Heapsort

Wednesday, April 2, 2025 12:25 PM

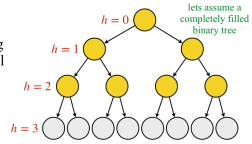
Heapsort

Practice

- Build a max-heap from the following array using BuildHeap
  - 10 42 25 13 17 33 45 50

Analysis

- Total cost
  - sum of the costs of calling **downHeap** for all internal nodes (sum of heights)



$$T(n) = 1(h) + 2(h-1) + 4(h-2) + \ldots + 2^h(0)$$
$$= \sum_{i=0}^{h} 2^i(h-i)$$
$$= h\sum_{i=0}^{h} 2^i - \sum_{i=0}^{h} i2^i = \ldots\ \ldots\ \ldots = \Theta(n)$$

Performance

Method	Unsorted Array	Sorted Array	Binary Heap
Enqueue	O(1)	O(n)	O(log n)
Dequeue	O(n)	O(1)	O(log n)
Max	O(n)	O(1)	O(1)
Size	O(1)	O(1)	O(1)
IsEmpty	O(1)	O(1)	O(1)
Enqueue N	O(n)	O(n²)	O(n)**

(**) assuming we use **buildHeap**

Practice

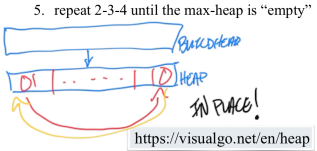
- What is this function doing?
- what is the time complexity?
  - depends on the running time of the priority queue operations

```
void foo(std::vector<int>& vec) {
    int n = vec.size();
    std::priority_queue<int> pq;

    for (auto& elem : vec)
        pq.push(elem);

    while (!pq.empty()) {
        vec[--n] = pq.top();
        pq.pop();
    }
}
```

heapSort

heapSort

- Algorithm
  1. use **buildHeap** to create a max-heap from the input array
  2. swap the root with the last element in the array
  3. "remove" the last element from the array (decrement the size)
  4. apply **downHeap** to the new root to restore the heap property
  5. repeat 2-3-4 until the max-heap is "empty"



https://visualgo.net/en/heap

Practice

- Apply heapSort to the following array
  - 10 42 25 13 17 33 45 50 20

Analysis

- Step 1
  - cost of **buildHeap** => $\Theta(n)$
- Steps 2-5
  - we apply **downHeap** O(n) times
  - cost => $\Theta(n \log n)$
- Heapsort cost => $\Theta(n \log n)$