Discrete Elastic Rod Simulation of Shoelace Knots and Strength Project Proposal

Mechanics of Flexible Structures and Soft Robots - MAE 263F

Esther Gérard, Evelyn Kim, Noah Shamsai

***Abstract*— In the field of flexible structure and soft robotics, analysing an elastic rod is an important fundamental subject with future potential and practical application. Recently, many innovative approaches have continued to emerge, such as [8], [4], and [1], enabling effective numerical interpretation and advanced simulation.**
Among the various topics, knot tying is the most representative target for analysis, which can serve as a foundational structure with the potential to extend to various forms of soft robotics.
In this project, we aim to advance our analysis beyond basic knot structures by conducting a comprehensive simulation study focusing on shoelace knots. Leveraging the Discrete Elastic Rod (D.E.R.) algorithm, we will simulate a shoelace knot structure with varying properties to compare knot strengths. We assume dynamic equilibrium under constraints for the implementation the Implicit Model Contact (IMC) model.
Through this analyses and simulation, we hope to not only acquire knowledge in the simulation of flexible structures, but also a deeper understanding of the the affects of model parameters and their impacts on both simulation accuracy and speed.

I. INTRODUCTION

Our group project goal is to simulate the tying and untying of a shoelace knot, and to study the impact of friction and elastic parameters on the strength of the knot. Our work will be based on the Discrete Elastic Rod algorithm and on the Implicit Contact Model, which are detailed below. We will implement our algorithm in C++ in order to take advantage of its computational efficiency.

II. DISCRETE ELASTIC ROD ALGORITHM

The Discrete Elastic Rod (D.E.R.) algorithm is useful in 3D simulation of elastic rod. The rod discretized and made to be at dynamic equilibrium under applied constraints. My changing the constraints over time (i.e. end positions of the ”shoelace”), we will be able to manipulate the rod into a tied and then subsequently untied shoelace knot. The D.E.R. algorithm is extremely useful to us because it models the rod with all elastic and external forces at equilibrium.

General implementation of the D.E.R. Algorithm can be followed in Figure 1 [6]. The first appearance of the D.E.R. algorithm is in [2]. Since, it has been used in several research projects, especially to study knot tying [5]. The reason for that is the strength of this model on 1-D bodies in large deformation.

Kirchhoff-Love plate theory states that thin elastic plates undergo linear deformation with negligible transverse shear deformations when subjected to mechanical loads. When adapted to model rods by considering them as effectively 1-D structures, we can predict the deformation and stress distribution along the rod's length, allowing for the accurate analysis of slender structures under various loading conditions. Key elements of Kirchoff-Love plate theory include linear elastic behavior, linear strain-displacement relations, and the assumption that shear deformations are negligible.

Algorithm 1 Discrete Elastic Rods

Require: $\mathbf{q}(t_j), \dot{\mathbf{q}}(t_j)$ ▷ DOFs and velocities at $t = t_j$
Require: $\left(\mathbf{a}_1^k(t_j), \mathbf{a}_2^k(t_j), \mathbf{t}^k(t_j)\right)$ ▷ Reference frame at $t = t_j$
Require: `free_index` ▷ Index of the free DOFs
Ensure: $\mathbf{q}(t_{j+1}), \dot{\mathbf{q}}(t_{j+1})$ ▷ DOFs and velocities at $t = t_{j+1}$
Ensure: $\left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)$ ▷ Reference frame at $t = t_{j+1}$

function DISCRETE_ELASTIC_RODS($\mathbf{q}, \dot{\mathbf{q}}(t_j), \left(\mathbf{a}_1^k(t_j), \mathbf{a}_2^k(t_j), \mathbf{t}^k(t_j)\right)$)
  Guess: $\mathbf{q}^{(1)}(t_{j+1}) \leftarrow \mathbf{q}(t_j)$
  $n \leftarrow 1$
  while error > tolerance **do**
    Compute reference frame $\left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)^{(n)}$ using $\mathbf{q}^{(n)}(t_{j+1})$
    Compute reference twist $\Delta m_{k,\text{ref}}^{(n)}$ $(k = 2, \ldots, N-1)$
    Compute material frame $\left(\mathbf{m}_1^k(t_{j+1}), \mathbf{m}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)^{(n)}$
    Compute $\mathbf{f}$ and $\mathbb{J}$ ▷ Eqs. 7.1 and 7.2; see Algorithm 2
    $\mathbf{f}_{\text{free}} \leftarrow \mathbf{f}$ (`free_index`)
    $\mathbb{J}_{\text{free}} \leftarrow \mathbb{J}$ (`free_index, free_index`)
    $\Delta\mathbf{q}_{\text{free}} \leftarrow \mathbb{J}_{\text{free}} \backslash \mathbf{f}_{\text{free}}$
    $\mathbf{q}^{(n+1)}$ (`free_index`) $\leftarrow \mathbf{q}^{(n)}$ (`free_index`) $-\Delta\mathbf{q}_{\text{free}}$ ▷ Update free DOFs
    error $\leftarrow$ `sum ( abs (` $\mathbf{f}_{\text{free}}$ `) )`
    $n \leftarrow n + 1$
  end while

  $\mathbf{q}(t_{j+1}) \leftarrow \mathbf{q}^{(n)}(t_{j+1})$
  $\dot{\mathbf{q}}(t_{j+1}) \leftarrow \frac{\mathbf{q}(t_{j+1}) - \mathbf{q}(t_j)}{\Delta t}$
  $\left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right) \leftarrow \left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)^{(n)}$
  return $\mathbf{q}(t_{j+1}), \dot{\mathbf{q}}(t_{j+1}), \left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)$
end function

Fig. 1. D.E.R. Algorithm Pseudo-code, from [6]

Since the publication of [2], which relied principally an space-parallel transport frames to compute elastic rod twist, it was discovered that significant computational time advantages could be had if time-parallel transport was used. Hence this is the implementation that we will be using.

III. IMPLICIT CONTACT MODEL

The D.E.R. algorithm is constructed to handle the elastic forces of the body. But in knot tying, another physical phenomenon is very important: friction. That is why a physical friction model was necessary to conduct a numerical study of knots. If a lot of numerical friction models exist, we chose the IMC model, for its physical relevancy and its computational efficiency.

The IMC model is presented in [9] and [3]. The main point of this method is to compute implicitly a forces equilibrium, including contact forces, by choosing a smoothed (and so differentiable) formulation of the contact potential. The contact potential is calculated based on an analytical segment-segment distance between the discrete edges of the rod, instead of on the distance between nodes, to obtain more realistic results. This differentiable formulation of the contact energy allows to compute its gradient and hessian, and therefore to include it in the implicit formulation. The contact potential in [9] is defined by:

$$E(\Delta, \delta) = (2h - \Delta)^2 \quad \text{if } \Delta \in [0, 2h - \delta]$$
$$E(\Delta, \delta) = \left(\frac{1}{K_1} log(1 + e^{K_1(2h-\Delta)})\right)^2 \quad \text{if } \Delta \in [2h - \delta, 2h + \delta]$$
$$E(\Delta, \delta) = 0 \quad \text{if } \Delta \geq 2h + \delta$$

In this formula, K_1 is the stiffness of the energy curve, h is the radius of the rod, Δ is the distance between the two edges in contact, and δ is a tolerance value on the distance. Using this expression, the curves shown in figure 2 are obtained.

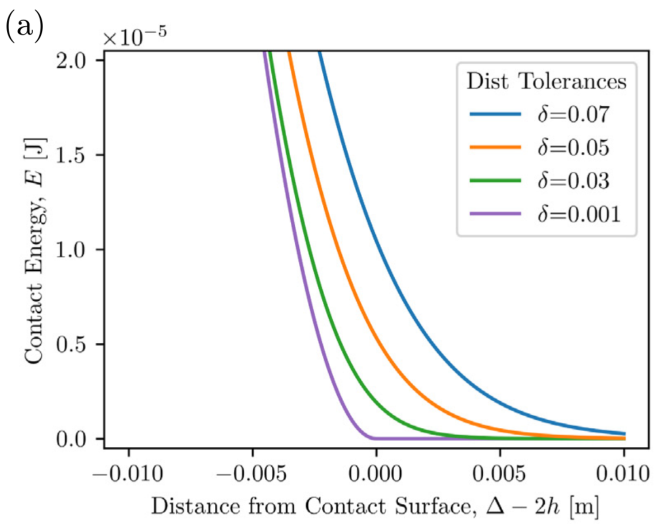

Fig. 2. Contact energy curve, this figure is extracted from [9]

The Implicit Model Contact was used in [3] to study the three-foil knot tying phenomena. To do so, the IMC was implemented in a D.E.R. algorithm. The simulation gave very good results when compared to theoretical results, or to another accurate model for contact in knot tying, the Spillmann and Teschner model (SPT). By comparison, IMC method gives smoother pull forces than SPT.

IV. Our Problem

Our goal is to apply this approach to another type of knot: the reef knot. To do so, we will use the latest IMC algorithm in date [9], and modify the initial configuration to simulate the untying of a reef knot.

The initial configuration will be obtained by a step by step tying of the knot, thanks to a progressive modification of the boundary condition. The point is to mimic an effective hand-tying of this knot numerically.

We will then be able to study the influence of the knot parameters and especially the influence of friction and elastic parameters. Each knot is indeed characterised by topological parameters, as shown in [7]. But our goal is to link this knot characterisation to a physical measurement of the knot stability and of the impact of friction and elastic parameters.

V. Conclusions

In this project, we will study the behavior of elastic rods using a series of simulations, with a specific focus on knot tying. Our project employs advanced simulations that closely refer to real-world scenarios, particularly with respect to the structure of shoelace knots. We leverage the Discrete Elastic Rod (D.E.R.) algorithm and the Implicit Model Contact (IMC) model to ensure a comprehensive analysis, all while considering the dynamic equilibrium under constraints. This endeavor will not only provide us with valuable practical experience in the implementation of soft robotics technology but also enable us to deepen our comprehension of the numerous factors that impact the results.

References

[1] Costanza Armanini et al. "Soft robots modeling: A structured overview". In: *IEEE Transactions on Robotics* (2023).

[2] Miklos Bergou et al. "Discrete Elastic Rods". In: *ACM Transactions on Graphics* 27 (2008).

[3] Andrew Choi et al. "Implicit contact model for discrete elastic rods in knot tying". In: *Journal of Applied Mechanics* 88.5 (2021).

[4] Xiaonan Huang et al. "Design and Closed-Loop Motion Planning of an Untethered Swimming Soft Robot Using 2D Discrete Elastic Rods Simulations". In: *Advanced Intelligent Systems* 4.10 (2022), p. 2200163.

[5] M Khalid Jawed et al. "Untangling the Mechanics and Topology in the Frictional Response of Long Overhand Elastic Knots". In: *American Physical Society* 115 (2015), p. 118302.

[6] M. Khalid Jawed and Sangmin Lim. *Discrete Simulation of Slender Structures*. Los Angeles: University of California, Los Angeles, 2022.

[7] Vishal P. Patil et al. "Topological Mechanics of Knots and Tangles". In: *Science* 367 (2020).

[8] Longhui Qin et al. "Modeling and simulation of dynamics in soft robotics: a review of numerical approaches". In: *Current Robotics Reports* (2023).

[9] Dezhong Tong et al. "A fully implicit method for robust frictional contact handling in elastic rods". In: *Extreme Mechanics Letters* 58 (2023), p. 101924.