Problem Set for Dynamical Systems

January 26, 2016

Problem 1

Suppose a vat contains 100 liters of liquid. An impressive stir bar sits at the bottom of the vat and keeps the solution inside well-mixed at all times. Now, suppose that there are two pipes (pipe A and pipe B) that pump liquid into this vat, and one pipe (pipe C) that drains liquid from the vat. Sugar water at a concentration of 30 grams/liter enters the vat at 2 liters/min through pipe A, and sugar water at a concentration of 45 grams/liter enters the vat at 1 liter/min through pipe B. Finally, sugar water leaves pipe C at a rate of 3 liters/min.

A) Write the differential equation describing the rate of change of sugar ($\frac{dS}{dt}$, where S is the amount of sugar in grams) in this vat over time.

B) Solve this differential equation. Find a particular solution using $S(0) = 0$ as your initial condition. What happens to the amount of sugar in the vat over time?

Problem 2

Simulate a leaky integrate-and-fire neuron using the numerical method (Euler's method) described in class. You can do this by completing the MATLAB script available on the website.

A) Plot your result.

Problem 3

The quadratic integrate-and-fire model is given by the following differential equation:

$$\tau \frac{dv}{dt} = \alpha(v - v_{rest})(v - v_{crit}) + RI$$

where $\alpha > 0$, $v_{rest} < v_{crit}$, and I corresponds to the amount of applied current. Similar to the leaky integrate-and-fire model neuron we discussed in class, a threshold is placed on the voltage such that if v reaches v_{thresh}, v is reset back to v_{reset}.

A) Assume (for now) that I = 0. Qualitatively analyze this differential equation. What happens if $v = v_{rest}$? Or if $v = v_{crit}$? Or if $v_{rest} < v < v_{crit}$? How does this compare to the leaky integrate and fire model neuron?

B) OPTIONAL: How do the fixed points (specifically, the number of fixed points and their stability) change with the amount of applied current? In the dynamical systems field, a qualitative change in the behavior of a system due to a change in a parameters is called a bifurcation.