The future of fresh groundwater in coastal megacities: A review of the Henry saltwater intrusion problem for steady-state transport of saltwater into freshwater coastal aquifers and the role of dispersion in freshwater and saltwater mixing.

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Abstract

The urban water supply in rapidly growing coastal megacities is severely threatened by anthropogenic forcings and climate change. Unprecedented population growth, as well as mass migration from rural to urban areas, has led to increased population density and expansion of coastal megacities. Changes in landuse and landcover associated with rapid development and urbanization, such as the transition from permeable (i.e. soils) to impermeable (i.e. pavement) surfaces, have also increased runoff and limited natural recharge of aquifers. Increased stress on groundwater supplies without proper recharge can lead to lower water tables, aquifer depletion and consequent land subsidence. Climate change induced sea level rise, coupled with the latter impacts, can result in imbalances between freshwater and saltwater at the interface and increased saltwater intrusion. An improved understanding of the saltwater-freshwater interface in coastal aquifers may help mitigate the severity and extent of impacts from seawater intrusion into groundwater resources in low-lying coastal areas. The Henry saltwater intrusion problem is a considered a benchmark analysis for testing density-dependent groundwater flow models for coastal aquifers. Improvements in these models can help us better understand the underlying mechanisms and dynamics of saltwater and freshwater interactions, especially at the interface and within the zone of dispersion, and can be utilized in creating more effective resilience plans for coastal megacities.

1. Introduction

1.1 Background

Steady flow of seaward freshwater is needed to maintain a consistent and constant freshwater head to counter the landward flow of seawater, to combat intrusion of saltwater into aquifers (Henry,1959). This is especially important for coastal aquifers underlain by saltwater. Recharge, the process in which surface water infiltrates and percolates into the ground and eventually into aquifers, is one of the major ways to maintain a steady flow. However, increased demands on limited water supplies due to population growth and expansion of coastal megacities (UN, 2014; UN Habitat, 2016) along with increases in impervious surface cover in cities, limits the ability of aquifers to recharge, leading to water stress conditions and rapid depletion of aquifers (Essink, 2001). Water table levels typically fluctuate due to seasonal and tidal variations, but are dropping to very low levels due to chronic overextraction without time or ability to recharge (Timmerman and White, 1997). Depletion of groundwater sources and lowering of piezometric heads can also induce land subsidence as subsurface stability decreases and soils compact and collapse (Essink, 2001). Another major problem facing coastal megacities today is global sea level rise (Nicholls and Cazenave, 2010; Chang et al., 2011). The elevation of the freshwater-seawater interface can move due to land subsidence, sea level rise and lowering of the water table relative to sea level, which can lead to saltwater intrusion into the fresh water supply.

The fresh-water/saltwater interface in coastal aquifers has been troubling scientists for decades. Understanding the dynamics of saltwater intrusion and the exact location of the zone of brackish water is essential to water resource planning and groundwater monitoring (Werner et al., 2013). It is imperative for low-lying coastal areas to know the physical processes involved in saltwater intrusion into freshwater aquifers, as well as the factors that exacerbate its impact, to try and protect the local water supply.

1.2 The Henry Saltwater Intrusion Problem

The problem of saltwater intrusion into fresh-water aquifers was proposed by Henry in 1959, as part of his PhD research, and is considered the benchmark analysis for testing density-dependent groundwater flow models. Henry (1959) developed analytical solutions to describe two-dimensional horizontal, confined coastal aquifers for two cases. The first case was for vertical outflow faces, and the second for horizontal outflow faces. He solved both cases assuming finite conditions, then modified to solve for infinite extension to the left, and then infinite extension to the left and infinite depth. Henry's initial solutions assumed a steady-state condition, with unchanging (or negligible changes in) conditions over time. It was also assumed that salt and freshwater were immiscible, there was no fingering or tidal action, and the aquifer was homogenous.

Previous analyses assumed a sharp, well-defined interface (Hubbert, 1940; Glover, 1959; Henry, 1959). But it was realized that a "zone of dispersion" exists where the saltwater mechanically mixes with freshwater gradually, aided by chemical diffusion (Henry,1960). The width of this zone is dependent on pore characteristics in the aquifer, water particle movement due to tidal action and fluctuations due to recharge (Cooper,1959). An extensive dispersion zone requires the dispersion process and saltwater entrainment in freshwater to be accounted for in the analysis (Kohout, 1960).

Early studies found that the dispersion coefficient, D, is generally proportional to the velocity and much greater in the direction of the velocity vector, as compared to the lateral direction (Ogata,1958; Simpson,1960; Rifai et. al.,1956). Cooper (1959) used this proportionality, with the assumption of constant water density, to account for the impact of tidal dispersion on flow patterns. He reasoned that neither dispersion or diffusion could be the main mode of transport of salts from the sea to the zone of diffusion due to the absence of a concentration gradient extending all the way to the sea floor. However, he theorized that the dispersion produced by tidal motion was great enough for seaward flowing freshwater to entrain large amounts of saltwater and induce the movement of oceanic saltwater to the diffusion zone. This cyclic motion was confirmed in the field by Kohout (1960).

As part of his doctoral dissertation, under the assumptions of steady flow and a constant dispersion coefficient, Henry attempted to describe the zone of dispersion and its corresponding flow patterns with the inclusion of cyclic flow postulated by Cooper. He concluded that dispersion plays a role in reducing salt intrusion and causes saltwater to recirculate in the bottom of the aquifer near the ocean (Henry,1964). This analytical solution was the first attempt to account for dispersion effects and density dependent fluid flow on saltwater encroachment in confined coastal aquifers (Reilly and Goodman, 1985). This approach used the advection-diffusion equation for miscible fluids and quantitatively accounted for hydrodynamic dispersion in the mixing zone, rather than applying the sharp-interface approach for immiscible fluids.

This paper will explore the theoretical framework and assumptions used by Henry (1964) to formulate his problem, the characteristics of the freshwater-saltwater dispersive mixing zone, and how these can be related to coastal aquifer problems today.

2. Formulation of the Henry Saltwater Intrusion Problem

2.1 Governing Equations

To account for dispersion, Darcy's equation of motion for fluid flow must retain density as a variable since it can change with salt concentration. The equation is given by (Henry, 1964):

$$\vec{q} = -\frac{k}{\mu}(\nabla p - \rho \vec{g}) \tag{1}$$

Where $\vec{q}$ is the velocity vector for the fluid, k is the permeability of the porous medium, μ is the viscosity of water, p is the pressure, ρ is the density of the solution, and $\vec{g}$ is the vector acceleration due to gravity. For steady flow, the conservation of mass for water (2a) and salt in solution (2b) are:

$$\nabla \cdot \rho_w \vec{q} = 0 \tag{2a}$$

$$\nabla \cdot c\overrightarrow{q_e} = 0 \tag{2b}$$

Where ρ_w is the mass of pure water per unit volume of the solution, c is the mass of salt per unit volume, and $\overrightarrow{q_e}$ is the effective vector velocity of salt motion. Equation 2a satisfies the continuity equation for water, but the effective vector velocity combines the fluid velocity with dispersion effects resulting in the continuity equation for salt:

$$\nabla \cdot c\vec{q} - \nabla \cdot D\nabla c = 0 \tag{3}$$

Where D is the dispersion coefficient, which should be chosen to represent the average unsteady impacts due to tides and recharge, imposed on the steady flow of fluid seaward.

A few important dimensionless quantities need to be considered to evaluate the first three equations with respect to the confines of the aquifer:

$$u' = \frac{ud}{Q},\ \ v' = \frac{vd}{Q},\ \ x' = \frac{x}{d},\ \ y' = \frac{y}{d},\ \ c' = \frac{c}{c_s},\ \ and\ \rho' = \frac{\rho-\rho_o}{\rho_s-\rho_o} \tag{4}$$

Where u is the horizontal velocity, v is the vertical velocity, x and y and the coordinate values, c_s is the concentration of salt in sea water, ρ_0 is the density of freshwater, ρ_s is the density of salt water, d is the thickness of the aquifer and Q is the net discharge of freshwater per unit length of coast (beach). An empirical relationship relating the density of pure water to that of a mixture (Baxter and Wallace, 1916) can be used to simplify equation 2a to:

$$\rho = \rho_w + c = \rho_o + (1-E)c \tag{5}$$

Where E is a constant approximately equal to 0.3 for mixture with concentrations similar to that of sea water. Substituting the simplified equation into the continuity equation for water (Equation 2a) results in:

$$\nabla \cdot q - \frac{Ec_s}{\rho_o}(\nabla \cdot c'q) = 0 \tag{6}$$

$\nabla \cdot q$ and $\nabla \cdot c'q$ have the same order of magnitude, but $\frac{Ec_s}{\rho_o}$ is very small (~0.008), making the order of magnitude of the second term much smaller than the first. We can then consider the second term to be negligible and it can be dropped out of the final equation, leaving $\nabla \cdot q = 0$. This equation indicates a stream function, which can be defined as a dimensionless quantity:

$$u' = \frac{\partial \psi'}{\partial y'}, \quad v' = -\frac{\partial \psi'}{\partial x'} \tag{7}$$

Where ψ' is the dimensionless stream function. By writing the scalar form of equation 1, and differentiating u(y) and v(x), we can remove pressure by subtraction. We can then substitute the quantities given in equation 7 and replace u and v with the stream function. Finally, we know from equation 5 that $\rho' = c'$, which leaves us with:

$$\nabla^2 \psi' = \frac{k_1 d}{Q} \frac{\partial c'}{\partial x'} \tag{8}$$

In this approach, we assume a D that is an average representative value of all of the unsteady terms which is a constant scalar throughout the field, since a dispersion coefficient as a function of velocity (seen in Equation 3) cannot be determined without a solution to the problem first. This also simplifies the details of the analysis without having major impacts on the essential aspects of the problem itself. With a constant D and u and v substitutions in terms of the stream function, we get a dimensionless form of Equation 3:

$$\frac{D}{Q} \nabla^2 c' = \frac{\partial \psi'}{\partial y'} \frac{\partial c'}{\partial x'} - \frac{\partial \psi'}{\partial x'} \frac{\partial c'}{\partial y'} \tag{9}$$

Henry defined his model as "... a rectangular confined aquifer in contact with a fresh-water reservoir on one end and the ocean on the other," (Henry, 1964, C76). The overlaying top and underlying bottom boundaries are considered to be impermeable, with velocity normal to the boundaries equal to 0 and a constant stream function at the boundaries. The movement of salt through diffusion must also be equal to 0, so that $\frac{\partial c'}{\partial y'} = 0$. This is only possible if the isochlors are vertical as they approach the aquifer base. This is consistent with observations of the "salt-water wedge" in the lower part of the aquifer, where the saltwater meets the freshwater at the "zone of diffusion". Pressure distributions along the vertical boundaries are assumed to be hydrostatic, with $\frac{\partial\psi'}{\partial x'} = 0$. The freshwater boundary is also assumed to have completely pure water distribution with $c' = 0$, while the saltwater boundary is assumed to have a salt concentration equal to 1.

2.2 Summary of Boundary Conditions

1) At $y' = 0$: $\psi' = 0, \frac{\partial c'}{\partial y'} = 0$

2) At $y' = 1$: $\psi' = 1, \frac{\partial c'}{\partial y'} = 0$

3) At $x' = 0$: $\frac{\partial\psi'}{\partial x'} = 0, c' = 0$

4) At $x' = 1$: $\frac{\partial\psi'}{\partial x'} = 0, c' = 1$

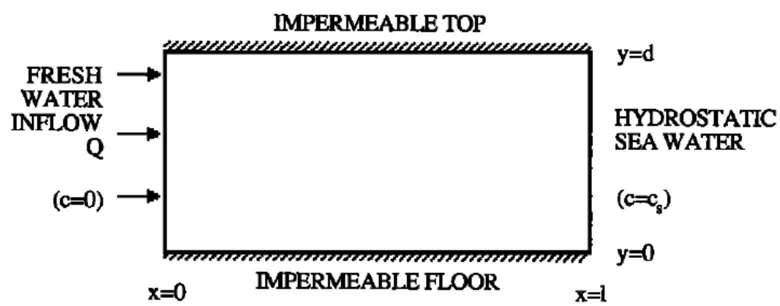

Figure 1. The Henry problem.

Figure reproduced from Croucher & O'Sullivan, 1995

Henry used the Fourier-Galerkin solution to satisfy the boundary conditions and solve the nonlinear boundary value problem set up in the preceding section. The details of the solution and results, however, will not be further discussed in this paper.

3. Discussion of the Henry Problem

3.1 Saltwater Intrusion Transport Mechanisms and Processes in the Zone of Dispersion

The Henry saltwater intrusion problem was a benchmark in studies of the zone of dispersion (mixing zone). Many interactions take place simultaneously between the saltwater and freshwater due to dispersive mixing, tides, differences in density, hydrological conditions at the surface and below, geology, etc. (Werner et al., 2013). The convective circulation described by Cooper (1959) and analyzed in the Henry problem is also essential in our understanding or seawater movement and its potential to intrude into freshwater aquifers. The number and complexity of the involved processes and interactions makes it difficult to fully understand the characteristics of the freshwater-saltwater mixing zone and the transport of seawater through and across it. In this paper, we mainly focus on the role of dispersion in this zone.

Hydrodynamic dispersion occurs in groundwater flow and is the effect due to a combination of mechanical dispersion and molecular diffusion. Mechanical dispersion dominates and is caused by variations in velocity, and molecular diffusion depends on concentration gradients, and fluid properties, which cause molecular movement throughout a fluid. The magnitude of hydrodynamic dispersion is important, yet difficult to determine exactly (Essink, 2001). However, it does play a role in determining the thickness of mixing zone for the homogenous aquifer in the steady state case, unchanging with time (Volker and Rushton, 1982). Wider mixing zones are more common with higher dispersion coefficients, less discharge of freshwater flowing seaward and greater contrasts in density between the saltwater and freshwater fronts across the interface (Volker and Rushton, 1982). On the other hand, narrow mixing zones tend to have characteristically low dispersion coefficients in the transverse direction (Paster and Dagan, 2007). Abarca et. al. analyzed the Henry problem with modifications to account for anisotropic and dispersive effects on the mixing zone. The study showed that the mixing zone thickness is sensitive to both longitudinal and transverse dispersive effects (Abarca et. al., 2006). Accounting for dispersive processes is essential, and the lack of dispersion simulation is one of the major limitations of the Henry problem (Werner et. al., 2013).

3.2 Improvements and Model Development

Over the years, many attempts have been made to improve the Henry problem (Pinder and Cooper, 1970; Lee and Cheng 1974; Segol et. al., 1975, Frind, 1982; Huyakorn et. al., 1987; Voss and Souza 1987, Croucher and O'Sullivan, 1995) and test its validity and usefulness in representing the actual processes that occur at the freshwater and saltwater interface. Finite element models were created and tested in both two and three dimensions (Huyakorn et. al., 1987; Abd-Elhamid and Javadi, 2009). Overall, there is general disagreement about some of the numerical solutions to the original problem that modify the Diffusion coefficient and/or original boundary conditions (Simpson and Clement, 2003). In the initial Henry problem, the boundary conditions assume homogenous conditions with a pure freshwater and saltwater boundary, assuming no mixing of the two at initialization. Many subsequent studies such as Segol et. al. (1975) chose a partial freshwater, partial seawater boundary to better represent actual conditions. However, results with heterogenous boundary conditions cannot be properly compared against Henry's analytical solution for homogenous boundaries (Simpson and Clement, 2003). The approach taken by Frind (1982) is considered a more realistic modification of the Henry problem for inclusion of a mixed boundary and was used as the benchmark for Simpson and Clement (2003) in their analysis of the Henry problem as a benchmark for density-dependent groundwater flow models. Their analysis considered both the coupled and uncoupled solutions of the problem and determined that the distribution of saline water is dependent mostly upon the boundary forcings rather than density coupled flow and transport processes (Simpson and Clement, 2003). A follow-up study applied this finding to modify the Henry problem by reducing freshwater recharge, and create a better test case as a benchmark for density-dependent groundwater flow models, while still being able to compare the results with the original analytical solution presented by Henry (Simpson and Clement, 2004).

4. Conclusion

Although there is a lot of room for improvement and increased understanding of the processes involving saltwater intrusion, it is undoubtedly an important issue today and will become an even greater issue in the future. Since the development of the Henry saltwater intrusion problem in the early 1960's, many have attempted to improve the general model for coastal aquifers and make it more realistic especially in zone of dispersion. Some have also tried to test existing problems and models with experiments in the lab (Goswami and Clement, 2007) and in the field (Barlow and Reichard, 2009) to confirm the validity of numerical and analytical solutions currently available and to provide more realistic numbers and data based on actual observations and measurements. However, the general coastal aquifer itself is changing, along with some oceanic processes due to anthropogenic stresses and climate change. The Henry problem today needs to be understood in these changing conditions as extreme events become more common, aquifers are depleted, land subsides and sea levels rise. This is imperative for coastal megacities, where a large portion of the population resides. It is likely that the number of climate refugees will rise, along with general population increases, and the resources of cities will not be able to compete with the demands placed on them. It is important to not only perform case studies for specific aquifers in coastal megacities that are projected to see growth, but also to come up with solution to counter these potential impacts. Improvements in understanding of the system dynamics of saltwater transport across the saltwater-freshwater interface and the role of external climate and anthropogenic forcings on changes in the location of that interface, and models to accurately represent these changes, will be important in urban planning for the future of local water supplies.

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