- Due Sunday by 11:59pm
- Points 100
- Submitting an external tool
- Available after Feb 22 at 12am

MAS3114

MATLAB Assignment 3

MATLAB (https://ufl.instructure.com/courses/497092/modules/items/10717393)

Introduction

This assignment must be completed individually in MATLAB and submitted on Gradescope. It is worth 100 points (+5 extra credit points).

Before Starting the Assignment

1. Download the ZIP (https://ufl.instructure.com/courses/497092/files/85615899?wrap=1) (https://ufl.instructure.com/courses/497092/files/85615899/download?download_frd=1) of the template files, upload it to your M-drive, and un-ZIP it. You should have the following files:
   - Assignment3_Output.mlx [Used to Run the Exercises & EC]
   - CramersRule.m [Function Template]
   - CramersRule3x3.m [Pre-written Function]
   - Exercise1.m [Exercise Template File]
   - Exercise2.m [Exercise Template File]
   - Exercise3.m [Exercise Template File & EC]
   - ParticularSolution.m [Pre-written Function]
2. Enter your Name & UFID on all template files (see below warning)!

> **WARNING:** If you do not enter your name and UF ID correctly, you will receive a zero for this assignment.

MATLAB 3

There are three exercises as part of this assignment. Exercise 1 is worth 40 points, and exercises 2 and 3 are worth 30 points.

Exercise 1 | Exercise 2 | Exercise 3

Solving a Linear System Ax = b, where A is 3x3

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40 points [10 Points Per Part]
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When A is invertible, the linear system A**x** = **b** has a unique solution. We have learned three ways to solve a linear system, and we will observe three different methods in this exercise when A is 3 x 3.

- Method 1 -- Use RREF (use `ParticularSolution.m`)
- Method 2 -- Use $\mathbf{x} = A^{-1}\mathbf{b}$ (use `A\b` or `inv(A)*b`)
- Method 3 -- Use Cramer's rule (use `CramersRule3x3.m`)

A. Complete the following:

- (1) For Method 2, we can use either `A\b` or `inv(A)*b` in Live Editor to get a unique solution when A is invertible.
    - We will use the functions `tic` and `toc` to measure elapsed time for both commands `A\b` and `inv(A)*b`. Running the file will print the elapsed times.
    - Do research online to compare both commands and include the following in your report.
        - When A is invertible, which command is faster and which command is more accurate when calculating the unique solution?
        - The command `A\b` does not always return the same output as `inv(A)*b`. What does `A\b` return in general?
- (2) Examine the m-file function `ParticularSolution.m` first and we treat the determinant as zero if $|\det(A)| \leq 10^{-8}$. Let's consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -5 & -6 \\ 7 & 8 & 9 \end{bmatrix}$.
    - Create matrix A.
    - Use `rref(A)` to display the RREF of A and observe the output of the `rref(A)`. Why A is not invertible? (give the reason using concepts from Linear Algebra based on the RREF of A) What's the determinant of A?

- Use `det(A)` and display the determinant of A (It seems that there are round-off errors when MATLAB calculates `det(A)`). You can also compute determinants using symbolic scalar variables, `det(sym(A))`, which should give you the symbolic 0.
  Make sure to comment out this line before submitting (`sym` will cause your exercise to fail)!
- Use MATLAB code to find the determinant of A^T. Find the relation between det(A) and det(A^T). What can you say about the invertibility of A^T when A is not invertible?

B. Consider the system $\begin{cases} x_1 + x_2 - 2x_3 = 1 \\ x_1 - 2x_2 + 4x_3 = -2 \\ x_2 - 2x_3 = 3 \end{cases}$ and express it as a matrix equation $A_1\mathbf{x} = \mathbf{b}_1$.

- Create the coefficient matrix A1 and vector b1
- [sol_1_partic] Call `ParticularSolution.m`
- [sol_1_matlab] Use `A\b` (Note: You may see a warning and ignore it for now.)
- [sol_1_cramer] Call `CramersRule3x3.m`
- Conclusion: Is the system consistent? Determine the solution of the system. If there are infinitely many solutions, provide the solution set in parametric form (e.g. "x = [1; 2; 3] + [4; 5; 6] * t, t in R").

C. Consider the system $\begin{cases} x_1 + x_2 - 2x_3 = 1 \\ x_1 - 2x_2 + 4x_3 = -2 \\ x_2 - 2x_3 = 1 \end{cases}$ and express it as a matrix equation $A_2\mathbf{x} = \mathbf{b}_2$.

- Create the coefficient matrix A2 and vector b2
- [sol_2_partic] Call `ParticularSolution.m`
- [sol_2_matlab] Use `A\b` (Note: You may see a warning and ignore it for now.)
- [sol_2_cramer] Call `CramersRule3x3.m`
- Conclusion: Is the system consistent? Determine the solution of the system. If there are infinitely many solutions, provide the solution set in parametric form (e.g. "x = [1; 2; 3] + [4; 5; 6] * t, t in R").
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D. Consider the system $\begin{cases} x_1 + x_2 - 2x_3 = 1 \\ x_1 - 2x_2 - 2x_3 = -2 \\ x_2 + 2x_3 = 1 \end{cases}$ and express it as a matrix equation $A_3\mathbf{x} = \mathbf{b}_3$.

- Create the coefficient matrix A3 and vector b3

- [sol_3_partic] Call `ParticularSolution.m`
- [sol_3_matlab] Use `A\b` (Note: You may see a warning and ignore it for now.)
- [sol_3_cramer] Call `CramersRule3x3.m`
- Conclusion: Is the system consistent? Determine the solution of the system. If there are infinitely many solutions, provide the solution set in parametric form (e.g. "x = [1; 2; 3] + [4; 5; 6] * t, t in R").

Solving a Linear System Ax = b, where A is m x n

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30 Points [10 Points Per Part]
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In this exercise, we are still working on solving a linear system but A is an m x n coefficient matrix.

A. Modify the m-file function `CramersRule.m` to get an improved (generalized) Cramer's Rule function that works for all square matrices. You must include the following in your m-file function:
   - You must check for and return NaN if   (i.e. A is non-square) or the magnitude of   (i.e. A is singular).
   - When A is invertible, use a for loop to calculate each x(i), i = 1:n.
   - Modify the comments and include your name and UFID in your m-file `CramersRule.m`.

B. Create three random systems (with integer entries between -7 and 7) and solve each system.
   - Create a 5 x 3 random matrix A1 and a vector 5 x 1 b1; repeat the three methods to solve the system
   - Create a 3 x 5 random matrix A2 and a vector 3 x 1 b2; repeat the three methods to solve the system
   - Create a 5 x 5 random matrix A3 and a vector 5 x 1 b3; repeat the three methods to solve the system

C. Observe all results from both exercises 1 and 2. Answer the following in your report.
   - When A is invertible, which method(s) returns a unique solution **x**?
   - When A**x** = **b** is consistent where A is an n x n singular matrix, which method(s) returns a correct solution **x**?
   - When A**x** = **b** is consistent where A is m x n (not a square matrix), which method(s) returns a correct solution **x**?

The Rank Theorem

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30 Points [10 Points Per Part] + EC 5 points
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This exercise will use MATLAB to verify the rank theorem for matrices A1, A2, and A3 from Exercise 2.

A. Find bases for the null space, column space, and row space of matrix A1
   - `null(A1)` -- each column represents a vector in a basis for Nul A1
   - Recall the process of finding the column space of A -- When `[B, pivcol] = rref(A1)` is used, write a single command* to find a basis for the column space of A1 so that each column represents a vector in a basis for Col A1. (DO NOT use the command `colspace`)
   - Now, think about how we find a basis for the row space. Write a single command* to find a basis for the row space of A1 so that each row represents a vector in a basis for Row A1.

B. Let Repeat the process to

   - find bases for the null space, column space, and row space of matrices A2 and A3
   - find bases for the null space, column space, and row space of matrix A4 and verify if your code for finding the bases for the column space and row space is correct
   - Describe the null space column space of A4 geometrically
     - The null space of A4 is _______ in (find the value of m)
     - The column space of A4 is _______ in (find the value of n)

C. For each matrix (A1, A2, A3, and A4), find the rank (use the command `rank`) and verify the rank theorem in your report.

*** The command for Col A and Row A must work for all matrices A.**

Extra Credit (5 points)

Consider the case that you have found 3 linearly independent solutions to the system A**x** = **0** where A is a 20 x 23 coefficient matrix. Also, these solutions can construct every other possible solution to the system by forming some linear combination of the set of them.
(a) What is dim Nul A? What about dim Col A?
(b) Can you be certain that every non-homogenous system A**x** = **b** has a solution? Why or why not (provide valid reasoning/proof)?

Guidelines and Grading

This assignment must be submitted to Gradescope for grading. You must upload the following files:

- Exercise1.m
- Exercise2.m
- Exercise3.m
- CramersRule.m

Before submission, be sure to check that your Name & UFID is on all template files.

After submission, wait till the autograder runs and verify that your submission passes test cases "0.1) Check files were submitted" and "0.2) Check files run" case. If you did not, you may have not uploaded all the necessary files or your files do not run correctly. Make sure to fix any issues and resubmit before the deadline.

Note: You will not see the Autograded portion of your score until after grades are released.

> **WARNING:** There is a late penalty of a 25-point deduction per day, and therefore, we do not accept any submission after four days.