Math Tools for Neuroscience
Handout 1: Linear Algebra

January 11, 2017

1 Overview

These exercises are designed to give you some familiarity with matrix multiplication.

2 Dot products

Prompt. *Complete the following matrix multiplications. Before computing the multiplication, first write out what the dimensionality of the final product should be.*

$$x^T y = \begin{pmatrix} 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \qquad Wx = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad XX^T = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{pmatrix} \cdot \begin{pmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \\ x_{13} & x_{23} \end{pmatrix}$$

3 Outer products

Prompt. *The first matrix multiplication we learned was the element-wise multiplication, where as an example we scaled each variable by its standard deviation,*

$$\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} ./ \begin{pmatrix} \sigma_x & \sigma_x \\ \sigma_y & \sigma_y \end{pmatrix} = \begin{pmatrix} x_1/\sigma_x & x_2/\sigma_x \\ y_1/\sigma_y & y_2/\sigma_y \end{pmatrix}.$$

How can we quickly construct $\Sigma = \begin{pmatrix} \sigma_x & \sigma_x \\ \sigma_y & \sigma_y \end{pmatrix}$ *in MATLAB using an outer product? Given a 2 dimensional column vector of standard deviations (*`std(A')'` $= (\sigma_x \ \sigma_y)^T$ *in MATLAB), construct* Σ *using an outer product instead of a* `for` *loop.*

4 Rank

Prompt. *What is the rank of the following matrices?*

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$