Mechanics of Flexible Structures and Soft Robots
Homework 2

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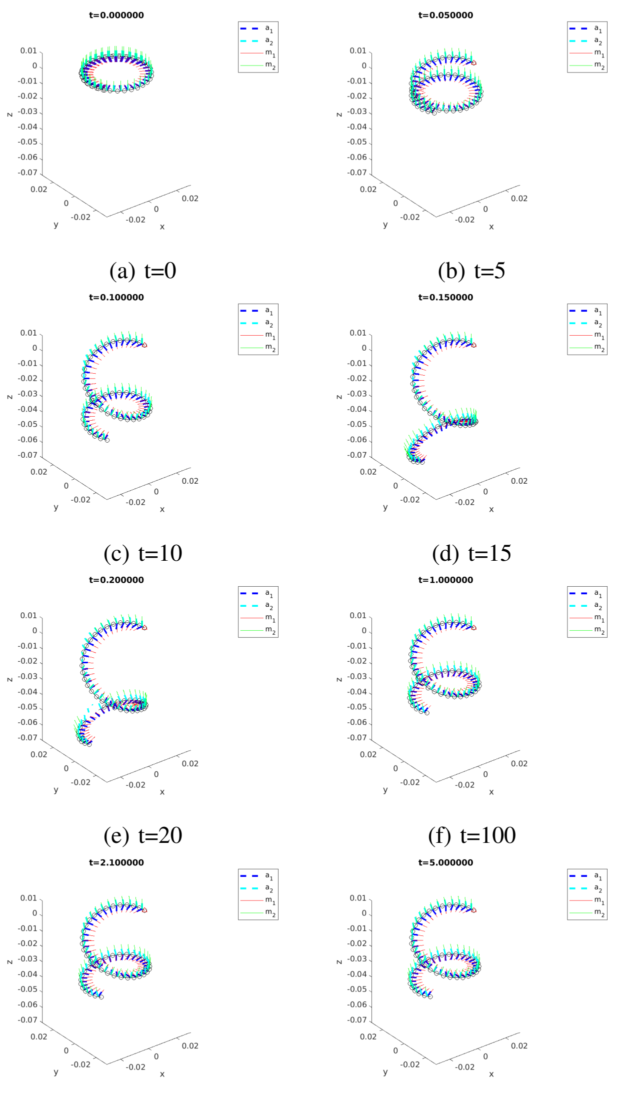
Github: https://github.com/lmcr136a/MAE263F

I. Chapter 7. Write a computer program that simulates the deformation of this rod under gravity from t = 0 to t = 5 s.



(a) t=0 (b) t=5 (c) t=10 (d) t=15 (e) t=20 (f) t=100 (g) t=210 (h) t=500

Fig. 1: The shape of the structure at t={0, 0.01, 0.05, 0.10, 1.0, 10.0}s in the implicit simulation.

The rod deformation simulation is visualized in Figure 1.

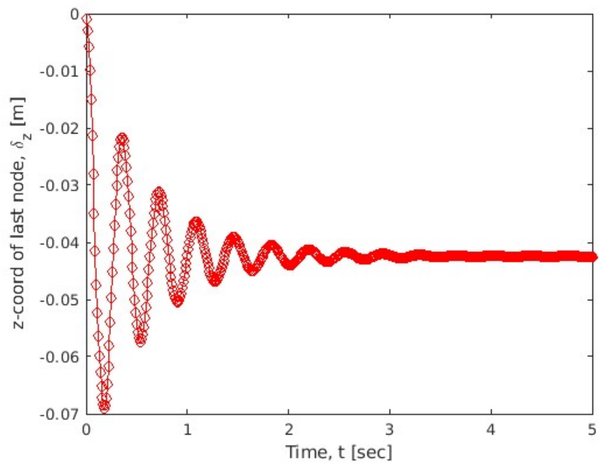


Fig. 2: z coordination of the last node in $t = [0, 5]$.

II. Chapter 7. Plot the z-coordinate of the last node (x N) with time.

The last node z coordinate is represented in Figure 2. Initially, it shows a large displacement, but from 3.5 seconds, it stabilizes at around 0.043.

III. Chapter 6. Assignment 1: Compute the integrated twist in the rod.

The integrated twist of the rod in Assignment1 can be calculated as follows:

$$\begin{aligned}
&e^k = x_{k+1} - x_k \\
&t^k = e^k/|e^k| \\
&b^1 = t^1 \times t^2 = [0.0.0.707] \\
&\hat{b^1} = b^1/|b^1| = [0., 0., 1.] \\
&n_1 = t^1 \times \hat{b^1} = [0., -1., 0.] \\
&n_2 = t^2 \times \hat{b^1} = [0.707, -0.707, 0.] \\
&d^1 = (u \cdot t^1) * t^2 + (u \cdot n_1) * n_2 + (u \cdot \hat{b^1}) * \hat{b^1} = [0., 0., 1.] \\
&\tau_2 = 0.
\end{aligned} \tag{1}$$

..
$$
\begin{aligned}
&b^2 = t^2 \times t^3 = [0.5, -0.5, 0.5] \\
&\hat{b^2} = b^2/|b^2| = [0.577, -0.577, 0.577] \\
&n_1 = t^2 \times \hat{b^2} = [0.408, -0.408, -0.816] \\
&n_2 = t^3 \times \hat{b^2} = [0.816, 0.408, -0.408] \\
&d^2 = (u \cdot t^2) * t^3 + (u \cdot n_1) * n_2 + (u \cdot \hat{b^2}) * \hat{b^2} = [-0.33, -0.67, 0.67] \\
&\tau_3 = 0.34
\end{aligned} \tag{2}
$$

Integrated twists on node 2 and node 3 are $\tau_2 = 0$, $\tau_3 = 0.33$ respectively.

IV. Chapter 6. Assignment 2: Space-parallel reference frame and evaluation

A. *compute the three directors of the frame at the subsequent edges*

$$
\begin{aligned}
&u^1 = [0, 0, 1] \\
&v^1 = t^1 u^1 = [0, -1, 0] \\
&u^2 = P_1^2(u^1, t^1, t^2) = [0, 0, 1] \\
&v^2 = t^2 u^2 = [0, -1, 0] \\
&u^3 = P_2^3(u^2, t^2, t^3) = [-0.333, -0.667, 0.667] \\
&v^3 = t^3 u^3 = [0.471, -0.471, -0.236]
\end{aligned} \tag{3}
$$

B. *Evaluate the angle ν^k at each edge.*

$$
\begin{aligned}
&\nu^1 = \nu^2 = 0 \\
&\nu^3 = arccos([0, -1/\sqrt{2}, 1/\sqrt{2}] \cdot [-1/3, -2/3, 2/3]) = 0.339
\end{aligned} \tag{4}
$$

C. *Compute twist*

$$
\begin{aligned}
\tau^1 &= \nu^1 \\
\tau^2 &= \nu^2 \\
\tau^3 &= \nu^3
\end{aligned} \tag{5}
$$

Since the edge's twist ν^k are same as the integrated twists τ^k, we can conclude that the two approaches drive into the same result.

V. Chapter 6. Assignment 3: Reference frames

Tangent vectors:

$$
\begin{aligned}
t^1 &= [-0.585, 0.811, 0] \\
t^2 &= [-0.954, 0.3, 0] \\
t^3 &= [-0.017, 1, 0] \\
t^4 &= [-0.9436, -0.331, 0]
\end{aligned} \tag{6}
$$

Also, we can calculate u^k and v^k as in previous problems, so we obtain tangent vectors and reference directors. The time-parallel reference frame can be calculated by the equation: $a_1^k(t_{j+1})$ =Parallel-transport($a_1^k(t_j), t^k(t_{j+1}), t^k(t_j)$)

The numerical values are represented as follows:
Initially)

$$
\begin{aligned}
a_1^1 &= [-0.811, -0.585, 0.] \\
a_1^2 &= [0.2995, -0.954, 0.] \\
a_1^3 &= [1, -0.166, 0.] \\
a_1^4 &= [0.331, 0.944, 0.]
\end{aligned} \tag{7}
$$

t=0.05s)

$$
\begin{aligned}
a_1^1 &= [-0.811, -0.585, 0.] \\
a_1^2 &= [0.2839, -0.959, -d0.003] \\
a_1^3 &= [1, 0.758, 0.005] \\
a_1^4 &= [0.371, 0.929, 0.0003]
\end{aligned} \tag{8}
$$

t=0.1s)

$$
\begin{aligned}
a_1^1 &= [-0.811, -0.585, 0.] \\
a_1^2 &= [0.253, -0.967, -0.02] \\
a_1^3 &= [0.998, 0.048, 0.04] \\
a_1^4 &= [0.421, 0.907, -0.0006]
\end{aligned} \tag{9}
$$

To calculate the reference twist and material frames, we first need to obtain twist θ of each time. Then, by obtaining the difference between the values before and after values, we can get the reference twist of each node. The reference twist values can be represented as follows:
Initially)

$$
\begin{aligned}
\Delta m_{2,ref} &= 0 \\
\Delta m_{3,ref} &= 0 \\
\Delta m_{4,ref} &= 0
\end{aligned} \tag{10}
$$

t=0.05s)

$$
\begin{aligned}
\Delta m_{2,ref} &= -0.267 \\
\Delta m_{3,ref} &= -0.565 \\
\Delta m_{4,ref} &= -0.29
\end{aligned} \tag{11}
$$

t-0.1s)

$$
\begin{aligned}
\Delta m_{2,ref} &= -0.505 \\
\Delta m_{3,ref} &= -1.03 \\
\Delta m_{4,ref} &= -0.665
\end{aligned} \tag{12}
$$

Integrated twist τ_k is calculated by: $\tau_k = \nu^k - \nu^{k-1} = (\theta^k + m_{ref}^k) - (\theta^{k-1} + m_{ref}^{k-1})$. We have already obtained all required values, and the results are as follows.
Initially)

$$
\begin{aligned}
\tau_2 &= 0 \\
\tau_3 &= 0 \\
\tau_4 &= 0
\end{aligned} \tag{13}
$$

t=0.05s)

$$
\begin{aligned}
\tau_2 &= -0.467 \\
\tau_3 &= -0.145 \\
\tau_4 &= -0.036
\end{aligned} \tag{14}
$$

t=0.1s)

$$\begin{aligned} \tau_2 &= -0.896 \\ \tau_3 &= -0.28 \\ \tau_4 &= -0.061 \end{aligned} \tag{15}$$