Convergence rate of $X^N - X$ for McKean equations

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1 What is this?

An adaptation of [1, Proposition 3.1] for the case of McKean SDEs proposed in [2].

The proof builds on a number of results presented in the sections below.

Definition 1 *For any real-valued continuous semi-martingale, the local time at zero $L_t^0(\bar{Y})$ is defined as*

$$L_t^0(\bar{Y}) = \lim_{\epsilon \to 0} \frac{1}{2\epsilon} \int_0^t \mathbb{1}_{\{|\bar{Y}| \leq \epsilon\}} d\langle \bar{Y} \rangle_s, \ \mathbb{P}\text{-}a.s. \tag{1}$$

For all $t \geq 0$.

2 Lemma 5.1

The first result, [1, Lemma 5.1], is not necessary to prove for this particular setting since the result holds for any semi-martingale, I include it here for self-containment reasons.

Lemma 1 (Lemma 5.1) *For any $\epsilon \in (0, 1)$ and any real-valued, continuous semi-martingale*

3 Lemma 5.2

Lemma 5.2 builds on the following result:

Proposition 1 *h*

Proposition 2 (Adaptation of Lemma 5.2) *Let $\beta \in (0, 1/2)$ and $b \in C_T\mathcal{C}^{-\beta}$. Let $u, u^N \in C_T\mathcal{C}^{(1+\beta)+}$ be (mild) solutions to the Kolmogorov equations*

$$\begin{cases} \partial u_i + \frac{1}{2} b_i \Delta u_i = \lambda u_i - b_i \\ u_i(T) = 0, \end{cases} \tag{2}$$

and

$$\begin{cases} \partial u_i^N + \frac{1}{2} b_i^N \Delta u_i^N = \lambda u_i^N - b_i^N \\ u_i^N(T) = 0. \end{cases} \tag{3}$$

From $\alpha > \beta$

4 Lemma 5.3

5 Proposition 5.4

6 Proposition 3.1 (main result)

References

[1] Tiziano De Angelis, Maximilien Germain, and Elena Issoglio. “A Numerical Scheme for Stochastic Differential Equations with Distributional Drift”. In: *arXiv:1906.11026 [cs, math]* (Oct. 22, 2020). arXiv: `1906.11026`.

[2] Elena Issoglio and Francesco Russo. “McKean SDEs with singular coefficients”. In: *arXiv:2107.14453 [math]* (July 30, 2021). arXiv: `2107.14453`.