Reflection coefficient of a slightly conductive panel

Alberto Prados Pérez, Julio Ureña González, Israel A. Callejas Vidoy

University of Granada

May 5, 2021

Lossy dielectric medium

Allowing for a non-null conductivity ($\sigma \neq 0$) leads to the attenuation of the propagating electromagnetic energy.

$$\left.\begin{array}{r}\epsilon_r\epsilon_0\dfrac{\partial\vec{E}}{\partial t} = \nabla\times\vec{H} - \vec{J} \\ \dfrac{\partial H}{\partial t} = -\dfrac{1}{\mu_0}\nabla\times \mathrm{E}\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\dfrac{\partial\tilde{E}_x(t)}{\partial t} = -\dfrac{1}{\epsilon_r\sqrt{\mu_0\epsilon_0}}\dfrac{\partial H_y(t)}{\partial z} - \dfrac{\sigma}{\epsilon_r\epsilon_0}\tilde{E}_x(t) \\ \dfrac{\partial H_y(t)}{\partial t} = -\dfrac{1}{\sqrt{\mu_0\epsilon_0}}\dfrac{\partial\tilde{E}_x(t)}{\partial z}\end{array}\right. \tag{1}$$

Where we have assumed the unidimensional propagation of the electromagnetic field. Furthermore, we have considered the electric field rescaling $\vec{\tilde{E}} = \sqrt{\epsilon_0/\mu_0}\vec{E}$.

Numerical method

Introducing the centered difference and average operators, and using the Courant condition $c_0 \Delta t / \Delta x = \Delta t / (\Delta x \sqrt{\epsilon_0 \mu_0}) = 1/2$, the FDTD scheme in this case turns out to be:

$$
\begin{aligned}
(\tilde{E}_x)_i^{n+1/2} &= \frac{1 - \frac{\sigma \Delta t}{2\epsilon_r \epsilon_0}}{1 + \frac{\sigma \Delta t}{2\epsilon_r \epsilon_0}} (\tilde{E}_x)_i^{n-1/2} - \frac{1}{2\epsilon_r \left(1 + \frac{\sigma \Delta t}{2\epsilon_r \epsilon_0}\right)} \left[(H_y)_{i+1/2}^n - (H_y)_{i-1/2}^n \right] \\
(H_y)_{i+1/2}^{n+1} &= (H_y)_{i+1/2}^n - \frac{1}{2} \left[(\tilde{E}_x)_{i+1}^{n+1/2} - (\tilde{E}_x)_i^{n+1/2} \right]
\end{aligned}
\tag{2}
$$

Reflectance and Transmittance

When a plane wave strikes a medium at *normal incidence* a fraction given by **R** is reflected and a fraction is transmitted (**T**). In terms of the impedances $\eta = \sqrt{\frac{\mu}{\epsilon_c}}$, with $\epsilon_c = \epsilon - j\frac{\sigma}{\omega}$ being the complex permittivity, the coefficients are:

$$R = \frac{E_{ref}}{E_{inc}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \tag{3}$$

$$T = \frac{E_{trans}}{E_{inc}} = \frac{2\eta_2}{\eta_2 + \eta_1} \tag{4}$$

Using conservation of energy ($\sigma = 0$), $\frac{I_R}{I_I} + \frac{I_T}{I_I} = 1$, this equation can be expressed in terms of **R** and **T**:

$$R^2 + T^2 = 1 \tag{5}$$