



RESEARCH ARTICLE

REVISED Qualitative analysis of HIV and AIDS disease transmission: impact of awareness, testing and effective follow up [version 2; peer review: 3 approved]

Oluwakemi E. Abiodun¹, Olukayode Adebimpe², James Ndako¹,
Olajumoke Oludoun¹, Benedicta Aladeitan¹, Michael Adeniyi³

¹Physical Sciences, Landmark University, Omu Aran, Kwara, 251101, Nigeria

²Mathematics and Statistic, First Technical University, Ibadan, Nigeria, 200103, Nigeria

³Mathematics and Statistic, Lagos State Polytechnic, Lagos, Lagos, Nigeria

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Abstract

Background: Since the early 1980s, human immunodeficiency virus (HIV) and its accompanying acquired immunodeficiency syndrome (AIDS) have spread worldwide, becoming one of the world's major global health issues. From the beginning of the epidemic until 2020, about 79.3 million people became infected, with 36.3 million deaths due to AIDS illnesses. This huge figure is a result of those unaware of their status due to stigmatization and invariably spreading the virus unknowingly.

Methods: Qualitative analysis through a mathematical model that will address HIV unaware individuals and the effect of an increasing defaulter on the dynamics of HIV/AIDS was investigated. The impact of treatment and the effect of inefficient follow-up on the transmission of HIV/AIDS were examined. The threshold for the effective reduction of the unaware status of HIV through testing, in response to awareness, and the significance of effective non-defaulting in treatment commonly called defaulters loss to follow-up as these individuals contribute immensely to the spread of the virus due to their increase in CD4+ count was determined in this study. Stability analysis of equilibrium points is performed using the basic reproduction number R_0 , an epidemiological threshold that determines disease eradication or persistence in viral populations. We tested the most sensitive parameters in the basic reproduction numbers.

Results: The results portray that early identification and treatment only are inadequate for the illness to be eradicated, but effectively used of condom, strict adherence to treatment and counseling of and testing of pregnant women contribute to a decrease in infected HIV individuals.

Conclusions: Other control techniques, such as treatment adherence

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1. **Adedapo Loyinmi** , Tai Solarin University of Education, Ijebu ode, Nigeria
2. **Fatmawati Fatmawati** , Universitas Airlangga, Surabaya, Indonesia
3. **Ropo Ebenzer Ogunsakin**, University of KwaZulu-Natal, Durban, South Africa

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and effective condom usage, and reduction in vertical transmission cannot be over-emphasis to lessen the disease's burden. Policymakers must address these strategies through a series of public awareness campaigns about the dangers of not adhering to treatment procedures and patterns.

Keywords

HIV/AIDS, infection-free equilibrium, defaulter lost to follow-up, endemic equilibrium, next generation matrix, basic reproduction number, stability.



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Corresponding author: Oluwakemi E. Abiodun (abiodun.oluwakemi@lmu.edu.ng)

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REVISED Amendments from Version 1

Based on reviewer feedback, the abstract section's result and conclusion were rewritten. New literature were incorporated, and the study's uniqueness was re-examined for more clarity. The reproduction number was investigated once more, and the relationship between the reproduction number and the stability analysis of the disease free equilibrium was demonstrated. The effect of infants born to infected mothers was lengthened to three steps (@ 0.02, 0.04, and 0.06). The conclusion was paraphrased, and the major finding was highlighted as the reviewer advised. Grammatical errors were also verified.

Any further responses from the reviewers can be found at the end of the article

1. Introduction

Human immunodeficiency virus (HIV) is a sexually transmitted infection (STI) and a blood-borne illness in humans with a wide range of clinical manifestations.^{1,2} HIV and its accompanying acquired immune deficiency syndrome (AIDS) have spread rapidly around the world since its discovery in the early 1980s, and it remains the world's most serious global health and development challenges. There is, however, a global devotion to avoiding new infections and making sure that all patients diagnosed have access to treatment. In addition, 79.3 million individuals have been infected with HIV since the pandemic began, with 36.3 million people dying due to AIDS diseases. About five million individuals contracted HIV for the first time in 2003, the largest number in any one year since the pandemic began.³ Globally, the figure of persons living with HIV/AIDS has risen from 35 million in 2001 to 37.7 million in 2020, with around 3 million people dying from the illness in that year.^{4,5} Around 84 percent [68 – 98 percent] of HIV-positive persons in the globe know their status in 2020, and the remaining 16 percent (about 6 million people) [4.8 million-7.1 million] need to be tested for HIV. HIV testing is an important initial step in HIV prevention, treatment, care, and support.^{6,7} Under Sustainable Development Goal 3, the international community pledged to work to end the AIDS pandemic by 2030. While progress has been made, it has been inconsistent, and the intermediate targets of “90-90-90” have been missed.^{7,8} New diseases continue to wreak havoc on communities and undermine vital socioeconomic infrastructure all across the planet. According to the United Nations Joint Program on HIV and AIDS, the number of HIV-positive people in 2021 was 37.6 million, up from 33.2 million in 2010.⁹ 1.5 million [1.1 million-2.1 million] people contracted HIV for the first time in 2020, 690,000 [480,000-1 million] people died of AIDS-related illnesses, and antiretroviral medication was available to 27.4 million [26.5 million-27.7 million] patients in December 2020, up from 7.8 million [6.9 million-7.9 million] in 2010.⁹⁻¹¹ HIV can be spread horizontally or vertically from one infected individual to another. Horizontal HIV transmission occurs when an individual comes into direct contact with an HIV-positive person, including sexual contact, or when they use a needle and syringe that has recently been utilized by an HIV-positive individual. Contrastingly, vertical transmission occurs when the virus is passed directly from an infected mother to her pregnant or newborn child.¹² HIV/AIDS transmission dynamics has piqued the interest of applied mathematicians, epidemiologists¹³⁻¹⁶ and biologists¹⁷⁻²² due to the disease's worldwide menace. Various improvements have been made to May and Anderson's early models,²³⁻²⁵ and particular issues have been discussed by researchers.^{12,26-48} In Lu *et al.* 2020²⁷ fostered a compartmental model for the yearly revealed HIV/AIDS MSM in the Zhejiang Region of China between 2007 to 2019 and anticipated that 90 percent of people tested for HIV/AIDS will have received treatment by 2020 neglecting those that fall out of treatment, while the screened extent will remain as low as 40 percent, and that antiretroviral treatment (ART) can control the transmission of HIV, even within the sight of medication opposition. Rana and Sharma, 2020³⁰ presented a simple Likely to be exposed-Infected (i.e.SI) form of the HIV/AIDS mathematical model, given the supposition that changing from an AIDS-infected to an HIV-infected individual is conceivable, to understand disease dynamics and develop strategies to reduce or control disease transmission among individual. In Ref. 30, HIV positive individual were not placed on treatment until they developed full blown AIDS. Mushanyu³² built a mathematical model for HIV acquisition using nonlinear ordinary differential equations to analyse the influence of delayed HIV diagnosis on the transmission of HIV in the year 2020. To prevent HIV from spreading further, the researchers advocated for early HIV treatment and the expansion of HIV self-testing initiatives, which would allow more people who have not been tested for HIV to learn their status. Teng¹² proposed and investigated a time-delay compartmental framework for HIV transmission in a sexually active cohort with press coverage, a disease that can result in a developed phase of infection known as acquired immunodeficiency syndrome (AIDS), as well as vertical transmission in the enrollment of people infected in 2019.³³ Saad *et al.* (2019) developed and considered an HIV^+ mathematical model with the next-generation matrix, the infection-free and endemic equilibrium points were identified, and the basic reproduction ratio R_0 was determined. The Lyapunov function was utilized to analyze the equilibria's global stability, and it was observed that the equilibria's stability is reliant on the magnitude of the fundamental reproduction ratio.³⁷ Developed an HIV/AIDS epidemic model with a generic nonlinear rate of occurrence and therapy and was able to obtain the basic reproductive number R_0 using the next generating matrix technique.

Researchers have employed numerous tools to manage and eradicate HIV/AIDS diseases.^{3,11,12} In order to study the combined effects of three control measures,⁴⁹ presented an innovative and workable human-bat (host-vector) model that predicts the spread and severity of the Ebola virus from bats to humans. To demonstrate the model's epidemiological viability, among other things, disease-free equilibrium, the endemic equilibrium, local and global stability, positivity, and boundedness were established. In order to understand the dynamics of the disease's transmission and its long-term repercussions,⁵⁰ created a novel deterministic model of Lassa Hemorrhagic fever (LHF) with nonlinear force of infection. The Cauchy's differential theorem, Birkhoff & Rota's theorem, and other well-known approaches used in the qualitative studies of the model both confirm and make clear how well-posed it is.

These studies revealed that awareness creation/information can help to control the disease burden but cannot eliminate the disease. Furthermore, there are other techniques and tools available that can be applied to study the dynamics of disease transmission and to provide suitable control interventions. The use of mathematical modeling is foremost among these techniques.^{16–19} Although many articles²⁰ have studied the impact of different controls; however, none of them have incorporated human behaviour in response to information. Hence, the aim of this study is to identifies the threshold for effective reduction of HIV/AIDS, as a result of HIV-unaware individuals becoming aware with counseling and testing and placed on treatment afterwards. The consequent effective follow-up in the use of treatment, immediate treatment of HIV positive diagnosed, the inclusion of effective condoms usage by susceptible and infectious individual on the transmission dynamics cannot be over-emphasis in the eradication of HIV/AIDS as proposed by WHO and UNAID come 2030.

According to the literature, no one has proposed a model that takes into account both vertical and horizontal transmission of susceptible into unaware populations, as well as undiagnosed and diagnosed HIV/AIDS-infected individuals, with preventive measures on transmission dynamics, screening, treatment controlling mechanisms, and the consideration of non-adherence to therapy. This model takes into account the rate of becoming conscious and unaware as a consequence of counselling and testing as variables rather than constants, as well as the rising effect of non-adherence to treatment. Thus, according to the author's knowledge, this work is unique.

The following is the structure of the paper: **Section 2** describes the model, while **Section 3** examines the model's basic features, the basic reproduction number, and equilibrium points. **Section 4** employs a parameter sensitivity index on the reproduction number to conduct a stability study of the equilibria (local and global), and the findings are generated from numerical simulations of data from previously published studies in **Section 5**. Finally, the research is examined and completed in **Section 6**.

2. Model formulation and description

A mathematical model on the mechanisms of horizontal and vertical transmission of HIV/AIDS was developed, by incorporating the effect of testing, defaulter loss to follow-up on treatment, and effective use of condoms on the existing model. The model is available from [GitHub](#) and is archived with [Zenodo](#).⁶⁸ The model is depicted schematically in **Figure 1**. The model contains six (6) state variables, namely: Susceptible (S), representing people who are likely to

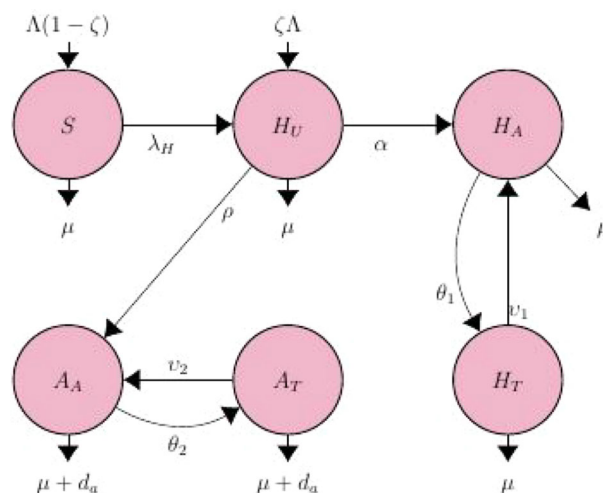


Figure 1. HIV/AIDS compartmental flow diagram.

become infected with HIV; Unaware HIV infectives (H_U), Aware HIV infectives (H_A), Treated HIV infectives (H_T); AIDS individuals (A_A) and AIDS on treatment individuals (A_T). The rate of effective contact with HIV-positive people either by immigration or emigration is given by Λ . A percentage of newborns get infected with HIV during birth at a rate of $(1 - \zeta)$ and are therefore directly enrolled into the unaware infected population H_U , at a rate $\zeta\Lambda$, with $0 \leq \zeta \leq 1$. $\lambda_H = c(1 - \psi\xi)\frac{\beta_1 H_U + \beta_2 H_A + \beta_3 A_A}{N}$ is the HIV transmission contact rate. Parameter c represents the average number of sexual partners acquired by people who are vulnerable to HIV annually. To simulate the influence of condom usage as a significant preventive intervention, the amount of condom protection (usage and effectiveness) is given as $\psi\xi[0, 1]$ based on assumption. If $\xi = 0$, condom use provides no protection, but $\xi = 1$ denotes complete protection, where ψ is the condom use. The parameters β_1, β_2 and β_3 account for the HIV transfer rates between persons at risk and (HIV unaware, HIV aware and full blown AIDS) infectives individuals, respectively. Both the HIV-infected and the AIDS-infected groups are thought to be active in the spread of HIV/AIDS amongst susceptible.⁵¹ Because infected patients with AIDS symptoms have a greater viral load than HIV-positive people (pre-AIDS) in the H_U and H_A classes, and because viral load and infectiousness have a positive connection, we must have $\beta_1 < \beta_2 < \beta_3$. There is evidence to suggest that individuals who know their HIV status H_A change their sexual behaviour (i.e. adopt safer-sex practices), resulting in reduced transmission.²⁵ Most HIV pandemic models disregard the role of AIDS patients in HIV transmission by applying simplistic assumptions such as AIDS death being immediate or AIDS patients being incapable of mingling and gaining new sex partners. However, epidemiological data show that AIDS patients participate in hazardous sexual activities, such as seldom wearing condoms or having several sex partners.⁵² As shown in the findings of Ref. 21 research of HIV-1-infected transfusion men and their women sex partners, severe AIDS patients are more likely to infect their partners than non-advanced immuno-compromised receivers.^{51,53} Also reported similar findings. HIV-positive individuals with and without AIDS signs are likely to have access to antiretroviral therapy (ART). Unaware HIV-infected persons, H_U , progress to the category of aware HIV infection H_A , after testing at a rate of α , while an unaware infected individuals who did not go for testing progress to stage IV of AIDS, A_A ; at a rate of ρ . HIV-infected aware people with no symptoms of AIDS; H_A , proceed to the group of HIV infection under ART therapy, H_T , whereas HIV-infected people with AIDS symptoms, A_A , are treated for AIDS at a rate of θ_2 on reaching the class of A_T . We presume that HIV-infected people on treatment do not spread the virus.^{54,55} HIV-infected people who are receiving therapy but do not have AIDS symptoms, H_T , who default during treatment and become resistant to the drug, will return to the HIV-infected aware individuals, H_A , and that HIV-infected persons with AIDS symptoms, A_A , who default during treatment in class A_T , become re-infected with HIV with symptoms of AIDS individuals, A_A , at a rate v_1 and v_2 respectively.⁵⁶ It is assumed that only HIV-infected people with AIDS symptoms, A_A and A_T , die of AIDS-related causes at a rate of d_a . The following mathematical model is based on these assumptions and that the system has a natural death in each class at a rate μ .

In order to contribute to the arduous aim of ending HIV/AIDS by 2030 there is need to foresee the epidemic's behaviour. One of the most significant tools we'll utilize to attain our aim is mathematical modeling of HIV infection. Based on Ref. 57, the following model was developed by the inclusion of AIDS on treatment compartment (by considering treatment of both individual not showing and showing symptoms of AIDS), individual who fall-out of treatment, considering AIDS individual are able to transmit infection, condom use to control transmission rate and average number of sexual partners acquired on force of infection. A system of ordinary differential equations (ODEs) can be used to express the mathematical equations that correspond to the schematic diagram:

$$\begin{aligned}
 \frac{dS}{dt} &= \Lambda(1 - \zeta H_U) - (\lambda_H + \mu)S & (a) \\
 \frac{dH_U}{dt} &= \Lambda\zeta H_U + \lambda_H S - (\alpha + \rho + \mu)H_U & (b) \\
 \frac{dH_A}{dt} &= \alpha H_U + v_1 H_T - (\theta_1 + \mu)H_A & (c) \\
 \frac{dH_T}{dt} &= \theta_1 H_A - (v_1 + \mu)H_T & (d) \\
 \frac{dA_A}{dt} &= \rho H_U + v_2 A_T - (\theta_2 + d_a + \mu)A_A & (e) \\
 \frac{dA_T}{dt} &= \theta_2 A_A - (v_2 + d_a + \mu)A_T & (f)
 \end{aligned} \tag{1}$$

with the positive initial conditions given as:

$$S(0) = S_0, H_U(0) = H_{U0}, H_A(0) = H_{A0}, H_T(0) = H_{T0}, A_A(0) = A_{A0}, A_T(0) = A_{T0} \tag{2}$$

3. Model investigation

3.1 Region of invariant

All of the parameters in the model are considered to be non-negative. System (1), on the other hand, keeps track of the human populace, hence, the state variables are always positive for all time $t \geq 0$. Thus, the total human populace is given as

$$N(t) = S(t) + H_U(t) + H_A(t) + H_T(t) + A_A(t) + A_T(t) \quad (3)$$

Here equation (1) is changing at a rate

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dH_U}{dt} + \frac{dH_A}{dt} + \frac{dH_T}{dt} + \frac{dA_A}{dt} + \frac{dA_T}{dt} = \Lambda - \mu N - d_a A_A - d_a A_T + \phi H_U \quad (4)$$

In the non-existence of infection i.e. for $H_U = H_A = H_T = A_A = A_T = 0$ we have,

$$\frac{dN}{dt} \leq \Lambda - \mu \quad (5)$$

We must have (6) by separating the variables of differential inequality.

$$\frac{dN}{\Lambda - \mu N} \leq dt \quad (6)$$

Integrating the above equation we have

$$\Lambda - \mu N \geq C e^{-\mu t}$$

where C is a constant to which to be determined. Let at $t = 0$, $N = N_0$. So we have,

$$C = \Lambda - \mu N_0 \quad (7)$$

From (7) we have

$$\begin{aligned} \Lambda - \mu N &\geq (\Lambda - \mu N_0) e^{-\mu t} \\ \Rightarrow N(t) &\leq \frac{\Lambda}{\mu} - \left[\frac{\Lambda - \mu N_0}{\mu} \right] e^{-\mu t} \end{aligned}$$

$$\text{As } t \rightarrow \infty, 0 \leq N(t) \leq \frac{\Lambda}{\mu}$$

As a result, the system (1) feasible solutions set enters the region.

$$\Omega = \left\{ (S, H_U, H_A, H_T, A_A, A_T) \in \mathcal{R}^6_+ : 0 \leq N \leq \frac{\Lambda}{\mu} \right\}$$

when $N \leq \frac{\Lambda}{\mu}$ every solution with an initial condition in \mathcal{R}^6_+ stays in that region for $t > 0$. As a result, the model is well posed and epidemiologically relevant in the domain Ω .

3.2 Non-negativity of solutions

This section discusses the positivity of the solutions, which describes the system's non-negativity of solutions (1).

Lemma 1: $S(t) \geq 0$, $H_U(t) \geq 0$, $H_A(t) \geq 0$, $H_T(t) \geq 0$, $A_A(t) \geq 0$, $A_T(t) \geq 0$ and $N(t) \geq 0$ satisfied by the solutions of system (1) with initial conditions (2) for all $t \geq 0$. The region $\Omega \subset \mathcal{R}^6_+$ is positively invariant and attracts in terms of system (1).

Proof: Take a look at the first equation (a) in (1)

$$\frac{dS}{dt} = \Lambda(1 - \zeta H_U) - (\lambda_H + \mu)S$$

we have;

$$\frac{dS}{dt} \geq -(\Lambda\zeta H_U + \lambda_H + \mu)S \int \frac{1}{S} dS - (\Lambda\zeta H_U + \lambda_H + \mu)dt$$

$$S \geq S_0 e^{-(\Lambda\zeta H_U + \lambda_H + \mu)t} \geq 0$$

provided $(\Lambda\zeta H_U + \lambda_H + \mu) < \infty$

As a result, $S \geq 0$

Likewise, for system (1)'s second equation (b), we have

$$\frac{dH_U}{dt} = \Lambda\zeta H_U + \lambda_H S - (\alpha + \rho + \mu)H_U$$

$$\frac{dH_U}{dt} \geq -(\alpha + \rho + \mu)H_U \int \frac{1}{H_U} dH_U - (\alpha + \rho + \mu)dt$$

$$H_U \geq H_{U0} e^{-(\alpha + \rho + \mu)t} \geq 0$$

provided $(\alpha + \rho + \mu) < \infty$

Hence, $H_U \geq 0$

Similarly it can be shown that $H_A \geq 0, H_T \geq 0, A_A \geq 0, A_T \geq 0$ for all $t > 0$.

Thus, the solutions $S, H_U, H_A, H_T, A_A, A_T$ remain positive forever.

3.3 Equilibrium point and basic reproduction number; R_0

The model (1) has exactly one disease-free equilibrium (DFE) point and the equilibrium point E_0 is given by $(S_0, H_{U0}, H_{A0}, H_{T0}, A_{A0}, A_{T0}) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0\right)$. In the absence of infection, the total population changes in proportion to the ratio of recruitment rate to the death rate.

The total population dynamics can be altered when an individual with an HIV/AIDS is introduced into a population. For the endemic equilibrium, there is an existence of infection hence $H_U \neq H_A \neq H_T \neq A_A \neq A_T \neq 0$. It is denoted by E_* . Setting equation (1a-1f) equal to zero which exist when $R_0 > 1$ we have

$$S_* = \frac{(M_1 - \zeta\Lambda)(M_4 M_5 - v_2 \theta_2)}{\lambda \rho M_5} A_A^* \quad (8)$$

$$H_{U*} = \frac{(M_4 M_5 - v_2 \theta_2)}{\rho M_5} A_A^* \quad (9)$$

$$H_{A*} = \frac{\alpha M_3 (-v_2 \theta_2 + M_4 M_5)}{(M_2 M_3 - v_1 \theta_1)} A_A^* \quad (10)$$

$$H_{T*} = \frac{\theta_1 \alpha M_3 (v_2 \theta_2 - M_4 M_5)}{M_3 (M_2 M_3 - v_1 \theta_1)} A_A^* \quad (11)$$

$$A_{A*} = \frac{\Lambda \rho M_5 \lambda}{M_1 (M_4 M_5 - v_2 \theta_2) \lambda + \mu (M_1 - \zeta\Lambda) (M_4 M_5 - v_4 \theta_2)} \quad (12)$$

$$A_{T*} = \frac{\theta_2}{M_5} A_{A*} \quad (13)$$

$$M_1 = \alpha + \rho + \mu, M_2 = \theta_1 + \mu, M_3 = v_1 + \mu, M_4 = \theta_2 + d_a + \mu, M_5 = v_2 + d_a + \mu.$$

Theorem 1: There exists a positive endemic equilibrium if $R_0 > 1$

Reference 58 presented a better method for determining R_0 which was an improved technique of solving the reproduction number firstly developed by Ref. 59 that is widely accepted because it represents the biological meaning of R_0 . By considering only the infective classes, we were able to obtain the system's (1) basic reproduction number, R_0 , which is the spectral radius (ρ) of the next generation matrix, NGM , i.e. $R_0 = \rho(FV^{-1})$. The rate of emergence of new infections in compartments i , while V denotes the rate of transfer of individual into and out of the compartment i by all other means. Where F and V are the $m \times m$ matrices defined as:

$$F = \frac{\partial F_{i(x)}}{\partial x_j} \text{ and } V = \frac{\partial V_{i(x)}}{\partial x_j} \text{ with } i \leq j \leq m$$

F is non-negative and V is non-singular matrix.

Then,

$$F = \begin{pmatrix} c(1-\psi\xi)\beta_1 & c(1-\psi\xi)\beta_2 & 0 & c(1-\psi\xi)\beta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} M_1 - \Lambda\zeta & 0 & 0 & 0 & 0 \\ -\alpha & M_2 & -v_1 & 0 & 0 \\ 0 & -\theta_1 & M_3 & 0 & 0 \\ -\rho & 0 & 0 & M_4 & -v_2 \\ 0 & 0 & 0 & -\theta_2 & M_5 \end{pmatrix}$$

$$V^{-1} = \begin{bmatrix} -\frac{1}{\Lambda\zeta - M_1} & 0 & 0 & 0 & 0 \\ -\frac{\alpha M_3}{\Lambda\zeta M_2 M_3 - \Lambda\zeta \theta_1 v_1 - M_1 M_2 M_3 + M_1 \theta_1 v_1} & \frac{M_3}{M_2 M_3 - \theta_1 v_1} & \frac{v_1}{M_2 M_3 - \theta_1 v_1} & 0 & 0 \\ -\frac{\alpha \theta_1}{\Lambda\zeta M_2 M_3 - \Lambda\zeta \theta_1 v_1 - M_1 M_2 M_3 + M_1 \theta_1 v_1} & \frac{\theta_1}{M_2 M_3 - \theta_1 v_1} & \frac{M_2}{M_2 M_3 - \theta_1 v_1} & 0 & 0 \\ -\frac{\rho M_5}{\Lambda\zeta M_4 M_5 - \Lambda\zeta \theta_2 v_2 - M_1 M_4 M_5 + M_1 \theta_2 v_2} & 0 & 0 & \frac{M_5}{M_4 M_5 - \theta_2 v_2} & \frac{v_2}{M_4 M_5 - \theta_2 v_2} \\ -\frac{\rho \theta_2}{\Lambda\zeta M_4 M_5 - \Lambda\zeta \theta_2 v_2 - M_1 M_4 M_5 + M_1 \theta_2 v_2} & 0 & 0 & \frac{\theta_2}{M_4 M_5 - \theta_2 v_2} & \frac{M_4}{M_4 M_5 - \theta_2 v_2} \end{bmatrix}$$

$$FV^{-1} = \begin{bmatrix} \frac{c(1-\psi\xi)\beta_1}{\Lambda\zeta - M_1} + \frac{c(1-\psi\xi)\beta_2 \alpha M_3}{\Lambda\zeta M_2 M_3 - \Lambda\zeta \theta_1 v_1 - M_1 M_2 M_3 + M_1 \theta_1 v_1} + \frac{c(1-\psi\xi)\beta_3 \rho M_5}{\Lambda\zeta M_4 M_5 - \Lambda\zeta \theta_2 v_2 - M_1 M_4 M_5 + M_1 \theta_2 v_2} & \frac{c(1-\psi\xi)\beta_2 M_3}{M_2 M_3 - \theta_1 v_1} & \frac{c(1-\psi\xi)\beta_2 v_1}{M_2 M_3 - \theta_1 v_1} & \frac{c(1-\psi\xi)\beta_3 M_5}{M_4 M_5 - \theta_2 v_2} & \frac{c(1-\psi\xi)\beta_3 v_2}{M_4 M_5 - \theta_2 v_2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

where $M_1 = \alpha + \mu + \rho$, $M_2 = \theta_1 + \mu$, $M_3 = v_1 + \mu$, $M_4 = \theta_2 + d_a + \mu$, $M_5 = v_2 + d_a + \mu$

The model reproduction number, denoted by R_0 is thus given by $R_0 = \rho(FV^{-1}) = R = R_1 + R_2 + R_3$, the spectral radius of the NGM FV^{-1} .

Here,

$$\begin{aligned} R_1 &= \frac{c(1-\psi\xi)\beta_1}{\zeta\Lambda - M_1} \\ R_2 &= \frac{c(1-\psi\xi)\beta_2\alpha M_3}{(M_2M_3 - \theta_1v_1)(\zeta\Lambda - M_1)} \\ R_3 &= \frac{c(1-\psi\xi)\beta_3\rho M_5}{(M_4M_5 - \theta_2v_2)(\zeta\Lambda - M_1)} \end{aligned}$$

Equivalently,

$$R_0 = \rho(FV^{-1}) = \frac{c(1-\psi\xi)(\alpha\beta_2M_3M_4M_5 - \alpha\beta_2M_3\theta_2v_2 + \beta_3\rho M_2M_3M_5 - \beta_3\rho M_5\theta_1v_1 + M_2M_3M_4M_5\beta_1 - M_2M_3\beta_1\theta_2v_2 - M_4M_5\beta_1\theta_1v_1 + \beta_1\theta_1\theta_2v_1v_2)}{(M_4M_5 - \theta_2v_2)(M_2M_3 - \theta_1v_1)(\Lambda\zeta - M_1)} \quad (*)$$

4. Equilibria stability analysis

4.1 Disease-free equilibrium stability on a local and global scale, E_0

Theorem 2: For all R_0 , the disease-free equilibrium E_0 exists, and it is locally asymptotically stable for $R_0 < 1$ and unstable otherwise.

Proof: The resulting matrix from linearized model $\frac{dx}{dt} = AX$, where $X = (x_1, x_2, x_3, x_4, x_5, x_6)^T$, $(x_1, x_2, x_3, x_4, x_5, x_6) \in R_+^6$, and

$$A = \begin{pmatrix} g1 - \mu & g2 - \Lambda\zeta & g5 & \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2} & g7 & \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2} \\ g3 & \zeta\Lambda - \alpha + g4 - \mu - \rho & g6 & -\frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2} & g8 & -\frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2} \\ 0 & \alpha & -\theta_1 - \mu & v_1 & 0 & 0 \\ 0 & 0 & \theta_1 & -v_1 - \mu & 0 & 0 \\ 0 & \rho & 0 & 0 & -\theta_2 - d_a - \mu & v_2 \\ 0 & 0 & 0 & 0 & \theta_2 & -v_2 - d_a - \mu \end{pmatrix} \quad (15)$$

$$g_1 = \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2} - \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)}{S + H_U + H_A + H_T + A_A + A_T},$$

$$g_2 = \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2} - \frac{c(1-\psi\xi)\beta_1S}{S + H_U + H_A + H_T + A_A + A_T},$$

$$g_3 = \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)}{S + H_U + H_A + H_T + A_A + A_T} - \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2},$$

$$g_4 = \frac{c(1-\psi\xi)\beta_1S}{S + H_U + H_A + H_T + A_A + A_T} - \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2},$$

$$g_5 = \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2} - \frac{c(1-\psi\xi)\beta_2S}{S + H_U + H_A + H_T + A_A + A_T},$$

$$g_6 = \frac{c(1-\psi\xi)\beta_2S}{S + H_U + H_A + H_T + A_A + A_T} - \frac{c(1-\psi\xi)(\beta_2H_A + \beta_3A_A + H_U\beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2}$$

$$g_7 = \frac{c(1-\psi\xi)(\beta_2 H_A + \beta_3 A_A + H_U \beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2} - \frac{c(1-\psi\xi)\beta_3 S}{S + H_U + H_A + H_T + A_A + A_T},$$

$$g_8 = \frac{c(1-\psi\xi)\beta_3 S}{S + H_U + H_A + H_T + A_A + A_T} - \frac{c(1-\psi\xi)(\beta_2 H_A + \beta_3 A_A + H_U \beta_1)S}{(S + H_U + H_A + H_T + A_A + A_T)^2}$$

The resulting Jacobian matrix of (15) at E_0 is

$$|A - \lambda I| = \begin{vmatrix} -\mu - \lambda & -\Lambda\zeta - c(1-\psi\xi)\beta_1 & -c(1-\psi\xi)\beta_2 & 0 & -c(1-\psi\xi)\beta_3 & 0 \\ 0 & \Lambda\zeta + c(1-\psi\xi)\beta_1 - \alpha - \rho - \mu - \lambda & c(1-\psi\xi)\beta_2 & 0 & c(1-\psi\xi)\beta_3 & 0 \\ 0 & \alpha & -\theta_1 - \mu - \lambda & v_1 & 0 & 0 \\ 0 & 0 & \theta_1 & -v_1 - \mu - \lambda & 0 & 0 \\ 0 & \rho & 0 & 0 & -\theta_2 - d_a - \mu - \lambda & v_2 \\ 0 & 0 & 0 & 0 & \theta_2 & -v_2 - d_a - \mu - \lambda \end{vmatrix} \quad (16)$$

from (16) the first eigenvalue is given as $\lambda_1 = -\mu$ then the matrix reduces to the 5×5 matrix below

$$|A - \lambda I| = \begin{vmatrix} \Lambda\zeta + c(1-\psi\xi)\beta_1 - M_1 - \lambda & c(1-\psi\xi)\beta_2 & 0 & c(1-\psi\xi)\beta_3 & 0 \\ \alpha & -M_2 - \lambda & v_1 & 0 & 0 \\ 0 & \theta_1 & -M_3 - \lambda & 0 & 0 \\ \rho & 0 & 0 & -M_4 - \lambda & v_2 \\ 0 & 0 & 0 & \theta_2 & -M_5 - \lambda \end{vmatrix} = 0$$

and the characteristics equation of the above matrix is obtained as:

$$\begin{aligned} f(\lambda) = & \lambda^5 + (c\psi\xi\beta_1 - \Lambda\zeta - c\beta_1 + M_1 + M_2 + M_3 + M_4 + M_5)\lambda^4 + (\alpha\beta_2 c\psi\xi + \beta_3 c\psi\rho\xi + c\psi\xi M_2\beta_1 + c\psi\xi M_3\beta_1 \\ & + c\psi\xi M_4\beta_1 + c\psi\xi M_5\beta_1 - \Lambda\zeta M_2 - \Lambda\zeta M_3 - \Lambda\zeta M_4 - \Lambda\zeta M_5 - \alpha\beta_2 c - \beta_3 c\rho - cM_2\beta_1 - cM_3\beta_1 - cM_4\beta_1 - cM_5\beta_1 \\ & + M_1 M_2 + M_1 M_3 + M_1 M_4 + M_1 M_5 + M_2 M_3 + M_2 M_4 + M_2 M_5 + M_3 M_4 + M_3 M_5 + M_4 M_5 - \theta_1 v_1 - \theta_2 v_2)\lambda^3 \\ & + (\alpha\beta_2 c\psi\xi M_3 + \alpha\beta_2 c\psi\xi M_4 + \alpha\beta_2 c\psi\xi M_5 + \beta_3 c\psi\rho\xi M_2 + \beta_3 c\psi\rho\xi M_3 + \beta_3 c\psi\rho\xi M_5 + c\psi\xi M_2 M_3\beta_1 + c\psi\xi M_2 M_4\beta_1 \\ & + c\psi\xi M_2 M_5\beta_1 + c\psi\xi M_3 M_4\beta_1 + c\psi\xi M_3 M_5\beta_1 + c\psi\xi M_4 M_5\beta_1 - c\psi\xi\beta_1\theta_1 v_1 - c\psi\xi\beta_1\theta_2 v_2 - \Lambda\zeta M_2 M_3 - \Lambda\zeta M_2 M_4 \\ & - \Lambda\zeta M_2 M_5 - \Lambda\zeta M_3 M_4 - \Lambda\zeta M_3 M_5 - \Lambda\zeta M_4 M_5 + \Lambda\zeta\theta_1 v_1 + \Lambda\zeta\theta_2 v_2 - \alpha\beta_2 cM_3 - \alpha\beta_2 cM_4 - \alpha\beta_2 cM_5 - \beta_3 c\rho M_2 \\ & - \beta_3 c\rho M_3 - \beta_3 c\rho M_5 - cM_2 M_3\beta_1 - cM_2 M_4\beta_1 - cM_2 M_5\beta_1 - cM_3 M_4\beta_1 - cM_3 M_5\beta_1 - cM_4 M_5\beta_1 + c\beta_1\theta_1 v_1 + c\beta_1\theta_2 v_2 \\ & + M_1 M_2 M_3 + M_1 M_2 M_4 + M_1 M_2 M_5 + M_1 M_3 M_4 + M_1 M_3 M_5 + M_1 M_4 M_5 - M_1\theta_1 v_1 - M_1\theta_2 v_2 + M_2 M_3 M_4 + M_2 M_3 M_5 \\ & + M_2 M_4 M_5 - M_2\theta_2 v_2 + M_3 M_4 M_5 + M_3\theta_2 v_2 - M_4\theta_1 v_1 - M_5\theta_1 v_1)\lambda^2 + (\alpha\beta_2 c\psi\xi M_3 M_4 + \alpha\beta_2 c\psi\xi M_3 M_5 \\ & + \alpha\beta_2 c\psi\xi M_4 M_5 - \alpha\beta_2 c\psi\xi\theta_2 v_2 + \beta_3 c\psi\rho\xi M_2 M_3 + \beta_3 c\psi\rho\xi M_2 M_5 + \beta_3 c\psi\rho\xi M_3 M_5 - \beta_3 c\psi\rho\xi\theta_1 v_1 \\ & + c\psi\xi M_2 M_3 M_4\beta_1 + c\psi\xi M_2 M_3 M_5\beta_1 + c\psi\xi M_2 M_4 M_5\beta_1 - c\psi\xi M_2\beta_1\theta_2 v_2 + c\psi\xi M_3 M_4 M_5\beta_1 - c\psi\xi M_3\beta_1\theta_2 v_2 \\ & - c\psi\xi M_4\beta_1\theta_1 v_1 - c\psi\xi M_5\beta_1\theta_1 v_1 - \Lambda\zeta M_2 M_3 M_4 - \Lambda\zeta M_2 M_3 M_5 - \Lambda\zeta M_2 M_4 M_5 + \Lambda\zeta M_2\theta_2 v_2 - \Lambda\zeta M_3 M_4 M_5 \\ & + \Lambda\zeta M_3\theta_2 v_2 + \Lambda\zeta M_4\theta_1 v_1 + \Lambda\zeta M_5\theta_1 v_1 - \alpha\beta_2 cM_3 M_4 - \alpha\beta_2 cM_3 M_5 - \alpha\beta_2 cM_4 M_5 + \alpha\beta_2 c\theta_2 v_2 - \beta_3 c\rho M_2 M_3 \\ & - \beta_3 c\rho M_2 M_5 - \beta_3 c\rho M_3 M_5 + \beta_3 c\rho\theta_1 v_1 - cM_2 M_3 M_4\beta_1 - cM_2 M_3 M_5\beta_1 - cM_2 M_4 M_5\beta_1 + cM_2\beta_1\theta_2 v_2 - cM_3 M_4 M_5\beta_1 \\ & + cM_3\beta_1\theta_2 v_2 + cM_4\beta_1\theta_1 v_1 + cM_5\beta_1\theta_1 v_1 + M_1 M_2 M_3 M_4 + M_1 M_2 M_3 M_5 + M_1 M_2 M_4 M_5 - M_1 M_2\theta_2 v_2 + M_1 M_3 M_4 M_5 \\ & - M_1 M_3\theta_2 v_2 - M_1 M_4\theta_1 v_1 - M_1 M_5\theta_1 v_1 + M_2 M_3 M_4 M_5 - M_2 M_3\theta_2 v_2 - M_4 M_5\theta_1 v_1 + \theta_1\theta_2 v_1 v_2)\lambda + \alpha\beta_2 cM_3\theta_2 v_2 \\ & - \alpha\beta_2 cM_3 M_4 M_5 - \beta_3 c\rho M_2 M_3 M_5 + \beta_3 c\rho M_5\theta_1 v_1 - cM_2 M_3 M_4 M_5\beta_1 + cM_2 M_3\beta_1\theta_2 v_2 + cM_4 M_5\beta_1\theta_1 v_1 - \Lambda\zeta\theta_1\theta_2 v_1 v_2 \\ & - c\beta_1\theta_1\theta_2 v_1 v_2 + \beta_3 c\psi\rho\xi M_2 M_3 M_5 - \beta_3 c\psi\rho\xi M_5\theta_1 v_1 + c\psi\xi M_2 M_3 M_4 M_5\beta_1 - c\psi\xi M_2 M_3\beta_1\theta_2 v_2 - c\psi\xi M_4 M_5\beta_1\theta_1 v_1 \\ & + \alpha\beta_2 c\psi\xi M_3 M_4 M_5 - \alpha\beta_2 c\psi\xi M_3\theta_2 v_2 + c\psi\xi\beta_1\theta_1\theta_2 v_1 v_2 + M_1 M_2 M_3 M_4 M_5 - M_1 M_2 M_3\theta_2 v_2 - M_1 M_4 M_5\theta_1 v_1 \\ & + M_1\theta_1\theta_2 v_1 v_2 - \Lambda\zeta M_2 M_3 M_4 M_5 + \Lambda\zeta M_2 M_3\theta_2 v_2 + \Lambda\zeta M_4 M_5\theta_1 v_1 = 0 \end{aligned} \quad (17)$$

The above characteristic equation in (17) is of the form

$$f(\lambda) = a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4$$

$$\begin{aligned}
& \alpha\beta_2 cM_3\theta_2v_2 - \alpha\beta_2 cM_3M_4M_5 - \beta_3 c\rho M_2M_3M_5 + \beta_3 c\rho M_5\theta_1v_1 - cM_2M_3M_4M_5\beta_1 + cM_2M_3\beta_1\theta_2v_2 \\
& + cM_4M_5\beta_1\theta_1v_1 - \Lambda\zeta\theta_1\theta_2v_1v_2 - c\beta_1\theta_1\theta_2v_1v_2 + \beta_3 c\psi\rho\xi M_2M_3M_5 - \beta_3 c\psi\rho\xi M_5\theta_1v_1 \\
& + c\psi\xi M_2M_3M_4M_5\beta_1 - c\psi\xi M_2M_3\beta_1\theta_2v_2 - c\psi\xi M_4M_5\beta_1\theta_1v_1 + \alpha\beta_2 c\psi\xi M_3M_4M_5 - \alpha\beta_2 c\psi\xi M_3\theta_2v_2 \\
& + c\psi\xi\beta_1\theta_1\theta_2v_1v_2 + M_1M_2M_3M_4M_5 - M_1M_2M_3\theta_2v_2 - M_1M_4M_5\theta_1v_1 + M_1\theta_1\theta_2v_1v_2 - \Lambda\zeta M_2M_3M_4M_5 \\
& + \Lambda\zeta M_2M_3\theta_2v_2 + \Lambda\zeta M_4M_5\theta_1v_1 = 0
\end{aligned}$$

By Descartes rule of sign

$$\begin{aligned}
& c(1-\psi\xi)(\alpha\beta_2 M_3M_4M_5 - \alpha\beta_2 M_3\theta_2v_2 + \beta_3\rho M_2M_3M_5 - \beta_3\rho M_5\theta_1v_1 + M_2M_3M_4M_5\beta_1 - M_2M_3\beta_1\theta_2v_2 \\
& - M_4M_5\beta_1\theta_1v_1 + \beta_1\theta_1\theta_2v_1v_2) < (M_4M_5 - \theta_2v_2)(M_2M_3 - \theta_1v_1)(\Lambda\zeta - M_1)
\end{aligned}$$

$$1 < \frac{c(1-\psi\xi)(\alpha\beta_2 M_3M_4M_5 - \alpha\beta_2 M_3\theta_2v_2 + \beta_3\rho M_2M_3M_5 - \beta_3\rho M_5\theta_1v_1 + M_2M_3M_4M_5\beta_1 - M_2M_3\beta_1\theta_2v_2 - M_4M_5\beta_1\theta_1v_1 + \beta_1\theta_1\theta_2v_1v_2)}{(M_4M_5 - \theta_2v_2)(M_2M_3 - \theta_1v_1)(\Lambda\zeta - M_1)}$$

Let

$$R_0 = 1 < \frac{c(1-\psi\xi)(\alpha\beta_2 M_3M_4M_5 - \alpha\beta_2 M_3\theta_2v_2 + \beta_3\rho M_2M_3M_5 - \beta_3\rho M_5\theta_1v_1 + M_2M_3M_4M_5\beta_1 - M_2M_3\beta_1\theta_2v_2 - M_4M_5\beta_1\theta_1v_1 + \beta_1\theta_1\theta_2v_1v_2)}{(M_4M_5 - \theta_2v_2)(M_2M_3 - \theta_1v_1)(\Lambda\zeta - M_1)}$$

Then,

$$R_0 = \frac{c(1-\psi\xi)(\alpha\beta_2 M_3M_4M_5 - \alpha\beta_2 M_3\theta_2v_2 + \beta_3\rho M_2M_3M_5 - \beta_3\rho M_5\theta_1v_1 + M_2M_3M_4M_5\beta_1 - M_2M_3\beta_1\theta_2v_2 - M_4M_5\beta_1\theta_1v_1 + \beta_1\theta_1\theta_2v_1v_2)}{(M_4M_5 - \theta_2v_2)(M_2M_3 - \theta_1v_1)(\Lambda\zeta - M_1)}$$

Because all parameters of the model are assumed to be positive, $\lambda_2 < 0, \lambda_3 < 0, \lambda_4 < 0, \lambda_5 < 0, \lambda_6 < 0$. Evidently, if $R_0 < 1$, the roots of $f(\lambda)$ have negative real parts, implying that E_0 is locally asymptotically stable (LAS) when $R_0 < 1$; if $R_0 > 1$, the roots of $f(\lambda)$ are real and some are positive, implying that E_0 is unstable.

Theorem 3: If $R_0 < 1$, the disease free equilibrium is asymptotically stable globally for system (1).

Proof: The comparison theorem, as demonstrated by Ref. 60 proves the global stability of the disease-free equilibrium. We rename the infected class: $\frac{dx}{dt} = (F - V)X - JX, X = (H_U, H_A, H_T, A_A, A_T)$ where,

$$F = \begin{pmatrix} c(1-\psi\xi)\beta_1 & c(1-\psi\xi)\beta_2 & 0 & c(1-\psi\xi)\beta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, V = \begin{pmatrix} M_1 - \Lambda\zeta & 0 & 0 & 0 & 0 \\ -\alpha & M_2 & -v_1 & 0 & 0 \\ 0 & -\theta_1 & M_3 & 0 & 0 \\ -\rho & 0 & 0 & M_4 & -v_2 \\ 0 & 0 & 0 & -\theta_2 & M_5 \end{pmatrix} \quad (18)$$

Then all of the matrix $F - V$ eigenvalues have negative real parts, i.e. so that

$$J = \left(1 - \frac{S}{N}\right) \begin{vmatrix} c(1-\psi\xi)\beta_1 + \Lambda\zeta - M_1 - \lambda & c(1-\psi\xi)\beta_2 & 0 & c(1-\psi\xi)\beta_3 & 0 \\ \alpha & -M_2 - \lambda & v_1 & 0 & 0 \\ 0 & \theta_1 & -M_3 - \lambda & 0 & 0 \\ \rho & 0 & 0 & -M_4 - \lambda & v_2 \\ 0 & 0 & 0 & \theta_2 & -M_5 - \lambda \end{vmatrix} = 0 \quad (19)$$

$$\begin{aligned}
& \lambda^5 - (c\beta_1 - c\psi\xi\beta_1 + \Lambda\zeta - M_1 - M_2 - M_3 - M_4 - M_5)\lambda^4 - (\Lambda\zeta M_2 + \Lambda\zeta M_3 + \Lambda\zeta M_4 + \Lambda\zeta M_5 - \xi c\psi\beta_2\alpha - c\psi\rho\xi\beta_3 \\
& - c\psi\xi M_2\beta_1 - c\psi\xi M_3\beta_1 - c\psi\xi M_4\beta_1 - c\psi\xi M_5\beta_1 + c\beta_2\alpha + \rho c\beta_3 + cM_2\beta_1 + cM_3\beta_1 + cM_4\beta_1 + cM_5\beta_1 - M_2M_1 - M_3M_1 \\
& - M_4M_1 - M_1M_5 - M_3M_2 - M_2M_4 - M_2M_5 - M_3M_4 - M_3M_5 - M_4M_5 + v_1\theta_1 + \theta_2v_2)\lambda^3 - (\Lambda\zeta M_2M_3 + \Lambda\zeta M_2M_4 \\
& + \Lambda\zeta M_2M_5 + \Lambda\zeta M_3M_4 + \Lambda\zeta M_3M_5 + \Lambda\zeta M_4M_5 - \alpha c\psi\xi M_3\beta_2 - \alpha c\psi\xi M_4\beta_2 - \alpha c\psi\xi M_5\beta_2 - c\psi\rho\xi M_2\beta_3 - c\psi\rho\xi M_3\beta_3 \\
& - c\psi\rho\xi M_5\beta_3 - c\psi\xi M_2M_3\beta_1 - c\psi\xi M_2M_4\beta_1 - c\psi\xi M_2M_5\beta_1 - c\psi\xi M_3M_4\beta_1 - c\psi\xi M_3M_5\beta_1 - c\psi\xi M_4M_5\beta_1 + c\psi\xi\beta_1\theta_1v_1 \\
& + c\psi\xi\beta_1\theta_2v_2 - \Lambda\zeta\theta_1v_1 - \Lambda\zeta\theta_2v_2 + \alpha cM_3\beta_2 + \alpha cM_4\beta_2 + \alpha cM_5\beta_2 + c\rho M_2\beta_3 + c\rho M_3\beta_3 + c\rho M_5\beta_3 + cM_2M_3\beta_1 \\
& + cM_2M_4\beta_1 + cM_2M_5\beta_1 + cM_3M_4\beta_1 + cM_3M_5\beta_1 + cM_4M_5\beta_1 - c\beta_1\theta_1v_1 - c\beta_1\theta_2v_2 - M_1M_2M_3 - M_1M_2M_4 - M_1M_2M_5 \\
& - M_1M_3M_4 - M_1M_3M_5 - M_1M_4M_5 + M_1\theta_1v_1 + M_1\theta_2v_2 - M_2M_3M_4 - M_2M_3M_5 - M_2M_4M_5 + M_2\theta_2v_2 - M_3M_4M_5 \\
& + M_3\theta_2v_2 + M_4\theta_1v_1 + M_5\theta_1v_1)\lambda^2 - (\alpha c\psi\xi\beta_2\theta_2v_2 - \alpha c\psi\xi M_3M_4\beta_2 - \alpha c\psi\xi M_3M_5\beta_2 - \alpha c\psi\xi M_4M_5\beta_2 - c\psi\rho\xi M_2M_3\beta_3 \\
& - c\psi\rho\xi M_2M_5\beta_3 - c\psi\rho\xi M_3M_5\beta_3 + c\psi\rho\xi\beta_3\theta_1v_1 - c\psi\xi M_2M_3M_4\beta_1 - c\psi\xi M_2M_3M_5\beta_1 - c\psi\xi M_2M_4M_5\beta_1 + c\psi\xi M_2\beta_1\theta_2v_2 \\
& - c\psi\xi M_3M_4M_5\beta_1 + c\psi\xi M_3\beta_1\theta_2v_2 + c\psi\xi M_4\beta_1\theta_1v_1 + c\psi\xi M_5\beta_1\theta_1v_1 + \Lambda\zeta M_2M_3M_4 + \Lambda\zeta M_2M_3M_5 + \Lambda\zeta M_2M_4M_5 \\
& - \Lambda\zeta M_2\theta_2v_2 + \Lambda\zeta M_3M_4M_5 - \Lambda\zeta M_3\theta_2v_2 - \Lambda\zeta M_4\theta_1v_1 - \Lambda\zeta M_5\theta_1v_1 + \alpha cM_3M_4\beta_2 + \alpha cM_3M_5\beta_2 + \alpha cM_4M_5\beta_2 \\
& - \alpha c\beta_2\theta_2v_2 + c\rho M_2M_3\beta_3 + c\rho M_2M_5\beta_3 + c\rho M_3M_5\beta_3 - c\rho\beta_3\theta_1v_1 cM_2M_3M_4\beta_1 + cM_2M_3M_5\beta_1 + cM_2M_4M_5\beta_1 \\
& - cM_2\beta_1\theta_2v_2 + cM_3M_4M_5\beta_1 - cM_3\beta_1\theta_2v_2 - cM_4\beta_1\theta_1v_1 - cM_5\beta_1\theta_1v_1 - M_1M_2M_3M_4 - M_1M_2M_3M_5 - M_1M_2M_4M_5 \\
& + M_1M_2\theta_2v_2 - M_1M_3M_4M_5 + M_1M_3\theta_2v_2 + M_1M_4\theta_1v_1 + M_1M_5\theta_1v_1 M_2M_3M_4M_5 + M_2M_3\theta_2v_2 + M_4M_5\theta_1v_1 \\
& - \theta_1\theta_2v_1v_2)\lambda + \alpha c\psi\xi M_3M_4M_5\beta_2 - \alpha c\psi\xi M_3\beta_2\theta_2v_2 + c\psi\rho\xi M_2M_3M_5\beta_3 c\psi\rho\xi M_5\beta_3\theta_1v_1 + c\psi\xi M_2M_3M_4M_5\beta_1 \\
& - c\psi\xi M_2M_3\beta_1\theta_2v_2 - c\psi\xi M_4M_5\beta_1\theta_1v_1 + c\psi\xi\beta_1\theta_1\theta_2v_1v_2 - \Lambda\zeta M_2M_3M_4M_5 + \Lambda\zeta M_2M_3\theta_2v_2 + \Lambda\zeta M_4M_5\theta_1v_1 \\
& - \Lambda\zeta\theta_1\theta_2v_1v_2 - \alpha cM_3M_4M_5\beta_2 + \alpha cM_3\beta_2\theta_2v_2 - c\rho M_2M_3M_5\beta_3 + c\rho M_5\beta_3\theta_1v_1 - cM_2M_3M_4M_5\beta_1 \\
& + cM_2M_3\beta_1\theta_2v_2 + cM_4M_5\beta_1\theta_1v_1 - c\beta_1\theta_1\theta_2v_1v_2 + M_1M_2M_3M_4M_5 - M_1M_2M_3\theta_2v_2 - M_1M_4M_5\theta_1v_1 + M_1\theta_1\theta_2v_1v_2 \\
& \quad \quad \quad (20)
\end{aligned}$$

Equation (20) has four (4) negative roots by Descartes rule of signs if

$$\begin{aligned}
& (\alpha c\psi\xi M_3M_4M_5\beta_2 - \alpha c\psi\xi M_3\beta_2\theta_2v_2 + c\psi\rho\xi M_2M_3M_5\beta_3 - c\psi\rho\xi M_5\beta_3\theta_1v_1 + c\psi\xi M_2M_3M_4M_5\beta_1 - c\psi\xi M_2M_3\beta_1\theta_2v_2 \\
& - c\psi\xi M_4M_5\beta_1\theta_1v_1 + c\psi\xi\beta_1\theta_1\theta_2v_1v_2 - \Lambda\zeta M_2M_3M_4M_5 + \Lambda\zeta M_2M_3\theta_2v_2 + \Lambda\zeta M_4M_5\theta_1v_1 - \Lambda\zeta\theta_1\theta_2v_1v_2 - \alpha cM_3M_4M_5\beta_2 \\
& + \alpha cM_3\beta_2\theta_2v_2 - c\rho M_2M_3M_5\beta_3 + c\rho M_5\beta_3\theta_1v_1 - cM_2M_3M_4M_5\beta_1 + cM_2M_3\beta_1\theta_2v_2 + cM_4M_5\beta_1\theta_1v_1 - c\beta_1\theta_1\theta_2v_1v_2 \\
& + M_1M_2M_3M_4M_5 - M_1M_2M_3\theta_2v_2 - M_1M_4M_5\theta_1v_1 + M_1\theta_1\theta_2v_1v_2) < [(c\beta_1 - c\psi\xi\beta_1 + \Lambda\zeta - M_1 - M_2 - M_3 - M_4 - M_5) \\
& \times (\Lambda\zeta M_2 + \Lambda\zeta M_3 + \Lambda\zeta M_4 + \Lambda\zeta M_5 - \xi c\psi\beta_2\alpha - c\psi\rho\xi\beta_3 - c\psi\xi M_2\beta_1 - c\psi\xi M_3\beta_1 - c\psi\xi M_4\beta_1 - c\psi\xi M_5\beta_1 + c\beta_2\alpha + \rho c\beta_3 \\
& + cM_2\beta_1 + cM_3\beta_1 + cM_4\beta_1 + cM_5\beta_1 - M_2M_1 - M_3M_1 - M_4M_1 - M_1M_5 - M_3M_2 - M_2M_4 - M_2M_5 - M_3M_4 - M_3M_5 \\
& - M_4M_5 + v_1\theta_1 + \theta_2v_2)(\Lambda\zeta M_2M_3 + \Lambda\zeta M_2M_4 + \Lambda\zeta M_2M_5 + \Lambda\zeta M_3M_4 + \Lambda\zeta M_3M_5 + \Lambda\zeta M_4M_5 - \alpha c\psi\xi M_3\beta_2 - \alpha c\psi\xi M_4\beta_2 \\
& - \alpha c\psi\xi M_5\beta_2 - c\psi\rho\xi M_2\beta_3 - c\psi\rho\xi M_3\beta_3 - c\psi\rho\xi M_5\beta_3 - c\psi\xi M_2M_3\beta_1 - c\psi\xi M_2M_4\beta_1 - c\psi\xi M_2M_5\beta_1 - c\psi\xi M_3M_4\beta_1 \\
& - c\psi\xi M_3M_5\beta_1 - c\psi\xi M_4M_5\beta_1 + c\psi\xi\beta_1\theta_1v_1 + c\psi\xi\beta_1\theta_2v_2 - \Lambda\zeta\theta_1v_1 - \Lambda\zeta\theta_2v_2 + \alpha cM_3\beta_2 + \alpha cM_4\beta_2 + \alpha cM_5\beta_2 + c\rho M_2\beta_3 \\
& + c\rho M_3\beta_3 + c\rho M_5\beta_3 + cM_2M_3\beta_1 + cM_2M_4\beta_1 + cM_2M_5\beta_1 + cM_3M_4\beta_1 + cM_3M_5\beta_1 + cM_4M_5\beta_1 - c\beta_1\theta_1v_1 - c\beta_1\theta_2v_2 \\
& - M_1M_2M_3 - M_1M_2M_4 - M_1M_2M_5 - M_1M_3M_4 - M_1M_3M_5 - M_1M_4M_5 + M_1\theta_1v_1 + M_1\theta_2v_2 - M_2M_3M_4 - M_2M_3M_5 \\
& - M_2M_4M_5 + M_2\theta_2v_2 - M_3M_4M_5 + M_3\theta_2v_2 + M_4\theta_1v_1 + M_5\theta_1v_1)(\alpha c\psi\xi\beta_2\theta_2v_2 - \alpha c\psi\xi M_3M_4\beta_2 - \alpha c\psi\xi M_3M_5\beta_2 \\
& - \alpha c\psi\xi M_4M_5\beta_2 - c\psi\rho\xi M_2M_3\beta_3 - c\psi\rho\xi M_2M_5\beta_3 - c\psi\rho\xi M_3M_5\beta_3 + c\psi\rho\xi\beta_3\theta_1v_1 - c\psi\xi M_2M_3M_4\beta_1 - c\psi\xi M_2M_3M_5\beta_1 \\
& - c\psi\xi M_2M_4M_5\beta_1 + c\psi\xi M_2\beta_1\theta_2v_2 - c\psi\xi M_3M_4M_5\beta_1 + c\psi\xi M_3\beta_1\theta_2v_2 + c\psi\xi M_4\beta_1\theta_1v_1 + c\psi\xi M_5\beta_1\theta_1v_1 + \Lambda\zeta M_2M_3M_4 \\
& + \Lambda\zeta M_2M_3M_5 + \Lambda\zeta M_2M_4M_5 - \Lambda\zeta M_2\theta_2v_2 + \Lambda\zeta M_3M_4M_5 - \Lambda\zeta M_3\theta_2v_2 - \Lambda\zeta M_4\theta_1v_1 - \Lambda\zeta M_5\theta_1v_1 + \alpha cM_3M_4\beta_2 \\
& + \alpha cM_3M_5\beta_2 + \alpha cM_4M_5\beta_2 - \alpha c\beta_2\theta_2v_2 + c\rho M_2M_3\beta_3 + c\rho M_2M_5\beta_3 + c\rho M_3M_5\beta_3 - c\rho\beta_3\theta_1v_1 + cM_2M_3M_4\beta_1 + cM_2M_3M_5\beta_1 \\
& + cM_2M_4M_5\beta_1 - cM_2\beta_1\theta_2v_2 + cM_3M_4M_5\beta_1 - cM_3\beta_1\theta_2v_2 - cM_4\beta_1\theta_1v_1 - cM_5\beta_1\theta_1v_1 - M_1M_2M_3M_4 - M_1M_2M_3M_5 \\
& - M_1M_2M_4M_5 + M_1M_2\theta_2v_2 - M_1M_3M_4M_5 + M_1M_3\theta_2v_2 + M_1M_4\theta_1v_1 + M_1M_5\theta_1v_1 - M_2M_3M_4M_5 + M_2M_3\theta_2v_2 \\
& + M_4M_5\theta_1v_1 - \theta_1\theta_2v_1v_2)]
\end{aligned}$$

Since $S(t) \leq \frac{\Delta}{\mu}$ in the invariant set, J is a non-negative matrix. Hence, it follows that

$$\frac{dx}{dt} \leq (F - V)X$$

When $R_0 < 1$, the eigenvalues of the matrix $F - V$ are negative. As a result, the linearized. Since $(H_U, H_A, H_T, A_A, A_T) \rightarrow (0, 0, 0, 0, 0)$ as $t \rightarrow \infty$. According to the comparison theorem, $(H_U, H_A, H_T, A_A, A_T) \rightarrow (0, 0, 0, 0, 0)$ as $t \rightarrow \infty$. Substituting $H_U = H_A = H_T = A_A = A_T = 0$ in (1) gives $S(t) \rightarrow S_0$ as $t \rightarrow \infty$. Thus, $(S, H_U, H_A, H_T, A_A, A_T) \rightarrow (S_0, 0, 0, 0, 0, 0)$ as $t \rightarrow \infty$ for $R_0 < 1$.

4.2 The endemic equilibrium's local and global stability; E^*

Theorem 4: The endemic steady state $E^*(S^*, H_U^*, H_A^*, H_T^*, A_A^*, A_T^*)$ of the model is locally asymptotically stable (LAS). If $R_0 > 1$.

Proof: We must now demonstrate the local stability of the endemic steady state. Assume $R_0 > 1$.

The Jacobian matrix for the variables of system (1) is computed in the proof of Theorem 2 as in (14).

Hence, for the endemic equilibrium $(S^*, H_U^*, H_A^*, H_T^*, A_A^*, A_T^*)$, the Jacobian matrix and the determinantal equation at the endemic equilibrium is given as matrix in (15)

Clearly, the equation reduces to:

$$(-\theta_1 - \mu - \lambda)(-v_1 - \mu - \lambda)(-v_2 - d_a - \mu - \lambda)(-\theta_2 - d_a - \mu - \lambda) \begin{vmatrix} g_1 - \mu - \lambda & -\Lambda\zeta + g_2 \\ g_3 & \Lambda\zeta - \alpha + g_4 - \mu - \rho - \lambda \end{vmatrix} = 0 \quad (21)$$

The first four eigenvalues of (21) are given as:

$$\lambda_1 = -(\theta_1 + \mu), \lambda_2 = -(v_1 + \mu), \lambda_3 = -(v_2 + d_a + \mu), \lambda_4 = -(\theta_2 + d_a + \mu)$$

The eigenvalue of the remaining 2×2 is obtained from the characteristics equation below:

$$\lambda^2 + (\alpha - \Lambda\zeta - g_1 - g_4 + 2\mu + \rho)\lambda + \Lambda\zeta g_1 + \Lambda\zeta g_3 - \Lambda\zeta\mu - \alpha g_1 + \alpha\mu + g_1 g_4 - g_1\mu - g_1\rho - g_2 g_3 - g_4\mu + \mu^2 + \mu\rho \quad (22)$$

The determinants of the characteristic polynomial from (22) yield the following result:

$$f(\lambda) = \lambda^2 + a_1\lambda + a_0.$$

Polynomials of order 2 satisfy the Routh-Hurwitz criterion, We know that $f(\lambda) = 0$ using Routh-Hurwitz criterion polynomials of order 2 is stable if and only if both coefficients in (22) satisfy the following conditions: $a_i > 0$ From Eq. (22) the condition is satisfied. Therefore, EE is locally asymptotically stable.

Theorem 5: When $R_0 < 1$, the equations of the model have a positive distinct endemic equilibrium, which is said to be globally asymptotically stable.

Proof: Considering the Lyapunov function, which is defined as

$$L(S^*, H_U^*, H_A^*, H_T^*, A_A^*, A_T^*) = \left(S - S^* \ln \left(\frac{S}{S^*} \right) \right) + \left(H_U - H_U^* \ln \left(\frac{H_U}{H_U^*} \right) \right) + \left(H_A - H_A^* \ln \left(\frac{H_A}{H_A^*} \right) \right) \\ + \left(H_T - H_T^* \ln \left(\frac{H_T}{H_T^*} \right) \right) + \left(A_A - A_A^* \ln \left(\frac{A_A}{A_A^*} \right) \right) + \left(A_T - A_T^* \ln \left(\frac{A_T}{A_T^*} \right) \right)$$

where L directly takes its derivative along the system as:

$$\frac{dL}{dt} = \left(1 - \frac{S^*}{S} \right) \frac{dS}{dt} + \left(1 - \frac{H_U^*}{H_U} \right) \frac{dH_U}{dt} + \left(1 - \frac{H_A^*}{H_A} \right) \frac{dH_A}{dt} + \left(1 - \frac{H_T^*}{H_T} \right) \frac{dH_T}{dt} + \left(1 - \frac{A_A^*}{A_A} \right) \frac{dA_A}{dt} + \left(1 - \frac{A_T^*}{A_T} \right) \frac{dA_T}{dt} \\ \frac{dL}{dt} = \left(1 - \frac{S^*}{S} \right) \left\langle \Lambda(1 - \zeta H_U) - \left(\frac{cb_h(1 - \psi\xi)\beta_1 H_U + \beta_2 H_A + \beta_3 A_A}{N} \right) + \mu \right\rangle S \\ + \left(1 - \frac{H_U^*}{H_U} \right) \left\langle \left(\frac{cb_h(1 - \psi\xi)\beta_1 H_U + \beta_2 H_A + \beta_3 A_A}{N} \right) S - (\alpha + \rho + \mu) H_U + \Lambda\zeta H_U \right\rangle \\ + \left(1 - \frac{H_A^*}{H_A} \right) \langle \alpha H_U + v_1 H_T - (\theta_1 + \mu) H_A \rangle + \left(1 - \frac{H_T^*}{H_T} \right) \langle \theta_1 H_A - (v + \mu) H_T \rangle \\ + \left(1 - \frac{A_A^*}{A_A} \right) \langle \rho H_U + v_2 A_T - (\theta_2 + d_a + \mu) A_A \rangle + \left(1 - \frac{A_T^*}{A_T} \right) \langle \theta_2 A_A - (v_2 + d_a + \mu) A_T \rangle$$

At equilibrium

$$\Lambda(1 - \zeta H_U) = cb_h(1 - \psi\xi) \left(\frac{\beta_1 H_U^* + \beta_2 H_A^* + \beta_3 A_A^*}{N^*} \right) S^* + \mu S^*$$

$$(\alpha + \rho + \mu + \Lambda\xi) = cb_h(1 - \psi\xi) \left(\frac{\beta_1 H_U^* + \beta_2 H_A^* + \beta_3 A_A^*}{H_U^* N^*} \right) S^*$$

$$(\theta_1 + \mu) = \frac{\alpha H_U^* + v_1 H_T^*}{H_A^*}$$

$$(v_1 + \mu) = \frac{\theta_1 H_A^*}{H_T^*}$$

$$(\theta_2 + d_a + \mu) = \frac{\rho H_U^*}{A_A^*} + \frac{v_2 A_T^*}{A_A^*}$$

$$(v_2 + d_a + \mu) = \frac{\theta_2 A_A^*}{A_T^*}$$

$$\begin{aligned} \frac{dL}{dt} &= \left(1 - \frac{S^*}{S}\right) \left\langle \left(\frac{cb_h(1 - \psi\xi)\beta_1 H_U^* + \beta_2 H_A^* + \beta_3 A_A^*}{N^*} \right) S^* + \mu S^* - \left(\frac{cb_h(1 - \psi\xi)\beta_1 H_U + \beta_2 H_A + \beta_3 A_A}{N} \right) + \mu \right\rangle S \\ &\quad + \left(1 - \frac{H_U^*}{H_U}\right) \left\langle \left(\frac{cb_h(1 - \psi\xi)\beta_1 H_U + \beta_2 H_A + \beta_3 A_A}{N} \right) S - \left(\frac{cb_h(1 - \psi\xi)(\beta_1 H_U^* + \beta_2 H_A^* + \beta_3 A_A^*)}{H_U^* N^*} \right) S^* H_U \right\rangle \\ &\quad - \left(1 - \frac{H_A^*}{H_A}\right) \left\langle \alpha H_U + v_1 H_T - \left(\frac{\alpha H_U^*}{H_A^*} + \frac{v_1 H_T^*}{H_A^*} \right) H_A \right\rangle + \left(1 - \frac{H_T^*}{H_T}\right) \left\langle \theta_1 H_A - \left(\frac{\theta_1 H_A^*}{H_T} \right) H_T \right\rangle \\ &\quad + \left(1 - \frac{A_A^*}{A_A}\right) \left\langle \rho H_U + v_2 A_T - \left(\frac{\rho H_U^*}{A_A^*} + \frac{v_2 A_T^*}{A_A^*} \right) A_A \right\rangle + \left(1 - \frac{A_T^*}{A_T}\right) \left\langle \theta_2 A_A - \frac{\theta_2 A_A^*}{A_T^*} A_T \right\rangle \\ &= \left(1 - \frac{S^*}{S}\right) \left\langle \frac{cb_h(1 - \psi\xi)\beta_1 H_U^*}{N^*} S^* + \frac{cb_h(1 - \psi\xi)\beta_2 H_A^*}{N^*} S^* + \frac{cb_h(1 - \psi\xi)\beta_3 A_A^*}{N^*} S^* + \mu S^* - \frac{cb_h(1 - \psi\xi)\beta_1 H_U}{N} S \right. \\ &\quad \left. - \frac{cb_h(1 - \psi\xi)\beta_2 H_A}{N} S - \frac{cb_h(1 - \psi\xi)\beta_3 A_A}{N} S - \mu S \right\rangle + \left(1 - \frac{H_U^*}{H_U}\right) \left\langle \frac{cb_h(1 - \psi\xi)\beta_1 H_U}{N} S + \frac{cb_h(1 - \psi\xi)\beta_2 H_A}{N} S \right. \\ &\quad \left. + \frac{cb_h(1 - \psi\xi)\beta_3 A_A}{N} S - \frac{cb_h(1 - \psi\xi)\beta_1 H_U^* S^* H_U}{H_U^* N^*} - \frac{cb_h(1 - \psi\xi)\beta_2 H_A^* S^* H_U}{H_U^* N^*} - \frac{cb_h(1 - \psi\xi)\beta_3 A_A^* S^* H_U}{H_U^* N^*} S \right\rangle \\ &\quad + \left(1 - \frac{H_A^*}{H_A}\right) \left\langle \alpha H_U + v_1 H_T - \frac{\alpha H_U^* H_A}{H_A^*} - \frac{v_1 H_T^* H_A}{H_A^*} \right\rangle + \left(1 - \frac{H_T^*}{H_T}\right) \left\langle \theta_1 H_A - \frac{\theta_1 H_A^* H_T}{H_T} \right\rangle \\ &\quad + \left(1 - \frac{A_A^*}{A_A}\right) \left\langle \rho H_U + v_2 A_T - \frac{\rho H_U^* A_A}{A_A^*} - \frac{v_2 A_T^* A_A}{A_A^*} \right\rangle + \left(1 - \frac{A_T^*}{A_T}\right) \left\langle \theta_2 A_A - \frac{\theta_2 A_A^* A_T}{A_T} \right\rangle \\ &= \left(1 - \frac{S^*}{S}\right) \left\langle \frac{cb_h(1 - \psi\xi)\beta_1 H_U^* S}{N} \left(1 - \frac{H_U^* S^* N}{H_U S N^*}\right) - cb_h(1 - \psi\xi)\beta_2 H_A S \left(1 - \frac{H_A^* S^* N}{H_A S N^*}\right) \right. \\ &\quad \left. - cb_h(1 - \psi\xi)\beta_3 A_A S \left(1 - \frac{H_A^* S^* N}{A_A S N^*}\right) - \mu S \left(1 - \frac{S^*}{S}\right) + \left(1 - \frac{H_U^*}{H_U}\right) \right. \\ &\quad \left\langle \frac{cb_h(1 - \psi\xi)\beta_1 H_U S}{N} \left(1 - \frac{H_U^* S^* N}{H_U S N^*}\right) + \frac{cb_h(1 - \psi\xi)\beta_2 H_A S}{N} \left(1 - \frac{H_A^* S^* H_U}{H_A S H_U^* N^*}\right) + \frac{cb_h(1 - \psi\xi)\beta_3 A_A S}{N} \left(1 - \frac{A_A^* S^* H_U}{A_A S H_U^* N^*}\right) \right\rangle \\ &\quad + \left(1 - \frac{H_A^*}{H_A}\right) \left\langle \alpha H_U \left(1 - \frac{H_U^*}{H_U}\right) \left(1 - \frac{H_A^*}{H_A}\right) + v_1 H_T \left(1 - \frac{H_T^*}{H_T}\right) \left(1 - \frac{H_A^*}{H_A}\right) \right\rangle \\ &\quad + \left(1 - \frac{H_T^*}{H_T}\right) \left\langle \left(\theta_1 H_A \left(1 - \frac{H_A^*}{H_A}\right) \left(1 - \frac{H_T^*}{H_T}\right) \right) + \left(1 - \frac{A_A^*}{A_A}\right) \left\langle \rho H_U \left(1 - \frac{H_U^*}{H_U}\right) \left(1 - \frac{A_A^*}{A_A}\right) + v_2 A_T \left(1 - \frac{A_T^*}{A_T}\right) \left(1 - \frac{A_A^*}{A_A}\right) \right\rangle \right\rangle \\ &\quad + \left(1 - \frac{A_T^*}{A_T}\right) \left\langle \theta_2 A_A \left(1 - \frac{A_A^*}{A_A}\right) \left(1 - \frac{A_T^* A_T}{A_T} \right) \right\rangle = -\mu S \left(1 - \frac{S^*}{S}\right)^2 + P_1(S, H_U, H_A, H_T, A_A, A_T) + P_2(S, H_U, H_A, H_T, A_A, A_T) \end{aligned}$$

where

$$P_1(S, H_U, H_A, H_T, A_A, A_T) = -\frac{cb_h(1-\psi\xi)\beta_1 H_U S}{N} \left(1 - \frac{S^*}{S}\right) \left(1 - \frac{H_U^* S^* N}{H_U S N^*}\right) - \frac{cb_h(1-\psi\xi)\beta_2 H_A^* S^* N}{H_A S N^*} \left(1 - \frac{S^*}{S}\right) \left(1 - \frac{H_A^* S^* N}{H_A S N^*}\right) \\ - \frac{cb_h(1-\psi\xi)\beta_3 A_A S}{N} \left(1 - \frac{S^*}{S}\right) \left(1 - \frac{A_A^* S^* N}{A_A S N^*}\right)$$

$$P_2(S, H_U, H_A, H_T, A_A, A_T) = \text{All others}$$

$$P_1 \leq 0 \text{ whenever } H_U S N^* \geq H_U^* S^* N, H_A S N^* \geq H_A^* S^* N, A_A S N^* \geq A_A^* S^* N \quad (23)$$

$$P_2 \leq 0 \text{ whenever } H_U^* S^* N^* \geq H_U^* S^* N, H_A S H_U^* N^* \geq H_A^* S^* H_U, A_A S H_U^* N^* \geq A_A^* S^* H_U, H_U H_A^* \geq H_U^* H_A, H_T^* H_A, \quad (24)$$

Thus

$$\frac{dL}{dt} \leq 0$$

if (23) and (24) holds.

Hence, by Lasalle theorem, the equilibrium is globally asymptotically stable in the feasible region R_+^6 .

4.3 Sensitivity indices

Knowing the relative relevance of the different factors involved in HIV transmission and prevalence is vital for deciding how effectively to minimize human morbidity and mortality rate due to HIV infections. Sensitivity analysis is performed in this sub-section to assess the resilience of factors that have a strong impact on the basic reproduction number, R_0 , so that suitable intervention strategies may be implemented.

The effect of HIV testing and treatment on HIV/AIDS dynamics was studied using the elasticity of ReH with respect to α and θ . Using the method described in Refs. ^{61–64} to compute the elasticity⁶⁵ of R_{eH} with respect to α and θ as shown in Equation (25)

$$\frac{\alpha\theta}{R_{eH}} \frac{\partial R_{eH}}{\partial \alpha\theta} = \frac{c(1-\psi\xi)}{\zeta\Lambda - M_1} + \frac{c(1-\psi\xi)\alpha M_3}{(M_2 M_3 - \theta_1 v_1)(\zeta\Lambda - M_1)} + \frac{c(1-\psi\xi)\rho M_4}{(M_4^2 - \theta_2 v_2)(\zeta\Lambda - M_1)} \quad (25)$$

Interpretation of the sensitivity indices

Table 1's sensitivity indices are read as follows: Positive indices indicate that the corresponding basic reproduction number increases (decreases) as those parameters increase (decrease). Negative indices, on the other hand, indicate that increasing (decreasing) those parameters reduces the associated basic reproduction number (increases).

Table 1. Sensitivity indices of R_0 .

Parameter	Sensitivity index	Parameter	Sensitivity index
Λ	+2.403314920	α	-1.668175411
ζ	+2.403314919	μ	-0.6142854995
β_1	+0.6078964848	ρ	-0.7905498030
β_2	+0.2655262466	d_a	-0.002219932464
β_3	+0.1265772695	θ_1	-0.2354014199
c	+1	θ_2	-0.1120071499
u_1	+0.01834155833		
u_2	+0.0009827457838		

The endemicity of HIV infection increases when the values of β_i , $i = 1, 2, 3$, v , and c are increased; when the values of α and μ are decreased, the endemicity of HIV infection decreases.

As a result, interventions should aim to reduce the annual average number of sexual partners acquired, c , the number of defaulters lost to follow-up, v , and the likelihood of HIV transmission per sexual contact, β_i , $i = 1, 2, 3$, because the rate of progression from HIV to AIDS is increasing, ρ , indicates rapid progression to AIDS. In addition, effective condom use should be mandated as a precautionary measure to reduce the rate of HIV/AIDS transmission.

5. Numerical simulation

To affirm the model's theoretical prognosis, simulation studies of the system (22) are run with the estimated parameter values listed below:

Simulation 1. Take into account the parametric data in Table 2 $c = 3$, $\psi = 0$, $\xi = 0$, $\beta_1 = 0.050$, $\beta_2 = 0.055$, $\beta_3 = 0.060$, $\mu = 0.2$, $\Lambda = 29$, $\alpha = 0.7$, $\rho = 0.322$, $\zeta = 0.02$, $v_1 = 0.0169$, $v_2 = 0.0169$, $\theta_1 = 1.6949$, $\theta_2 = 1.6949$, $d_a = 0.0333$: Hence, $R_0 = 0.698$ and the infection-free equilibrium is (145;0;0;0;0;0): We can see in Figure 2 that by changing the initial values, the solution trajectories intersect to (145.00;0;0;0;0;0): This confirms the fact that if $R_0 < 1$, the virus-free equilibrium is globally asymptotically stable:

Simulation 2. Let $c = 6$, $\psi = 0$, $\xi = 0$, $\beta_1 = 0.080$, $\beta_2 = 0.085$, $\beta_3 = 0.090$, $\mu = 0.2$, $\Lambda = 29$, $\alpha = 0.7$, $\rho = 0.322$, $\zeta = 0.02$, $v_1 = 0.0169$, $v_2 = 0.0169$, $\theta_1 = 1.6949$, $\theta_2 = 1.6949$, $d_a = 0.0333$: Hence, $R_0 = 2.197$. Moreover, the endemic equilibrium is (64.197;13.225;5.251;41.035;2.348;15.905): We can see in Figure 3 that by changing the initial conditions, the solution trajectories intersect to (64.197;13.225;5.251;41.035; 2.348;15.905): This proves Theorem 5: if $R_0 > 1$, the endemic stability is globally stable.

Simulation 3 depicts the distribution of individual proportions over time in various classes where there are no new infected children ζ or recruitment Λ , and contact c i.e. taking $c = 0$, $\zeta = 0$, $\Lambda = 0$ when $\psi = 1$ and $\xi = 1$, (condom usage and effectiveness) i.e. when there is full protection keeping every other values at endemic equilibrium constant, the value of $R_0 = 0$.

The impact of perinatal transmission in the system, i.e. the incidence of new recruits of infected children directly into the infective group, is pointedly demonstrated in simulation 4.

Figure 5(a) shows that as the proportion of infected newborns (ζ) rises, so does the proportion of the general population who is unaware. Figure 5(b) shows that increasing the value of (ζ) causes the proportion of the AIDS population to decrease over time, then raise until it reaches its stable state. As a result, if newborns infected with the virus are treated, the total infective group will be better controlled, minimizing the AIDS individuals. Figure 5(c) shows that as the number of infected children born rises, so does the treated populace.

Table 2. Definition of Parameters values for the HIV model.

Parameters	Description	Parameters value	Source
Λ	Recruitment rate	29 yr^{-1}	³
ζ	Rate of newborns infected with HIV	0.02	[Assumed]
c	Contact rate	3 patners/yr	³
β_i , $i = 1, 2, 3$	Transmission rate for the infective HIV and AIDS	[0.050, 0.055, 0.060]	Assumed
μ	Natural mortality	0.2	[Assumed]
α	Testing rate	0.7	[Assumed]
ρ	Progression rate from Unaware HIV to AIDS	0.322	[Assumed]
u_i , $i = 1, 2$	HIV and AIDS defaulters from treatment	0.0169	⁵⁶
θ_i , $i = 1, 2$	HIV and AIDS treatment rate	1.6949	²⁷
d_a	Mortality due to AIDS	0.0333	[Assumed]
ψ	condom effectiveness	[0,1]	[Assumed]
ξ	condom usage	[0,1]	[Assumed]

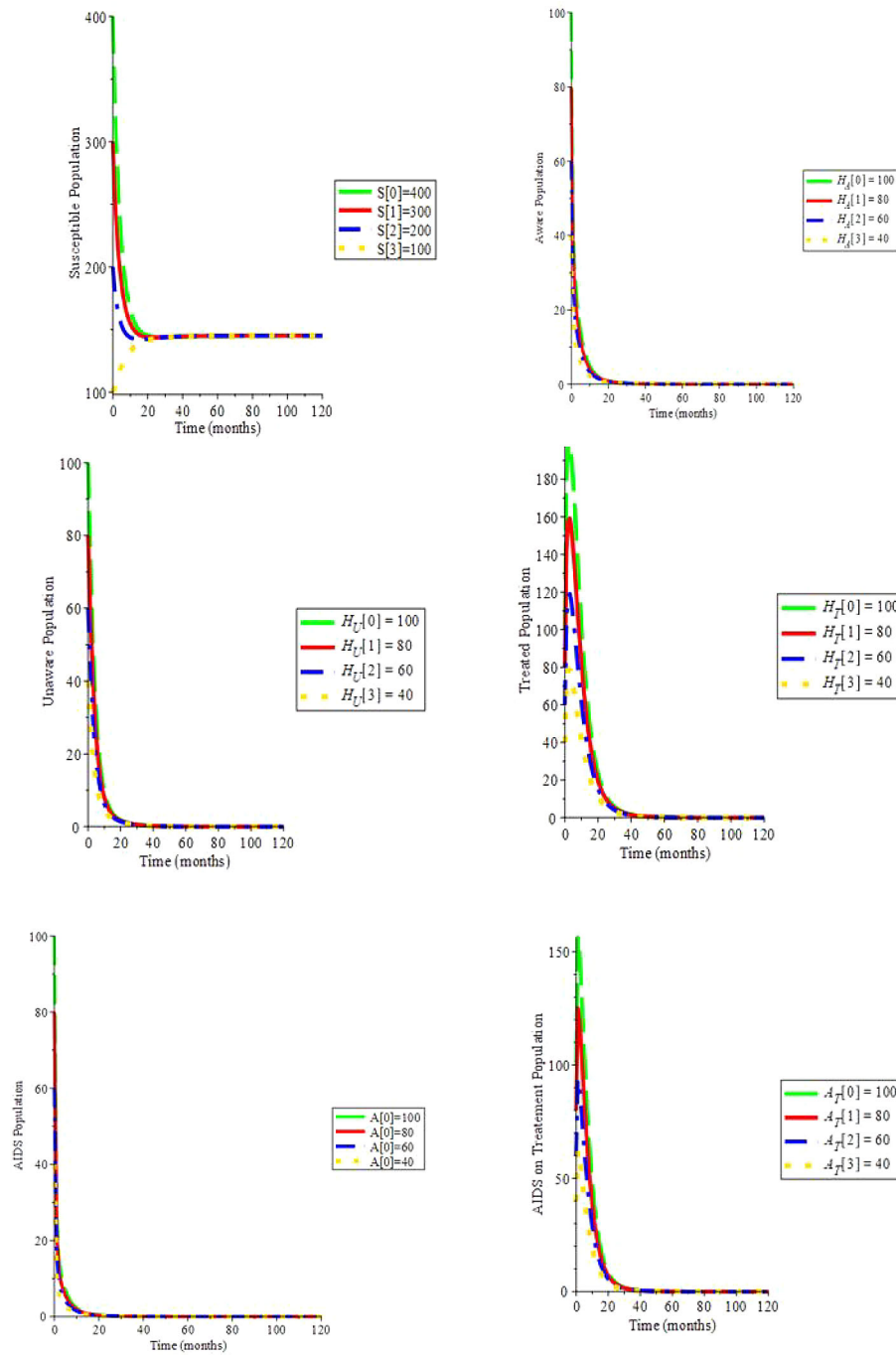


Figure 2. (Simulation 1) if $R_0 < 1$, the infection-free equilibrium is asymptotically stable.

The effect of defaulters on treatment lost to follow-up in the model is examined in simulation 5.

Figure 6(a) shows that as the rate of defaulters (v) increases, so does the proportion of the population that is aware, whereas the proportion of HIV patients on treatment decreases (b). Figure 6(c) shows how increasing ϵ causes the proportion of the AIDS population to increase over time while decreasing the proportion of the AIDS population on treatment until equilibrium is reached. As a result, if the HIV-aware infected population follows therapy, the infectious individual as a whole would then remain under control, lowering the HIV-aware and AIDS number of individuals.

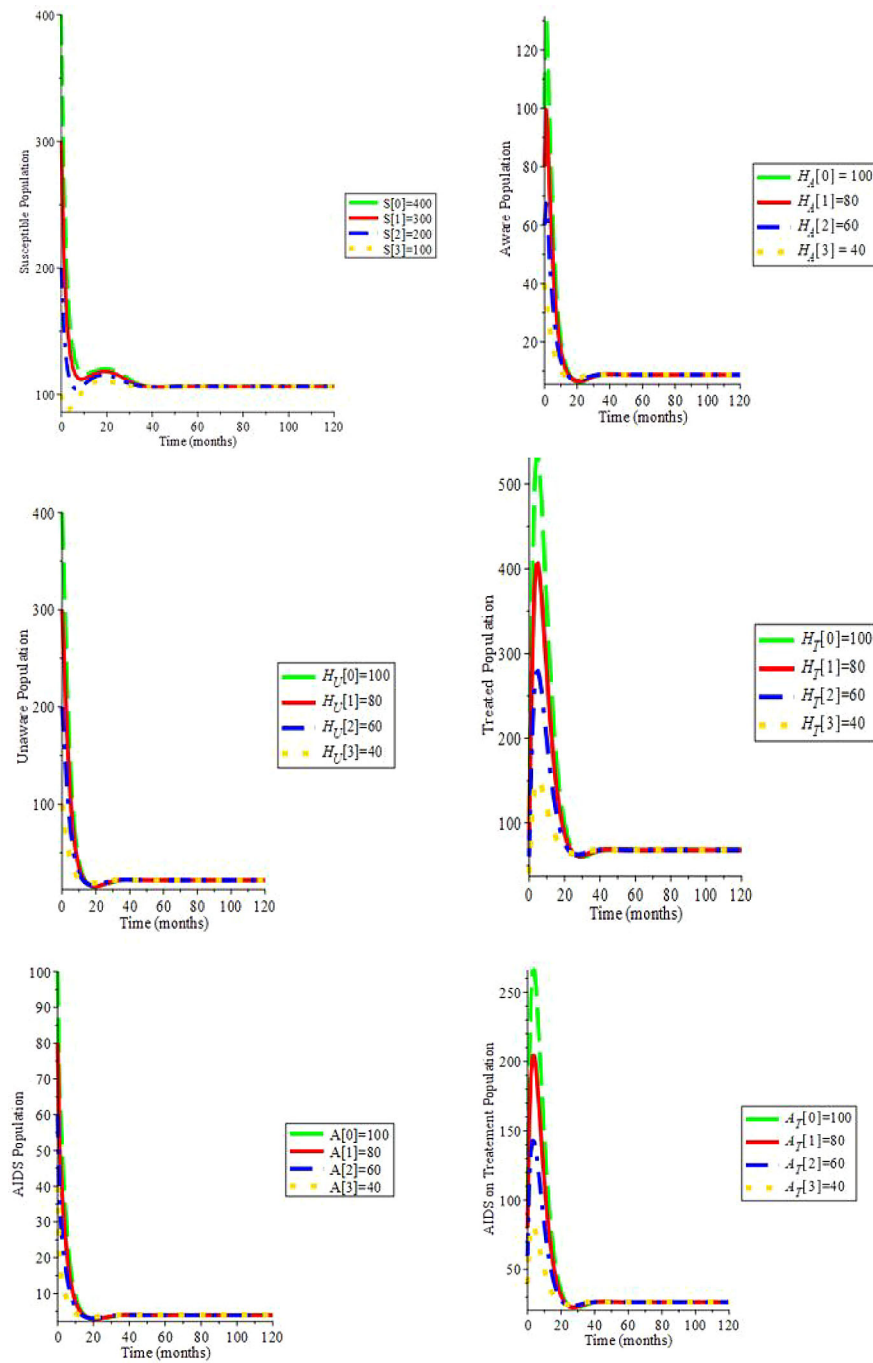


Figure 3. (Simulation 2) If $R_0 > 1$, the endemic stability is asymptotically stable.

The increasing effect of testing and treatment on the model is examined in simulation 6.

From Figure 7(a-d), it is observed that if testing rate and treatment rate is increase, the unaware HIV decrease, while aware HIV and AIDS individual decrease with time due to treatment. Furthermore, the susceptible individual increases, and as treatment increases, so does the population of HIV and AIDS patients on treatment. As a result, increasing HIV screening and treatment is the first procedure to UNAIDS' 90-90-90 aspirations.

Figure 8 shows the effect of treatment fall out on the reproduction number. When the number of infected individual on treatment that fallout is 19.8 percent then $R_0 = 0.041$. The linear graphical representation also revealed that if 40.1 percent of the population drops out of treatment, the reproduction number rises to 0.043. This simply means that, as defaulters lost

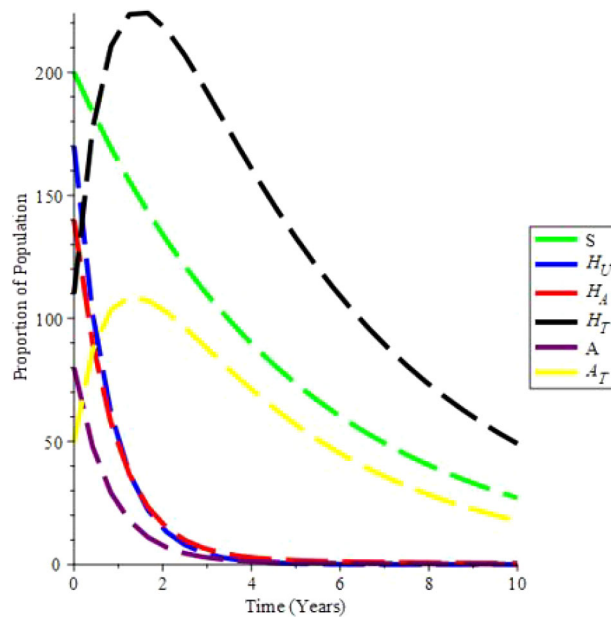


Figure 4. (Simulation 3) Take $\psi = 1$ and $\xi = 1$, to check the impact of condom use and effectiveness on the population when there's no contact.

to follow-up increase, the reproduction number also increases. Hence, reducing high-risk habits, mainly through education, is the most effective way to reduce the overall number of HIV/AIDS patients.

6. Conclusions and recommendations

This study investigated the effect of testing and ART on the vertical and horizontal transmission dynamics of HIV/AIDS infection using an improved compartmental model and the dynamics theory of SI infectious diseases.

Reducing high-risk behaviours, primarily through education on the importance of HIV/AIDS status awareness and treatment adherence, is the best option for reducing the total number of HIV/AIDS patients.

Increased HIV testing is the first step toward UNAIDS's 90-90-90 objectives, although many countries still face significant obstacles in attaining this goal. Early detection allows for prompt antiretroviral therapy, which lowers HIV viral load and hence slows the transmission of the virus. We believe that increasing HIV/AIDS diagnosis rates will increase the number of HIV/AIDS patients treated in the short term but decrease the number in the long term. WHO advises HIV self-testing as a complementary strategy,⁶⁶ which can improve the efficiency of HIV testing.⁶⁷

The following summarizes the paper's key findings:

- The discovery that increased screening rates will slow the spread of the disease suggests that the endemicity of the virus will rise in the absence of screening, which will lead to a constant rise in the number of people living with AIDS. When HIV infected people are aware of the consequences of not following treatment protocol strictly and taking preventative actions with their contacts in the community, the effects on the dynamics of HIV/AIDS are also examined.
- The current research showed that these intervention strategies are effective in combating the HIV/AIDS epidemic. This also emphasizes the need of behavioral and biologic therapies in preventing HIV transmission among pregnant women as neglected by Ref. 33.
- Finally, based on the study, it can be inferred that the best strategy to lower the infections rate and incidence level is to inform, counsel, and educate the public about HIV testing as well as the dangers of engaging in risky behavior such as having unsafe sex or failing to use condoms properly.

This study has flaws, as well. Homosexual transmission was not included in the model and certain characteristics were chosen on the basis of assumptions and may not reflect reality.

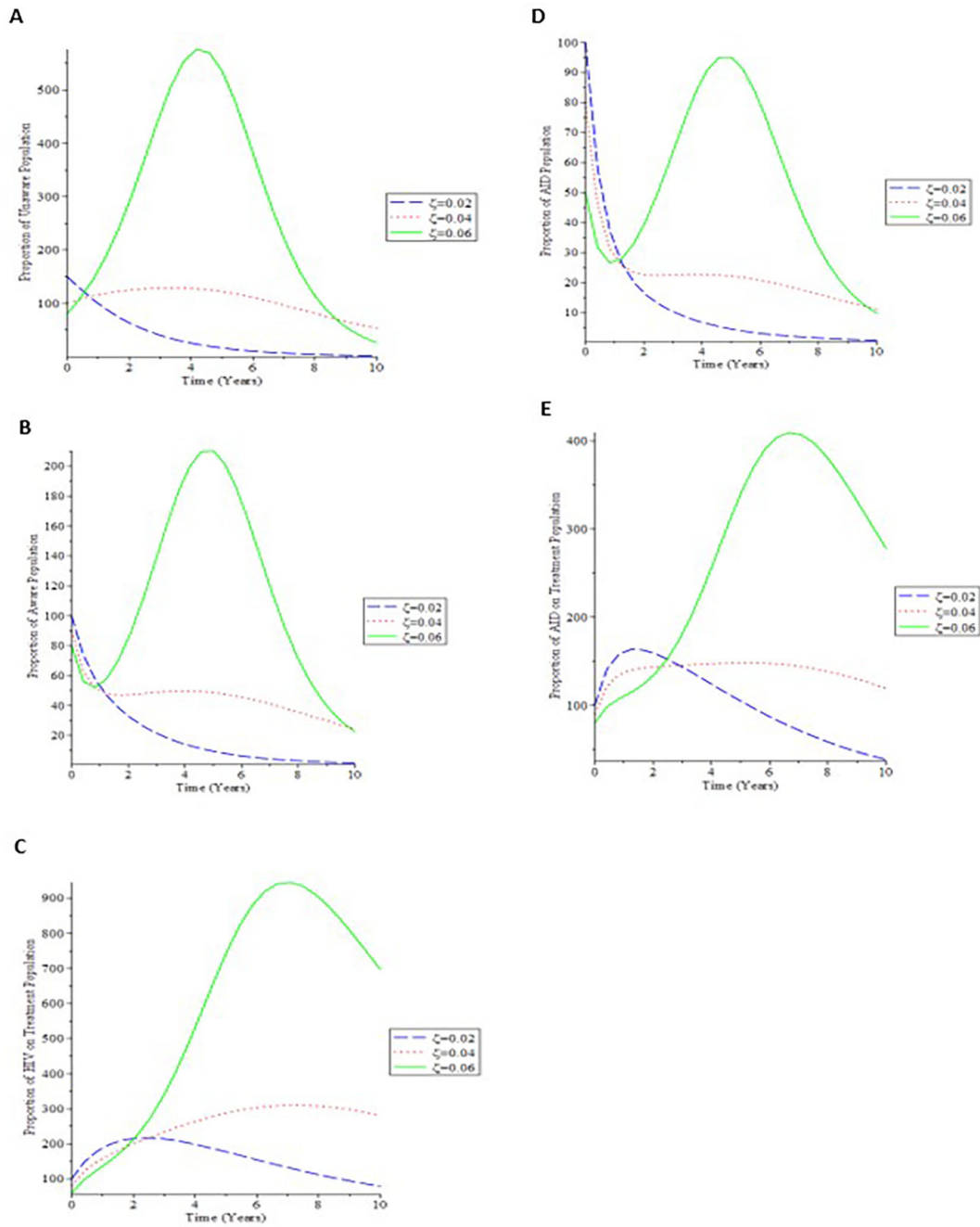


Figure 5. (Simulation 4) Variation in the infected individual for different ζ values. A. Variation of Unaware HIV population for different values of ζ . B. Variation of Aware HIV population for different values of ζ . C. Variation of HIV on Treatment population for different values of ζ . D. Variation of AIDS population for different values of ζ . E. Variation of AIDS on Treatment population for different values of ζ .

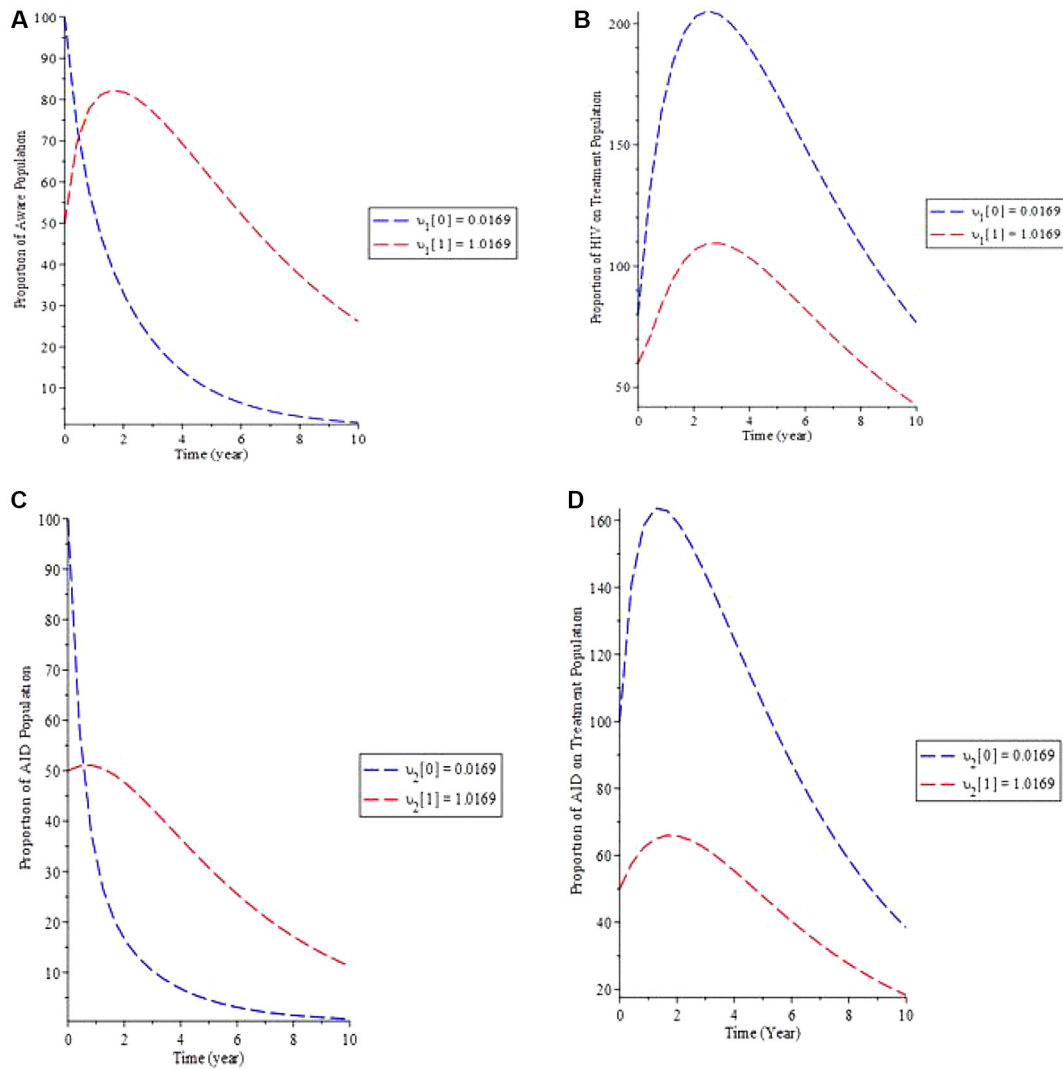


Figure 6. (Simulation 5) Variation of the infected individual for different fallout, u values. A. Variation of HIV Aware population for different values of u . B. Variation of HIV on Treatment population for different values of u . C. Variation of AIDS population for different values of u . D. Variation of AIDS on Treatment population for different values of u .

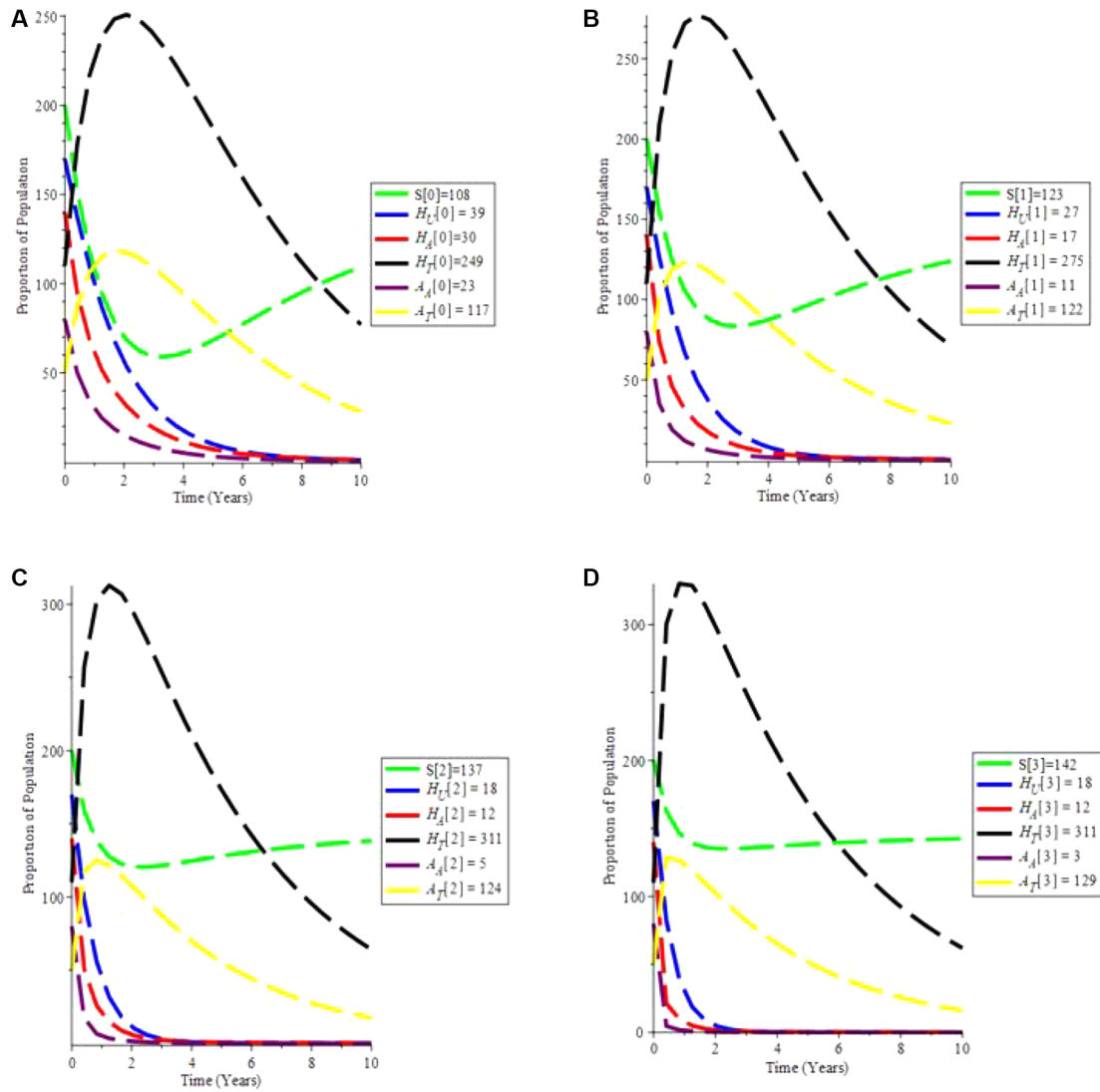


Figure 7. (Simulation 6) Proportion of different Population at the increased values of α and θ . A. Proportion of Population when $\alpha = 0.7$ and $\theta = 1.6949$. B. Proportion of Population when $\alpha = 0.9$ and $\theta = 2.6949$. C. Proportion of Population when $\alpha = 1.5$ and $\theta = 4.6949$. D. Proportion of Population when $\alpha = 1.9$ and $\theta = 9.6949$.

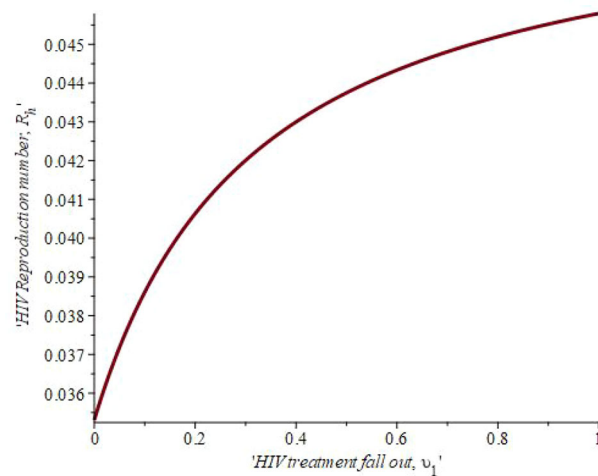


Figure 8. Impact of treatment fall out on HIV reproduction number.

In conclusion, the model implies that the most effective strategy to lower the incidence rate, in light of our study's findings, is through HIV counseling and testing, behavioural and biological strategies, effective condom use, controlling vertical transmission and stringent adherence to ART are required for HIV prevention among individuals and pregnant women. Even in the face of medication resistance, ART procedure adherence and effective condom use can successfully limit the transmission of HIV. The 90-90-90 strategy may not be sufficient on its own to end the global HIV/AIDS outbreak, but educational campaign by policy maker on strict adherence to treatment by HIV/AIDS positive on treatment and effective use of condom by those who are vulnerable to the virus and unaware individual would go a long way in meeting this eradication strategies by WHO.

Data availability

The data in this article come from Mukandavire *et al.*, 2010, Zu *et al.*, 2016, Lu *et al.*, 2020, and other assumed/estimated data.

Software availability

Source code available from: <https://github.com/OE-Abiodun/release/tag/v3.1.2>

Archived source code at the time of publication: <https://doi.org/10.5281/zenodo.6894864>.⁶⁸

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Fatmawati Fatmawati 

Department of Mathematics, Faculty of Science and Technology, Universitas Airlangga, Surabaya, Indonesia

The authors have addressed most of the issues raised in my report, and improved the paper accordingly.

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Mathematical modeling in life science, optimal control, fractional modelling.

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Reviewer Report 08 March 2023

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Adedapo Loinmi 

Department of Mathematics, Tai Solarin University of Education, Ijebu ode, Nigeria

All observed lapses are corrected. The article is now fit for indexing.

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Biological and computational Mathematics

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Version 1

Reviewer Report 25 January 2023

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**Ropo Ebenzer Ogunsakin**

¹ Discipline of Public Health Medicine, School of Nursing & Public Health, College of Health Sciences, University of KwaZulu-Natal, Durban, South Africa

² Discipline of Public Health Medicine, School of Nursing & Public Health, College of Health Sciences, University of KwaZulu-Natal, Durban, South Africa

In this paper, the authors developed a new mathematical model for the transmission dynamics of HIV and AIDS and the model was rigorously analysed. The authors considered the impact of three major strategies (impact of awareness, testing and follow up) which are important in controlling the transmission dynamics of HIV/AIDS. The six- compartmental model of HIV/AIDS presented are: Susceptible, (S), representing people who prone to be infected with HIV in engage in risk habits/factors, those who are unaware of their HIV status (HU), those who become aware after testing (HA), those who are placed on treatment after becoming aware (HT); the AIDS individuals (AA) due to non-adherence to treatment and the AIDS on treatment population (AT). The author showed that the model is positive through a given region in their analyses and the reproduction number of the model was correctly worked out. The effects of the strategies were graphically represented in the numerical simulations. The title is peaks a volume to the context of the article.

The work is well organized and approved for indexing. I have the following observation/comment on the paper:

1. The overall command of English looks good, however there are some grammatical errors in the paper. Also the authors should check for incomplete sentences in the paper.
2. Some representation can be made to reduce the size of the matrices as they are big or boxes should be adjusted.
3. It is necessary that the biological meaning of R_0 be properly stated.
4. In the sensitivity section, the authors can as well placed the real values of the indices.
5. The conclusion looks good, the novelty of the work is discussed.

Is the work clearly and accurately presented and does it cite the current literature?

Yes

Is the study design appropriate and is the work technically sound?

Yes

Are sufficient details of methods and analysis provided to allow replication by others?

Yes

If applicable, is the statistical analysis and its interpretation appropriate?

Partly

Are all the source data underlying the results available to ensure full reproducibility?

Yes

Are the conclusions drawn adequately supported by the results?

Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Statistician, Biological and Computational Mathematics

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard.

Reviewer Report 05 January 2023

<https://doi.org/10.5256/f1000research.135825.r158041>

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Fatmawati Fatmawati 

¹ Department of Mathematics, Faculty of Science and Technology, Universitas Airlangga, Surabaya, Indonesia

² Department of Mathematics, Faculty of Science and Technology, Universitas Airlangga, Surabaya, Indonesia

I have read this article. Some of my critical comments include:

1. Please explore the novelty of this article. What is the difference between this article and previous research that already exists.
2. I don't see the connection between the existence of the endemic equilibrium point and basic reproduction number (R_0). Likewise to prove the stability of the disease-free equilibrium. The authors only mention that if $R_0 < 1$ then the disease-free equilibrium is LAS, while depending on R_0 cannot be proven exactly
3. Please also check for proof of global stability of the disease-free equilibrium.

4. The authors must proofread very carefully the language of the manuscript.
5. Carefully check in whole manuscript, dot, and comma after each equation.
6. Then numerical results need to be discussed in more details.
7. The conclusion sections should highlight the main findings of this study.
8. References should be up to date and must follow the style correctly.

Is the work clearly and accurately presented and does it cite the current literature?

Partly

Is the study design appropriate and is the work technically sound?

Partly

Are sufficient details of methods and analysis provided to allow replication by others?

Partly

If applicable, is the statistical analysis and its interpretation appropriate?

Partly

Are all the source data underlying the results available to ensure full reproducibility?

No

Are the conclusions drawn adequately supported by the results?

Partly

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Mathematical modeling in life science, optimal control, fractional modelling.

I confirm that I have read this submission and believe that I have an appropriate level of expertise to state that I do not consider it to be of an acceptable scientific standard, for reasons outlined above.

Reviewer Report 18 November 2022

<https://doi.org/10.5256/f1000research.135825.r152688>

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Adedapo Loyinmi

¹ Department of Mathematics, Tai Solarin University of Education, Ijebu ode, Nigeria

² Department of Mathematics, Tai Solarin University of Education, Ijebu ode, Nigeria

The article proposed a mathematical model for the transmission dynamics of HIV and AIDS considering three control/preventive strategies: the impact of awareness, testing and follow up. The work presented a six- compartmental model of HIV/AIDS which are Susceptible, (*S*), representing people who are likely to become infected with HIV; Unaware HIV infective, (*HU*), Aware HIV infective (*HA*), Treated HIV infective, (*HT*); AIDS individuals (*AA*) and AIDS on treatment individuals (*AT*). Analysis of the model showed that the model is positively invariant. Also, the reproduction number of the model was accurately worked out. The stability of the model showed that the disease free and endemic equilibrium is stable if necessary conditions are satisfied. The numerical simulations graphically showed the effect of the control strategies. **The work is well organized and suitable for indexing.**

However, a few issues should be addressed:

1. The manuscript needs to be properly arranged as some of the matrices are too big. The boxes should be adjusted.
2. Under stability analysis of the DFE, the R_0 is not clearly shown. Since conclusion depends on the value of R_0 , it is necessary that R_0 is properly substituted in the equation.
3. In the sensitivity section, the sensitivity value need to be presented in order to know how sensitive each parameters are. The + and - signs are not enough
4. Graphs are not bold enough.
5. Revisit the conclusion under stability of DFE to drive home your findings
6. In figure 5, I suggest that more than two values of ζ be used.
7. In the conclusion, the novelty of the work is not elaborately discussed. Please rewrite this part to show what the contribution to knowledge is.
8. There should be a paragraph discussing earlier work on diseases such as Ebola, covid-19 and the likes threatening global health. Such articles like, but not limited to 'qualitative analysis and dynamical behaviour of a Lassa haemorrhagic fever model with exposed rodents and saturated incidence rate' would help.

Is the work clearly and accurately presented and does it cite the current literature?

Yes

Is the study design appropriate and is the work technically sound?

Yes

Are sufficient details of methods and analysis provided to allow replication by others?

Yes

If applicable, is the statistical analysis and its interpretation appropriate?

Yes

Are all the source data underlying the results available to ensure full reproducibility?

Yes

Are the conclusions drawn adequately supported by the results?

Yes

Competing Interests: No competing interests were disclosed.

Reviewer Expertise: Biological and computational Mathematics

I confirm that I have read this submission and believe that I have an appropriate level of expertise to confirm that it is of an acceptable scientific standard, however I have significant reservations, as outlined above.

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