Discrete Elastic Rod Simulation of Shoelace Knots and Strength

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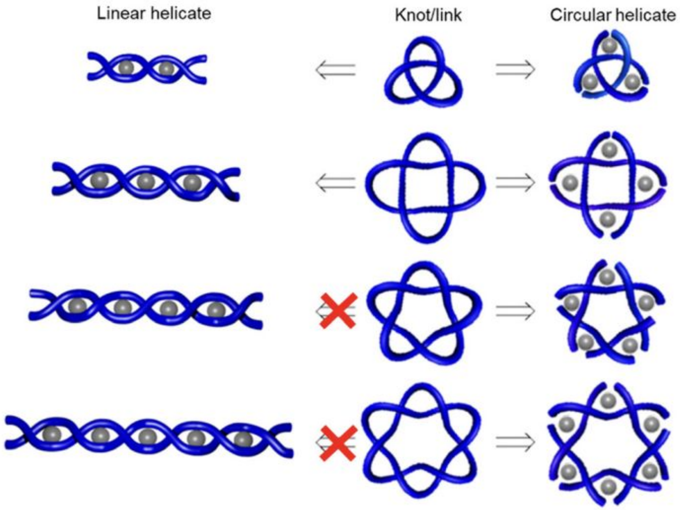
Motivation



Chisnall, R. (2021). An Analytical Review of Figure Eight Loops and Bowlines as Harness Tie-in and Anchoring Knots . Uluslararası Dağcılık ve Tırmanış Dergisi , 4 (2) , 43-59 . DOI: 10.36415/dagcilik.993072

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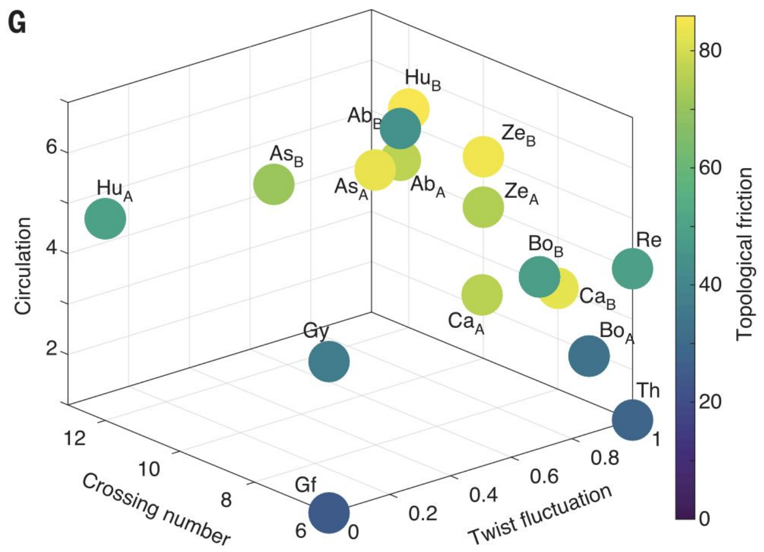
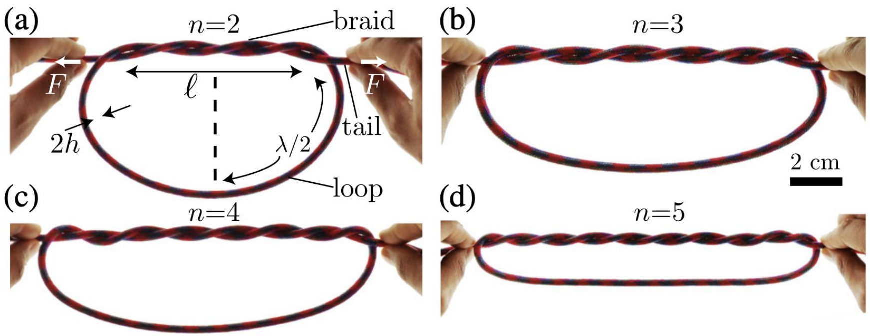
Background

Topological classification of knots



Vishal P. Patil et al. “Topological Mechanics of Knots and Tangles”. In: Science 367 (2020).

Mechanical study of n-foil knots



M Khalid Jawed et al. “Untangling the Mechanics and Topology in the Frictional Response of Long Overhand Elastic Knots”. In: American Physical Society 115 (2015), p. 118302.

Implementation

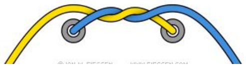
Algorithm 1 Discrete Elastic Rods

Require: $\mathbf{q}(t_j), \dot{\mathbf{q}}(t_j)$ ▷ DOFs and velocities at $t = t_j$
Require: $\left(\mathbf{a}_1^k(t_j), \mathbf{a}_2^k(t_j), \mathbf{t}^k(t_j)\right)$ ▷ Reference frame at $t = t_j$
Require: `free_index` ▷ Index of the free DOFs
Ensure: $\mathbf{q}(t_{j+1}), \dot{\mathbf{q}}(t_{j+1})$ ▷ DOFs and velocities at $t = t_{j+1}$
Ensure: $\left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)$ ▷ Reference frame at $t = t_{j+1}$

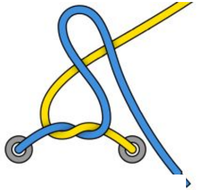
function DISCRETE_ELASTIC_RODS($\mathbf{q}, \dot{\mathbf{q}}(t_j), \left(\mathbf{a}_1^k(t_j), \mathbf{a}_2^k(t_j), \mathbf{t}^k(t_j)\right)$)
    Guess: $\mathbf{q}^{(1)}(t_{j+1}) \leftarrow \mathbf{q}(t_j)$
    $n \leftarrow 1$
    while `error` $>$ `tolerance` **do**
        Compute reference frame $\left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)^{(n)}$ using $\mathbf{q}^{(n)}(t_{j+1})$
        Compute reference twist $\Delta m_{k,\mathrm{ref}}^{(n)}$ $(k = 2, \ldots, N-1)$
        Compute material frame $\left(\mathbf{m}_1^k(t_{j+1}), \mathbf{m}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)^{(n)}$
        Compute $\mathbf{f}$ and $\mathbb{J}$ ▷ Eqs. 7.1 and 7.2;
        $\mathbf{f}_{\mathrm{free}} \leftarrow \mathbf{f}$ `(free_index)`
        $\mathbb{J}_{\mathrm{free}} \leftarrow \mathbb{J}$ `(free_index, free_index)`
        $\Delta \mathbf{q}_{\mathrm{free}} \leftarrow \mathbb{J}_{\mathrm{free}} \backslash \mathbf{f}_{\mathrm{free}}$
        $\mathbf{q}^{(n+1)}$ `(free_index)` $\leftarrow \mathbf{q}^{(n)}$ `(free_index)` $-\Delta \mathbf{q}_{\mathrm{free}}$ ▷ Update free DOFs
        `error` $\leftarrow$ `sum ( abs (` $\mathbf{f}_{\mathrm{free}}$ `) )`
        $n \leftarrow n + 1$
    end while

    $\mathbf{q}(t_{j+1}) \leftarrow \mathbf{q}^{(n)}(t_{j+1})$
    $\dot{\mathbf{q}}(t_{j+1}) \leftarrow \frac{\mathbf{q}(t_{j+1}) - \mathbf{q}(t_j)}{\Delta t}$
    $\left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right) \leftarrow \left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)^{(n)}$
    return $\mathbf{q}(t_{j+1}), \dot{\mathbf{q}}(t_{j+1}), \left(\mathbf{a}_1^k(t_{j+1}), \mathbf{a}_2^k(t_{j+1}), \mathbf{t}^k(t_{j+1})\right)$
end function

Discrete Elastic Rods (DER) algorithm with Implicit model

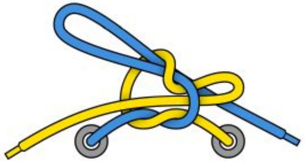


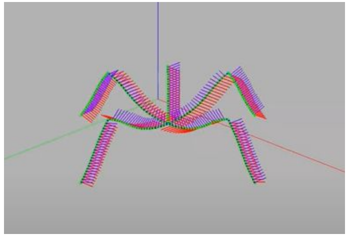
Tying the knot -> Untying the knot



$$f_i \equiv \frac{m_i}{\Delta t}\left[\frac{q_i(t_{j+1}) - q_i(t_j)}{\Delta t} - \dot{q}_i(t_j)\right] + \frac{\partial E_{\mathrm{elastic}}}{\partial q_i} - f_i^{\mathrm{ext}} = 0, \tag{7.1}$$

$$\mathbb{J}_{ij} = \frac{\partial f_i}{\partial q_j} = \mathbb{J}_{ij}^{\mathrm{inertia}} + \mathbb{J}_{ij}^{\mathrm{elastic}} + \mathbb{J}_{ij}^{\mathrm{ext}}, \tag{7.2}$$

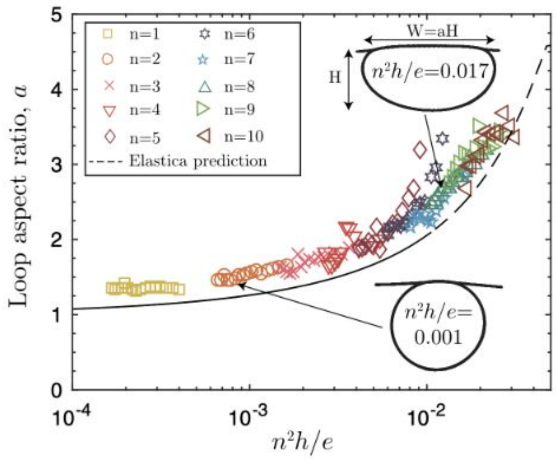


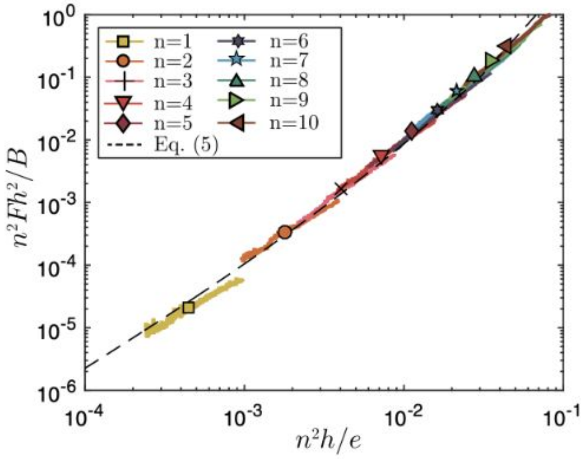


https://www.fieggen.com/shoelace/standardknot.htm

Desired Results

- Successful realistic simulation of reef knot tying & untying
- Implementation in C++
- Plots of
  - Shoelace stiffness vs. knot strength
  - Shoelace friction vs. knot strength





Thank you! Questions?