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When Allais meets Ulysses: Dynamic axioms and the common ratio effect

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Abstract We report experimental findings about subjects' behavior in dynamic decision problems involving multistage lotteries with different timings of resolution of uncertainty. Our within-subject design allows us to study violations of the independence and dynamic axioms: Dynamic Consistency, Consequentialism and Reduction of Compound Lotteries. We investigate the effects of changes in probability and outcome levels on the pattern of choices observed in the Common Ratio Effect (CRE) and in the Reverse Common Ratio Effect (RCRE) and on their dynamic counterparts. We find that the probability level plays an important role in violations of Reduction of Compound Lottery and Dynamic Consistency and the outcomes levels in violations of Consequentialism. Moreover, more than one quarter of our subjects satisfy the Independence axiom but violate two dynamic axioms. We thus suggest that there is a greater dissociation that might have been expected between preferences captured by dynamic axioms and those observed over single-stage lotteries.

Keywords Decision theory · Experiment · Independence axiom · Dynamic consistency · Consequentialism

JEL Classifications C91 · D81

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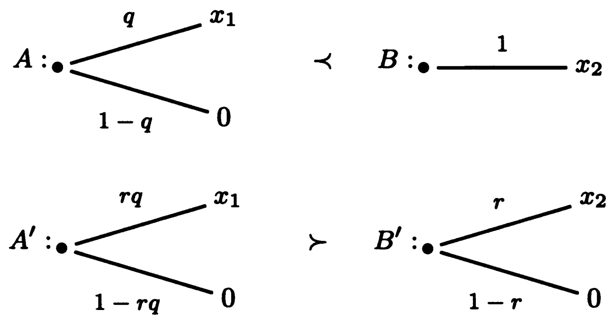


Fig. 1 Common Ratio Effect pattern of choices (Allais 1953)

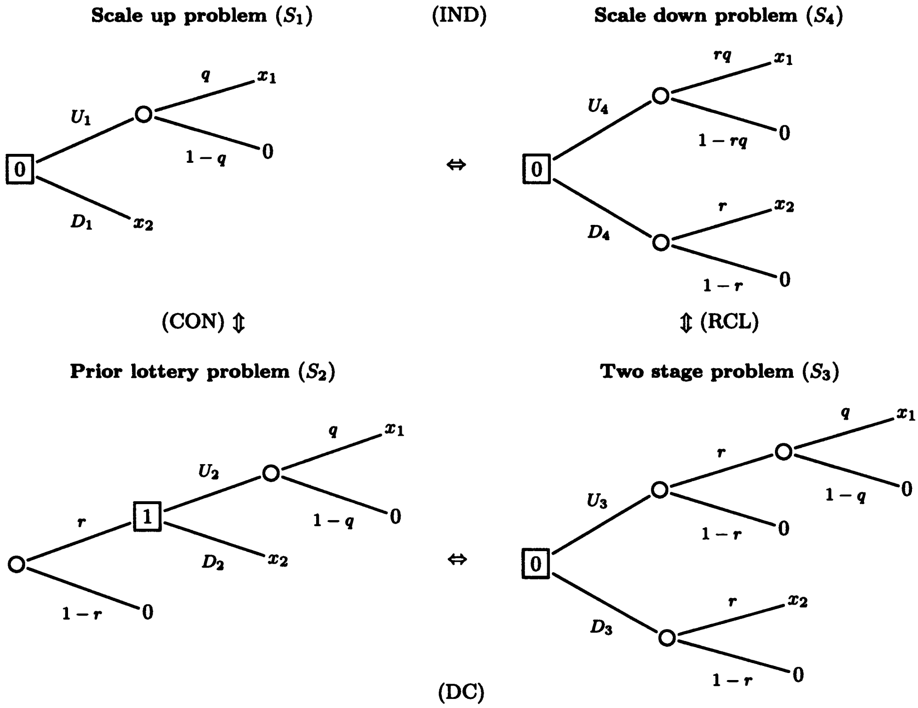
One of the most robust violations of Expected Utility Theory (EUT) is the Common Ratio Effect (CRE). CRE, exhibited in the pattern of choices shown in Fig. 1, is one of the main effects (with Common Consequence Effect) related to the Allais paradoxes[1] (Allais 1953; McCrimmon and Larsson 1979; Kahneman and Tversky 1979; Starmer and Sugden 1989) which cast doubt upon the descriptive adequacy of the independence axiom (IND). Nevertheless, the choice pattern $A \succ B$ and $B' \succ A'$ is also a violation of IND called Reverse Common Ratio Effect (RCRE). These violations of EUT led to new models of decision under risk (non-EU models) which account for these effects and thus have stronger descriptive power in a static set up.

In fact, IND is a property of preferences over one stage lotteries. It has however been connected with choice behavior in dynamic decision problems. The mixture operation involved in IND was originally interpreted in terms of composition of lotteries (von Neumann and Morgenstern 1947). Dynamically there are several ways of composing two lotteries. One can consider the compound lottery prior to the resolution of any uncertainty, or after the resolution of the uncertainty of the first lottery. This allows us to define the three dynamic axioms that are studied in this paper. Figure 2 gives a graphical representation of these axioms (see Cubitt et al. (1998) and Wakker (1999)) namely *Consequentialism* (CON), *Dynamic Consistency* (DC) and *Reduction of Compound Lotteries* (RCL). Formally, Burks (1977) and later Karni and Schmeidler (1991) showed that one satisfies these three dynamic principles only if the induced preference relation over one stage lotteries satisfies IND. Reciprocally, Volij (1994) made it clear that a Non-EU model of decision under risk that relaxes IND must specify which one of the three dynamic axioms mentioned above is not satisfied. Indeed, he showed that if two of the three aforementioned dynamic axioms are verified, then the remaining one is equivalent to the independence axiom.[2] In these

[1]The original parameters proposed by Allais are the following: $x_1 = 5M€$, $x_2 = 1M€$, $q = 0.9$ and $r = 0.1$.

[2]Logically, it means:

- If RCL and CON hold then DC $\Leftrightarrow$ IND. (Karni and Schmeidler 1991)
- If RCL and DC hold then CON $\Leftrightarrow$ IND. (Volij 1994)
- If DC and CON hold then RCL $\Leftrightarrow$ IND. (Volij 1994)



CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *independence axiom*.

Fig. 2 Dynamic axioms and independence axiom for two-outcome lotteries

theorems, the hypothesis, that only one dynamic axiom can be violated at a time, is implicit. The generalization of CRE and RCRE violations of IND for dynamic axioms is presented in Fig. 2 and will be explained in more details in Section 1.1. Finally, it is possible, from a descriptive perspective, to observe simultaneous violations of the dynamic axioms in a way that does not necessarily imply violations of the independence axiom.

In this paper, we build on the work of Cubitt et al. (1998) who provided valuable empirical information[3] about consequentialism (CON), dynamic consistency (DC)[4] and reduction of compound lotteries (RCL) using a between-subjects design. We propose an experimental design that allows two main innovations; first we test each dynamic axiom at an individual level (within-subject design), second we perform these within-subject tests for different values of the parameters. We can therefore

[3] The first experimental investigation of the decomposition of the independence axiom in a dynamic set up is due to Kahneman and Tversky (1979). Indeed, the *isolation effect* comes from the decomposition of IND between CON and DC+RCL.

[4] In this study, the authors refer to separability and timing independence for what we call consequentialism and dynamic consistency.

determine which dynamic principle is more prone to be violated and the direction of violation depending on the outcome and the ratio levels. We can also account for simultaneous violations of the dynamic axioms and test the association between IND and each axiom. We provide new insights about how the type of rejection of each axiom depends on the outcomes and probability (ratio) levels. Our results and findings are relevant in order to further our understanding of risky decisions in a dynamic context and of the impact of the timing of resolution of uncertainty on individual decision behavior. More precisely, we connect standard findings of choices between one stage lotteries to new observations of choice behavior in more sophisticated dynamic contexts. Because the timing and the probability of resolution of uncertainty as well as its consequences have strong behavioral implications and differ in many real-life situations, the results of our study should deepen our interpretation of many existing stylized facts concerning choice under risk.

Notably, we reproduce the benchmark results of Cubitt et al. (1998) and of McCrimmon and Larsson (1979). We confirm that for the independence axiom the smaller the probability level (ratio) and the higher the outcomes, the more frequently CRE is observed. For the dynamic axioms, we find that for RCL and DC, the rate of CRE violations is higher for small ratio values, but is not affected by the outcome level; whereas for CON, CRE violations are more frequently observed with high outcomes. These results are confirmed when subjects who violate more than one dynamic axioms are excluded from our sample. This category of subjects is of particular interest and is composed of a grand majority of individuals who satisfy IND but violate two dynamic axioms in an opposite direction. Interestingly, more than 75% of them exhibit CRE violations of CON and RCRE violations of RCL. This systematic pattern of violations constitutes an empirical contradiction to the implicit normative hypothesis (Karni and Schmeidler 1991; Volij 1994) that violations of dynamic axioms are necessarily connected to violations of independence. Finally, we find that CON is the best candidate as a dynamic version of independence (as suggested by Machina (1989)) since we find a significant association only between IND and CON.

Our paper proceeds as follows. Section 1 presents our notation, the tasks used in the experiment, and the way to detect acceptance or rejection of dynamic axioms from the patterns of choices in these tasks. It also discusses the relationship between the terminology used for the axioms and existing literature. Section 2 describes our experimental design. The results of the study are presented in Section 3. Finally Section 4 summarizes and discusses the experimental findings.

1 Preliminaries

We now present the three dynamic axioms. The terminology we use is in strict accordance with Karni and Schmeidler (1991).

1.1 Notation and decision problems

Let us introduce the decision tasks we used in the experiment. We restrict ourselves to the set L of two-outcome lotteries where outcomes can be three monetary values taken from $\{x_1, x_2, 0\}$ such that $x_1 > x_2 > 0$. Let $\succcurlyeq$ be a preference relation over L. We identify four types of choice problems (Cubitt et al. 1998): scaled down, scaled up, prior lottery and two-stage. In Fig. 2, they are represented following standard notation where circles correspond to chance nodes and squares to decision nodes. For decision nodes, we note by U_k (resp. D_k) the choice of up (resp. down) in problem $S_k, k = 1, \ldots, 4$.

First, in the scaled down and the scaled up problems, the choice patterns $[D_1/U_4]$ and $[U_1/D_4]$ contradict the independence axiom.[5] Similarly, in the scaled up and the prior lottery problems, choice patterns $[D_1/U_2]$ and $[U_1/D_2]$ contradict CON. In the prior lottery and the two-stage lottery problems, the choice patterns $[D_2/U_3]$ and $[U_2/D_3]$ are violations of DC. In the two-stage lottery and scaled down problems, choice patterns $[D_3/U_4]$ and $[U_3/D_4]$ contradict RCL. More specifically, the pattern $[D_1/U_4]$ corresponds to the common ratio effect. So we call the patterns $[D_1/U_2]$, $[D_2/U_3]$ and $[D_3/U_4]$ CRE violations (in opposition to the RCRE violations) of, respectively, CON, DC and RCL that correspond to CRE in a dynamic set up. By contrast, choice patterns $[D_i/D_j]$ and $[U_i/U_j]$ respect the corresponding axiom depending on i and j and are therefore acceptance patterns. These dynamic axioms can therefore be tested within the revealed preference paradigm in order to complement the existing theoretical research into these concepts.

1.2 Consequentialism (CON)

This term was first introduced in the formal Decision Theory literature by Hammond (1988,1989) and refers to the idea that acts are only valued by their consequences. Formulated in terms of decision trees, consequentialism "would be false if missed opportunities, regrets, sunk costs, etc. affected behaviour and yet were excluded from the domain of consequences". In this paper, we define consequentialism as shown in Fig. 2. This is a special case of Hammond's (1988) notion.[6] In fact, it corresponds to the separability condition in Machina (1989), Cubitt et al. (1998) and McClennen (1990) and to forgone event independence in Wakker (1999). We argue that CON means that choice behavior should not be influenced by an uncertainty already resolved. If CON is abandoned, then behavior is affected by events that are known not to have happened at the moment of decision and therefore involves counterfactual reasoning about outcomes that could have occurred but are revealed not to. Machina (1989) and McClennen (1990) argued that CON is an inappropriate property to impose on a Non-EU model because it is a dynamic version of separability which constitutes, for them, the core of the independence axiom.

[5] Indeed, with $P = (x_2; 1)$, $Q = (x_1, 0; q)$, $R = (0; 1) \in L$ these patterns imply that: $\exists\ P, Q, R \in L, \exists r \in [0, 1]\, s.t.\, P \succ Q \not\Leftrightarrow rP + (1-r)R \succ rQ + (1-r)R$ which is the negation of IND which is formally stated as: $\forall\ P, Q, R \in L, \forall r \in [0, 1],\ P \succcurlyeq Q \Leftrightarrow rP + (1-r)R \succcurlyeq rQ + (1-r)R$. Subsequently, r is called ratio to recall the common ratio effect that contradicts this axiom ($r = 0.1$ in the original paradox of Allais (1953)).

[6] Consequentialism in the sense of Hammond is the conjunction of the two axioms we call CON and RCL.

1.3 Dynamic consistency (DC)

Karni and Safra (1989,1990) used the term Dynamic Consistency for strategies in a sequential decision problem. A strategy is dynamically consistent if the ex-ante plan is actually implemented at each step (decision node) of the sequential problem. We restrict this definition to preferences towards dynamic single decision problems and consider that, when ex-post and ex-ante preferences relative to the resolution of an uncertainty correspond, DC is verified. This is motivated by the approach used by Cubitt et al. (1998) with the difference being that we merge what they called "timing independence" and "frame independence" under the same axiom, dynamic consistency (DC). Violation of DC has great implications when dealing with sequential decision problems because it renders the use of backward induction reasoning ineffective and therefore requires the use of alternative sequential strategies. There exists more empirical research into violations of DC than for CON (Busemeyer et al. 2000; Barkan and Busemeyer 2003; Hey and Panaccione 2011).

1.4 Reduction of compound lotteries (RCL)

There is a consensus about the definition of Reduction of Compound Lotteries in the literature. This axiom describes the ability to compute probabilities according to the definition of conditional probability. Segal (1987,1990) argued that multiple stage gambles should be distinguished from single stage gambles and described the dynamic behavior of an individual who does not satisfy this axiom. Bar-Hillel (1973), Carlin (1992), Budescu and Fischer (2001) provide empirical evidence of violation of RCL where, most of the time, the multiple stage gamble is preferred to the reduced single stage one. Therefore according to this axiom the preferences observed in a choice between a two-stage prospect and another option should be the same than the one observed if this two-stage gamble is replaced by its reduced single stage counterpart. Therefore, if choices in the two-stage and in the scale down problem are not the same, then RCL is not satisfied.

2 Experimental design

2.1 Architecture

The experiment was conducted at the LEEM[7] experimental lab in Montpellier (France). A total of 114 participants,[8] graduate and undergraduate students from various disciplines, took part in a computerized experiment. A typical session lasted for about 1 hour and was composed of 30 questions of 4 different kinds[9](presented as S_i

[7]Laboratoire d'Economie Expérimentale de Montpellier (France)

[8]We ran 6 sessions of 19 participants each.

[9]For every problem type, participants were given instructions and a short questionnaire to check their understanding of the task.

in Fig. 2). We opt for a 2×2×2 design in order to control and test for the effects of the following three dimensions:

- We fixed two levels for the ratio: $r^L = 0.3$ and $r^H = 0.7$. For r^L, we chose a value close but slightly higher than the one ($r = 0.25$) used by Cubitt et al. (1998). For r^H, we chose a value slightly higher than the one ($r = 0.6$) used by Starmer and Sugden (1989) and for which they observe more RCRE than CRE.[10]
- We fixed two levels for the sure outcome $x_2 = 15$€ and $x_2 = 60$€. McCrimmon and Larsson (1979) found an increase in violations of the independence axiom, specifically CRE rejection, for smaller ratios and higher outcomes. Therefore, we decided to control for the effects of the ratio and outcomes levels over the rate of acceptance/rejection of dynamic axioms.
- For each of these two levels of x_2, we fixed two levels of maximal gain x_1. For $x_2 = 15$€, we choose $x_1 = 20$€ and $x_1 = 24$€ and for $x_2 = 60$€, we choose $x_1 = 80$€ and $x_1 = 95$€. We introduce this additional dimension in order to control for the heterogeneity of risk attitudes in our sample and to gain in statistical discriminative power.

With $q = 0.8$ for all the questions, this makes 8 questions per type of problem except for S_1 where there is no r. To sum up, the experiment[11] was divided as follows:

(i) Four scaled up problem (S_1) questions involving a choice between a sure amount of money (x_2; 1) and a lottery ($x_1, 0; q$)
(ii) Eight scaled down problem (S_4) questions involving a choice between 2 lotteries: $((x_2, 0; r)$, and lottery $(x_1, 0; rq))$. We added two questions to test for first-order stochastic dominance by proposing a choice between lottery $(x, 0; q)$ and lottery $(x^*, 0; q)$ with $x^* > x$.
(iii) Eight two-stage problem (S_3) questions involving a choice between a simple lottery $(x_2, 0; r)$, and the two-stage lottery $((x_1, 0; q), 0; r)$.
(iv) 8 prior lottery problem (S_2) questions where, first, participants had to manually activate a prior lottery. Then further instructions were displayed: depending on the outcome of the prior lottery, either (with probability $1 - r$) they get nothing and there was no choice to be made or (with probability r) they were told to choose between a sure amount $(x_2; 1)$ and a lottery $(x_1, 0; q)$. From an experimental point of view, an important aspect of this task is that $1 - r$ is the probability that the subject fails to reach the second stage and therefore the proportion of missing data for this question.

As explained in Section 1, our design allows us to test which, if any, axiom is violated for each participant and each combination of parameters: for the independence axiom we compare S_1 and S_4, for CON we compare S_1 and S_2, for DC we compare S_2 and S_3 and for RCL we compare S_3 and S_4.

[10] With the aim to study exclusively CRE violations of IND and of the dynamic axioms, it would have been more adequate to use ratio values between 0.1 and 0.5. This is exactly what is done in Nebout and Willinger (2013). However, in this study we are interested in both CRE and RCRE violations, so it is useful to have a ratio value over one half.

[11] Screenshots of each problem type are available in Appendix B (Figs. 3, 4, 5 and 6).

A pilot study revealed that subjects had difficulties in answering questions involving multi-stage lotteries, because of misunderstandings or task complexity. We therefore decided to introduce such lotteries step by step, starting with simple choices between a lottery and a sure outcome (S_1) followed by choices between two lotteries (S_4). For the question types S_2 and S_3, we control for possible order effects[12] as follows: half of the subjects were confronted with the task sequence $S_1/S_4/S_2/S_3$ and the other half with $S_1/S_4/S_3/S_2$.

2.2 Incentive system

The use of monetary incentives is the topic of an active debate among behavioral economists. Depending on the type of experiment, the chosen incentive scheme might have a significant impact on the results (Camerer and Hogarth 1999; Read 2005; Bardsley et al. 2010, chapter 6). The nature of our experimental design (multiple binary choices for each subject) raised the issue of the most appropriate incentive schemes. In this section, we present the advantages and drawbacks of each possible scheme and justify the choice of hypothetical payment that we implemented in our experiment.

We identified four possible incentive schedules: "play one pay one", "play all pay all", random incentive system (RIS) and hypothetical payment. There is no doubt that the "play one pay one" solution is more appropriate for an experimental protocol investigating dynamic preferences (Bardsley et al. 2010, p280). This requires forming several groups of subjects, each of them facing one problem for real and in isolation. This solution, chosen by Cubitt et al. (1998), is ideal in terms of incentives because it involves no risk of contamination between different choice tasks and no *prediction failure*. However such protocol consumes an important amount of time, money and subjects. Furthermore, with only one question asked to one subject, this methodology restricts the analysis to a between-subjects design and, by consequence, rules out any study of violations of dynamic axioms at an individual level. For such a study, multiple questions per subject are required and the following criticism, anticipated by Cubitt et al. (1998), can be made: "If a subject faces more than one decision problem in an experiment, then the experiment as a whole can be understood as a single problem of dynamic choice. In order to interpret the subject's responses to such an experiment as revealing her preferences over the options in the individual problems, it would be necessary to assume the truth of at least one of the dynamic choice principles which we wish to test." (p1372). This possibility of contamination is a risk that has to be taken in order to obtain results about dynamic preferences at an individual level. For this purpose three incentive schemes were available.

A first idea would be to use the "play all pay all" incentive scheme and thus to pay the cumulative gain at the end of the experiment. In order to keep the payments reasonable, it would be necessary to reduce the gain associated to each question or to introduce an exchange rate. That would force us to use maximal gains between 0.5 € and 2 € in each lottery. Such amounts of money are unlikely to motivate subjects. In

[12] We do not find such effect in our sample so we do not evoke this feature later in the paper.

addition, the difference between the outcome levels might be too small to detect any effect on the outcome dimension. There are also several drawbacks in incentivizing each question like contamination, income and house money effects. Since few studies use this method, it would hinder comparison of our results with the existing literature. We therefore discarded this solution.

Random incentive system (RIS) was another option. It consists in paying only one question randomly selected from the set of answered questions at the end of the experiment. For our purposes, Cubitt et al. (1998) argued that this option was not appropriate because the assumption justifying RIS is isolation which is exactly what is investigated here.[13] Although several studies show that RIS does not prevent subjects separating questions in a multiple tasks experiment (Hey and Lee 2005), we thought that this scheme was not adequate for the critical issues investigated in this paper.

We decided to use hypothetical incentives following Kahneman and Tversky (1979) who claimed that: "the method of hypothetical choices emerges as the simplest procedure by which a large number of theoretical questions can be investigated. The use of the method relies on the assumption that people often know how they would behave in actual situations of choice, and on the further assumption that the subjects have no special reason to disguise their true preferences". Thus, we payed subjects a flat fee of 15€ and compensated for travel costs with 5€ or 10€ depending on the journey required. Nevertheless, there are potential problems raised by this scheme. A first drawback is that it could bias participants' attitude towards risk in reducing their level of risk aversion (Beattie and Loomes 1997; Holt and Laury 2002). This effect is not critical in our protocol since the detection of violations of dynamic axioms is independent of the subjects' risk attitude. A second objection could be that the effect of the resolution of uncertainty in the prior lottery problems might have a limited impact on individuals' state of mind given the fact that these questions will not be paid. Subjects may not experience the disappointment or the relief related to the resolution of the prior uncertainty. This could undermine our findings concerning CON and DC. In fact, this "prediction failure" phenomenon might induce the subjects to give more thought to their answers rather than to give an answer whilst in the grip of their emotions or as a reaction to a "gut feeling". Consequently, we expect our subjects to be more rational and to satisfy DC and CON more frequently than they would with real incentives. In addition, it is likely that prediction failure will also occur with RIS because each question has a low probability of being selected which could dilute the emotions due to the resolution of uncertainty in a particular prior lottery problem.

[13]"Since the random lottery incentive system is widely used in experimental economics this points to a further motivation for testing dynamic choice principles. In any random lottery design, the subject makes precommitments to actions to be taken conditional on a chance event. Timing independence implies that these precommitments are in line with the actions which would be taken after the realisation of nature's move. Separability implies that the latter actions are identical to those which would have been taken had the relevant decision problems been faced in isolation and for real. Thus, timing independence (DC) and separability (CON) are jointly sufficient for the validity of the random lottery incentive system." Footnote 8 in Cubitt et al. (1998).

In conclusion, given that there is probably no perfect incentive scheme that goes with our within-subject protocol, we opted for the one that, in our opinion, minimizes the impact of all possible detrimental effects.

2.3 Statistical methodology of analysis

We present the method of aggregation of our data along two parameter dimensions which aims to improve the clarity and the statistical power of our study. First, we merge the samples for parameter combinations with a similar level of outcomes. More specifically, we define x^L as the merging of the samples for outcomes (20, 15, 0) & (24, 15, 0) and x^H as the merging for the samples for outcomes (80, 60, 0) & (95, 60, 0). Then, in order to investigate the influence of one dimension (ratio: r^L and r^H or outcome levels : x^L and x^H) on the rate of acceptance/rejection of a dynamic axiom, we merge the samples along the other dimension. Finally, we merge all 8 samples to obtain the most aggregated level of results. This aggregation is a convenient tool for dealing with the scarce data resulting from the loss of observations for CON and DC. For each axiom, we first study the acceptance versus rejection rates then we refine our understanding of the rejection behavior by testing the two possible types of rejection (CRE versus RCRE) against each other. In this part, we first compare our results to existing evidence in the literature, then we provide new results for each of the 3 dynamic axioms. We further exploit the within-subject characteristic of our data by studying the number of dynamic axioms violated by a subject within the same parameter set. Finally, we investigate the links between the independence axiom and each of these dynamic axioms.

3 Results

Three subjects out of 114 (2.6%) chose a strictly dominated lottery at least once.[14] We therefore excluded them from the analysis. In Tables 1 and 2, we give the overall experimental results for the remaining 111 subjects. Table 1 presents the rates of choices between U and D for each decision problem and each parameter set. As explained in the previous section, we choose two values for x_1 for the same x_2 in order to control for different levels of risk aversion in our sample. In fact, we observe more choice of the riskiest option, U, for the high levels of x_1 (24€ and 95€) than for the small levels (20€ and 80€). There is only one exception for S_1 where U is more frequently observed for 20€ than for 24€.

There are no significant differences between the results of our experiment and the ones of Cubitt et al. (1998).[15] The parameter profile ($r = 0.3, x_1 = 24€, x_2 = 15€, x_3 = 0$) is comparable with the one used by Cubitt et al. (1998) ($r = 0.25, x_1 = 16£,$

[14]This paucity of violations of dominance is re-assuring and suggests that a majority of subjects took the tasks seriously, despite hypothetical incentives.

[15]S_1: $\chi^2 = 0.157$ p-value $= 0.692$, S_2: $\chi^2 = 1.130$ p-value $= 0.288$, S_3: $\chi^2 = 0.792$ p-value $= 0.374$ and S_4: $\chi^2 = 0.577$ p-value $= 0.448$

Table 1 Frequency and percentage (in brackets) of U and D in each problem

		CSS*	$r^L = 0.3$				$r^H = 0.7$			
			x^L		x^H		x^L		x^H	
			20€	24€	80€	95€	20€	24€	80€	95€
S_1	U_1	19	56	37	17	47	56	37	17	47
		(38.00)	(50.45)	(33.33)	(15.32)	(42.34)	(50.45)	(33.33)	(15.32)	(42.34)
	D_1	31	55	74	94	64	55	74	94	64
		(62.00)	(49.55)	(66.67)	(84.68)	(57.66)	(49.55)	(66.67)	(84.68)	(57.66)
S_2	U_2	13	8	15	12	34	28	53	36	60
		(28.90)	(28.57)	(42.86)	(31.58)	(68)	(32.94)	(51.46)	(39.13)	(66.67)
	D_2	32	20	20	26	16	57	50	56	30
		(71.10)	(71.43)	(57.14)	(68.42)	(32)	(67.06)	(48.54)	(60.87)	(33.33)
S_3	U_3	32	43	64	57	80	34	57	53	81
		(66.70)	(38.74)	(57.66)	(51.35)	(72.07)	(30.63)	(51.35)	(47.75)	(72.97)
	D_3	16	68	47	54	31	77	54	58	30
		(33.30)	(61.26)	(42.34)	(48.65)	(27.93)	(69.37)	(48.65)	(52.25)	(27.03)
S_4	U_4	25	45	62	70	68	23	30	20	49
		(48.10)	(40.54)	(55.86)	(63.06)	(61.26)	(20.72)	(27.03)	(18.02)	(44.14)
	D_4	27	66	49	41	43	88	81	91	62
		(61.90)	(59.46)	(44.14)	(36.94)	(38.74)	(79.28)	(72.97)	(81.98)	(55.86)

* CSS corresponds to the observations of Cubitt et al. (1998)

$x_2 = 10£, x_3 = 0£$). Let us note that in our design each subject answers each decision problem while this is not the case in Cubitt et al.'s experiment. Hence we cannot use the same z-test in order to find a significant difference between the prior lottery and two-stage problems and to draw conclusions concerning the violation of DC.

Table 2 presents the statistics of each pattern of choice for the independence and all the dynamic axioms. First, we compare the results of these two tables with the standard results of the experimental literature on the common ratio effect and its dynamic extensions.

Result 1

(i) The value of the ratio affects the frequency of rejection of the independence axiom (IND),
(ii) The CRE violation of IND (D_1/U_4) is more frequently observed than RCRE with small ratio value and high outcome level whereas with high ratio value and low outcome level RCRE is more frequently observed.

At the aggregate level (Table 2) the IND axiom is rejected in 48.65% of the cases for the small ratio and in 36.71% for the high ratio. The difference is significant ($\chi^2 = 12.447$, p-value < 0.001). If we focus on the type of rejection (Table 3), D_1/U_4 is more frequently chosen than U_1/D_4 for r^L while the reverse is true for r^H. The difference between the two contexts is significant ($\chi^2 = 35.415$, p-value < 0.001). The difference is also significant at the low outcome level (for x^L, $\chi^2 = 15.319$ p-value < 0.001) and the higher one (for x^H, $\chi^2 = 16.251$ p-value < 0.001).

Table 2 Frequency and percentage (in brackets) of U and D in each problem

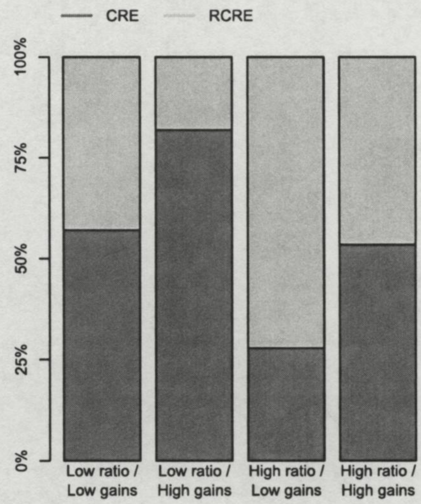
		$r^L = 0.3$				$r^H = 0.7$			
		x^L		x^H		x^L		x^H	
		20€	24€	80€	95€	20€	24€	80€	95€
	Obs.	28	35	38	50	85	103	92	90
CON	U_1/U_2	4 (14.29)	8 (22.86)	3 (7.89)	15 (30)	19 (22.35)	25 (24.27)	8 (8.7)	29 (32.22)
	D_1/D_2	9 (32.13)	18 (51.43)	24 (63.17)	13 (26)	30 (35.29)	39 (37.86)	50 (54.35)	22 (24.44)
	U_1/D_2	11 (39.29)	2 (5.71)	2 (5.26)	3 (6)	27 (31.76)	11 (10.68)	6 (6.52)	8 (8.89)
	D_1/U_2	4 (14.29)	7 (20)	9 (23.68)	19 (38)	9 (10.59)	28 (27.19)	28 (30.43)	31 (34.45)
DC	U_2/U_3	6 (21.43)	13 (37.14)	9 (23.68)	24 (48)	12 (14.12)	35 (33.98)	20 (21.74)	50 (55.56)
	D_2/D_3	11 (39.29)	10 (28.57)	10 (26.32)	7 (14)	42 (49.41)	34 (33.01)	31 (33.70)	17 (18.89)
	U_2/D_3	2 (7.14)	2 (5.71)	3 (7.89)	10 (20)	16 (18.82)	18 (17.48)	16 (17.39)	10 (11.11)
	D_2/U_3	9 (32.14)	10 (28.57)	16 (42.11)	9 (18)	15 (17.65)	16 (15.53)	25 (27.17)	13 (14.44)
	Obs.	111	111	111	111	111	111	111	111
RCL	U_3/U_4	21 (18.92)	42 (37.84)	40 (36.04)	53 (47.75)	8 (7.21)	21 (18.92)	10 (9.01)	43 (38.74)
	D_3/D_4	44 (39.64)	27 (24.32)	24 (21.62)	16 (14.42)	62 (55.86)	45 (40.54)	48 (43.24)	24 (21.62)
	U_3/D_4	22 (19.82)	22 (19.82)	17 (15.32)	27 (24.32)	26 (23.42)	36 (32.43)	43 (38.74)	38 (34.23)
	D_3/U_4	24 (21.62)	20 (18.02)	30 (27.02)	15 (13.51)	15 (13.51)	9 (8.11)	10 (9.01)	6 (5.41)
IND	U_1/U_4	21 (18.92)	29 (26.13)	13 (11.71)	30 (27.03)	13 (11.71)	15 (13.51)	6 (5.41)	24 (21.62)
	D_1/D_4	31 (27.93)	41 (36.94)	37 (33.33)	26 (23.42)	45 (40.54)	59 (53.16)	80 (72.07)	39 (35.14)
	U_1/D_4	35 (31.53)	8 (7.21)	4 (3.6)	17 (15.32)	43 (38.74)	22 (19.82)	11 (9.91)	23 (20.72)
	D_1/U_4	24 (21.62)	33 (29.73)	57 (51.35)	38 (34.23)	10 (9.01)	15 (13.51)	14 (12.61)	25 (22.52)

* CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *Independence Axiom*.
* First lines correspond to frequency and second lines (in brackets) to percentage.
* D/U corresponds to CRE and U/D to RCRE.

Table 3 Contingency table of violations of independence/ Frequencies of CRE vs RCRE violations of independence

		$r^L = 0.3$	$r^H = 0.7$
$x_1 = 20$	U_1/D_4	35	43
	D_1/U_4	24	10
$x_1 = 24$	U_1/D_4	8	22
	D_1/U_4	33	15
$x_1 = 80$	U_1/D_4	4	11
	D_1/U_4	57	14
$x_1 = 95$	U_1/D_4	17	23
	D_1/U_4	38	25
x^L	U_1/D_4	43	65
	D_1/U_4	57	25
x^H	U_1/D_4	21	34
	D_1/U_4	95	39
Total	U_1/D_4	64	99
	D_1/U_4	152	64

* D_1/U_4 corresponds to CRE.
* U_1/D_4 corresponds to RCRE.



If we consider the data at the x_1 level (with no merging), the frequency of $[D_1/U_4]$ choices for r^L is also significantly higher than for r^H, at all values except $x_1 = 95$ where the statistic is just above the 10% threshold (for $x_1 = 20$: $\chi^2 = 5.293$, p-value $= 0.021$, for $x_1 = 24$: $\chi^2 = 11.479$, p-value $= 0.001$, for $x_1 = 80$: $\chi^2 = 14.762$ p-value < 0.001, and for $x_1 = 95$: $\chi^2 = 2.446$ p-value $= 0.118$).

So, whatever level of aggregation is used, we find significantly more CRE violations for the small ratio value. For high ratio value, it is the other way around, we observe more frequently the U_1/D_4 pattern than D_1/U_4 which corresponds to the RCRE.[16] This last result is less known but was already found in Starmer and Sugden (1989). On the outcome dimension, we find that the criterion $[D_1/U_4]$ is more frequently observed than $[U_1/D_4]$ for x^H than for x^L ($\chi^2 = 28.628$, p-value< 0.001) when aggregating the samples over the ratio dimension. These two results about the rate of CRE violations with regards to the ratio and the outcome levels are in line with McCrimmon and Larsson (1979). In conclusion, these first descriptive statistics of the data are consistent with the benchmark studies we are building our experiment on.

3.1 Acceptance versus rejection

In this section, we provide results comparing the rejection versus the acceptance rate for each of the dynamic axioms. Acceptance of the axiom is obtained for the choice patterns U_i/U_j and D_i/D_j, rejection of the axioms for U_i/D_j and U_j/D_i. Table 4

[16] This case, where the risky option is chosen in the scale up problem and the safe option in the scale down problem, has been accounted for theoretically by Blavatskyy (2010).

Table 4 Aggregated frequencies

		r^L		r^H	
		x^L	x^H	x^L	x^H
IND	$Accept: U_1/U_4$ - D_1/D_4	54.95	47.75	59.46	67.12
	$Reject: U_1/D_4$ - D_1/U_4	45.05	52.25	40.54	32.88
CON	$Accept: U_1/U_2$ - D_1/D_2	61.90	62.50	60.11	59.89
	$Reject: U_1/D_2$ - D_1/U_2	38.10	37.50	39.89	40.11
DC	$Accept: U_2/U_3$ - D_2/D_3	63.49	56.82	65.43	64.84
	$Reject: U_2/D_3$ - D_2/U_3	36.51	43.18	34.57	35.16
RCL	$Accept: U_3/U_4$ - D_3/D_4	60.36	59.91	61.26	56.31
	$Reject: U_3/D_4$ - D_3/U_4	39.64	40.09	38.74	43.69

*CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *Independence Axiom*

presents the descriptive results and allows us to draw two conclusions. First, the rate of rejection does not differ among axioms. Second, for all three dynamic axioms, the rate of rejection is affected neither by the ratio level nor by the outcome levels. This result is based on a composite measure which aggregates violations in opposite directions. The impact of the different parameter sets on the type of violations are more specifically studied in Section 3.2.

Result 2 The frequencies of rejection of CON, DC and RCL axioms

(i) are not significantly different from each other whatever level is considered.
(ii) are affected neither by the ratio nor by the outcomes levels.

In Table 4, we observe that the rate of rejection is similar among the dynamic axioms at each level. None of the Chi-square values of the two by two tests of independence between each dynamic axioms are significant (Table 12 in Appendix A). We also observe that the rates of rejection are similar for each dynamic axiom at each level.[17] The fact that we found no particular effect in this result might appear

[17] Based on Table 4 for proportions and Table 13 in Appendix A for chi-square values:

- CON is rejected (both criteria pooled) in 37.75% of the cases for r^L and 40.00% for r^H. This difference is not significant ($\chi^2 = 0.143$ p-value $= 0.705$). When we aggregate over the ratios in Table 4, CON is rejected also in about 40% of the cases (more precisely 39.44% and 39.26% respectively for x^L and x^H, this difference being not significant, $\chi^2 = 0.002$ p-value $=0.963$).
- DC is rejected (both criteria pooled) in 35.06% of the cases for x^L and 37.78% for x^H, which is not significantly different ($\chi^2 = 0.306$ p-value $= 0.580$). DC is more frequently rejected for r^L (40.40%) than for r^H (34.86%), but the difference is not statistically significant ($\chi^2 = 1.188$ p-value $= 0.276$).
- RCL is rejected in 39.86% of the cases for r^L and 41.22% for r^H, a non-significant difference ($\chi^2 = 0.117$ p-value $= 0.733$). The difference of rejection frequency between x^L and x^H (i.e. when we aggregate over the ratios) is also non-significant (39.19% for x^L and 41.89% for x^H, $\chi^2 = 0.565$ p-value $= 0.452$).

unfortunate but is mainly due to the aggregation of each axiom's violation directions which masks possible asymmetry in the violations directions and therefore possible effects of the parameters. This more refined study is presented in the next section and is the one comparable with the existing literature because most of the empirical data available only focuses on CRE violations of axioms.

3.2 Relationship between the parameter set and the type of violation

In this section we focus on the two types of violations for each of the dynamic axioms. Recall that the patterns $[D_1/U_2]$, $[D_2/U_3]$ and $[D_3/U_4]$ correspond to dynamic versions of the CRE and will be denoted, for each axiom, as CRE violations. Our approach is systematic and studies the influence of each parameter level (ratio and outcome) on the rate of the two rejection types (CRE versus RCRE). For each result we present a contingency table for the parameter levels and all the tests are presented in Table 14 in Appendix A.

Result 3 The CRE violation of CON (D_1/U_2)

(i) is more frequently observed than RCRE with high outcome level whereas with low outcome level rates of CRE and RCRE violations are even.
(ii) is not affected by the ratio level

For i), when we aggregate over the ratio, the distribution of rejections is inverted between the two outcome levels: in Table 5, we observe 51.52% of U_1/D_2 and 48.48% of D_1/U_2 for low values of x_1 and 17.92% of U_1/D_2 and 82.08% of D_1/U_2 for high values of x_1. This difference is significant ($\chi^2 = 24.214$ p-value < 0.001). If we test for differences at the ratio level we also find significant differences (for r^L $\chi^2 = 8.066$ p-value $= 0.005$ and for r^H $\chi^2 = 14.743$ p-value < 0.001). For this result, the aggregation process between low outcomes values has an important influence given that, for $x_1 = 20$, we observe much more RCRE than CRE.

Proving ii), the distribution of the rejections between CRE and RCRE is very similar for both ratios at an aggregate level (U_1/D_2: 31.58% and 35.14%, D_1/U_2: 68.42% and 64.86% for respectively r^L and r^H, $\chi^2 = 0.100$, p-value $= 0.752$) and for each aggregated outcome level[18] (for x^L, $\chi^2 = 0.004$, p-value $= 0.949$ and for x^H, $\chi^2 = 0.052$, p-value $= 0.820$). To sum up, the ratio level (r^L or r^H) does not affect the frequency of CRE versus RCRE whether it be at an aggregate level or for each outcome level (Table 14).

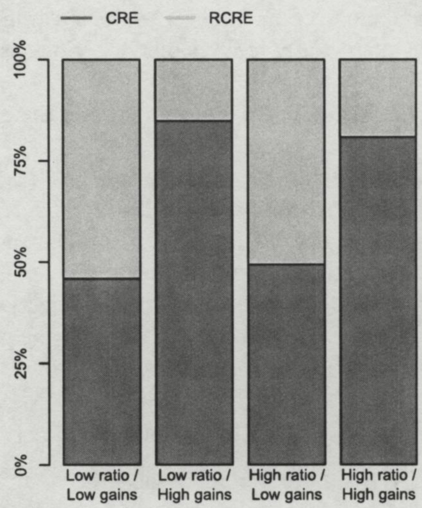
These two results reinforce the idea that the counterfactual reasoning involved in the prior lottery problem after the resolution of the prior risk focuses more on the final sure gains that could have been lost rather than on the probability of having lost

[18]It is also true for each outcome level, for $x_1 = 20$ $\chi^2 = 0.052$ p-value $= 0.820$, for $x_1 = 24$ $\chi^2 = 0.003$ p-value $= 0.959$, for $x_1 = 80$ $\chi^2 = 0.171$ p-value $= 0.679$ and for $x_1 = 95$ $\chi^2 = 0.105$ p-value $= 0.746$

Table 5 Contingency table of violations of consequentialism / Frequencies of CRE vs RCRE violations of consequentialism

		$r^L = 0.3$	$r^H = 0.7$
$x_1 = 20$	U_1/D_2	11	27
	D_1/U_2	4	9
$x_1 = 24$	U_1/D_2	2	11
	D_1/U_2	7	28
$x_1 = 80$	U_1/D_2	2	6
	D_1/U_2	9	28
$x_1 = 95$	U_1/D_2	3	8
	D_1/U_2	19	31
x^L	U_1/D_2	13	38
	D_1/U_2	11	37
x^H	U_1/D_2	5	14
	D_1/U_2	28	59
Total	U_1/D_2	18	52
	D_1/U_2	39	96

* D_1/U_2 corresponds to CRE.
* U_1/D_2 corresponds to RCRE.



these possible sure gains.[19] Consequently we observe that the outcomes instead of the probability dimension have a non neutral influence on the type of violation of CON.

Result 4 The CRE violation of DC (D_2/U_3)

(i) is more frequently observed than RCRE with the small ratio value.
(ii) is not affected by the outcome level

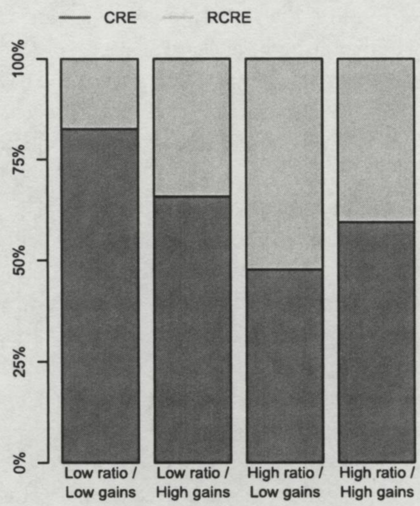
As shown in result 2, the DC axiom is more frequently rejected for r^L (40.40%) than for r^H (34.86%), but the difference is not statistically significant ($\chi^2 = 1.188$ p-value $= 0.276$). However 72.13% of the rejection cases for r^L are due to the D_2/U_3 type against 53.49% for r^H, a significant difference ($\chi^2 = 5.224$ p-value $= 0.022$). More precisely we observe in Table 6 that the ratio value affects the rejection type when the x_1 values are low ($\chi^2 = 7.079$ p-value $= 0.008$) but not when the x_1 values are high ($\chi^2 = 0.188$ p-value $= 0.664$). Moreover the distribution of the rejections between the two types is very similar for both outcomes levels at an aggregate level (D_2/U_3: 56.82% and 61.76%, U_2/D_3: 43.18% and 38.24% for respectively x^L and x^H, $\chi^2 = 0.296$, p-value $= 0.586$) or for both ratio levels (for r^L, $\chi^2 = 1.266$, p-value $= 0.260$ and for r^H, $\chi^2 = 1.331$, p-value $= 0.249$). So, the outcome level (x^L or x^H) does not affect the frequency of $[D_2/U_3]$ whether it be at an aggregate level or for each ratio level (Table 14).

[19] As a reminder, these sure gains are $x_2 = 15$€ for x^L and $x_2 = 60$€ for x^H.

Table 6 Contingency table of violations of dynamic consistency/ Frequencies of CRE vs RCRE violations of dynamic consistency

		$r^L = 0.3$	$r^H = 0.7$
$x_1 = 20$	U_2/D_3	2	16
	D_2/U_3	9	15
$x_1 = 24$	U_2/D_3	2	18
	D_2/U_3	10	16
$x_1 = 80$	U_2/D_3	3	16
	D_2/U_3	16	25
$x_1 = 95$	U_2/D_3	10	10
	D_2/U_3	9	13
x^L	U_2/D_3	4	34
	D_2/U_3	19	31
x^H	U_2/D_3	13	26
	D_2/U_3	25	38
Total	U_2/D_3	17	60
	D_2/U_3	44	69

* D_2/U_3 corresponds to CRE
* U_2/D_3 corresponds to RCRE



We also performed a within-subject test for those participants who answered the prior lottery problem, for both $r^L = 0.3$ and $r^H = 0.7$. Whatever the outcome level ($x_1 = 20, 24, 80$ or 95) we do not observe any significant difference[20] between the numbers of acceptances and rejections for r^L and r^H. There is also no significant difference[21] between CRE choices for r^L and r^H except for $x_1 = 20$. This emphasizes the fact that a dynamically inconsistent participant (in the CRE direction) for r^H is also dynamically inconsistent (in the CRE direction) for r^L.

These results show that violations of DC are driven by the ratio level. This may be explained by the fact that the only difference between the two-stage and the prior lottery problems is the timing of resolution of the first stage lottery. It seems therefore not too surprising that the key variable in terms of behavioral impact is its probability.

Result 5 The RCRE violation of RCL (U_3/D_4)

(i) is more frequently observed for high ratio values than CRE whereas with for low ratio values the rates of CRE and RCRE violations are even.
(ii) is not affected by the outcome level

At the aggregate level the ratio does not affect the frequency of rejection of the RCL axiom (Tables 4 and 13), since the rejection frequency is around 40% for both

[20]Mc Nemar change test, $x_1 = 20$: $\chi^2 = 0.750$ p-value $= 0.387$, $x_1 = 24$: $\chi^2 = 0$ p-value $= 1$, $x_1 = 80$: $\chi^2 = 0.842$ p-value $= 0.359$, $x_1 = 95$: $\chi^2 = 1.389$ p-value $= 0.239$

[21]Mc Nemar change test, $x_1 = 20$: $\chi^2 = 3.125$ p-value $=0.077$, $x_1 = 24$: $\chi^2 = 1.777$ p-value $= 0.182$, $x_1 = 80$: $\chi^2 = 1.388$ p-value $= 0.239$ and $x_1 = 95$: $\chi^2 = 0.100$ p-value $= 0.752$

samples ($\chi^2 = 0.117$, p-value $= 0.773$). However, the distribution of CRE versus RCRE[22] is very different depending on the ratio.[23] For r^L, CRE (D_3/U_4 pattern) represents 50.28% of the rejections (Table 7) while it only represents 21.86% for r^H.

Finally, the outcome level (x^L or x^H) does not affect the frequency of $[U_3/D_4]$ whether it be at an aggregate level ($\chi^2 = 1.283$ p-value $= 0.157$) or for each ratio level (for r^L, $\chi^2 = 0.006$ p-value $= 0.940$ and for r^H, $\chi^2 = 2.840$ p-value $= 0.092$).

3.3 Within subject analysis and robustness of the results

In the previous section we displayed aggregated results depending on the parameter dimension we aimed to study. If a within-subject protocol was necessary in order to obtain each axiom variable, this analysis was run in a between-subjects perspective. This technique allows us to maintain at a reasonable level the number of observations required to draw conclusions about the influence of each parameter dimension on the rate of CRE and RCRE violations for each axiom. However, it is possible to further exploit the within-subject characteristic of our protocol by evaluating for each set of parameters the number of violated dynamic axioms depending on whether or not the independence axiom is violated (1 or 3 if independence is violated, 0 or 2 when independence is verified). This allows us to define individual type depending on the number of switches between U and D from a decision task to another: EU subjects verifying IND and all the three dynamic axioms (O switch-type), coherent Non-EU subjects violating IND and only one dynamic axiom (1 switch-type), static EU subjects verifying IND but violating two dynamic axioms in opposite directions (2 switches-type) and random Non-EU subjects who violate IND and all three dynamic axioms for the same parameter set (3 switches-type).[24] In addition, we can evaluate the robustness of the results presented in the previous section by excluding anomalous subjects from our sample.

3.3.1 Multiple switches statistics

In Table 8, we present the number of switches observed for each parameter set on the sample of subjects that answered all four decision tasks. Over the sixteen possible patterns of choices in the four decision tasks, two correspond to 0 switch (U or D for all four problems), two correspond to 3 switches, six to 1 switch and six to 2 switches. Thus a proper comparison of frequencies should be made only between 0 and 3-switches types and 1 and 2-switches types.

[22] Note that this result is presented in terms of RCRE because, for RCL, this type of violation is more frequently observed than CRE.

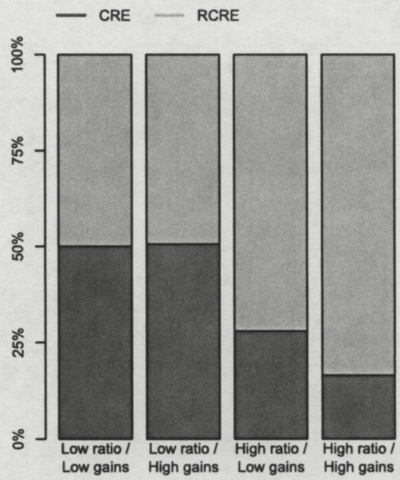
[23] This difference is significant at an aggregate level ($\chi^2 = 30.392$, p-value < 0.001). At the aggregated outcome level this difference is also significant (for x^L, $\chi^2 = 8.013$ p-value $= 0.005$ and for x^H, $\chi^2 = 22.919$ p-value < 0.001). Finally this result is also obtained for all outcome levels except for $x_1 = 20$, (for $x_1 = 20$: $\chi^2 = 1.546$ p-value $= 0.214$, for $x_1 = 24$: $\chi^2 = 6.266$ p-value $= 0.012$, for $x_1 = 80$: $\chi^2 = 19.151$ p-value < 0.001 and for $x_1 = 95$: $\chi^2 = 4.5432$ p-value $= 0.033$).

[24] We thank an anonymous referee for suggesting that we explore this dimension of our data set and that we run this fruitful analysis.

Table 7 Contingency table of reduction of compound lottery violations / Frequencies of CRE vs RCRE violations of reduction of compound lottery

		$r^L = 0.3$	$r^H = 0.7$
$x_1 = 20$	U_3/D_4	22	26
	D_3/U_4	24	15
$x_1 = 24$	U_3/D_4	22	36
	D_3/U_4	20	9
$x_1 = 80$	U_3/D_4	17	43
	D_3/U_4	30	10
$x_1 = 95$	U_3/D_4	27	38
	D_3/U_4	15	6
x^L	U_3/D_4	44	62
	D_3/U_4	44	24
x^H	U_3/D_4	44	81
	D_3/U_4	45	16
Total	U_3/D_4	88	143
	D_3/U_4	89	40

* D_3/U_4 corresponds to CRE
* U_3/D_4 corresponds to RCRE



Result 6 About two thirds of the subjects violate one or no dynamic axiom while one quarter of the subjects verify IND and violate two dynamic axioms in opposite directions.

First, the proportion of subjects that systematically switch from U to D from a decision problem to another is very low (around 6%) in comparison to the 0-switch type. This 3-switches type is hard to explain from a theoretical perspective and most likely reveals pure randomness in the subjects' answers. Between 25% and 35% of our subjects behave accordingly with the independence axiom and its dynamic extensions. They therefore always choose the same option (U or D) in each of the 4 decision tasks (0-switch type). They could be thought of as expected utility maximizers both in a static and a dynamic framework. This proportion is consistent with other experimental classification results between EU and Non-EU subjects (Bruhin et al. 2010). The most observed type in our sample is the one we are interested in, i.e. subjects who are not verifying the independence axiom and that consequently do not verify one and only one dynamic axiom. These subjects fit in the framework of the theorems of Karni and Schmeidler (1991) and Volij (1994). This type is observed more frequently for a small than for a high ratio. This is consistent with the fact that we observe more violations of independence for a small ratio. Finally, there is a last type that has been investigated neither in the theoretical nor in the experimental literature, i.e. subjects who satisfy the independence axiom but violate two dynamic axioms. This case is possible from a theoretical point of view and would suggest that preferences revealed for single stage prospects could be totally independent of the ones revealed for multiple stage prospects. Nevertheless, we observe

Table 8 Frequency and percentage (in brackets) of switches

# of switches	$r^L = 0.3$					$r^H = 0.7$					Total
	20	24	80	95	Total	20	24	80	95	Total	
0	5	13	6	13	37	24	34	25	26	109	146
	(17.86)	(37.14)	(15.79)	(26)	(24.50)	(28.24)	(33.01)	(27.17)	(28.89)	(29.46)	(28.02)
1	13	15	22	18	68	30	30	18	38	116	184
	(46.43)	(42.86)	(57.89)	(36)	(45.03)	(35.29)	(29.13)	(19.57)	(42.22)	(31.35)	(35.32)
2	7	6	9	15	37	24	34	44	22	124	161
	(25)	(17.14)	(23.68)	(30)	(24.50)	(28.24)	(33.01)	(47.83)	(24.44)	(33.51)	(30.90)
3	3	1	1	4	9	7	5	5	4	21	30
	(10.71)	(2.86)	(2.63)	(8)	(5.96)	(8.24)	(4.85)	(5.43)	(4.44)	(5.68)	(5.76)
NA	83	76	73	61	293	26	8	19	21	74	367

that between 25% and 30% of our subjects are of this 2-switches type. This is a significant proportion of our sample. Therefore it is important to determine if these two switches are random which would suggest that this type should be treated as noise like the 3-switches type or if these two switches are more systematic which would suggest that this type is behaviorally grounded. The next section investigates this question.

3.3.2 2-switches subjects

In this section, we isolated the 2-switches subjects for each parameter set. The 6 possible profiles are presented in Table 9. CRE violations correspond to profiles 3 and 4 for CON, profiles 2 and 5 for DC and profiles 1 and 6 for RCL. In Table 15 in Appendix A, we present the descriptive statistics of these six profiles for each parameter set.

Result 7 For the 2-switches types, CRE violations of CON and RCRE violations of RCL represent more than 75% of the subsample.

We observe that for all our parameter sets, profiles 3 and 5 represent more than 50% of the 2-switches types. These two profiles correspond to RCRE behavior in RCL (i.e. U_3/D_4). In particular, profile 5 corresponds to CRE behavior in DC. Table 15 in Appendix A shows that the profiles distribution within 2-switches subjects is not random as profiles 3, 4 and 5 contain more than 75% of the two-switches type for each parameter set.

In addition, an interesting feature of this within-subject analysis is that it sheds light on one of the between-subject results of Cubitt et al. (1998). Indeed, Cubitt et al. (1998) detect a significant difference between the frequencies of choices of U in S_2 and S_3 (28.9% versus 66.7%) suggesting a CRE violation of DC, but also between the frequencies of choices of U in S_3 and S_4 (66.7% versus 48.1%) suggesting, on

Table 9 Profiles of 2-switches subjects

Profiles	CON	DC	RCL	r^L	r^H	Total
$RCRE_{con}/CRE_{rcl}$	U_1/D_2	D_2/D_3	D_3/U_4	2	10	12
				(5.41)	(8.06)	(7.45)
$RCRE_{con}/CRE_{dc}$	U_1/D_2	D_2/U_3	U_3/U_4	6	6	12
				(16.22)	(4.84)	(7.45)
$CRE_{con}/RCRE_{rcl}$	D_1/U_2	U_2/U_3	U_3/D_4	8	37	45
				(21.62)	(29.84)	(27.95)
$CRE_{con}/RCRE_{dc}$	D_1/U_2	U_2/D_3	D_3/D_4	4	25	29
				(10.81)	(20.16)	(18.01)
$CRE_{dc}/RCRE_{rcl}$	D_1/D_2	D_2/U_3	U_3/D_4	13	43	56
				(35.14)	(34.68)	(34.78)
$RCRE_{dc}/CRE_{rcl}$	U_1/U_2	U_2/D_3	D_3/U_4	4	3	7
				(10.81)	(2.42)	(4.35)

*CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *Independence Axiom*

the contrary, a RCRE violation of RCL. These two results seem paradoxical with the hypothesis that a representative agent is either verifying independence or exhibiting CRE behavior and violating one and only one dynamic axiom. In fact, under this assumption, the frequencies of choices of U should be increasing from S_1 to S_4. Our study of 2-switches reconciles these two apparently contradictory results of Cubitt et al. (1998) because our within-subject protocol allows identification of subjects verifying independence but not some dynamic axioms whereas it is not possible with a between-subject protocol.

This section supports the idea that preferences towards dynamic prospects are not only a subset of preferences towards single stage prospects because they characterize more complex behavioral traits. This is consistent with the finding of subjects that verify the independence axiom in a static context but that can be influenced in opposite directions by the different characteristics of the timing of resolution of uncertainty (ex-post, ex ante or simultaneous). However, we cannot exclude the possibility of these 2-switches type subjects just being due to tremble or noise. Therefore it is necessary to check that the results we presented in Section 3.2 are robust to their exclusion from our analysis.

3.3.3 Robustness

In this section, we exclude the 2- and 3-switches types from our analysis and present in Table 10, the contingency tables of the rejection types for each dynamic axiom.

Result 8 All the results presented in Section 3.2 are preserved in this subsample and we detect stronger effects than on the whole sample.

Table 10 Frequency for each axiom, without 2- and 3-switches subjects

		CON		DC		RCL	
		r^L	r^H	r^L	r^H	r^L	r^H
x^L	$RCRE$	6	20	2	15	12	17
	CRE	6	4	9	5	23	9
x^H	$RCRE$	1	5	1	7	13	19
	CRE	15	20	13	4	32	8
Total	$RCRE$	7	25	3	22	25	36
	CRE	21	24	22	9	55	17

*CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *Independence Axiom*

For CON, CRE is observed more frequently for high outcomes than for low outcomes[25] (Result 3).

For DC (Result 4), CRE is observed more frequently than RCRE for a small ratio whereas it is the contrary for a high ratio ($\chi^2(1) = 17.159$ p-value < 0.001) but we detect no outcomes effect ($\chi^2(1) = 2.070$ p-value $= 0.150$).

For RCL (Result 4), CRE is observed more frequently than RCRE for a small ratio whereas it is the contrary for a high ratio ($\chi^2(1) = 15.824$ p-value < 0.001) but we detect no outcomes effect ($\chi^2(1) = 0.033$ p-value $= 0.855$).

3.4 Associations

Result 9 Whatever level is considered (global, aggregated over ratios and aggregated over outcomes) there is an association between the acceptance/rejection of IND and CON. For the other axioms, we find no association except with DC for x^L.

Table 11 reports the Chi-square values of the two by two tests of association between the two modality variables (Accept/Reject) for IND and each dynamic axiom. For CON, we find a systematic significant association between the two variables. For the other axioms, there is no significant association (except for DC for small outcomes values). This result for CON should be taken with caution since, excluding the 3-switches type from the analysis (Table 16 in Appendix A), significant association between IND and CON is only found at the most aggregated level.

[25]More precisely, 72.22% of U_2/D_3 and 27.78% of D_3/U_2 are observed for low outcomes, while the frequencies are respectively 14.63% and 85.37% for high outcomes and the difference is statistically significant ($\chi^2(1) = 23.857$ p-value <0.001). The difference is also significant for each ratio level (r_L: $\chi^2(1) = 4.861$ p-value $= 0.027$, r_H : $\chi^2(1) = 17.202$ p-value < 0.001). If we aggregate the outcome levels and test for the effects of the ratio, the frequencies of observation of (U_2/D_3, D_3/U_2) for r_L and r_H are respectively (25%, 75%) and (51.02%, 48.98%), a significant difference ($\chi^2(1) = 3.954$ p-value $= 0.047$).

Table 11 Chi-square values for association between axiom's rejection variables

		*IND * CON*	*IND * DC*	*IND * RCL*
	Total	16.519***	1.904^{ns}	1.496^{ns}
Ratios aggregated	x^L	7.550***	7.266***	0.161^{ns}
	x^H	8.236***	0.276^{ns}	1.514^{ns}
Outcomes aggregated	r^L	6.232**	0.631^{ns}	0.215^{ns}
	r^H	10.436***	0.712^{ns}	1.597^{ns}

*CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *Independence Axiom*

^{ns} not significant *** 1%, ** 5%

This result reinforces the idea defended in Section 3.3 that dynamic preferences are not necessarily connected to preferences over single stage prospects. It is therefore not surprising to find that CON is the axiom that relates the closest to IND as the prior lottery problem also involves choices between single stage prospects (although a prior uncertainty has been resolved). This result gives credit to the argument of (Machina 1989) saying that CON is a dynamic version of IND. However, multiple switches types play an important role in this association and were not predicted by Machina (1989)'s argument. Therefore, this result mainly suggests that choices involving multiple stage lotteries require other reasoning mechanisms than the one used in standard static choices.

4 Conclusion

This study provides new empirical evidence concerning individual dynamic preferences. Taking McCrimmon and Larsson (1979) and Cubitt et al. (1998) as a starting point, we go further than these studies insofar as our experimental design takes account of individuals' heterogeneity[26] in allowing multiple tests of behavioral axioms at the individual level. This design allows us to test for the acceptance/rejection of all three dynamic axioms (RCL, CON, and DC) and of the independence axiom at an individual level. On the one hand, it brings out the influence of the ratio value and of the outcome level on the rate of violation of these axioms and more specifically on the rate of violation corresponding to the Common Ratio Effect versus the Reverse Common Ratio Effect. On the other hand, it provides information at an individual level about the link between violations of the independence axiom and its dynamic extensions. In fact, we test the implicit theoretical assumption stating that the violation of a dynamic axiom necessarily implies the violation of independence and find that it is not always verified. Finally, we test the association between independence and each of the dynamic axioms.

[26]For a discussion of the differences between within- and between-subjects experiments, see for example Ballinger and Wilcox (1997).

Experimental investigation of individual attitudes toward dynamic prospects faces several methodological difficulties: complexity of the decision task, loss of data induced by the prior lottery problem and correct incentivization of the experiment. We construct our experimental design under these constraints and on this last issue, we prefer to use hypothetical incentives;[27] nevertheless our data is in line with the aforementioned benchmark studies on CRE and dynamic preferences.

Our study reveals that none of the dynamic axioms are more rejected than others and that their rejection rates are not affected by the ratio or outcome levels. However, for each dynamic axiom, the rate of CRE violations versus RCRE violations depends on the ratio and outcome values as follows: (i) for Consequentialism, the rate of CRE violations is higher than RCRE for large outcome levels but does not depend on the ratio level, (ii) for Dynamic Consistency, the frequency of CRE is higher than RCRE for large ratio levels but does not depend on the outcomes level and (iii) for the Reduction of Compound Lotteries the frequency of RCRE is higher than CRE for high ratio levels but does not depend on the outcomes level. For CON, this result could be explained by the fact that the counterfactual reasoning involved in the evaluation of the prior lottery problem by a Non-Consequentialist decision maker is probably less focused on the probability of occurrence of a forgone event than on the level of the outcome that could have been lost. This could plead in favor of an interpretation of Non-Consequentialist behavior in terms of changing the reference point as proposed by Barkan and Busemeyer (2003). For DC, as the main difference between the prior and the two-stage problems resides in the timing of resolution of the first stage lottery, it seems appropriate that its probability is the relevant variable to consider when looking at choice differences between the two problems.

In addition, our within-subject analysis allows us to distinguish between EU subjects verifying IND and all three dynamic axioms, coherent Non-EU subjects violating IND and only one dynamic axiom, static EU subjects verifying IND but violating two dynamic axioms in opposite directions and finally random Non-EU subjects who violate IND and all three dynamic axioms for the same parameter set. We find that about two thirds of the subjects violate one or no dynamic axioms (EU and coherent Non-EU) while one quarter of the subjects verify IND and violate two dynamic axioms in opposite directions. For this last subsample, we find that the two violations are not random as more than 75% of these subjects exhibit CRE violations of CON and RCRE violations of RCL. This suggests that static and dynamic preferences are not as intimately connected as maintained in the theoretical literature. This idea is reinforced by the fact that we only found an association between IND and CON when we crossed the acceptance/rejection variables between independence and each of the dynamic axioms. This result is consistent with Machina (1989)'s claim saying that: "consequentialism is essentially a dynamic version of the separability that non-expected utility maximizers reject", p1642.

This study on the violation of the independence axiom in the CRE fashion could also be carried out with the common consequence version of the Allais paradoxes.

[27] Nebout and Willinger (2013) use real incentives (RIS) and collected data that, for DC and CON, are consistent with the one presented in this paper.

Our study of independence under risk may also be extended to the case where the probabilities in the lotteries are unknown as in Maher and Kashima (1997) who build an experiment on the three colour Ellsberg paradox and also test the influence of the resolution of uncertainty on individual preferences in the case of uncertainty.

In conclusion, this experimental study is compatible with the revealed preference paradigm because each decision problem consists of a single binary choice between well-defined prospects. However the set of prospects over which these dynamic preferences are defined is more complicated than the set of single stage lotteries used in the standard models of decision under risk. This set contains lotteries that could have multiple stages and different timing of resolution of uncertainty. The experimental results presented here deepen our understanding of the independence axiom and of the probability mixture operation in a dynamic framework. Our findings may be relevant to the study of sequential decision making (i.e. where more than one decision is involved). Indeed, an interesting topic for future research in this direction would be to use, as a primitive, a preference relation that remains observable (unlike plans or strategies) and incorporates the dynamic characteristics that are relevant in sequential decision problems. For example, Nebout (2013), Nebout and Willinger (2013) use this approach in order to propose a categorization and a way to reveal strategies in sequential decision problems given the properties of the dynamic preferences (i.e. acceptance or rejection of DC, CON and RCL).

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Appendix A: Additional statistics

Table 12 Chi-square values of association between Accept/Reject variables

		$RCL * CON$	$RCL * DC$	$CON * DC$
	Global	0.148^{ns}	2.120^{ns}	0.799^{ns}
Ratios aggregated	x^L	0.600^{ns}	0.995^{ns}	0.852^{ns}
	x^H	0.378^{ns}	1.016^{ns}	0.070^{ns}
Outcomes aggregated	r^L	0.132^{ns}	0.000^{ns}	0.125^{ns}
	r^H	0.779^{ns}	3.181^{ns}	1.870^{ns}

*CON = *Consequentialism*, DC = *Dynamic Consistency* and RCL = *Reduction of Compound Lotteries*

^{ns} not significant *** 1%, ** 5%

Table 13 Chi-square values for Accept/Reject variables depending of the parameter levels

		IND	CON	DC	RCL
	Total	12.447***	0.143^{ns}	1.188^{ns}	0.117^{ns}
r^L vs. r^H	x^L	0.388^{ns}	0.011^{ns}	0.016^{ns}	0.010^{ns}
	x^H	16.251***	0.078^{ns}	1.299^{ns}	0.453^{ns}
	Total	0.000^{ns}	0.002^{ns}	0.306^{ns}	0.565^{ns}
x^L vs. x^H	r^L	2.029^{ns}	0.009^{ns}	0.430^{ns}	0.000^{ns}
	r^H	2.482^{ns}	0.004^{ns}	0.000^{ns}	0.930^{ns}

*CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *Independence Axiom*

^{ns} not significant *** 1%, ** 5%

Table 14 Chi-square values of the independence tests between CRE and RCRE for different parameter levels

		IND	CON	DC	RCL
	Total	35.415***	0.100^{ns}	5.224**	30.392***
r^L vs. r^H	x^L	15.319***	0.004^{ns}	7.079**	8.013**
	x^H	16.251***	0.052^{ns}	0.188^{ns}	22.919***
	Total	28.628***	24.214***	0.296^{ns}	1.283^{ns}
x^L vs. x^H	r^L	14.793***	8.066**	0.260^{ns}	0.006^{ns}
	r^H	10.068**	14.743***	1.331^{ns}	2.840^{ns}

*CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *Independence Axiom*

^{ns} not significant *** 1%, ** 5%

Table 15 Frequency and percentage (in brackets) of each profile for 2-switches subjects

Profile	$r^L = 0.3$					$r^H = 0.7$					Total
	20	24	80	95	Total	20	24	80	95	Total	
1	1	1	0	0	2	6	2	2	0	10	12
	(14.29)	(16.67)	(0)	(0)	(5.41)	(25)	(5.88)	(4.55)	(0)	(8.06)	(7.45)
2	2	1	1	2	6	1	1	1	3	6	12
	(28.57)	(16.67)	(11.11)	(13.33)	(16.22)	(4.17)	(2.94)	(2.27)	(13.64)	(4.84)	(7.45)
3	1	2	1	4	8	4	13	11	9	37	45
	(14.29)	(33.33)	(11.11)	(26.67)	(21.62)	(16.67)	(38.24)	(25)	(40.91)	(29.84)	(27.95)
4	0	0	1	3	4	4	8	8	5	25	29
	(0)	(0)	(11.11)	(20)	(10.81)	(16.67)	(23.53)	(18.18)	(22.73)	(20.16)	(18.01)
5	3	2	5	3	13	7	9	22	5	43	56
	(42.86)	(33.33)	(55.56)	(20)	(35.14)	(29.17)	(26.47)	(50)	(22.73)	(34.68)	(34.78)
6	0	0	1	3	4	2	1	0	0	3	7
	(0)	(0)	(11.11)	(20)	(10.81)	(8.33)	(2.94)	(0)	(0)	(2.42)	(4.35)

Table 16 Chi-square values for association between axiom's rejection variables without 3-switches type

		$IND * CON$	$IND * DC$	$IND * RCL$
	Total	4.518**	0.474^{ns}	6.062**
Ratios aggregated	x^L	1.553^{ns}	1.070^{ns}	3.625^{ns}
	x^H	2.606^{ns}	4.156**	2.151^{ns}
Outcomes aggregated	r^L	2.570^{ns}	0.020^{ns}	2.883^{ns}
	r^H	2.206^{ns}	1.168^{ns}	2.253^{ns}

*CON = *Consequentialism*, DC = *Dynamic Consistency*, RCL = *Reduction of Compound Lotteries* and IND = *Independence Axiom*

^{ns} not significant *** 1%, ** 5%

Appendix B: Screenshots

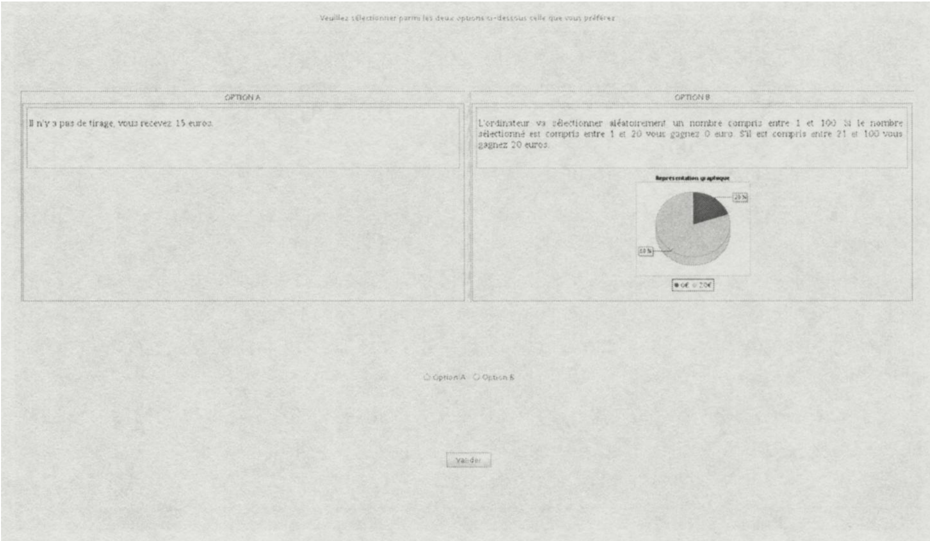

Fig. 3 Scaled up problem (S1)

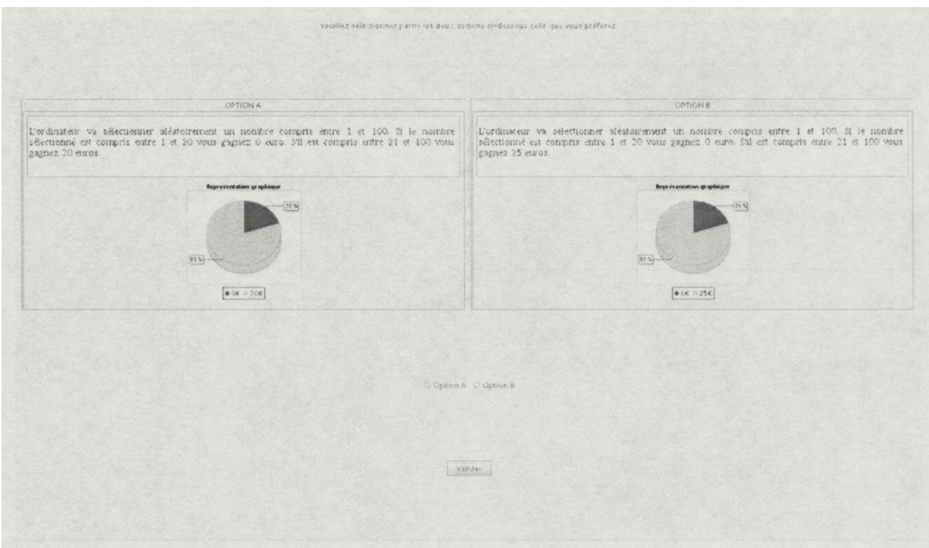

Fig. 4 Scaled down problem (S4)

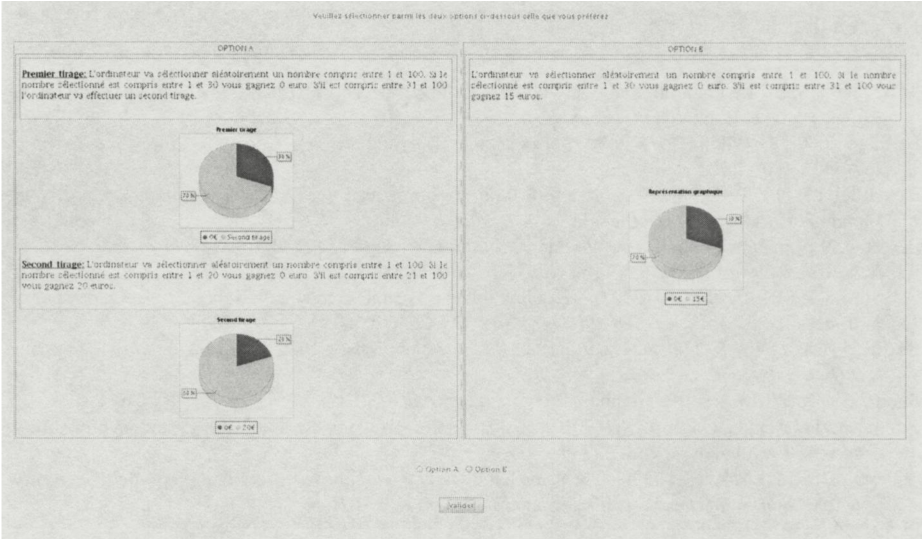

Fig. 5 Two stages problem (S3)

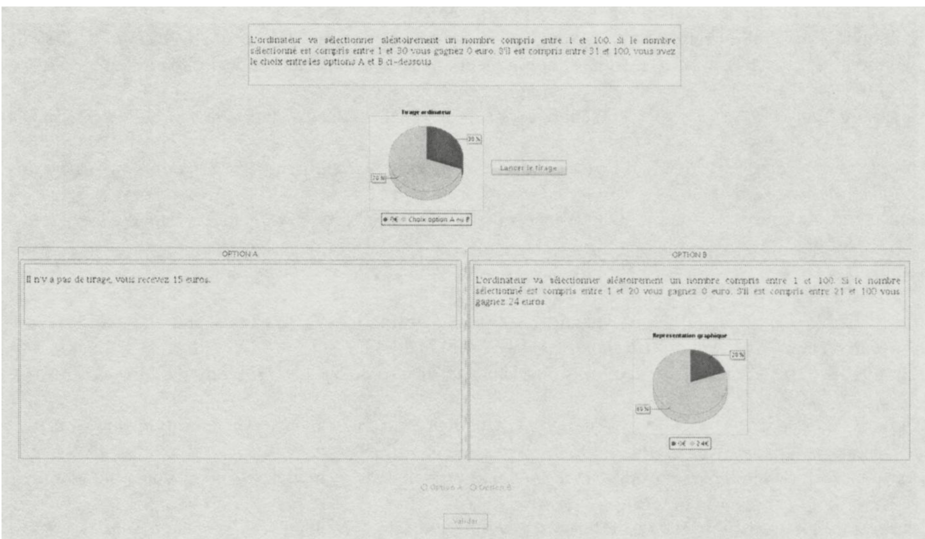

Fig. 6 Prior lottery problem (S2)

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