Problem Set 3 – Convolution, Filtering, and Fourier Spectra

1. Draw the power spectra for the following functions:

   (a) $f(t)=\sin(2\pi t) + 2\sin(4\pi t) + 3\sin(6\pi t)$
   (b) $f(t)=3\sin(2\pi t) + \sin(4\pi t) + 4\sin(6\pi t)$
   (c) $f(t)=2\sin(2\pi t) + 2\sin(4\pi t) + \sin(6\pi t)$

2. You have collected some data, and transformed the data into the frequency domain using the fourier transform function in matlab. Here's what you get at four different frequencies:

Frequency	Complex amplitude
2 kHz	1+2i
3 kHz	3+4i
4 kHz	2-4i
5 kHz	5-i

   a. Calculate the magnitude and the phase of the signal at each frequency.
   b. Which frequency contains the most power?

3. Without calculating anything, draw the following convolutions:
   a. The cross-correlation of a sine wave and a cosine wave
   b. The autocorrelation of a sine wave

4. You have a large dataset of calcium imaging movies, but due to low light the images are very grainy, which you suspect looks like additive white noise. To get a cleaner signal, you might want to try filtering with a:
   a. Bandpass, or biphasic filter that takes the derivative across local pixels
   b. High pass filter that keeps high frequencies
   c. Low pass filter that keeps low frequencies and averages across local pixels
   d. Notch filter to remove one particular frequency

5. In MATLAB, construct a 10,000 point time series using the `randn` function, and let the time axis go from 0 seconds to 10 seconds (so our sample rate is 1 kHz).
   a. Using the `conv` function, smooth this time series with a filter of your choice (could be Gaussian, exponential, box, etc.). Plot the time series.
   b. Determine the one-sided amplitude spectrum of this signal and report the mean of your time series just by examining the amplitude spectrum (i.e. you want to look at the amplitude at which frequency?).
   c. Get the one-sided **power spectrum** (the square of the amplitude spectrum) with appropriate units on the x-axis. Plot it.