CSC 211: Computer Programming

(Recursive) Backtracking

Michael Conti

Department of Computer Science and Statistics
University of Rhode Island

Spring 2025

THINK BIG WE DO℠

Recursion Reminder

- Problem solving technique in which we solve a task by reducing it to smaller tasks (of the same kind)
  - ✓ then use same approach to solve the smaller tasks
- Technically, a recursive function is one that calls itself
- General form:
  - ✓ **base case**
    - solution for a **trivial case**
    - it can be used to stop the recursion (prevents "*stack overflow*")
    - every recursive algorithm needs at least one base case
  - ✓ **recursive call(s)**
    - divide problem into **smaller instance(s)** of the **same structure**

Recursion Reminder

- Recursive Checklist:
  - ✓ **Find what information we need to keep track of.** What inputs/outputs are needed to solve the problem at each step?
  - ✓ **Find our base case(s).** What are the simplest (nonrecursive) instance(s) of this problem?
  - ✓ **Find our recursive step.** How can this problem be solved in terms of one or more simpler instances of the same problem that lead to a base case?
  - ✓ **Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?

Recursion Reminder

- Recursive Checklist:
  - ✓ **Find what information we need to keep track of.** What inputs/outputs are needed to solve the problem at each step?
  - ✓ **Find our base case(s).** What are the simplest (nonrecursive) instance(s) of this problem?
  - ✓ **Find our recursive step.** How can this problem be solved in terms of one or more simpler instances of the same problem that lead to a base case?
  - ✓ **Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?

Backtracking

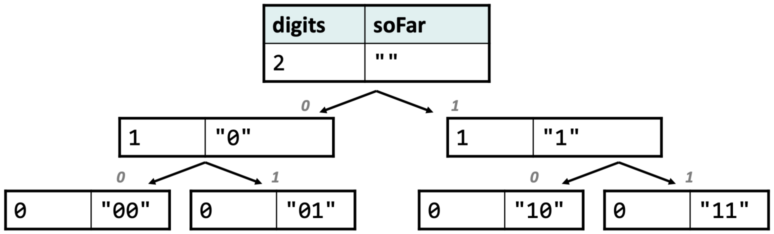
- Write a recursive function printAllBinary that accepts an integer number of digits and prints all binary numbers that have exactly that many digits, in ascending order, one per line

printAllBinary(2); **printAllBinary(3);**

```
00        000
01        001
10        010
11        011
          100
          101
          110
          111
```

Decision Trees

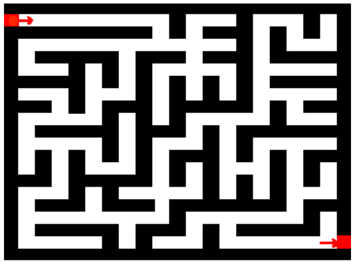
printAllBinary(2);



- This kind of diagram is called a **call tree** or **decision tree**
- Think of each call as a choice or decision made by the algorithm:
  - Should I choose 0 as the next digit?
  - Should I choose 1 as the next digit?
- The idea is to try every permutation. For every position, there are 2 options, either ‘0’ or ‘1’. **Backtracking** can be used in this approach to try every possibility or permutation to generate the correct set of strings.

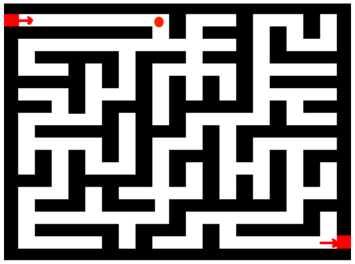
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable



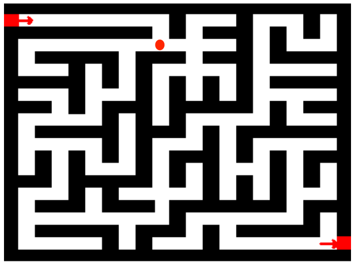
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable



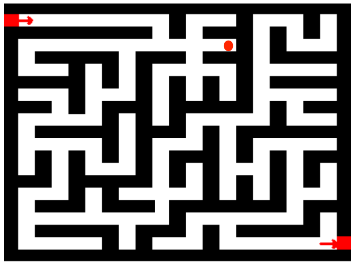
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable



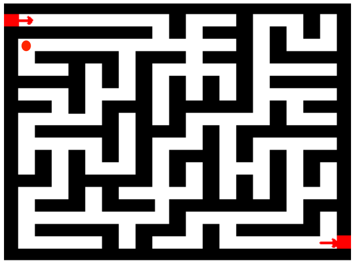
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable



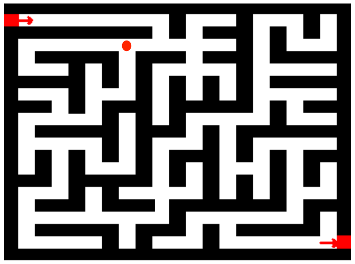
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable



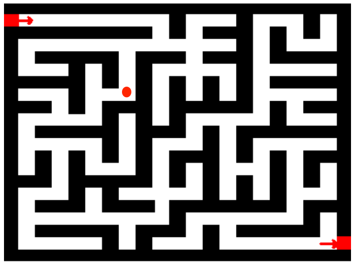
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable



Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable
-



Backtracking

- Let's take a look at a problem similar to the binarySequence problem.
- Write a recursive function diceRoll that accepts an integer representing a number of 6-sided dice to roll, and output all possible permutations of values that could appear on the dice.

diceRoll(2)

{1,1}	{3, 1}	{5, 1}
{1, 2}	{3, 2}	{5, 2}
{1, 3}	{3, 3}	{5, 3}
{1, 4}	{3, 4}	{5, 4}
{1, 5}	{3, 5}	{5, 5}
{1, 6}	{3, 6}	{5, 6}
{2, 1}	{4, 1}	{6, 1}
{2, 2}	{4, 2}	{6, 2}
{2, 3}	{4, 3}	{6, 3}
{2, 4}	{4, 4}	{6, 4}
{2, 5}	{4, 5}	{6, 5}
{2, 6}	{4, 6}	{6, 6}

Backtracking

- Backtracking Checklist:
  - ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

    For each valid choice:

    - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
    - **Undo it after exploring.** Restore everything to the way it was before making this choice.
  - ✓ **Find our base case(s).** What should we do when we are out of decisions?

Backtracking

- Backtracking Checklist:
  - ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

    For each valid choice:

    - **Make it and explore recurs** r a choice to the next recursive
    - **Undo it after exploring.** Restore everything to the way it was before making this choice.
  - ✓ **Find our base case(s).** What should we do when we are out of decisions?

What die value should I choose next?

Backtracking

- Backtracking Checklist:
  - ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

    For each valid choice:

    - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
    - **Undo it after exploring.** Restore everything to the way it was before making this choice.
  - ✓ **Find our base case(s).** What should we do when we are out of decisions?

We need to communicate the dice chosen so far to the next recursive call

Backtracking

- Backtracking Checklist:
  - ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

    For each valid choice:

    - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
    - **Undo it after exploring.** Restore everything to the way it was before making this choice.
  - ✓ **Find our base case(s).** What should we do when we are out of decisions?

We need to be able to remove the die we added to our first roll so far

Backtracking

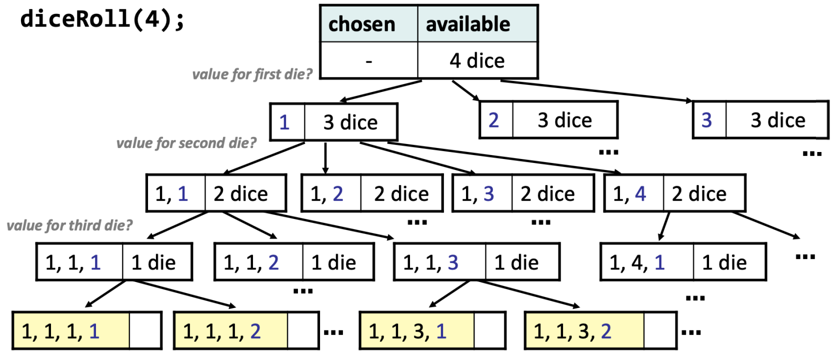
- Backtracking Checklist:
  - ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

    For each valid choice:

    - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
    - **Undo it after exploring.** Restore everything to the way it was before making this choice.
  - ✓ **Find our base case(s).** What should we do when we are out of decisions?

We have no dice left to choose, print them out

Backtracking



diceRoll(4);

chosen	available
-	4 dice

value for first die?

1 | 3 dice — 2 | 3 dice ... — 3 | 3 dice ...

value for second die?

1, 1 | 2 dice — 1, 2 | 2 dice ... — 1, 3 | 2 dice ... — 1, 4 | 2 dice ...

value for third die?

1, 1, 1 | 1 die — 1, 1, 2 | 1 die ... — 1, 1, 3 | 1 die — 1, 4, 1 | 1 die ...

1, 1, 1, 1 — 1, 1, 1, 2 ... — 1, 1, 3, 1 — 1, 1, 3, 2 ...

- Observations?
- This is a really big search space.
- Depending on approach, we can make wasteful decisions.
  Can we optimize it? Yes. Will we right now? No.

Backtracking

- Let’s us write flexible code, allowing us to make a decision and “backtrack” if we need to

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Backtracking

- **Pseudocode**

- function diceRolls(dice, chosenArr):

```
if dice == 0:
    Print current roll.
else:
    // handle all roll values for a single die; let recursion do the rest.
    for each die value i in range [1..6]:
        choose that the current die will have value i
        // explore the remaining dice
        diceRolls(dice-1, chosenArr)
        un-choose (backtrack) the value I
```

- Write a recursive function diceRoll that accepts an integer representing a number of 6-sided dice to roll, and output all possible combinations of values that could appear on the dice.

** Need to keep track of our choices somehow

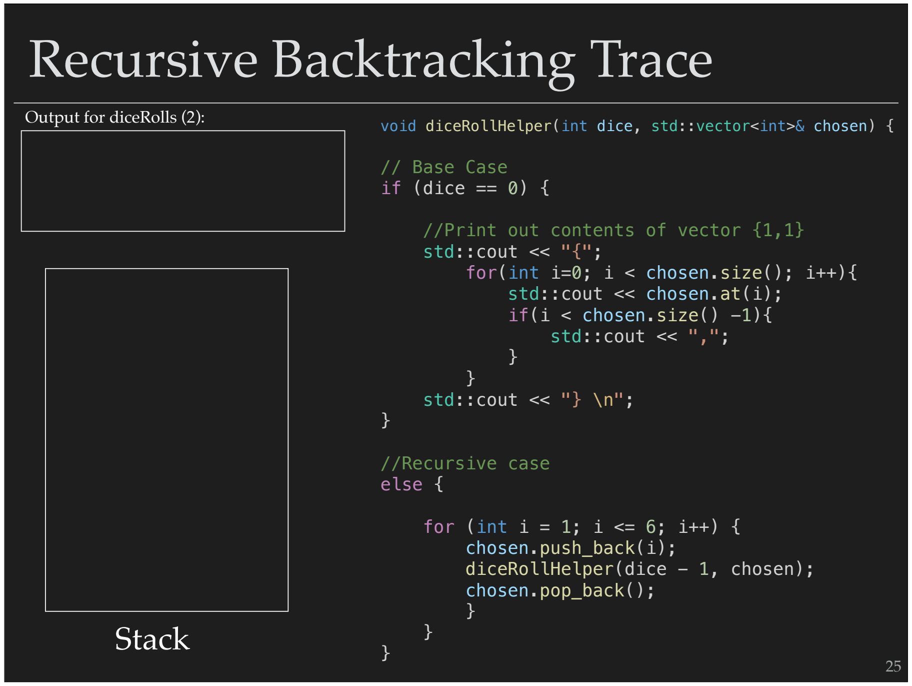
Code Demo

Recursive Backtracking Trace

Output for diceRolls (2):

```
//create vector and call driver function
void diceRolls(int dice) { // dice = 2
    std::vector<int> chosen; // chosen = {}
    diceRollHelper(dice, chosen);
}
```

Stack

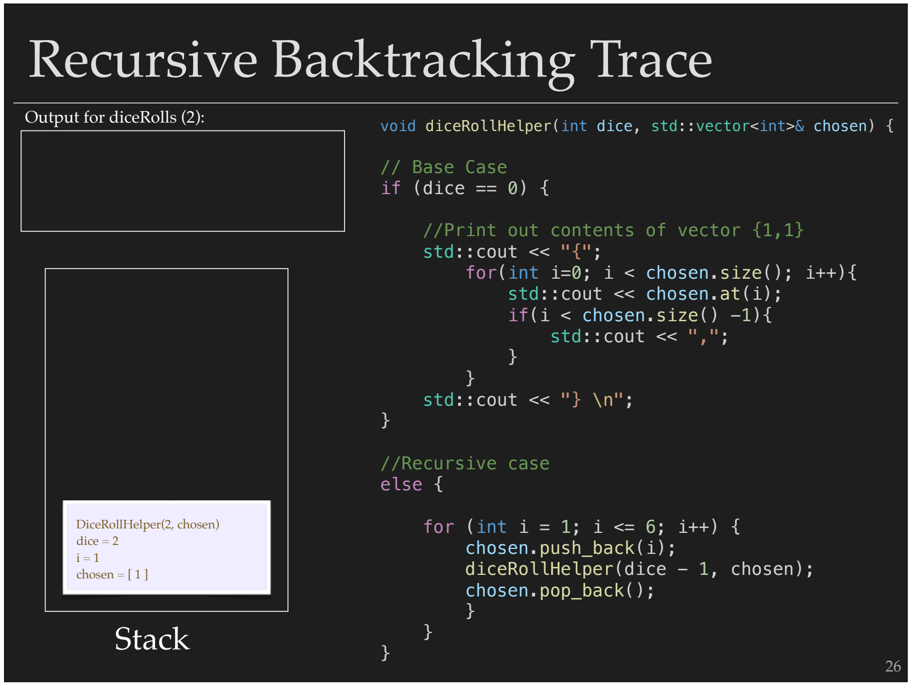


Recursive Backtracking Trace

Output for diceRolls (2):

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

Stack



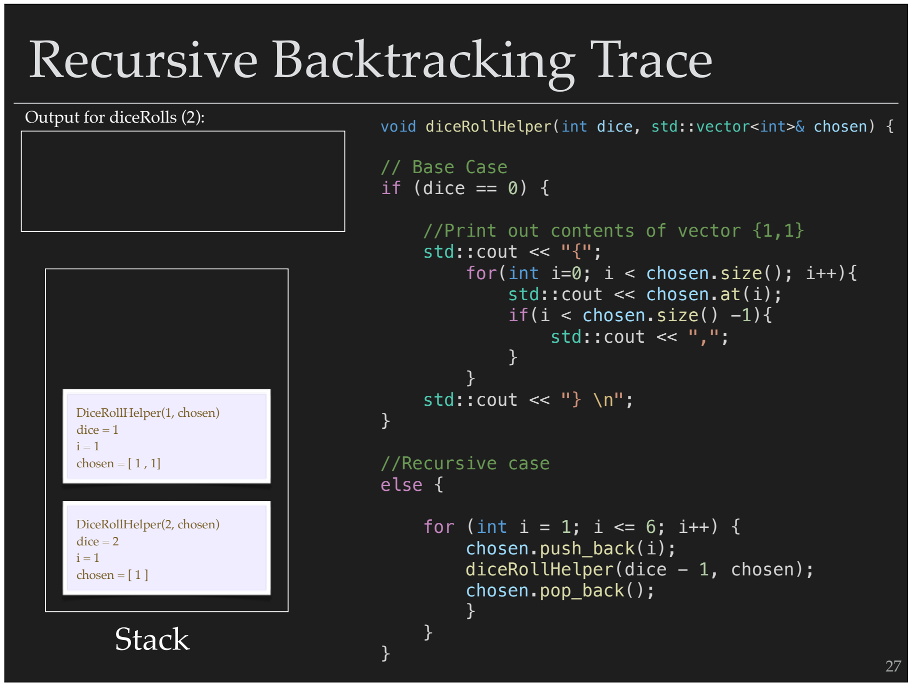
Recursive Backtracking Trace

Output for diceRolls (2):

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

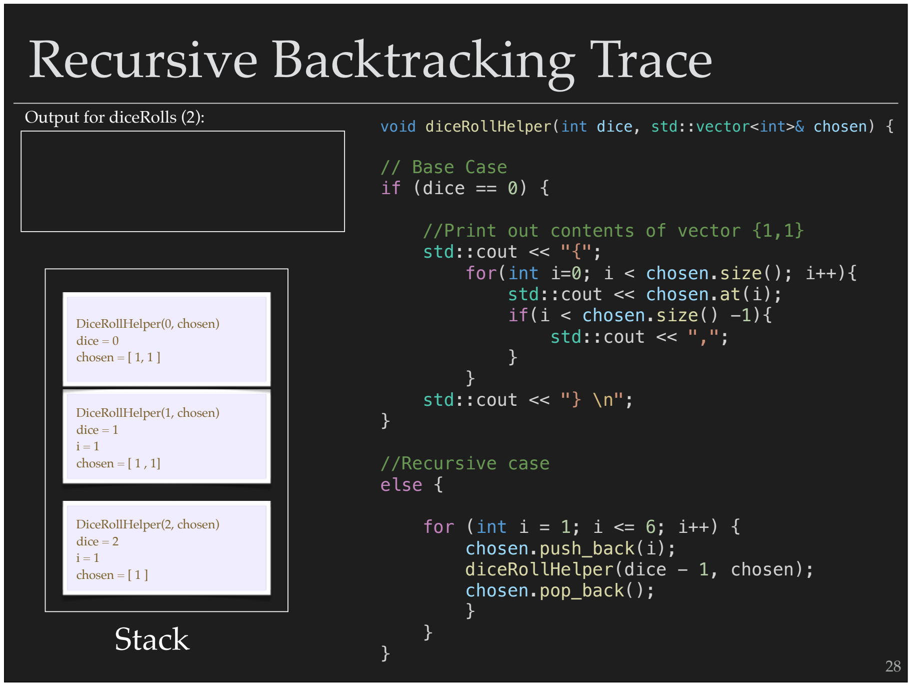
Output for diceRolls (2):

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1 , 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

Output for diceRolls (2):

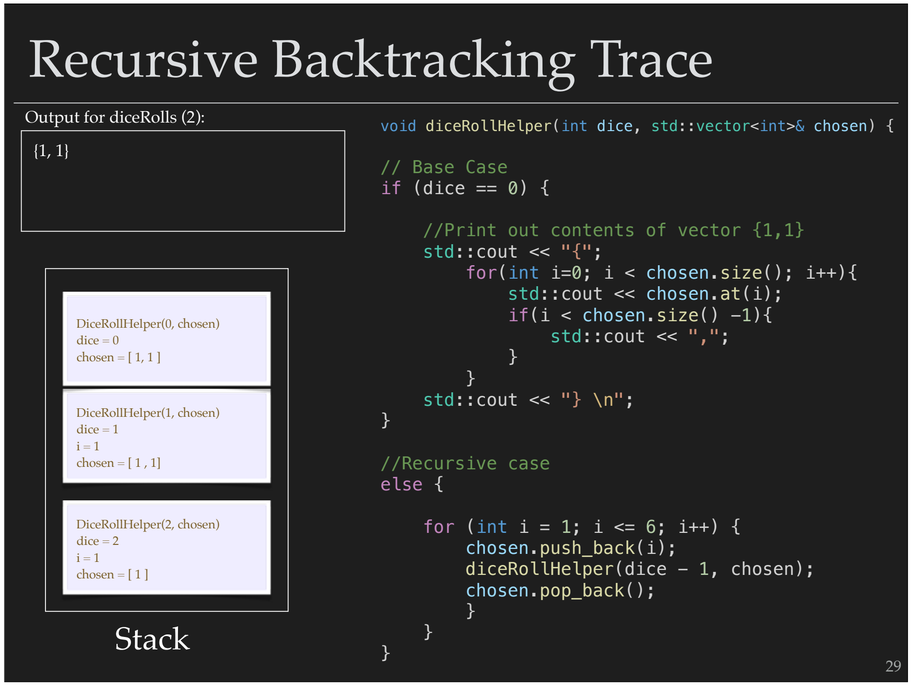
```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 1]

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1 , 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}

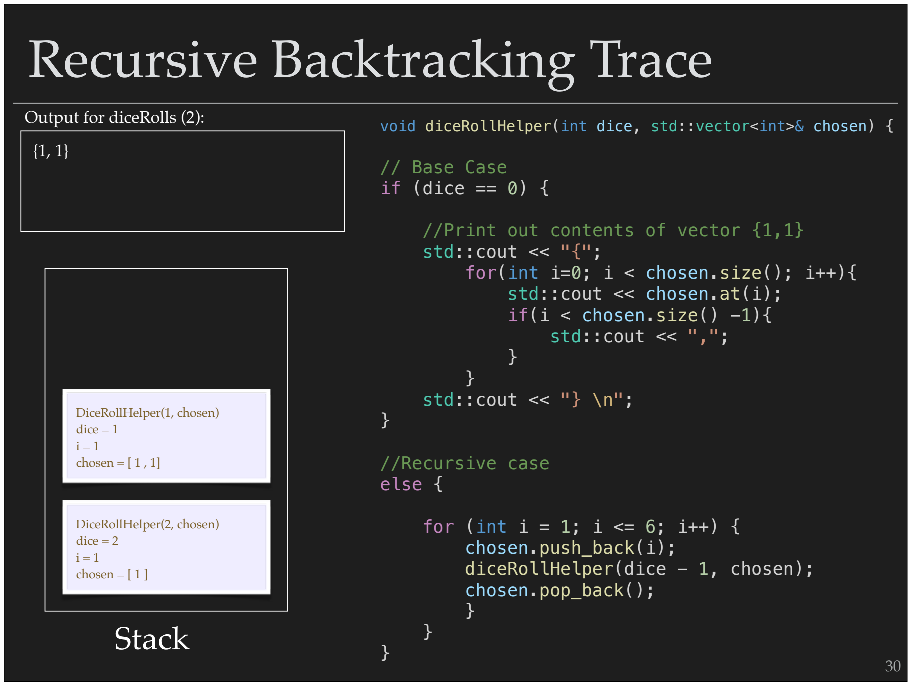
```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 1]

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1 , 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

Output for diceRolls (2):

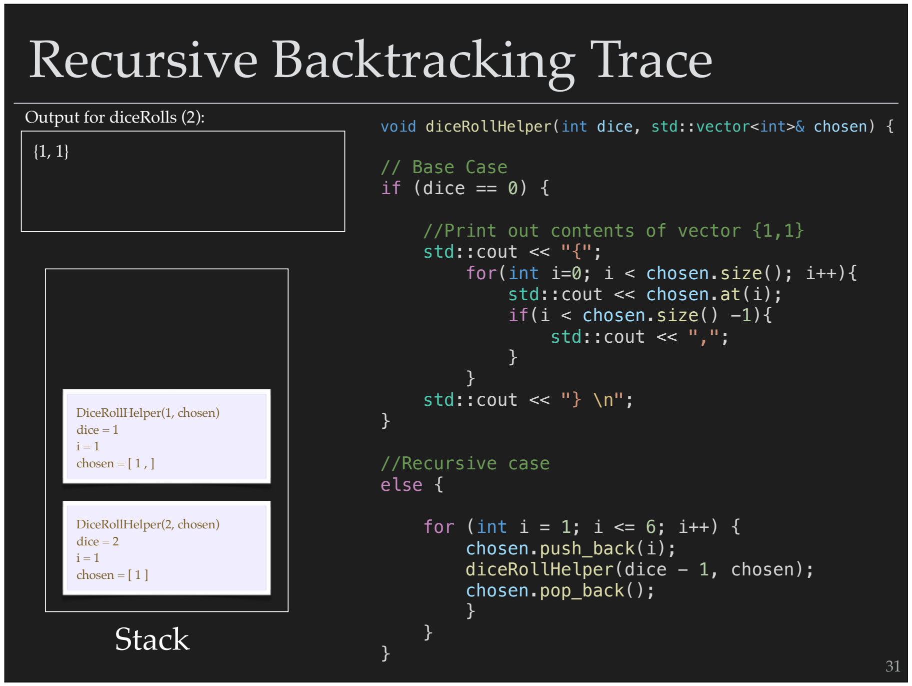
{1, 1}

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1 , 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

Output for diceRolls (2):

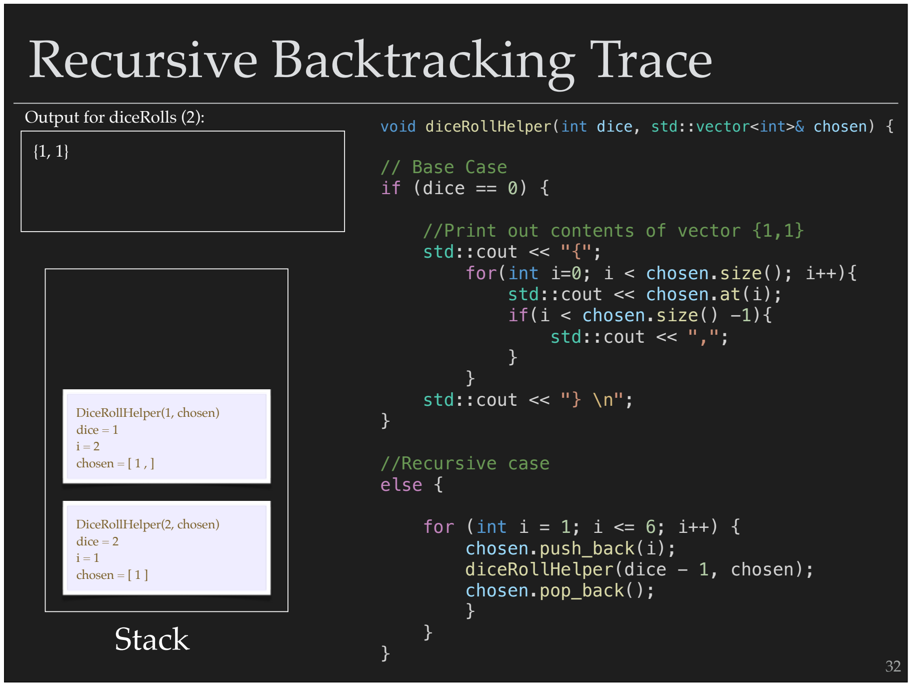
{1, 1}

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1 ,]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [1 ,]

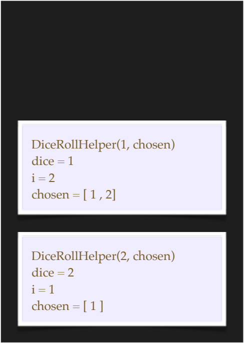
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
```



DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [1 , 2]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

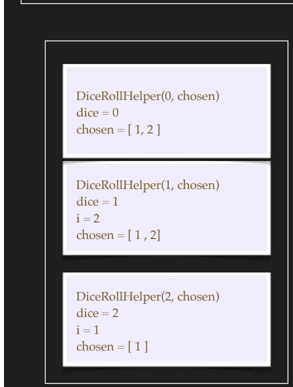
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice – 1, chosen);
        chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
```



DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 2]

DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [1 , 2]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

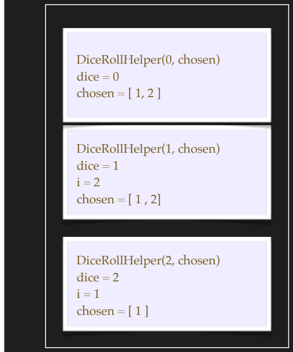
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice – 1, chosen);
        chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```



DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 2]

DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [1 , 2]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

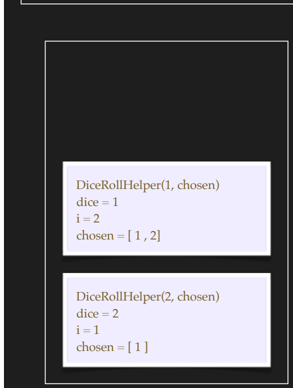
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice – 1, chosen);
        chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

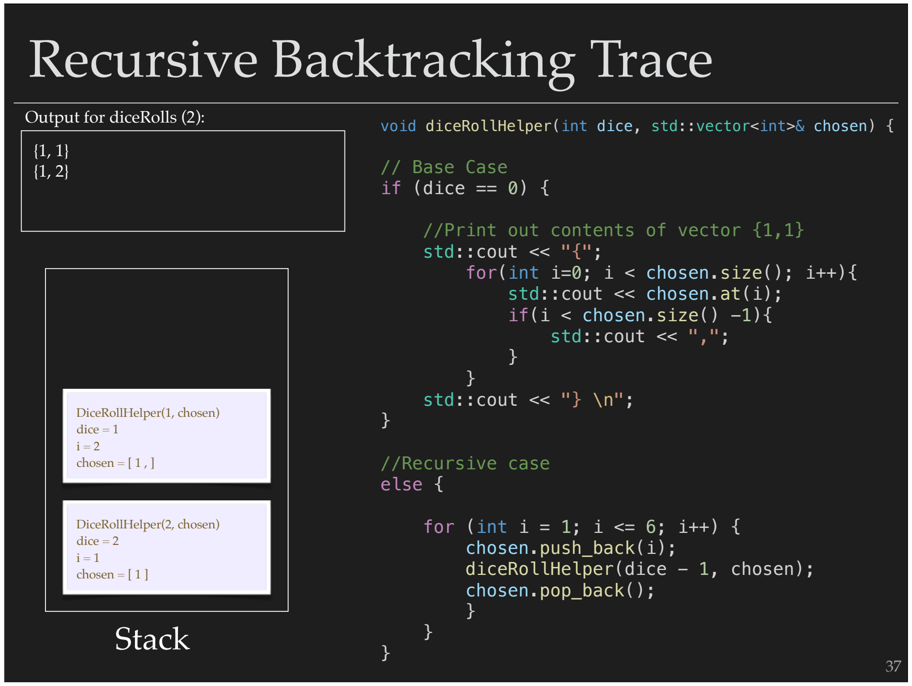


DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [1 , 2]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice – 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

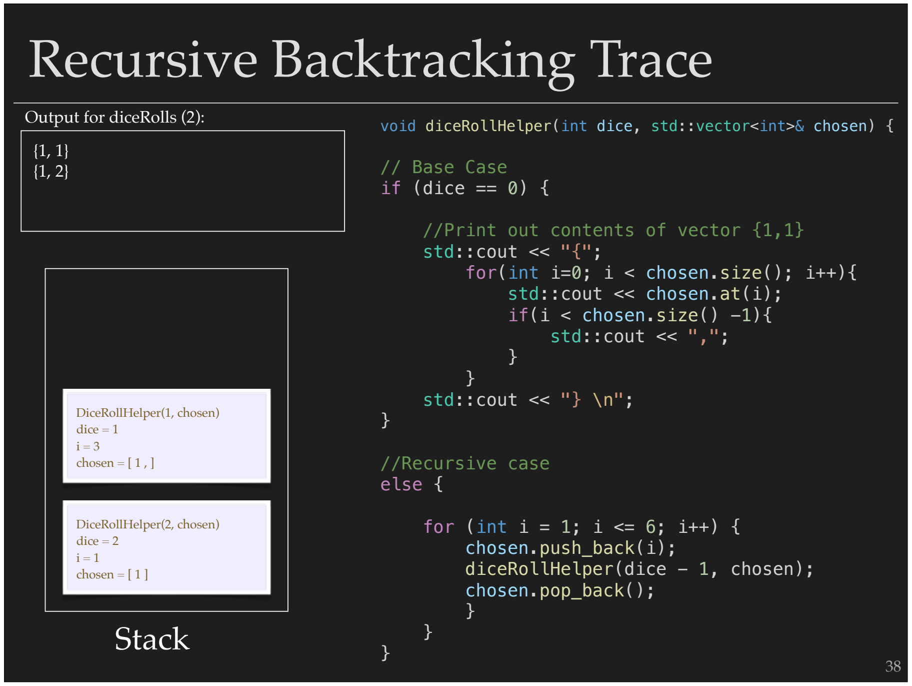
```
{1, 1}
{1, 2}
```

DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [1 ,]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

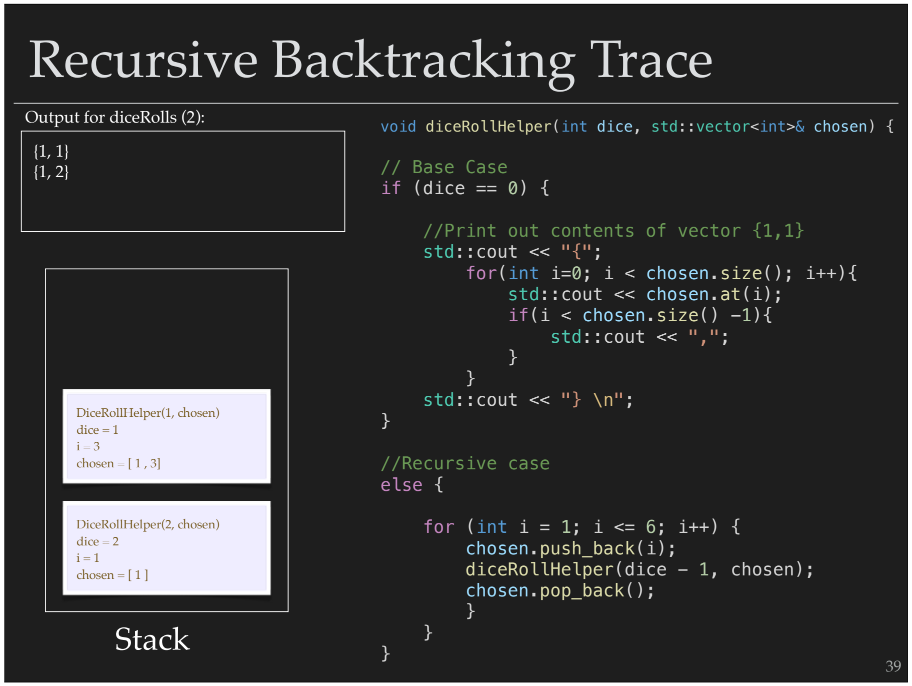
```
{1, 1}
{1, 2}
```

DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [1 ,]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

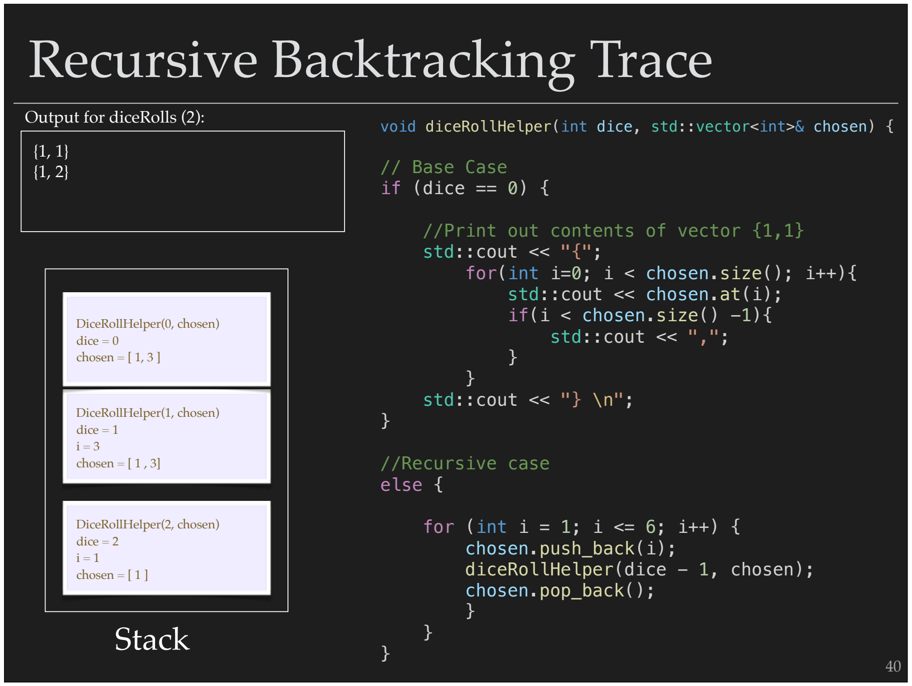
```
{1, 1}
{1, 2}
```

DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [1 , 3]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 3]

DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [1 , 3]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

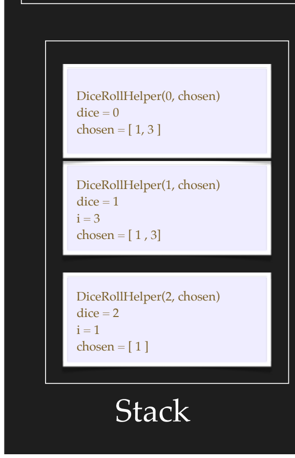
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
{1, 3}
```


DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 3]

DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [1 , 3]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

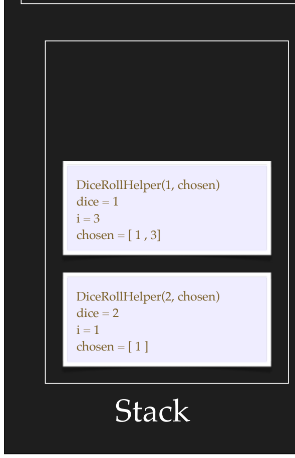
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
{1, 3}
```


DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [1 , 3]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

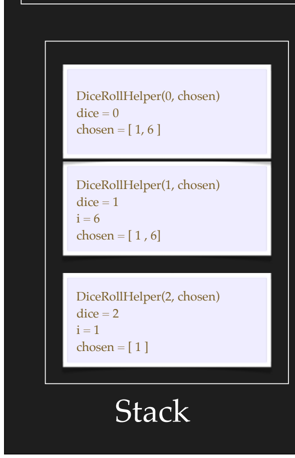
Recursive Backtracking Trace

Fastforward…

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}
{1, 3}
{1, 4}
```

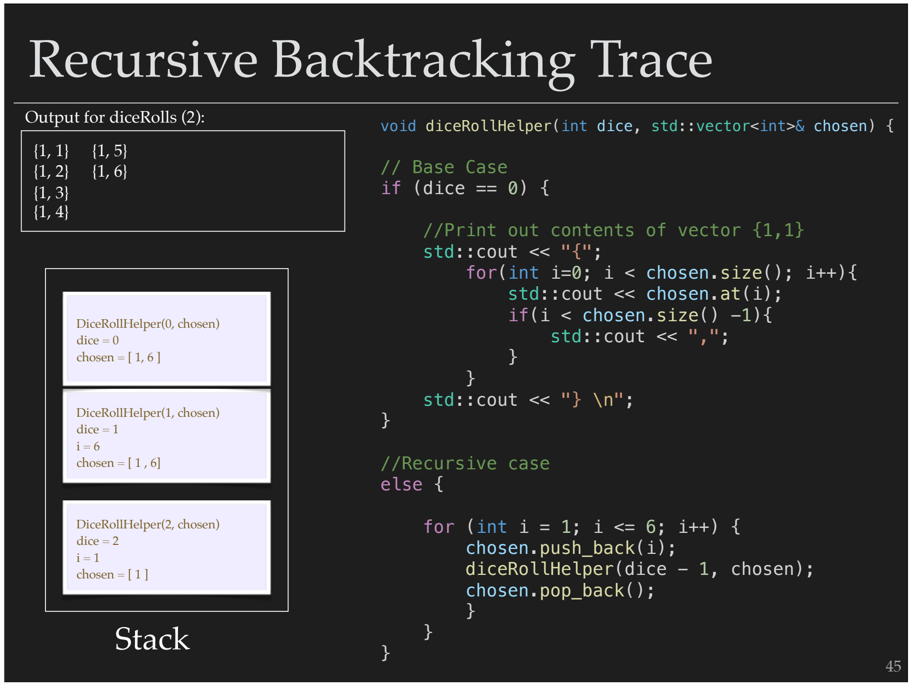

DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 6]

DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [1 , 6]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

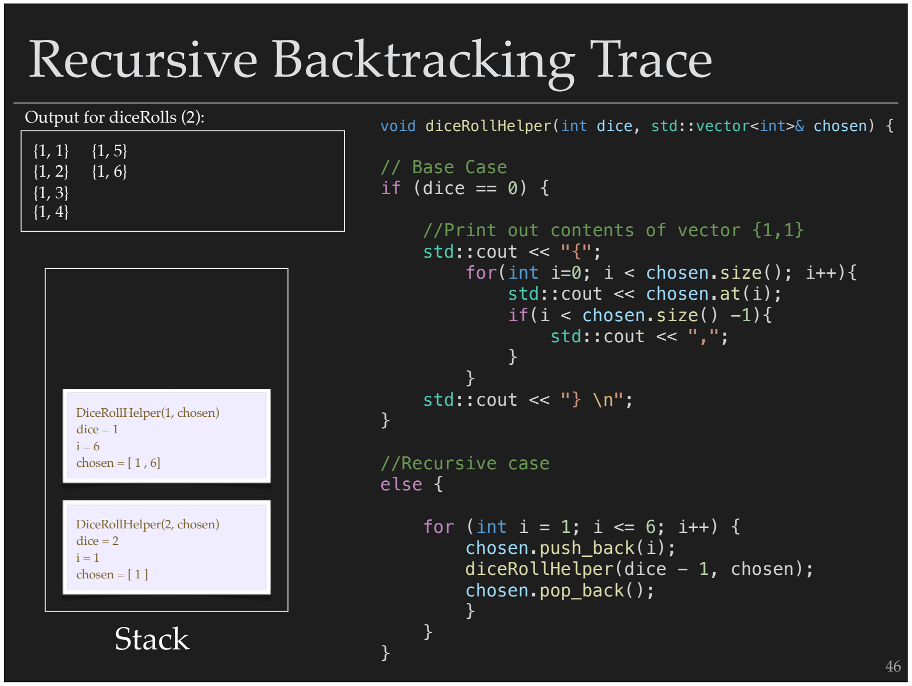
```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 6]

DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [1 , 6]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

Output for diceRolls (2):

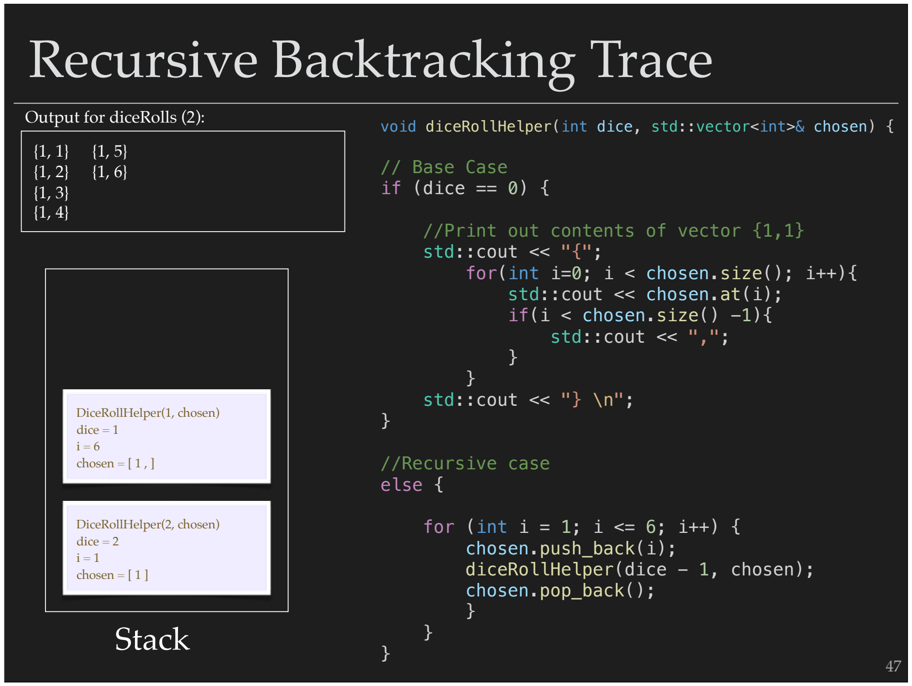
```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [1 , 6]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

Output for diceRolls (2):

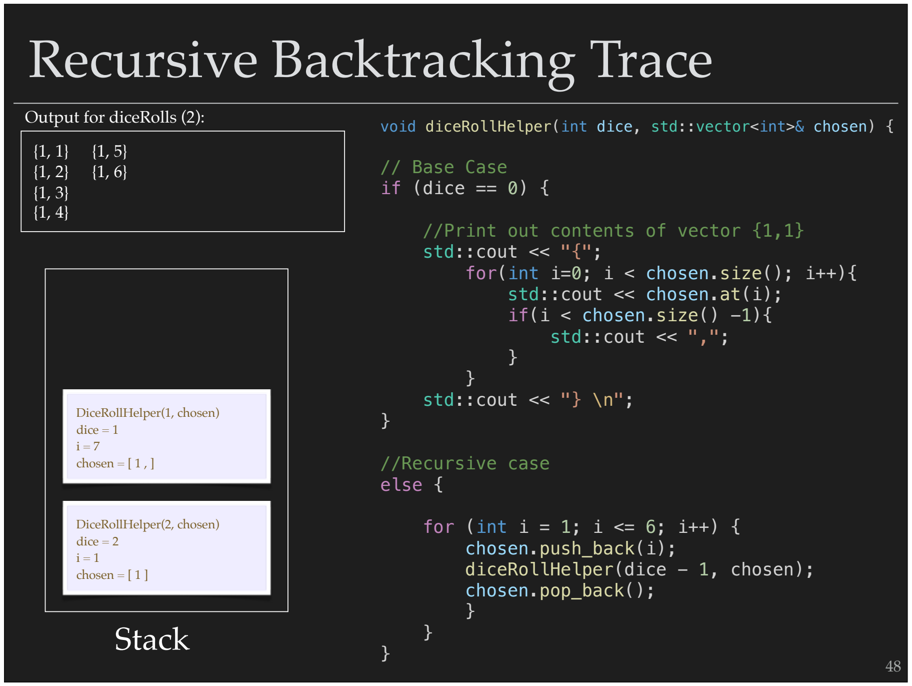
```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [1 ,]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack



Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```

DiceRollHelper(1, chosen)
dice = 1
i = 7
chosen = [1 ,]

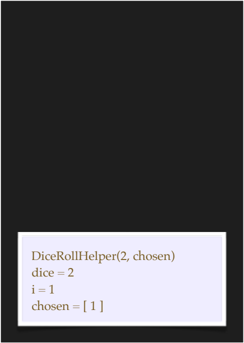
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```



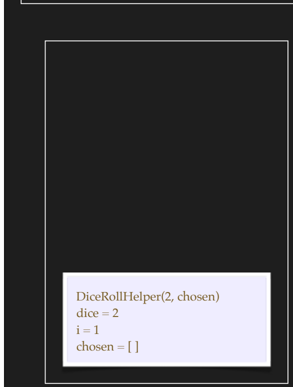
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

  //Print out contents of vector {1,1}
  std::cout << "{";
    for(int i=0; i < chosen.size(); i++){
      std::cout << chosen.at(i);
      if(i < chosen.size() -1){
        std::cout << ",";
      }
    }
  std::cout << "} \n";
}

//Recursive case
else {

  for (int i = 1; i <= 6; i++) {
    chosen.push_back(i);
    diceRollHelper(dice - 1, chosen);
    chosen.pop_back();
    }
  }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```



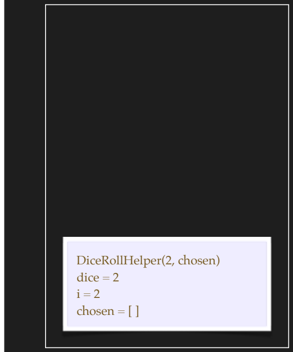
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

  //Print out contents of vector {1,1}
  std::cout << "{";
    for(int i=0; i < chosen.size(); i++){
      std::cout << chosen.at(i);
      if(i < chosen.size() -1){
        std::cout << ",";
      }
    }
  std::cout << "} \n";
}

//Recursive case
else {

  for (int i = 1; i <= 6; i++) {
    chosen.push_back(i);
    diceRollHelper(dice - 1, chosen);
    chosen.pop_back();
    }
  }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```



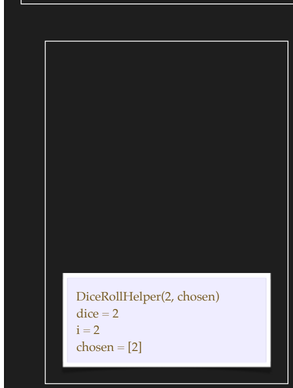
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

  //Print out contents of vector {1,1}
  std::cout << "{";
    for(int i=0; i < chosen.size(); i++){
      std::cout << chosen.at(i);
      if(i < chosen.size() -1){
        std::cout << ",";
      }
    }
  std::cout << "} \n";
}

//Recursive case
else {

  for (int i = 1; i <= 6; i++) {
    chosen.push_back(i);
    diceRollHelper(dice - 1, chosen);
    chosen.pop_back();
    }
  }
}
```

Recursive Backtracking Trace

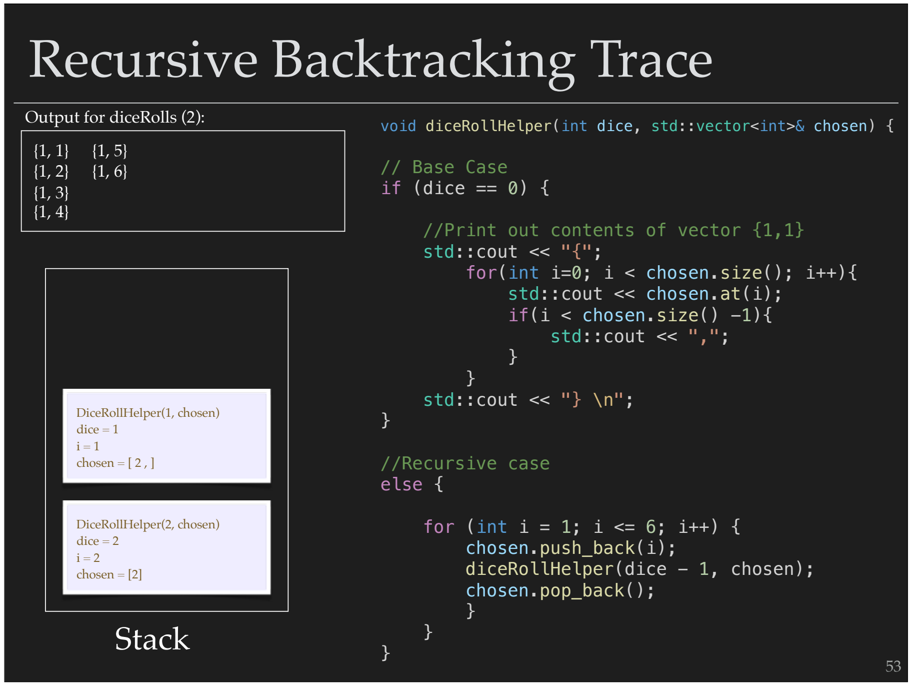
Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```



Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

  //Print out contents of vector {1,1}
  std::cout << "{";
    for(int i=0; i < chosen.size(); i++){
      std::cout << chosen.at(i);
      if(i < chosen.size() -1){
        std::cout << ",";
      }
    }
  std::cout << "} \n";
}

//Recursive case
else {

  for (int i = 1; i <= 6; i++) {
    chosen.push_back(i);
    diceRollHelper(dice - 1, chosen);
    chosen.pop_back();
    }
  }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

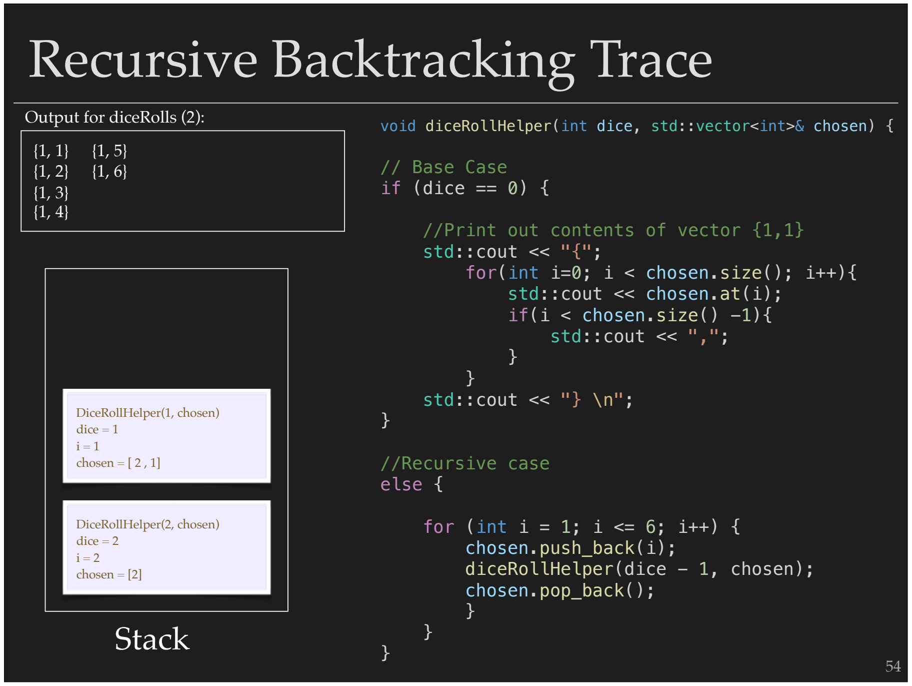
```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2 ,]

DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

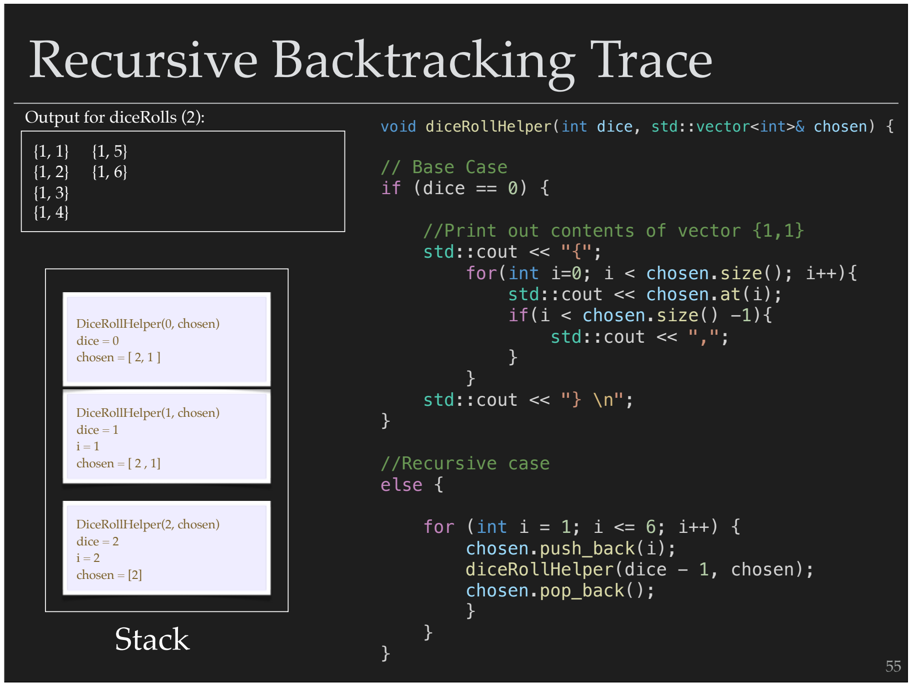
```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2 , 1]

DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

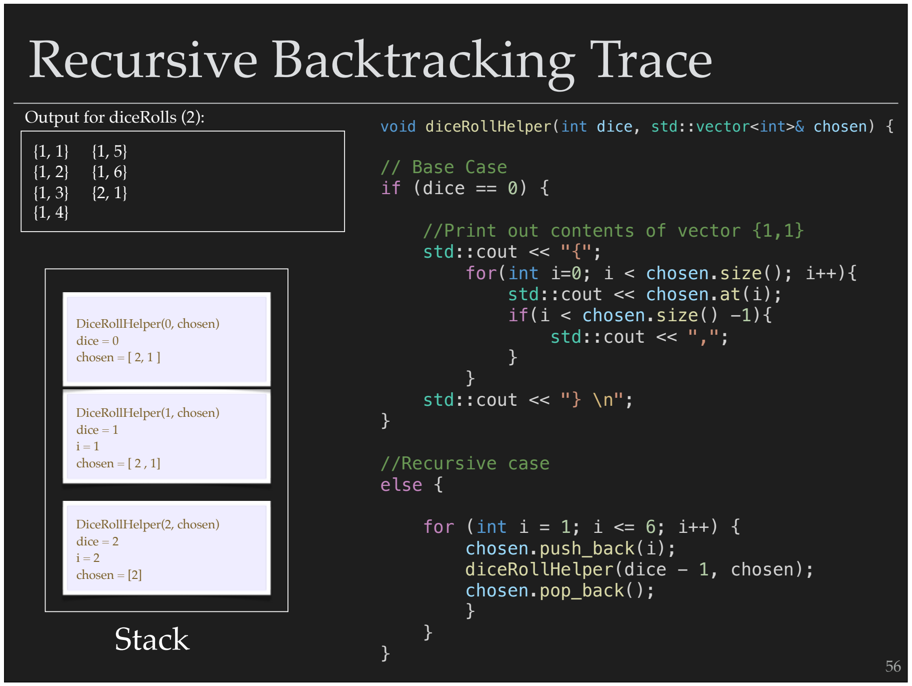
DiceRollHelper(0, chosen)
dice = 0
chosen = [2, 1]

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2 , 1]

DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}   {2, 1}
{1, 4}
```

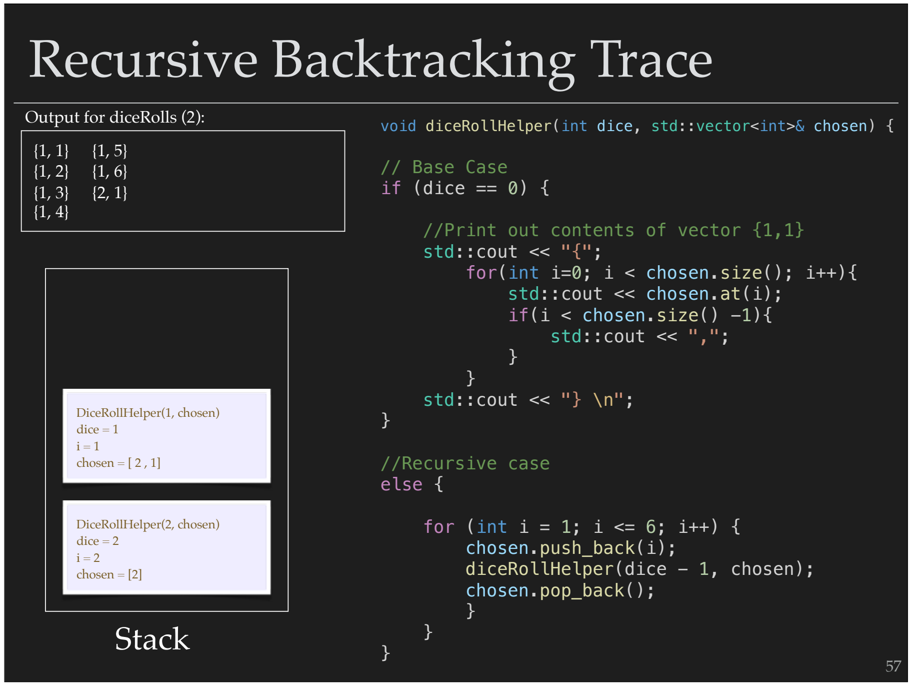
DiceRollHelper(0, chosen)
dice = 0
chosen = [2, 1]

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2 , 1]

DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```



Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}    {2, 1}
{1, 4}
```

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2 , 1]

DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
        }
    }
}
```