Mechanics of Flexible Structures and Soft Robots
Homework 1

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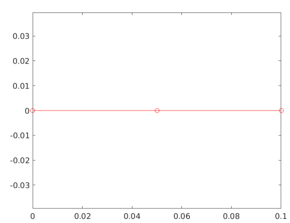
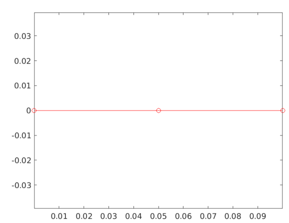
Github: https://github.com/lmcr136a/MAE263F

I. Problem 1: Simulation of the motion of three connected spheres falling inside viscous fluid

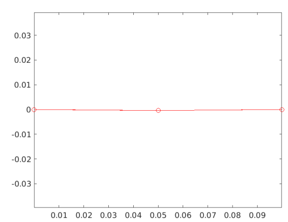
A. Show the structure's shape at t=0, 0.01, 0.05, 0.10, 1.0, 10.0 s. Plot the position and velocity (along the y-axis) of R2 as a function of time.



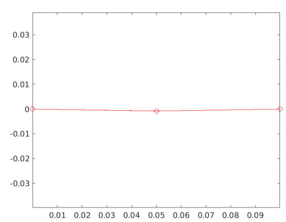
(a) t=0



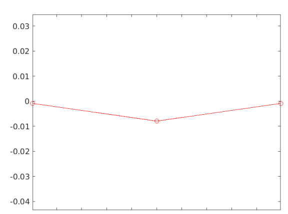
(b) t=0.01



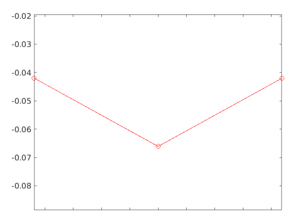
(c) t=0.05



(d) t=0.1

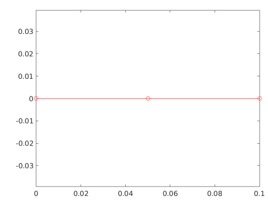


(e) t=1.0

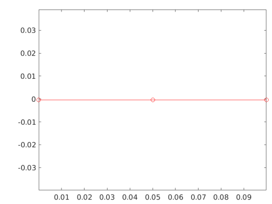


(f) t=10.0

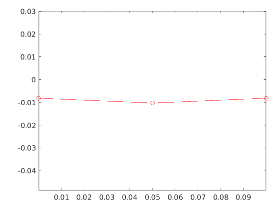
Fig. 1: The shape of the structure at t={0, 0.01, 0.05, 0.10, 1.0, 10.0}s in the implicit simulation.



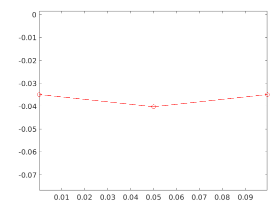
(a) t=0



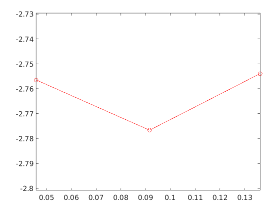
(b) t=0.01



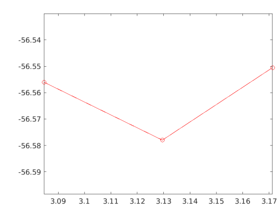
(c) t=0.05



(d) t=0.1

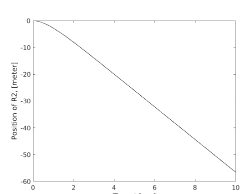


(e) t=1.0

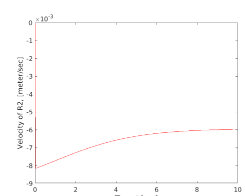


(f) t=10.0

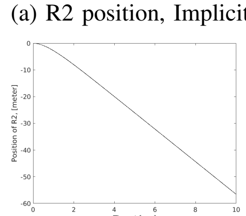
Fig. 2: The structure's shape at t={0, 0.01, 0.05, 0.10, 1.0, 10.0}s in the explicit simulation.



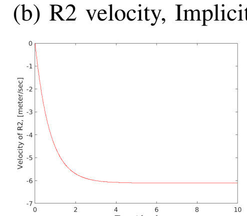
(a) R2 position, Implicit



(b) R2 velocity, Implicit



(c) R2 position, Explicit



(d) R2 velocity, Explicit

Fig. 3: The position and velocity (along the y-axis) of R2 as a function of time.

B. What is the terminal velocity (along the y-axis) of this system?

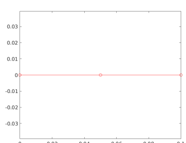
Implicit Sim Terminal Velocity:

R1=(0.000076, -0.005828), R2=(0.000000,-0.005966), R3=(-0.000076,-0.005828)

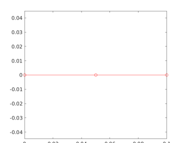
Explicit Sim Terminal Velocity:

R1=(0.000076, -0.005828), R2=(0.000000,-0.005966), R3=(-0.000076,-0.005828)

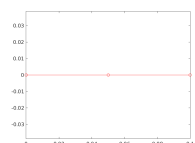
C. What happens to the turning angle if all the radii (R1, R2, R3) are the same? Does your simulation agree with your intuition?



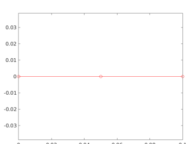
(a) t=0



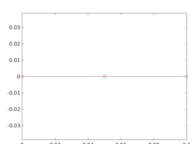
(b) t=0.01



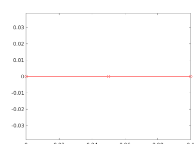
(c) t=0.05



(d) t=0.1

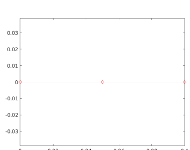


(e) t=1.0

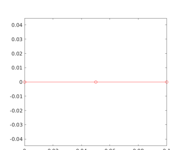


(f) t=10.0

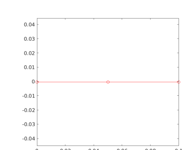
Fig. 4: The shape of the structure with **R1=R2=R3=0.005** at t={0, 0.01, 0.05, 0.10, 1.0, 10.0}s.



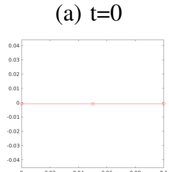
(a) t=0



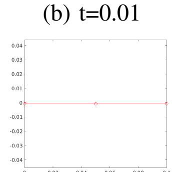
(b) t=0.01



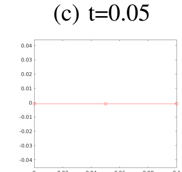
(c) t=0.05



(d) t=0.1



(e) t=1.0



(f) t=10.0

Fig. 5: The shape of the structure with **R1=R2=R3=0.025** at t={0, 0.01, 0.05, 0.10, 1.0, 10.0}s.

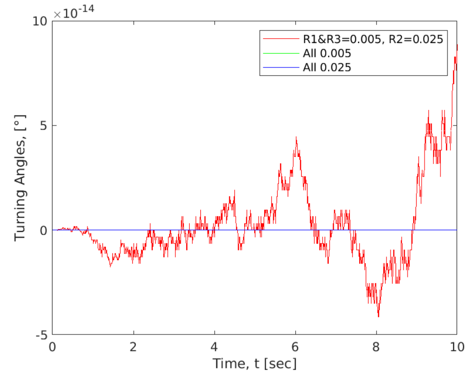


Fig. 6: Turning angles on various R conditions. If Rs are same, the turning angle is 0.

Yes. The simulation is aligned with my intuition. When the three radii are equal, the external force is the same for all three spheres, causing them to descend at the same y values. If R is larger, case of R=0.025, it can be observed that they descend more quickly than the smaller case R=0.005.

D. Try changing the time step size dt, particularly for your explicit simulation, and use the observation to elaborate on the benefits and drawbacks of the explicit and implicit approach.

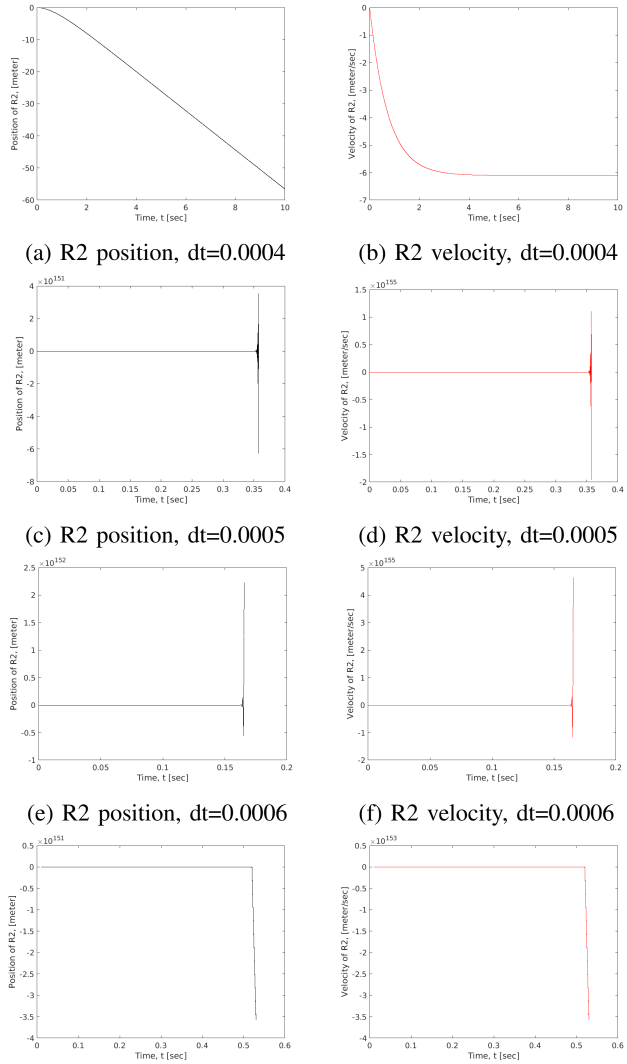
In the figure 7, it can be observed that when dt is greater than 0.0005, the simulation fails. The q and u values become increasingly unstable and eventually turn into NaN values as the iterations progress. From this, we can confirm that for the explicit method, a sufficiently small dt is required to obtain reliable results. Furthermore, to achieve accurate results with the explicit method, dt needs to be set more than 1000 times smaller than for the implicit method, which results in longer simulation times. On the other hand, the implicit method allows for much larger dt values while still obtaining accurate results, making it faster for simulations. However, implementing simulations using the implicit method can be considerably more complex, and each iteration involves significantly more calculations compared to the explicit method.

When the equations become overly complex for an implicit implementation, it might be advantageous to simplify the problem using an explicit simulation. If you aim for both accuracy and speed in your simulation, using the implicit method would be preferable over setting a very small dt for the explicit method.

II. Write a solver that simulates the system of the sphere as a function of time implicitly.

A. Include two plots showing the vertical position and velocity of the middle node with time. What is the terminal velocity?

The shape of the structure, the vertical position, and the velocity of the middle node are shown in Fig 13 as a function of time. The terminal velocity is -0.005834 [m/s].



(a) R2 position, dt=0.0004 (b) R2 velocity, dt=0.0004

(c) R2 position, dt=0.0005 (d) R2 velocity, dt=0.0005

(e) R2 position, dt=0.0006 (f) R2 velocity, dt=0.0006

(g) R2 position, dt=0.01 (h) R2 velocity, dt=0.01

Fig. 7: t=10 position and velocity of R2 in simulations with various time steps.

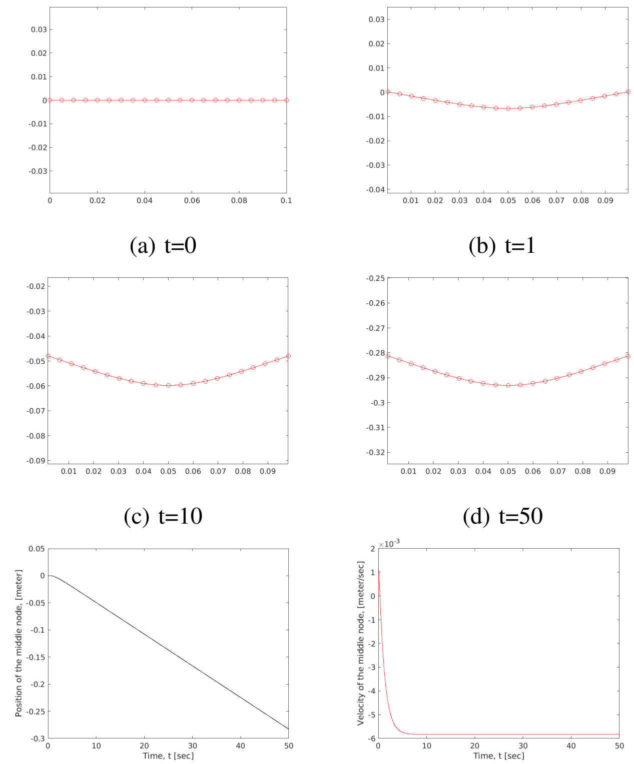
B. *Include the final deformed shape of the beam.*

It's shown in Fig 8d.

C. *Discuss the significance of spatial discretization (i.e. the number of nodes, N) and temporal discretization (i.e. time step size, dt). Any simulation should be sufficiently discretized such that the quantifiable metrics, e.g. terminal velocity, do not vary much if N is increased and dt is decreased. Include plots of terminal velocity vs. the number of nodes and terminal velocity vs. the time step size.*

If spatial discretization is large, it can represent the structure's shape more accurately, allowing for a more detailed observation of specific movements. However, if it's too small, only overly simplified movements can be observed.

On the other hand, if temporal discretization is large, it enables a more precise calculation of the structure's

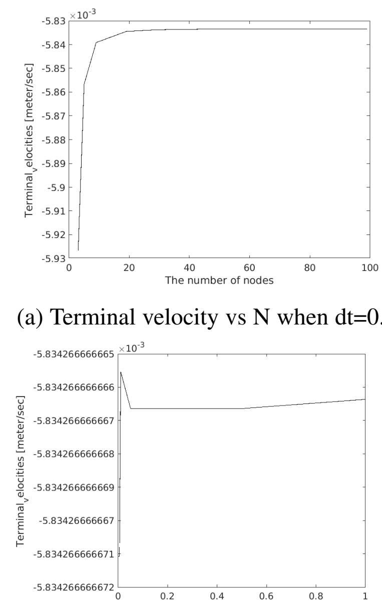


(a) t=0 (b) t=1

(c) t=10 (d) t=50

(e) Position of the middle node (f) Velocity of the middle node

Fig. 8: The shape of the structure at t={0, 1, 10, 50}s and R2's position and velocity. It can be observed that the velocity comes to a stop at terminal velocity before t=10.



(a) Terminal velocity vs N when dt=0.01

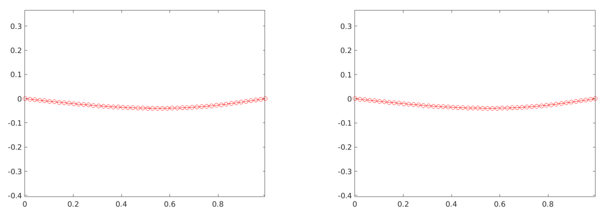
(b) Terminal velocity vs time step when N=21

Fig. 9: The termincal velocity with N = [3, 5, 9, 19, 29, 39, 49,59,69,79,89,99] and dt = [5e-4, 1e-3,5e-3, 1e-2,5e-2,1e-1,5e-1, 1e0].

movement for the given situation and allows for a detailed examination of motion over time. If temporal discretization is too small, dt is too large, the simulation's calculations may not proceed smoothly.

III. Write a solver that simulates the beam as a function of time implicitly.

A. Plot the maximum vertical displacement, y_{max}, of the beam as a function of time. Depending on your coordinate system, y_{max} may be negative. Does y_{max} eventually reach a steady value? Examine the accuracy of your simulation against the theoretical prediction from Euler beam theory.



(a) t=0.03 (b) t=1

Fig. 10: The shape of the structure at t={0, 1, 10, 50}s and R2's position and velocity. It can be observed that the velocity comes to a stop at terminal velocity before t=10.

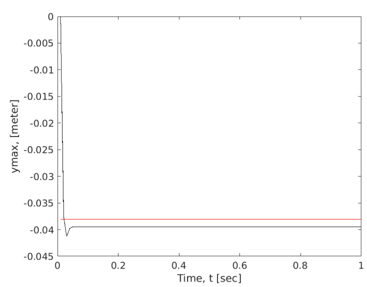
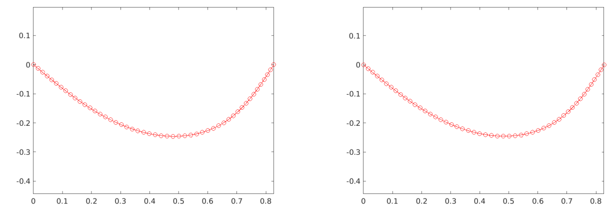


Fig. 11: The shape of the structure at t={0, 1, 10, 50}s and R2's position and velocity. It can be observed that the velocity comes to a stop at terminal velocity before t=10.

Yes, y_{max} reaches the steady value = -0.0395199332151966. The y_{max} value based on Euler beam theory is -0.038044915643450. The accuracy of the simulation is 96.123%.

B. What is the benefit of your simulation over the predictions from beam theory? To address this, consider a higher load P = 20000 such that the beam undergoes large deformation. Compare the simulated result against the prediction from beam theory in Eq. 4.21. Euler beam theory is only valid for small deformation whereas your simulation, if done correctly, should be able to handle large deformation. Optionally, you can make a plot of P vs. y max using data from both simulation and beam theory and quantify the value of P where the two solutions begin to diverge.

Through the simulation, I can interpret the y value depends on time and get more accurate value. From the figure 14,



(a) t=0.03 (b) t=1

Fig. 12: The shape of the structure at t={0, 1, 10, 50}s and R2's position and velocity. It can be observed that the velocity comes to a stop at terminal velocity before t=10.

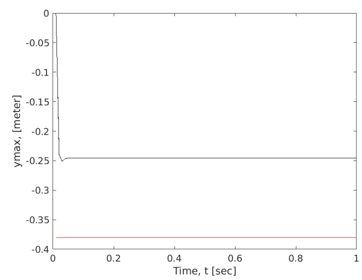


Fig. 13: The shape of the structure at t={0, 1, 10, 50}s and R2's position and velocity. It can be observed that the velocity comes to a stop at terminal velocity before t=10.

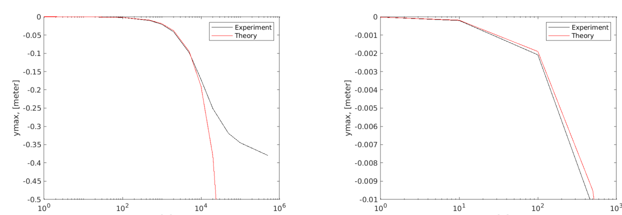


Fig. 14: y_{max} as a function of P. Notice that x-axis is log-scale. Experimented with $P = [1e0, 1e1, 1e2, 5e2, 1e3, 2e3, 5e3, 1e4, 2e4, 5e4, 1e5, 5e5]$. Both graphs are same but in the different scales.

it is evident that for small values of P, the theoretical and experimental results closely align. However, discrepancies between the two start to emerge around P values near 1e4. Therefore, it can be concluded that the theory applies primarily to small values.