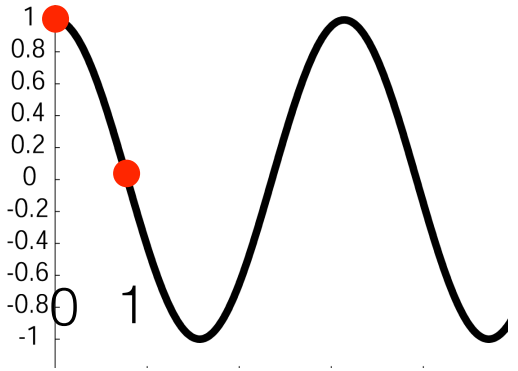
sine (amplitude 1, period 4)

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



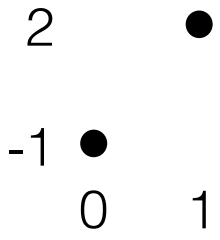
cosine (amplitude 1, period 4)

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

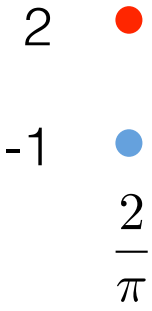


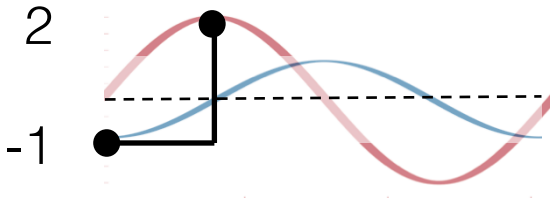
Moving to frequency space

signal measurements
vs
time



sinusoid coefficients
vs
frequency

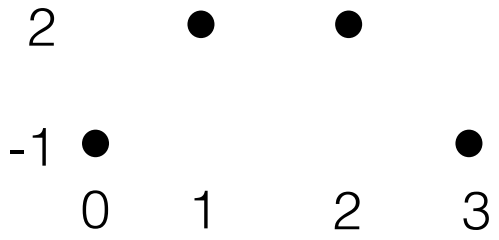




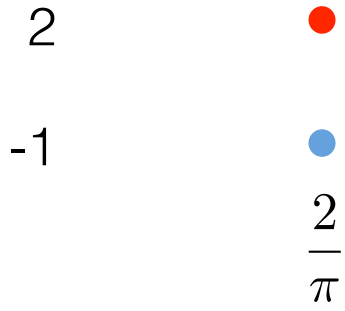
$$-1\cos(\frac{\pi}{2}x) + 2\sin(\frac{\pi}{2}x)$$

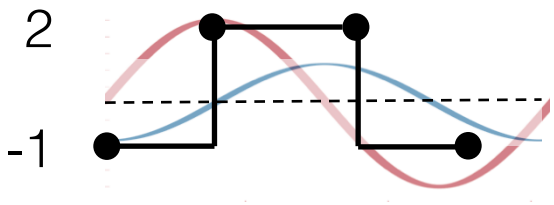
Moving to frequency space

signal measurements
vs
time



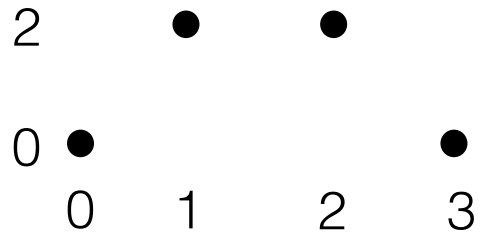
sinusoid coefficients
vs
frequency



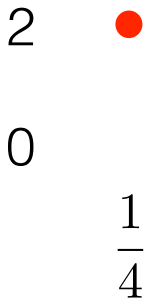


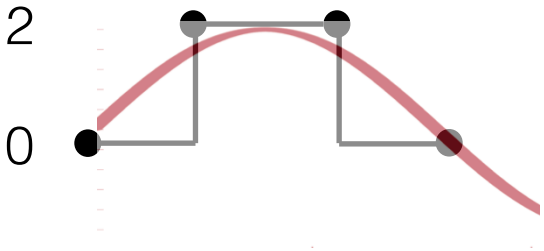
Moving to frequency space

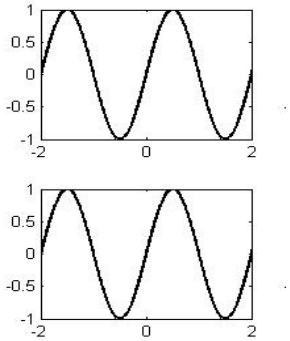
signal measurements
vs
time



sinusoid coefficients
vs
frequency

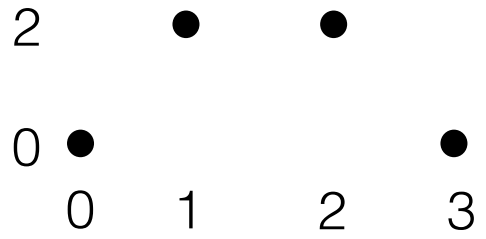




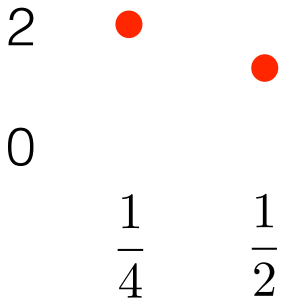


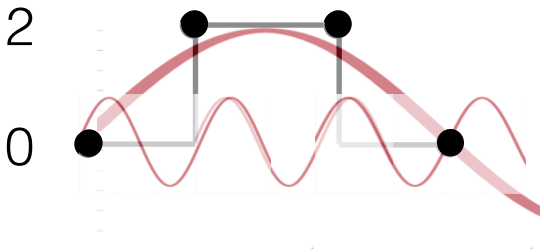
Moving to frequency space

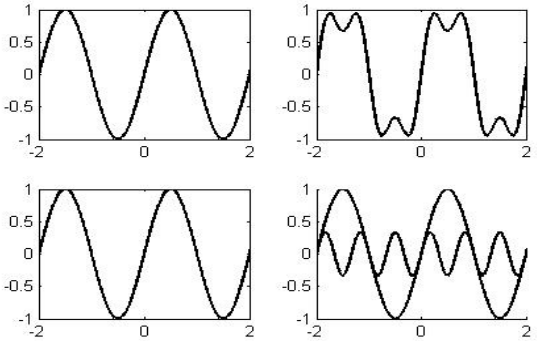
signal measurements
vs
time



sinusoid coefficients
vs
frequency

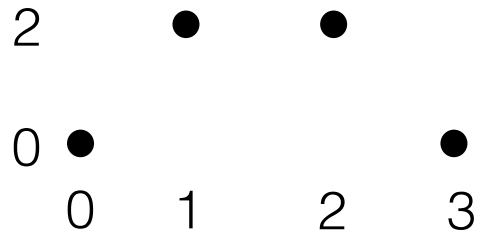




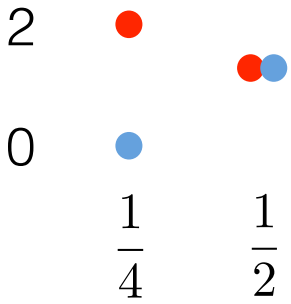


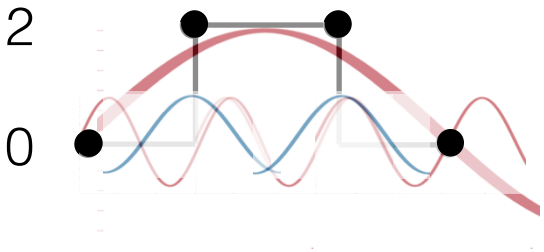
Moving to frequency space

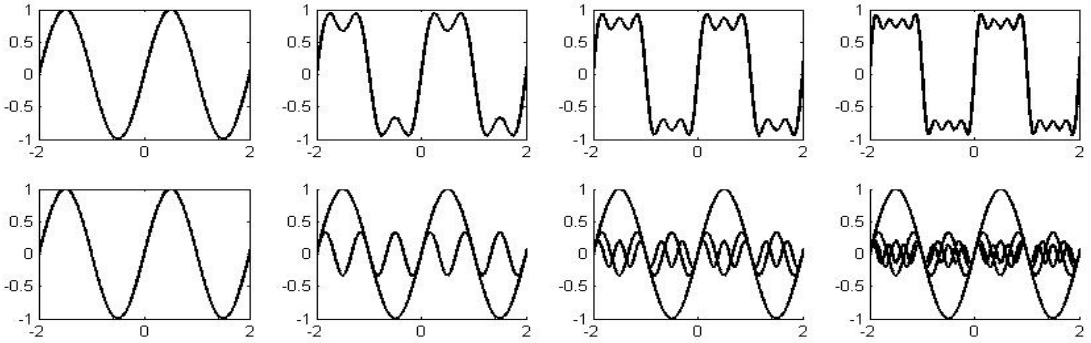
signal measurements
vs
time



sinusoid coefficients
vs
frequency

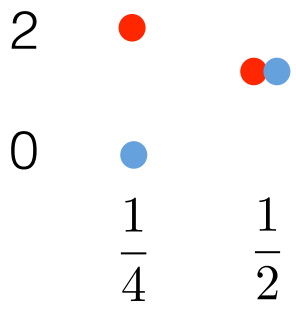






Fourier transform

Fourier series

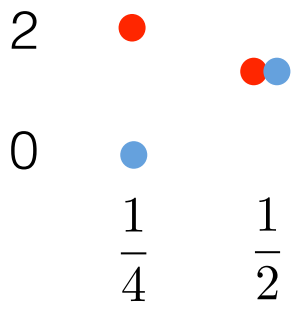


$$-1\cos(\frac{\pi}{2}x) + 2\sin(\frac{\pi}{2}x)$$

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Fourier transform

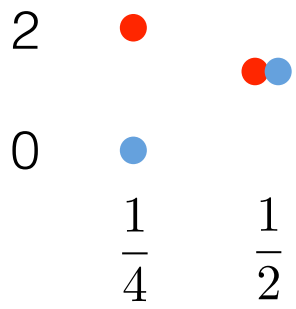
Fourier series



$$-1\cos(\frac{\pi}{2}x)+2\sin(\frac{\pi}{2}x)$$

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Fourier transform



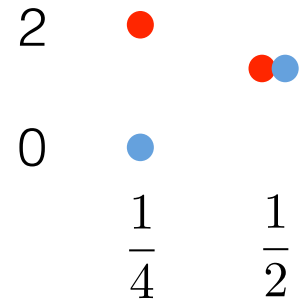
$$-1\cos(\frac{\pi}{2}x)-i\,2\sin(\frac{\pi}{2}x)$$

$$-1-2i$$

Fourier transform

$$F[\text{signal}]_f = \sum_{t}^{T-1} \text{signal}(t) \cdot e^{-i2\pi ft}$$

Fourier transform



$$-1\cos(\frac{\pi}{2}x) - i\,2\sin(\frac{\pi}{2}x)$$

$$-1 - 2i$$

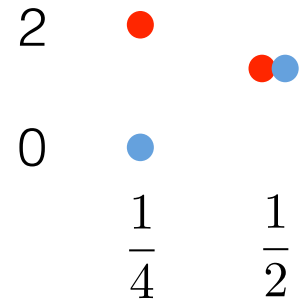
MATLAB: `fft(signal)`

Fourier transform

$$F[\text{signal}]_f = \sum_{t}^{T-1} \text{signal}(t) \cdot e^{-i2\pi ft}$$

$$e^{-i2\pi ft} = \cos(2\pi ft) - i\sin(2\pi ft)$$

Fourier transform



$$-1\cos(\frac{\pi}{2}x) - i\,2\sin(\frac{\pi}{2}x)$$

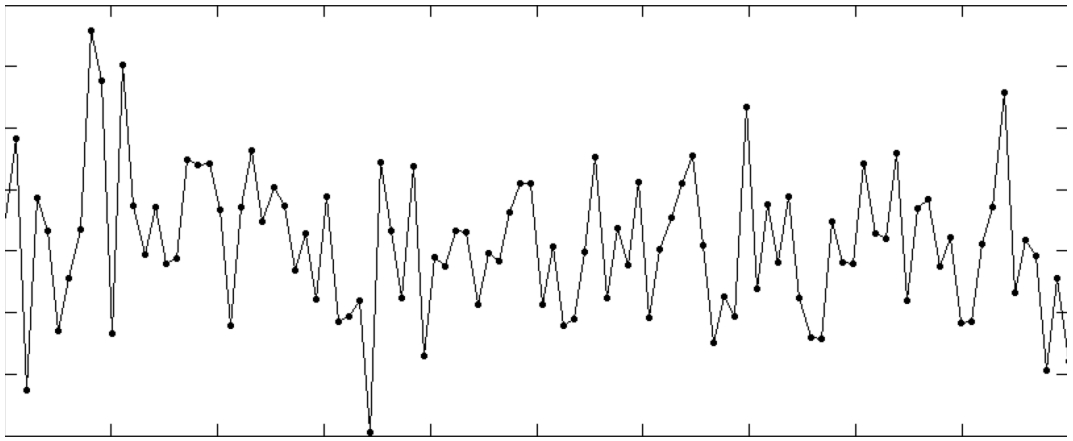
$$-1 - 2i$$

MATLAB: `fft(signal)`

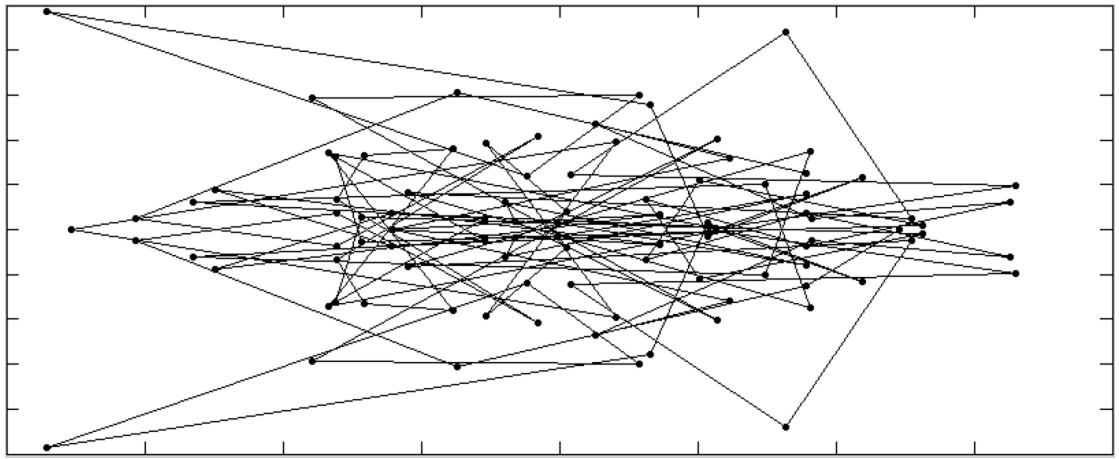
Fourier transform example

random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec



MATLAB: `fft(signal)`

Fourier transform example

random white noise
sampled at 10 Hz

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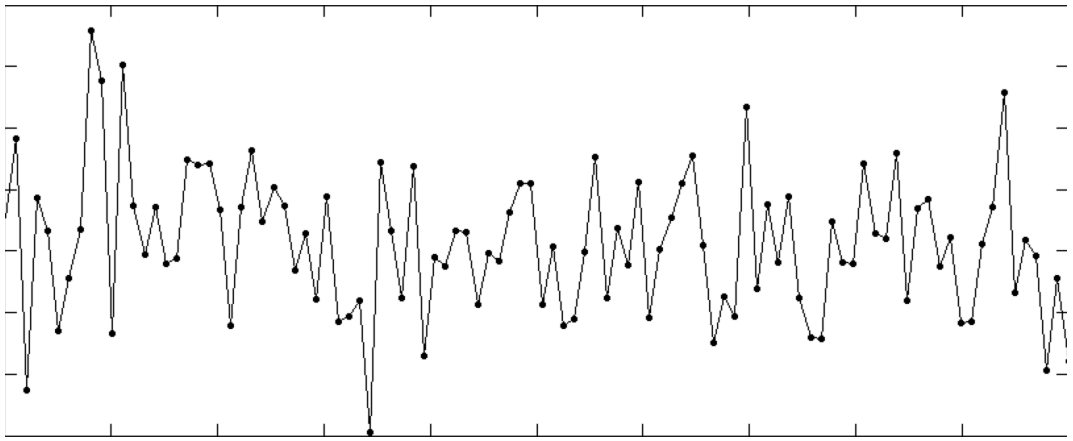
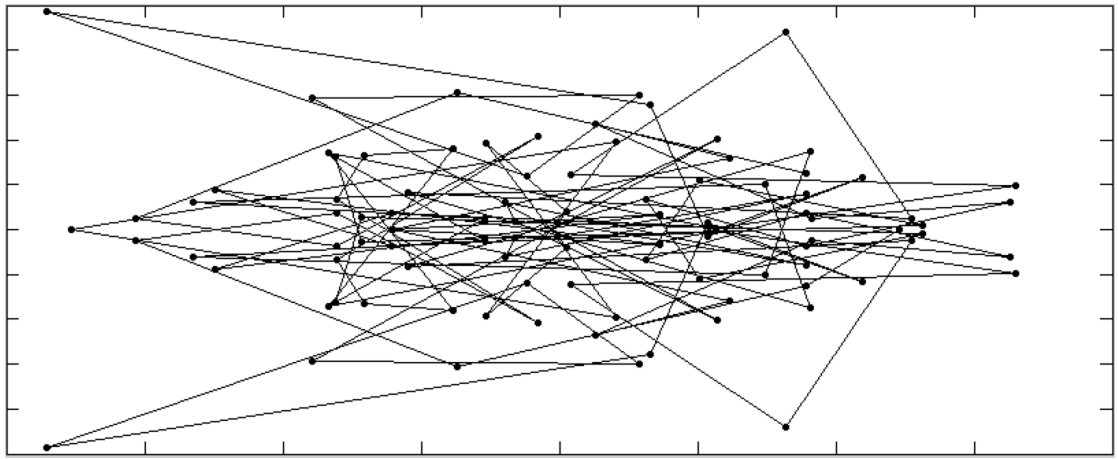


0 sec 10 sec

```
F    = fft(signal)
F(1) = 12.3081
F(2) = 8.95 - 6.25i
F(3) = -3.71 - 15.2i
...
```

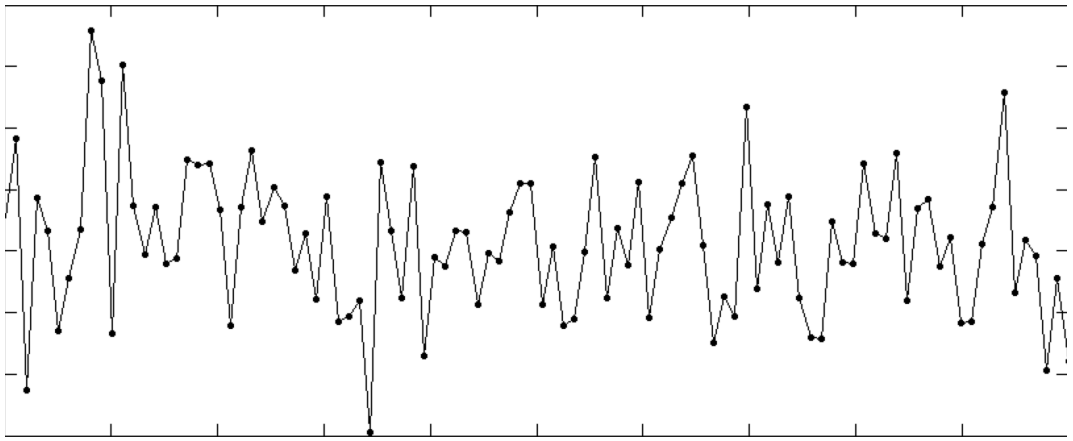


MATLAB: `fft(signal)`

Fourier transform example

random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec

```
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F(3) = -3.71 - 15.2i
...
```

$$F[\text{signal}]_f = \sum_{t}^{T-1} \text{signal}(t) \cdot e^{-i2\pi ft}$$

MATLAB: `fft(signal)`

Fourier transform example

random white noise
sampled at 10 Hz

```
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linspace(0,10,100)
```
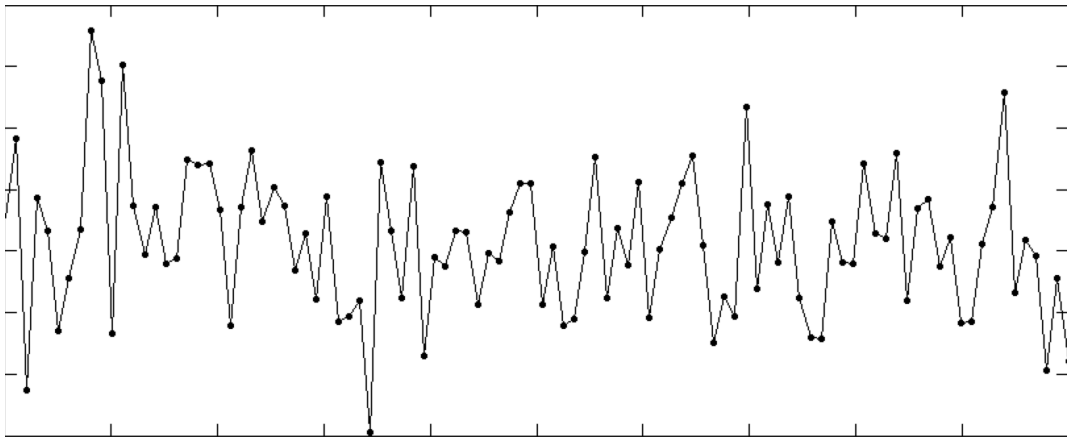



0 sec 10 sec

```
F    = fft(signal)
F(1) = 12.3081
F(2) = 8.95 - 6.25i
F(3) = -3.71 - 15.2i
...
```

$$F[\text{signal}]_f = \sum_t^{T-1} \text{signal}(t) \cdot e^{-i2\pi ft}$$

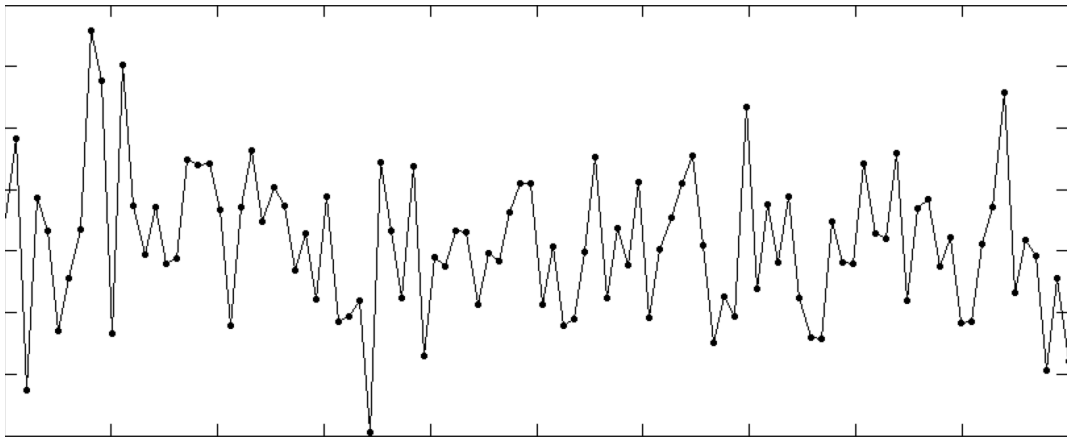
typical normalization so
F[0 Hz] = mean(signal)

MATLAB: `(1/length(signal))` * `fft(signal)`

Amplitude spectrum example

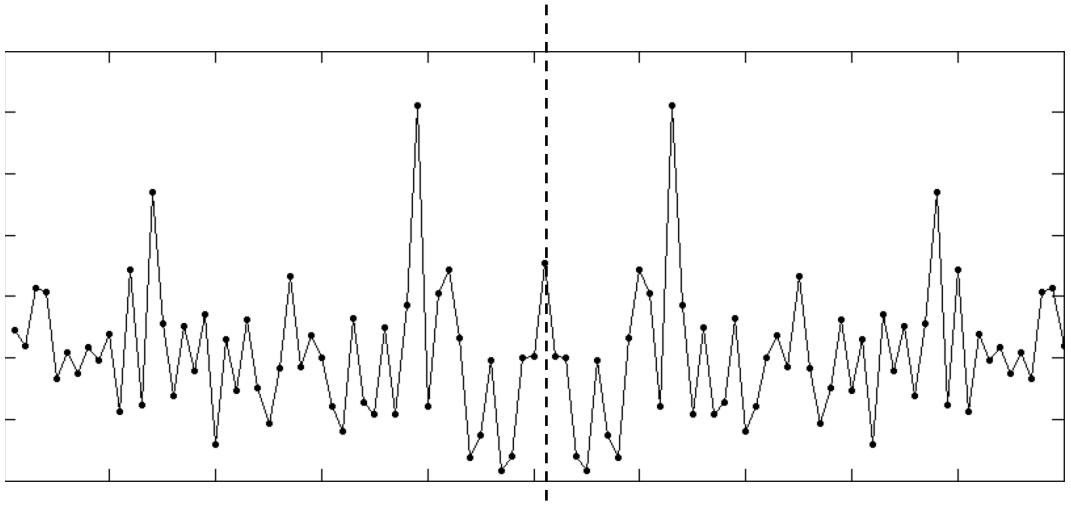
random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec

```
F    = fft(signal)
F(1) = 12.3081
F(2) = 10.9257
F(3) = 15.6638
...
```



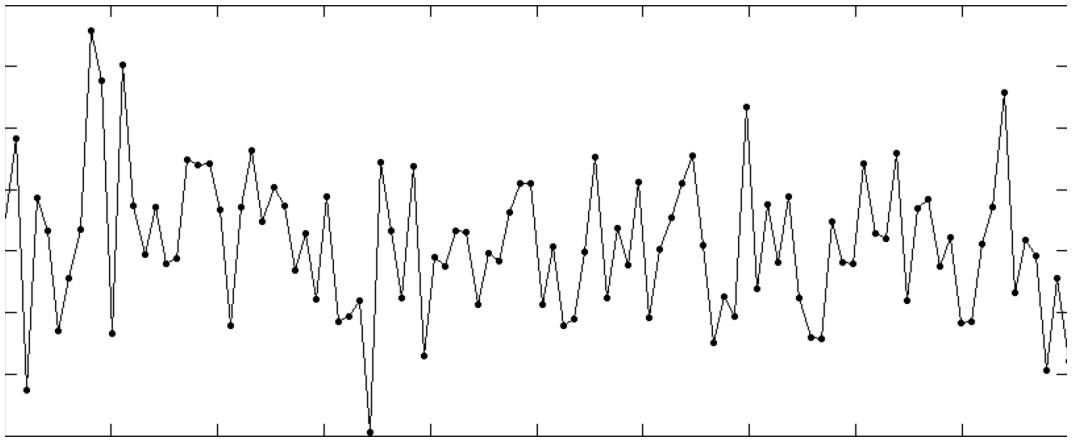
? Hz

MATLAB: `abs(fft(signal))`

Amplitude spectrum example

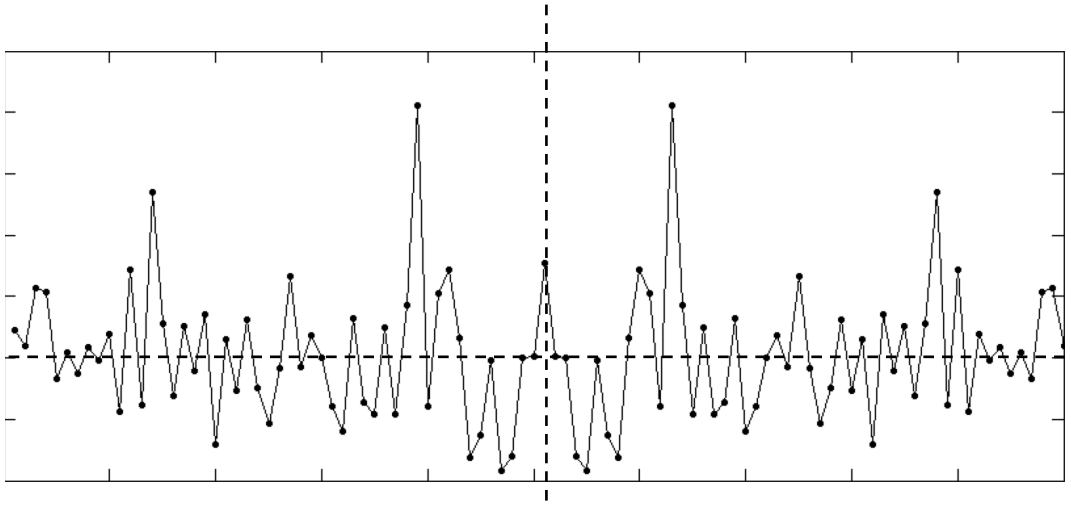
random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec

```
F    = fft(signal)
F(1) = 12.3081
F(2) = 10.9257
F(3) = 15.6638
...
```



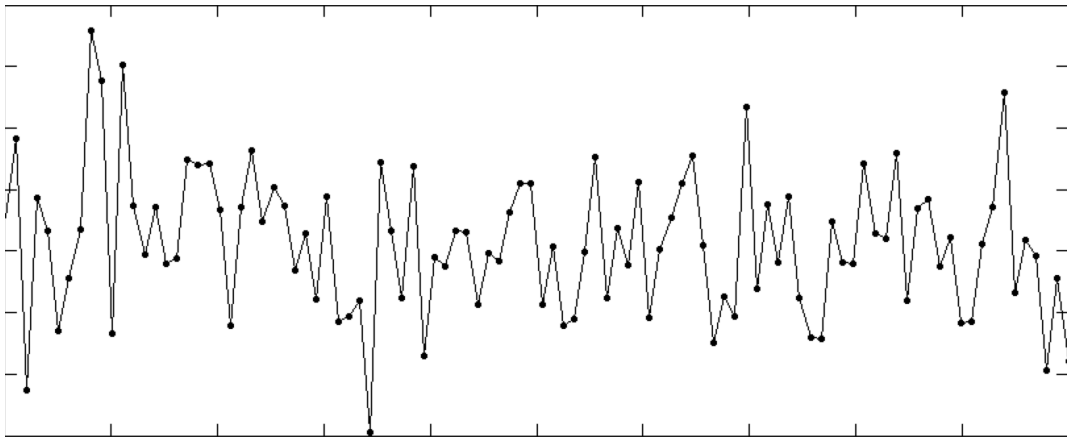
? Hz

MATLAB: `abs(fft(signal))`

Amplitude spectrum example

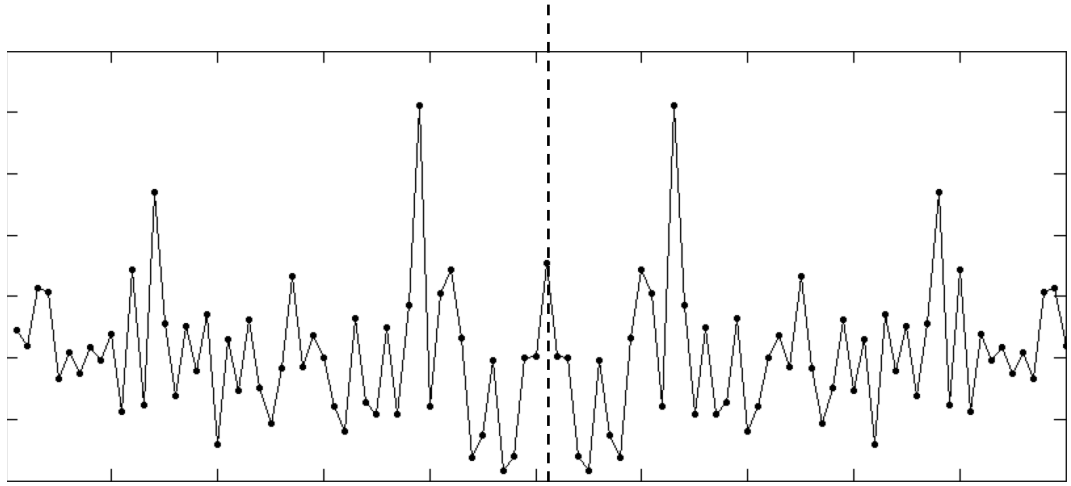
random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec

```
F    = fft(signal)
F(1) = 12.3081
F(2) = 10.9257
F(3) = 15.6638
...
```



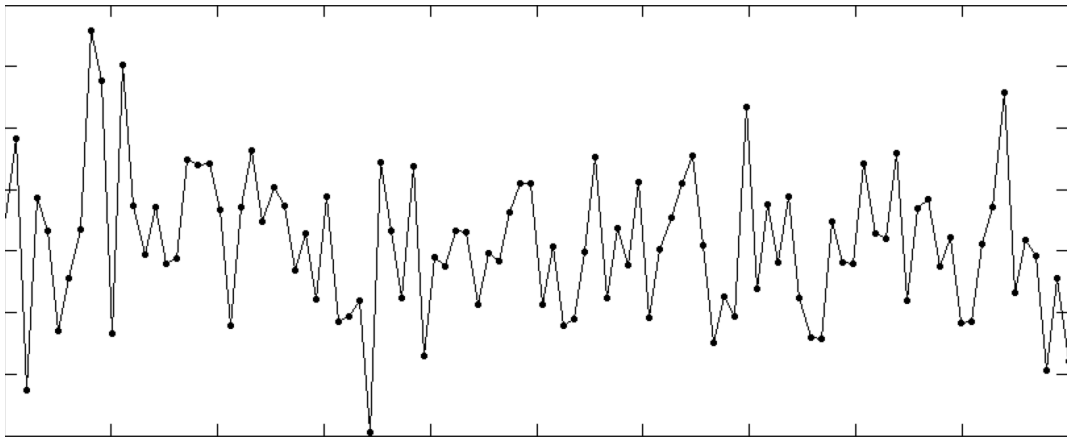
0 Hz ? Hz

MATLAB: `abs(fft(signal))`

Amplitude spectrum example

random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec

```
F    = fft(signal)
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...
```
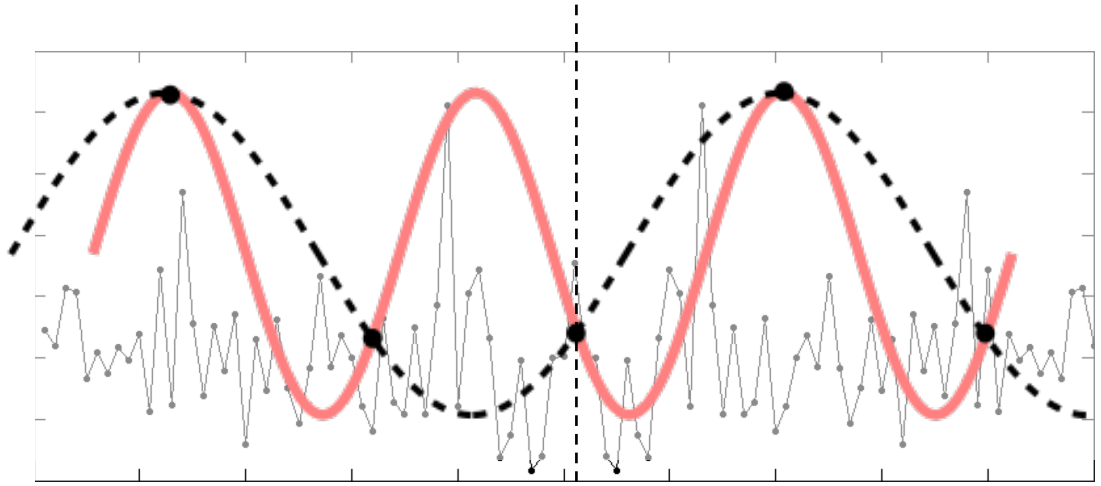



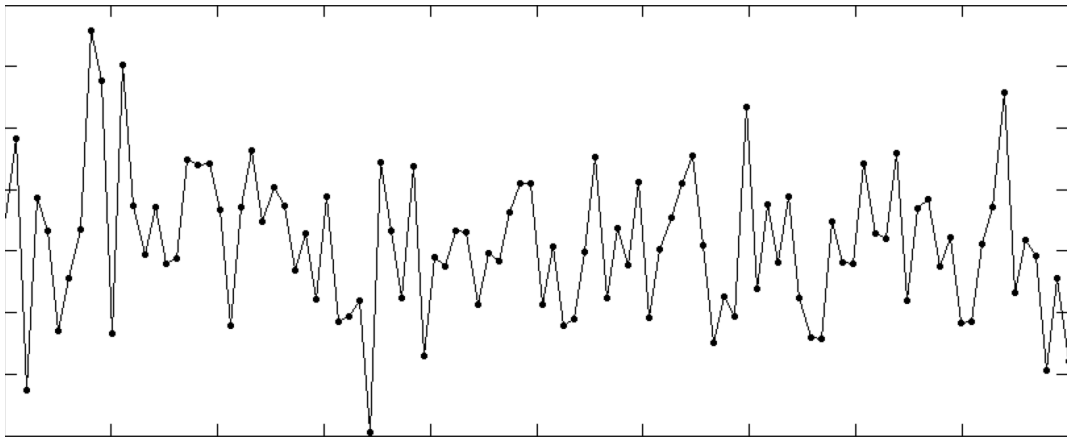
0 Hz 5 Hz Nyquist frequency is 0.5 * sample frequency

MATLAB: `abs(fft(signal))`

Amplitude spectrum example

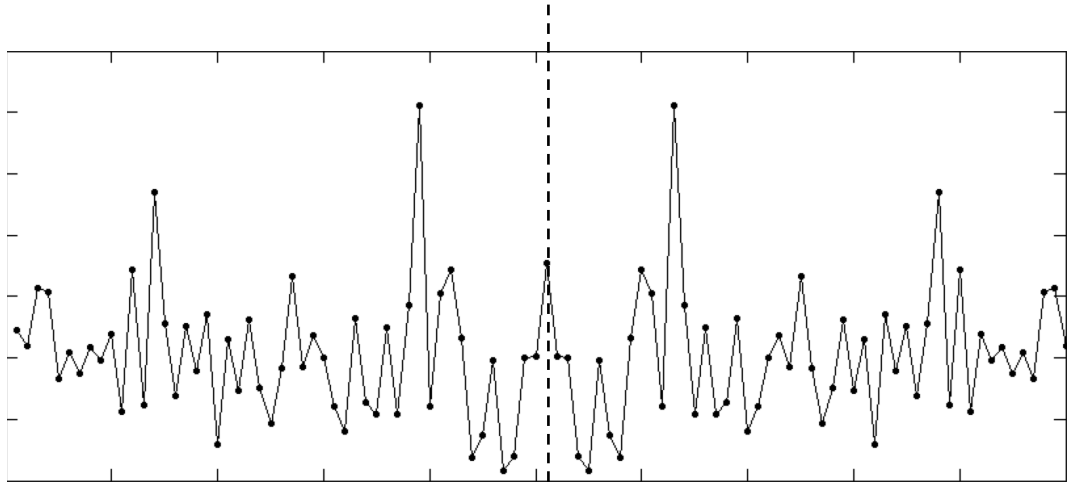
random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec

```
F    = fft(signal)
F(1) = 12.3081
F(2) = 10.9257
F(3) = 15.6638
...
```

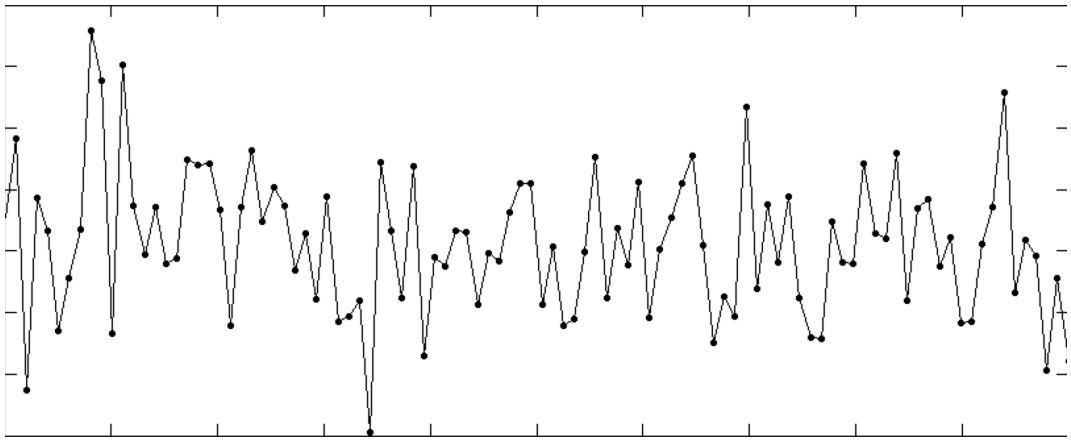


MATLAB: `abs(fft(signal))`

Power spectrum example

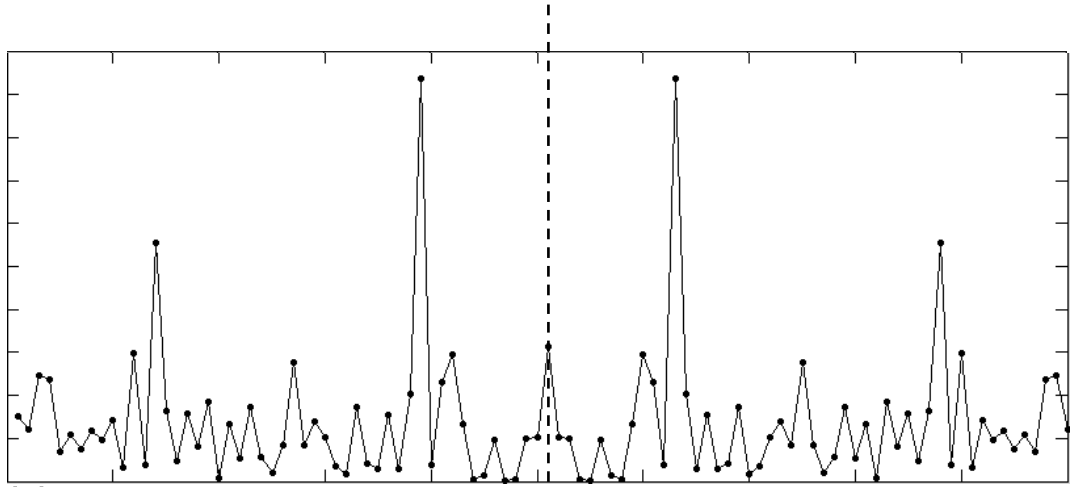
random white noise sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec

power spectrum is just the squared amplitude spectrum

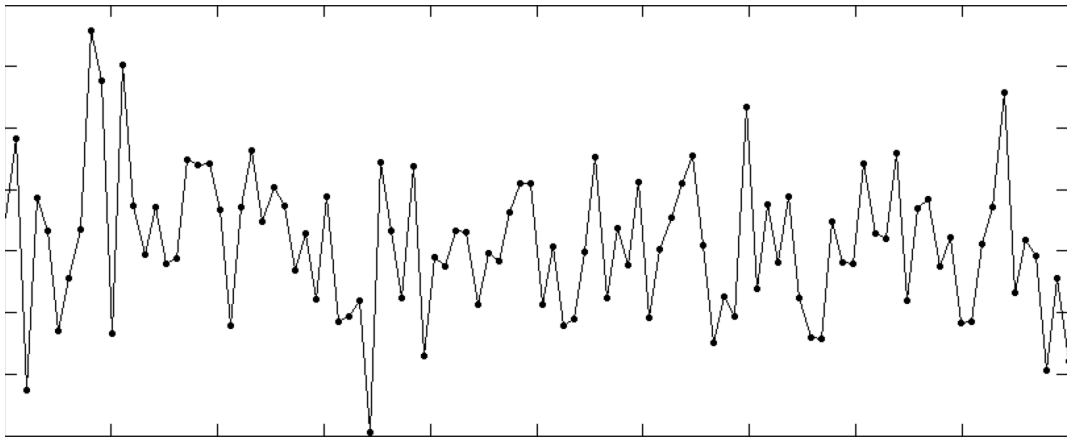


MATLAB: `abs(fft(signal)).^2`

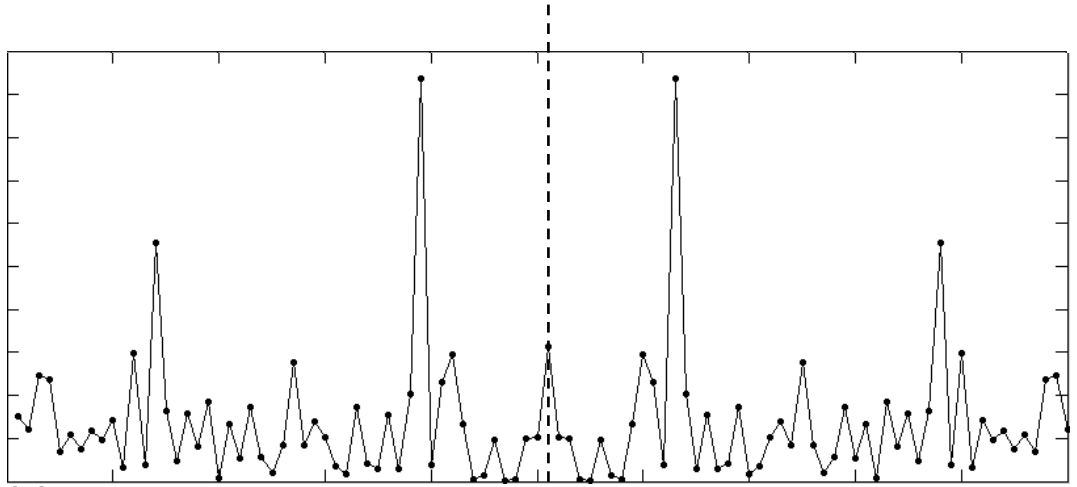
What part do I actually care about?

random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec



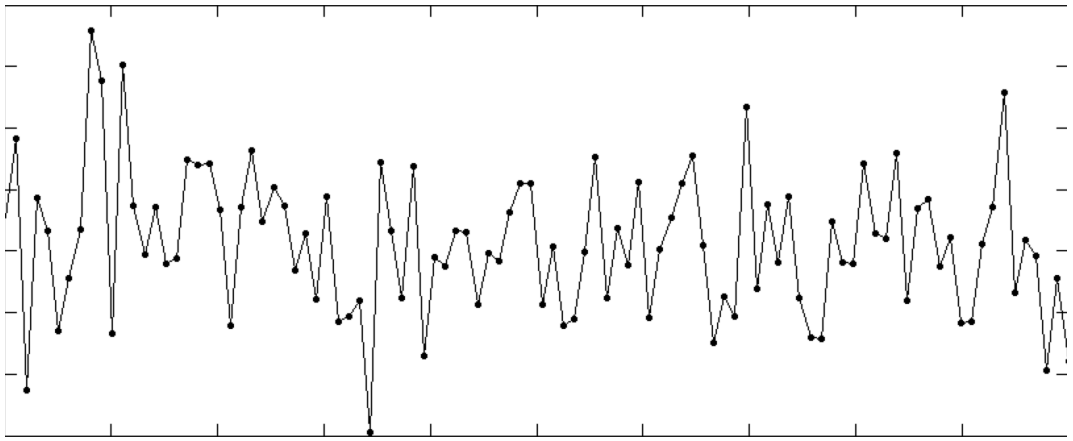
0 Hz 2.5 Hz 5 Hz -2.5 Hz -0.1 Hz

MATLAB: `abs(fft(signal)).^2`

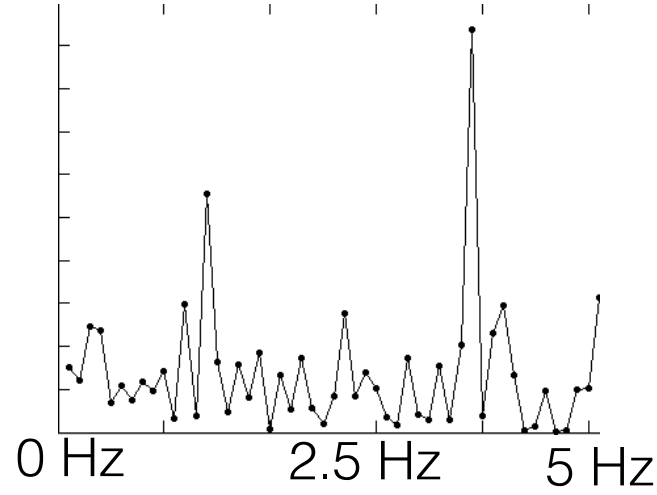
What part do I actually care about?

random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec



MATLAB (even length signal):

```
Ps = abs(fft(signal)).^2;
Ps = Ps(1:length(signal)/2+1)
```

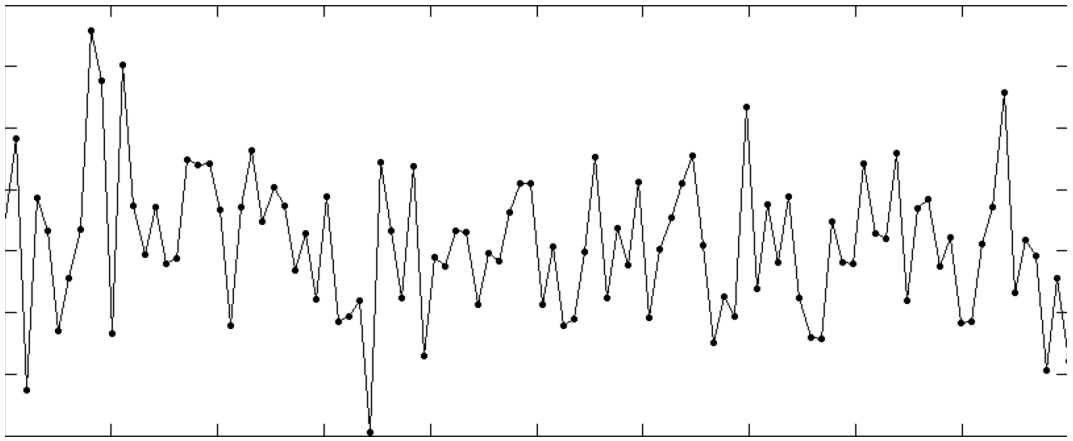
MATLAB (odd length signal):

```
Ps = abs(fft(signal)).^2;
Ps = Ps(1:(length(signal)-1)/2+1)
```

How to generate x-axis?

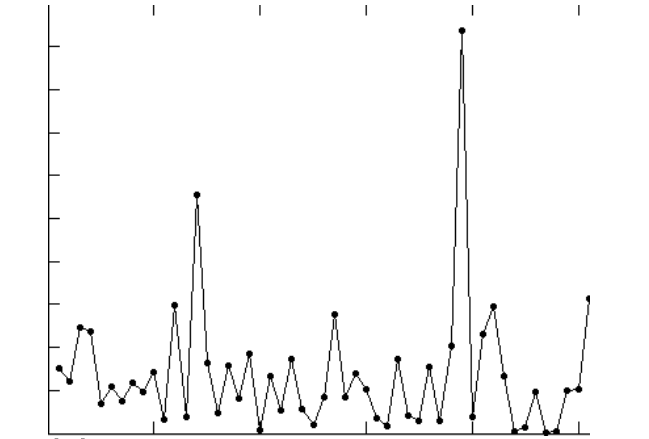
random white noise
sampled at 10 Hz

```
randn(100,1)
linspace(0,10,100)
```



0 sec 10 sec

```
%%     In general    %%
%% Fs is sample rate %%
freqs = linspace(0, Fs/2, length(Ps))

%% Our example %%
freqs = linspace(0, 5, 51)
```

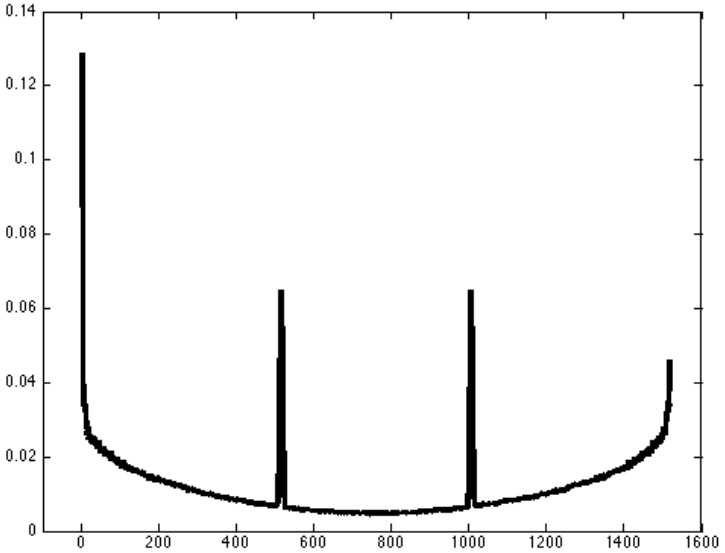


MATLAB (even length signal):

```
Ps = abs(fft(signal)).^2;
Ps = Ps(1:length(signal)/2+1)
```

Example: De-noising images

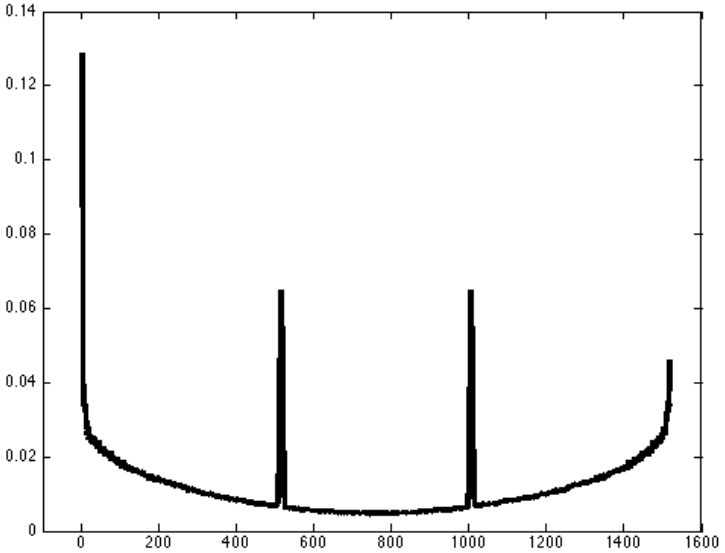
Example: De-noising images



MATLAB (odd length signal):

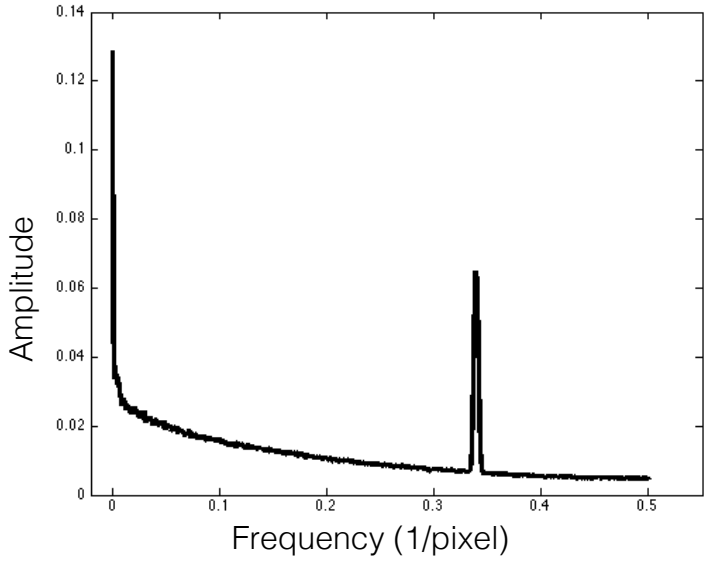
```
As_2d = (1/length(image(:)))*abs(fft2(image));
As_1d = mean(As_2d, 1) % average all horizontal spectra
```

Example: De-noising images

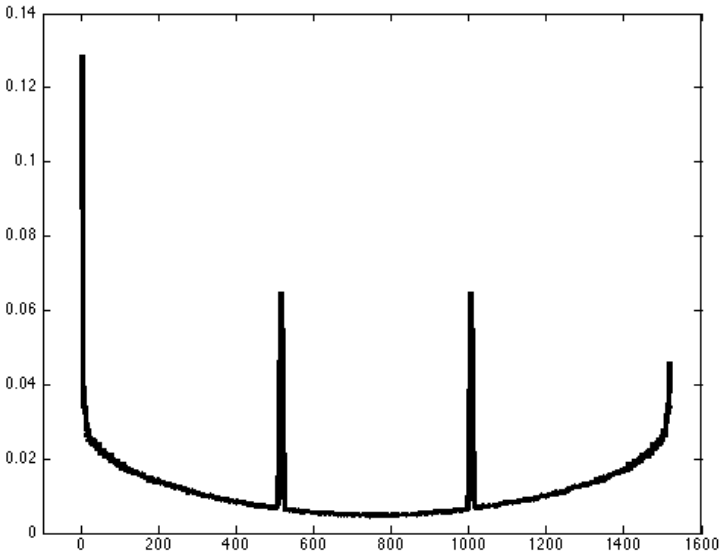


MATLAB (odd length signal):

```
As_2d = (1/length(image(:)))*abs(fft2(image));
As_1d = mean(As_2d, 1) % average all horizontal spectra
As    = As_1d(1:(length(As_1d)-1)/2+1)
freqs = linspace(0, 1/2, length(As))
```

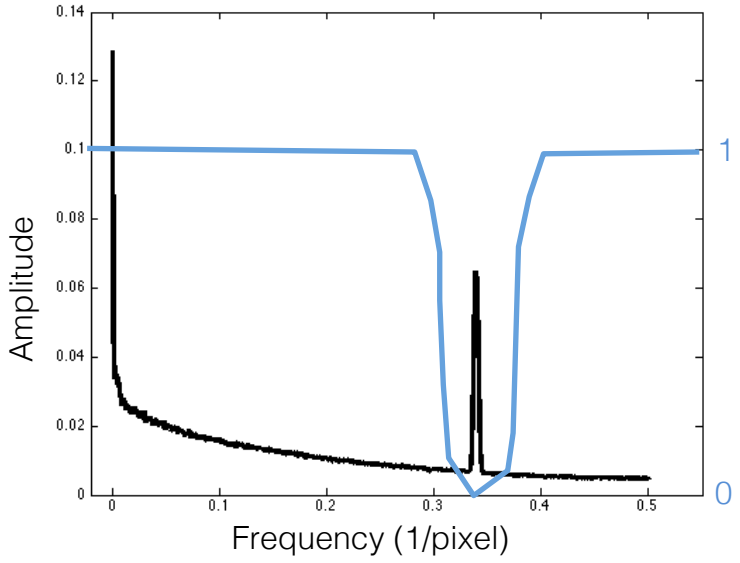


Example: notch filter

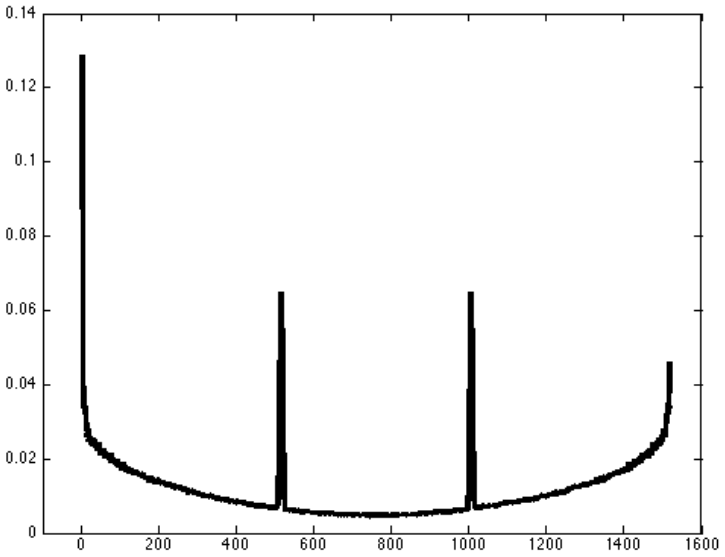


MATLAB (odd length signal):

```
As_2d = (1/length(image(:)))*abs(fft2(image));
As_1d = mean(As_2d, 1) % average all horizontal spectra
As    = As_1d(1:(length(As_1d)-1)/2+1)
freqs = linspace(0, 1/2, length(As))
```

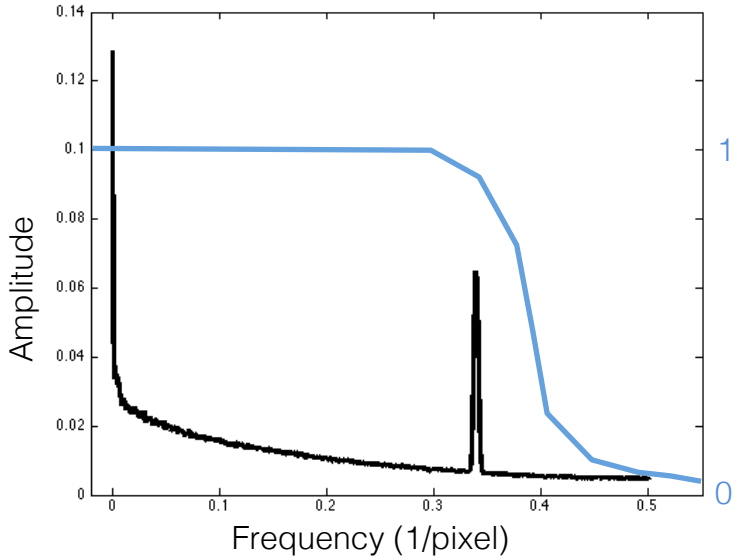


Example: low-pass filter

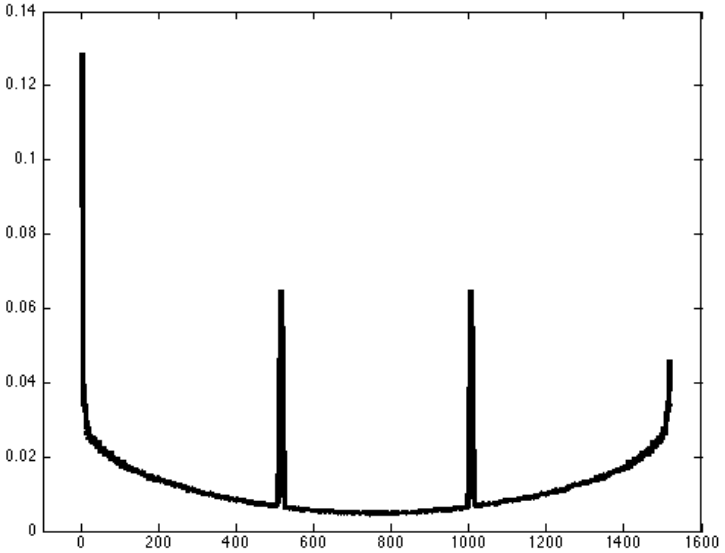


MATLAB (odd length signal):

```
As_2d = (1/length(image(:)))*abs(fft2(image));
As_1d = mean(As_2d, 1) % average all horizontal spectra
As    = As_1d(1:(length(As_1d)-1)/2+1)
freqs = linspace(0, 1/2, length(As))
```

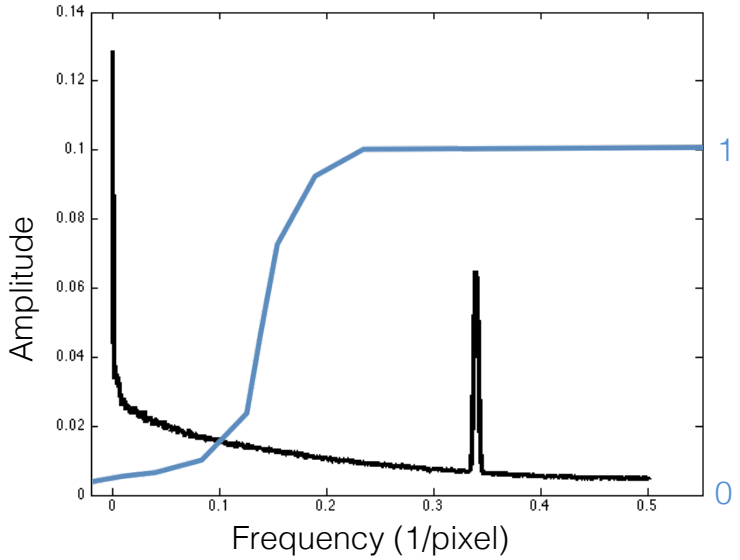


Example: high-pass filter

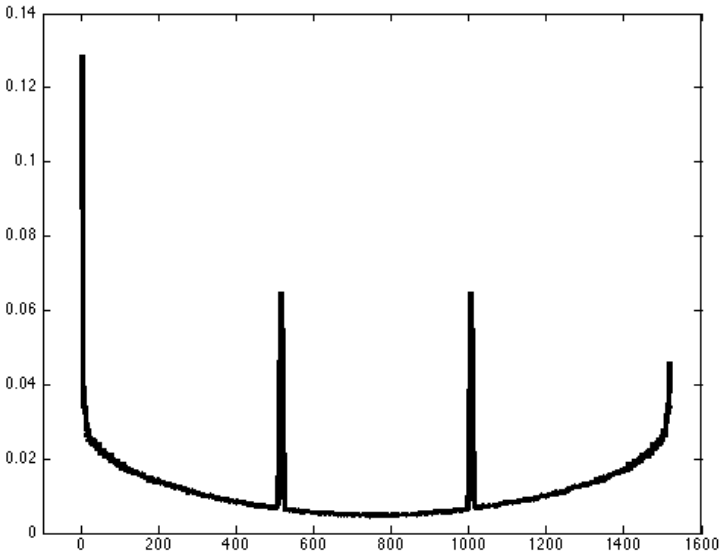


MATLAB (odd length signal):

```
As_2d = (1/length(image(:)))*abs(fft2(image));
As_1d = mean(As_2d, 1) % average all horizontal spectra
As    = As_1d(1:(length(As_1d)-1)/2+1)
freqs = linspace(0, 1/2, length(As))
```

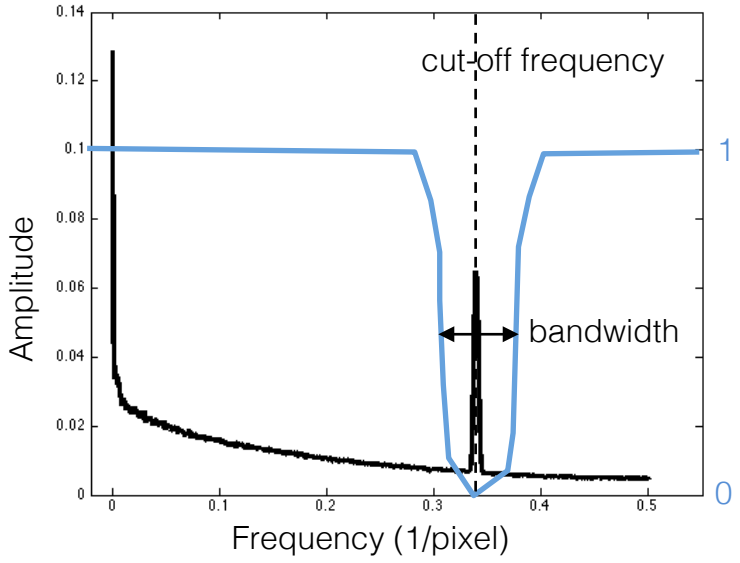


Example: notch filter

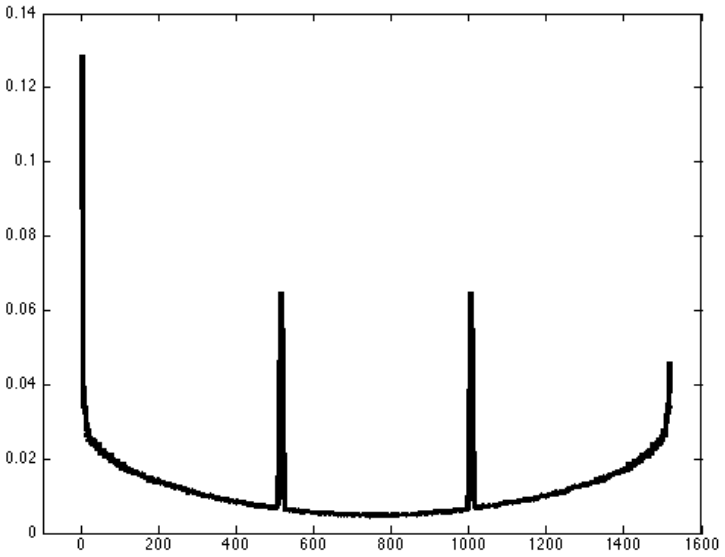


MATLAB (odd length signal):

```
As_2d = (1/length(image(:)))*abs(fft2(image));
As_1d = mean(As_2d, 1) % average all horizontal spectra
As    = As_1d(1:(length(As_1d)-1)/2+1)
freqs = linspace(0, 1/2, length(As))
```



Example: notch filter

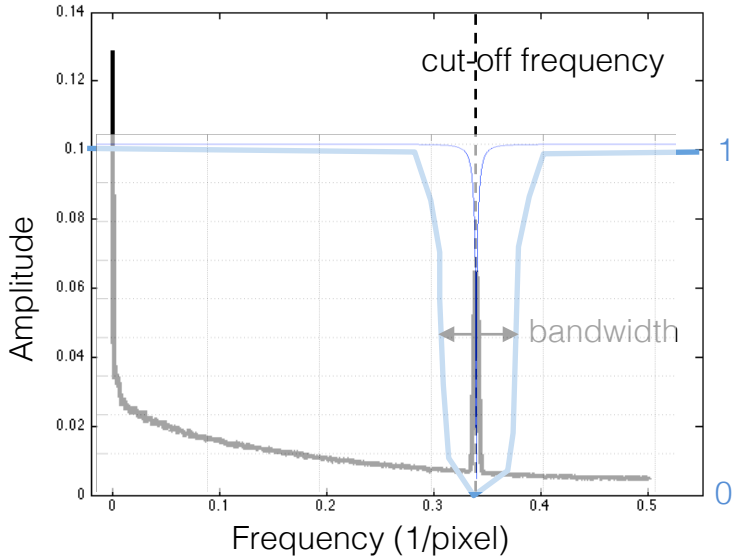


MATLAB (odd length signal):

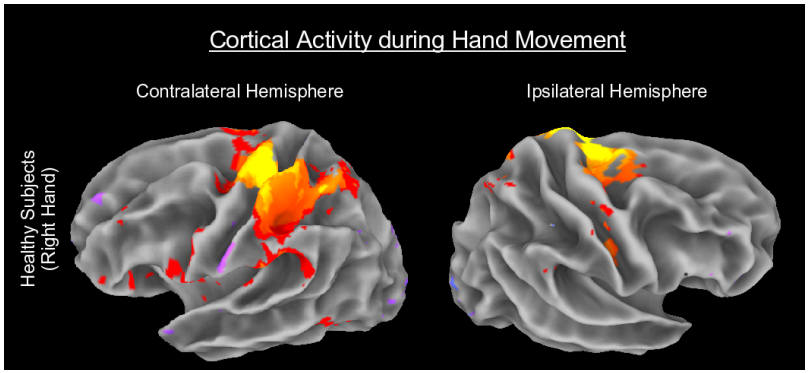
```
As_2d = (1/length(image(:)))*abs(fft2(image));
As_1d = mean(As_2d, 1) % average all horizontal spectra
As    = As_1d(1:(length(As_1d)-1)/2+1)
freqs = linspace(0, 1/2, length(As))
```

MATLAB (notch filter design):

```
cutoff      = 0.34
bandwidth   = .05
[num, den]  = iirnotch(cutoff, bandwidth)
denoised_im = filter(num, den, image)
```

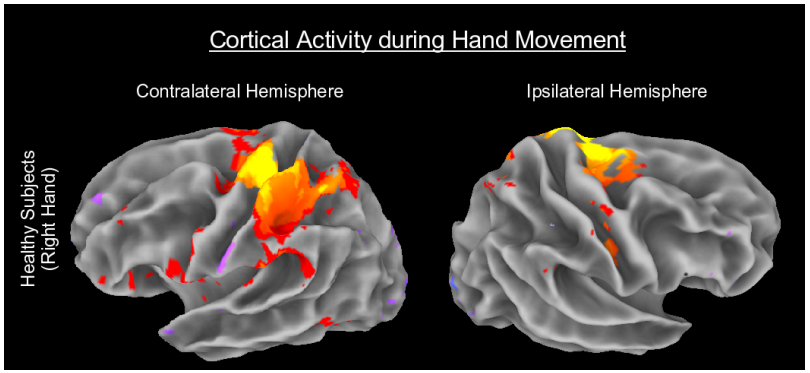


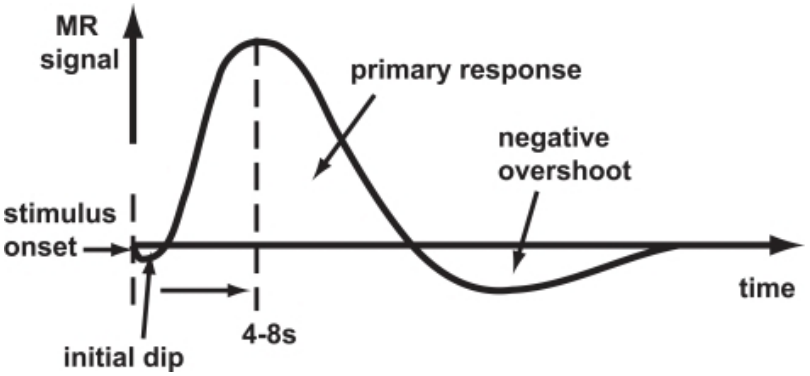
Example: Deconvolution and fMRI



fMRI

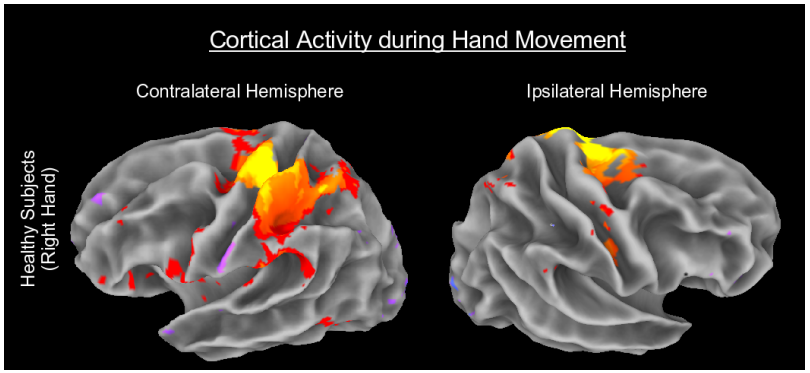
Example: Deconvolution and fMRI

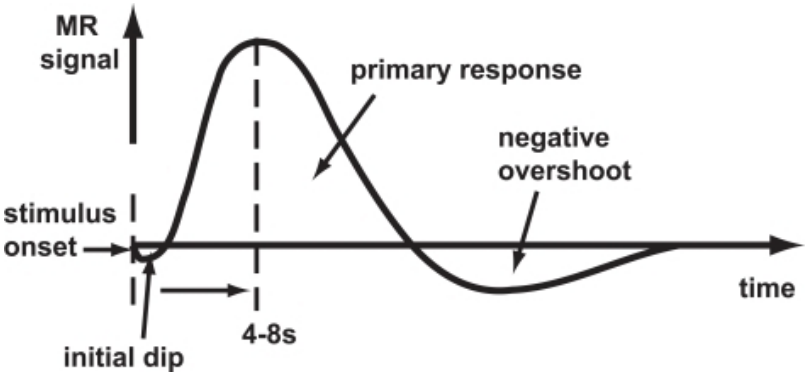




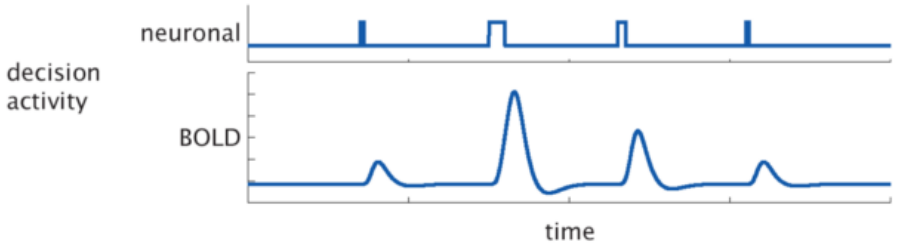
average BOLD response

Example: Deconvolution and fMRI





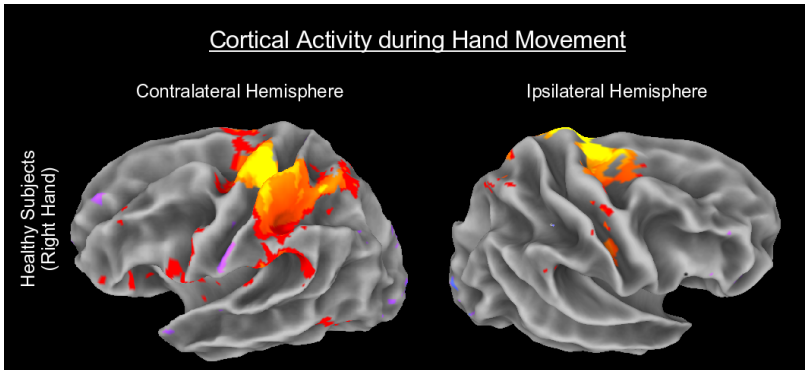
average BOLD response

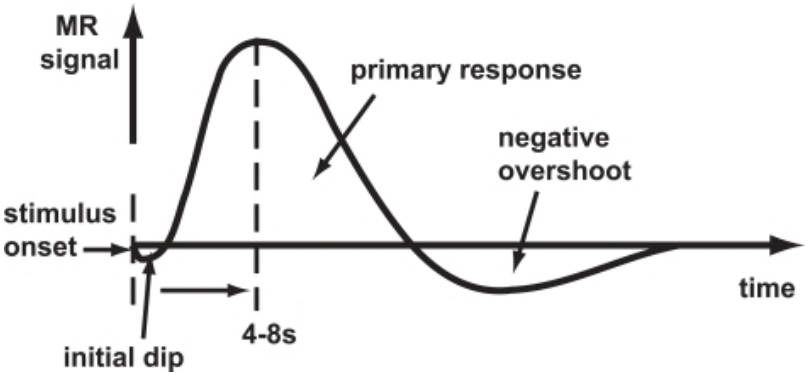


BOLD signal is the convolution of the original neuronal response and the average BOLD response

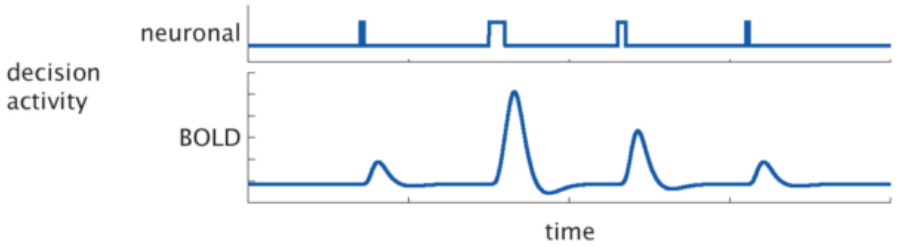
How do we get back the original neuronal response?

Example: Deconvolution and fMRI





average BOLD response



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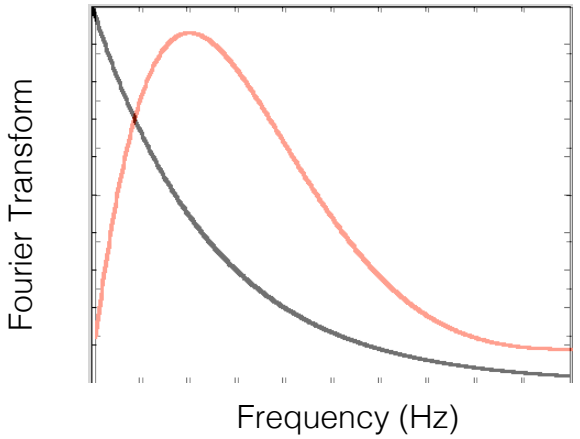
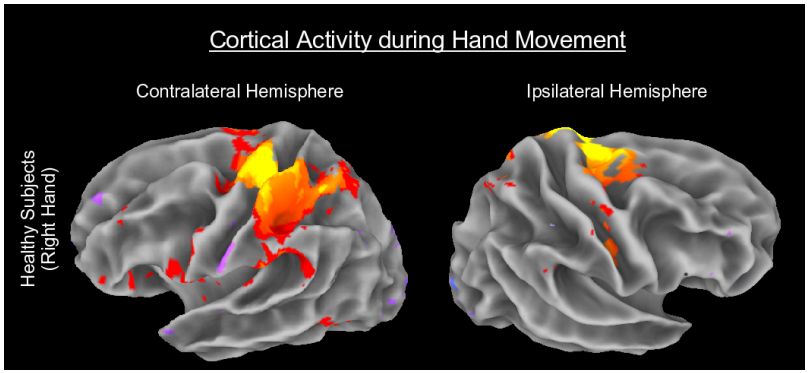
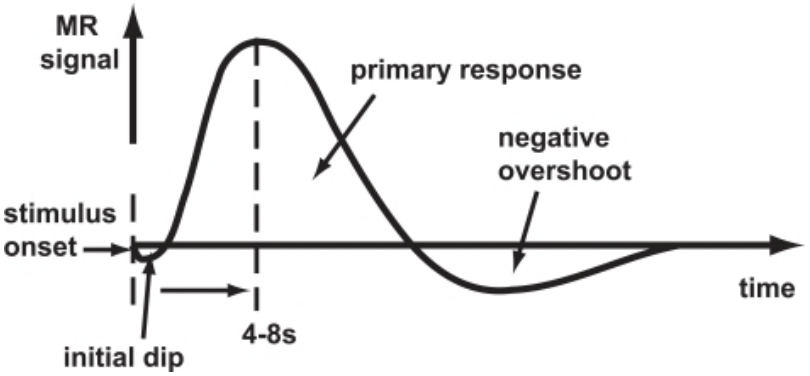
How do we get back the original neuronal response?

BOLD signal, average BOLD response

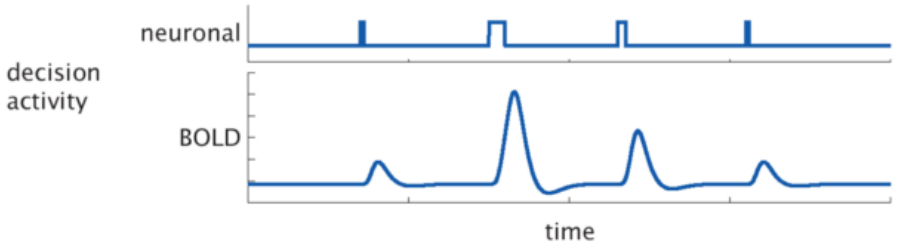


Example: Deconvolution and fMRI





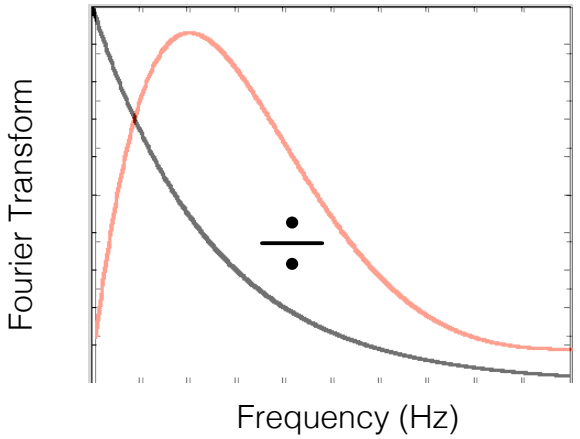
average BOLD response



BOLD signal is the convolution of the original neuronal response and the average BOLD response

How do we get back the original neuronal response?

BOLD signal, average BOLD response



MATLAB:

```
neuronal_fft  = signal_fft ./ avg_response_fft
neuronal      = ifft(neuronal_fft)

% alternatively, just do this:
neuronal = deconv(signal, avg_response)
```

Further Reading

Analyzing behavior

Long and Fee, 2008. “Using temperature to analyse temporal dynamics in the songbird motor pathway” *Nature*

Cortical circuits

Giocomo et al. 2007. “Temporal frequency of subtreshold oscillations scales with entorhinal grid cell field spacing” *Science*

Uhlhaas and Singer, 2010. “Abnormal neural oscillations and synchrony in schizophrenia.” *Nature Neuro Reviews*

Coding

Pollen and Ronner, 1982. “Visual cortical neurons as localized spatial frequency filters.” *IEEE*

van Hateren, 1992. “Theoretical predictions of spatiotemporal receptive fields of fly LMCs, and experimental validation” *Journal of Comp. Physiol.*

Acknowledgements

Some of the slides on Fourier transforms were adapted from the original Math Tools powerpoint slides