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[AppPhysics 181]: Activity 3: Inductors

Objectives

1. To observe the effects of adding an inductor in an AC circuit
2. To explore two applications of inductors in the circuits governed by frequencies

Solution Proper

1. Application 1: **RLC Parallel Bandpass Filter**

   In the spirit of an experiment, an ansatz was first established: *the order of the resonant frequency is the average of the absolute value of the orders of R, L, and C.*; and since the same materials were used for the two set-ups, *the calculated orders from both set-ups were defined as the lower and upper bound* when we search for the resonance or notch frequencies, narrowing the options for finding the experimental frequencies.

$$\begin{aligned}\text{Set-up 1:}\, \mathcal{O}[f_{\text{res/nch}}] &\sim \frac{|\mathcal{O}[R]|+|\mathcal{O}[L]|+|\mathcal{O}[C]|}{3}\\ &\sim \frac{|\mathcal{O}[100[\text{k-}]]|+|\mathcal{O}[0.01[\mu\text{-}]]|+|\mathcal{O}[12\text{m-}]|}{3}\\ &\sim \frac{|5|+|-8|+|-2|}{3}=\frac{5+8+2}{3}\\ &\sim \frac{15}{3}\\ &\sim 5\end{aligned}$$

$$\begin{aligned}\text{Set-up 2:}\, \mathcal{O}[f_{\text{res/nch}}] &\sim \frac{|\mathcal{O}[R]|+|\mathcal{O}[L]|+|\mathcal{O}[C]|}{3}\\ &\sim \frac{|\mathcal{O}[100[-]]|+|\mathcal{O}[0.01[\mu\text{-}]]|+|\mathcal{O}[12\text{m-}]|}{3}\\ &\sim \frac{|2|+|-8|+|-2|}{3}=\frac{2+8+2}{3}\\ &\sim \frac{12}{3}\\ &\sim 4\end{aligned}$$

$$\text{ANSATZ:}\quad \begin{aligned} f_{\text{res/nch}} &\in [10^4, 10^5]\ [\text{Hz}]\\ f_{\text{res/nch}} &\in [10, 100]\ [\text{kHz}]\end{aligned}$$

   Proceeding, circuits with these boundary frequencies as input frequencies were set-up. Below is the template of the first circuit (Circuit 1) used:

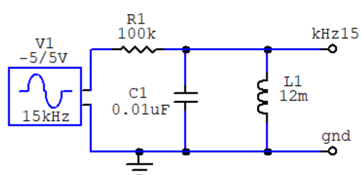

Figure 1: Template Circuit 1

   Upon observation, the circuit with 10 kHz (or order4 in Figure 2) gave a more "complete" wave than the 100 kHz's (or order5 in the Figure 2), as shown below:

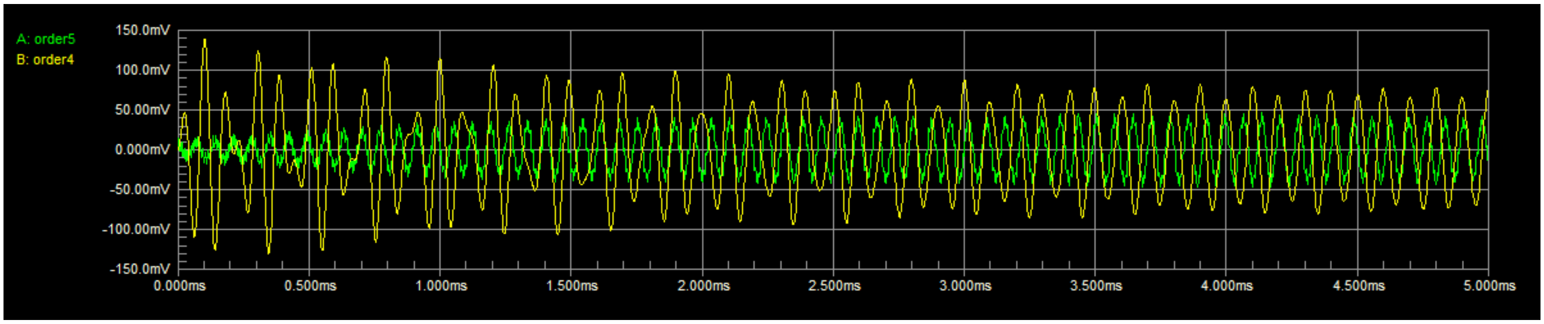

Figure 2: Output Waveforms of 10 kHz (yellow) and 100 kHz (green) in Circuit 1

This meant that the frequency is closer to 10kHz; thus, from 10 kHz, increments of 5 kHz were added to observe any changes in the waveforms while staying true in the ansatz.

After two increments, it is evident that the f_{res} is or very close to 15 kHz, as shown below. Given that the change of output waveform from 15 kHz to either 10 or 20 kHz is that significant, the frequency must be extremely close to 15 kHz. This can further be mathematically proved using a variant of Extreme Value Theorem, and loosely, Rolle's Theorem. Therefore, we will declare that this is our experimental resonance frequency: $f_{\text{res,exp}} = 15[\text{kHz}]$.

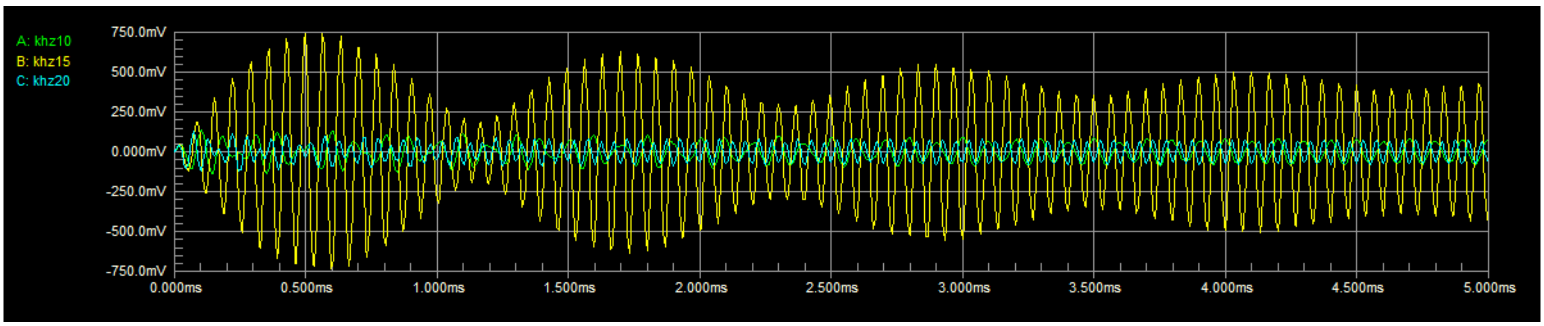

Figure 3: Output Waveforms of 10 kHz (green), 15 kHz (yellow), 20 kHz (cyan) in Circuit 1

Using the formula given, the theoretical resonance frequency is 14 529 Hz or 15 kHz (or $\frac{10^5}{\pi\sqrt{4.8}}$). Thus, the percent error is 3 %.

$$
\begin{aligned}
f_{\text{res,theo}} &= \frac{1}{2\pi\sqrt{LC}} \\
&= \frac{1}{\pi}\frac{1}{2\sqrt{0.01[\mu\text{-}]\cdot 12[\text{m-}]}}[\text{Hz}] \\
&= \frac{1}{\pi}\frac{1}{\sqrt{4\cdot 0.01\cdot 12}}\frac{1}{\sqrt{{}_E - 6 + {}_E - 3}} \\
&= \frac{1}{\pi}\frac{1}{\sqrt{0.48}_E}\left(\tfrac{1}{2}(6+3)\right)[\text{Hz}] \\
&= \frac{1}{\pi\sqrt{4.8}_E}\left(\tfrac{9}{2}+\tfrac{1}{2}\right) \\
&= \frac{1}{\pi\sqrt{4.8}_E}5[\text{Hz}] \\
f_{\text{res,theo}} &= \frac{10^5}{\pi\sqrt{4.8}}[\text{Hz}] \approx 14529[\text{Hz}]
\end{aligned}
$$

$$
\begin{aligned}
\%_{\text{err}} &= \left|\frac{f_{\text{res,theo}} - f_{\text{res,exp}}}{f_{\text{res,theo}}}\right| \\
&= \left|\frac{14529 - 15000}{14529}\frac{[\text{Hz}]}{[\text{Hz}]}\right| \\
\%_{\text{err}} &= 0.03 \sim 3\%
\end{aligned}
$$

This can be further confirmed by comparing the Bode plots of the experimental and theoretical:

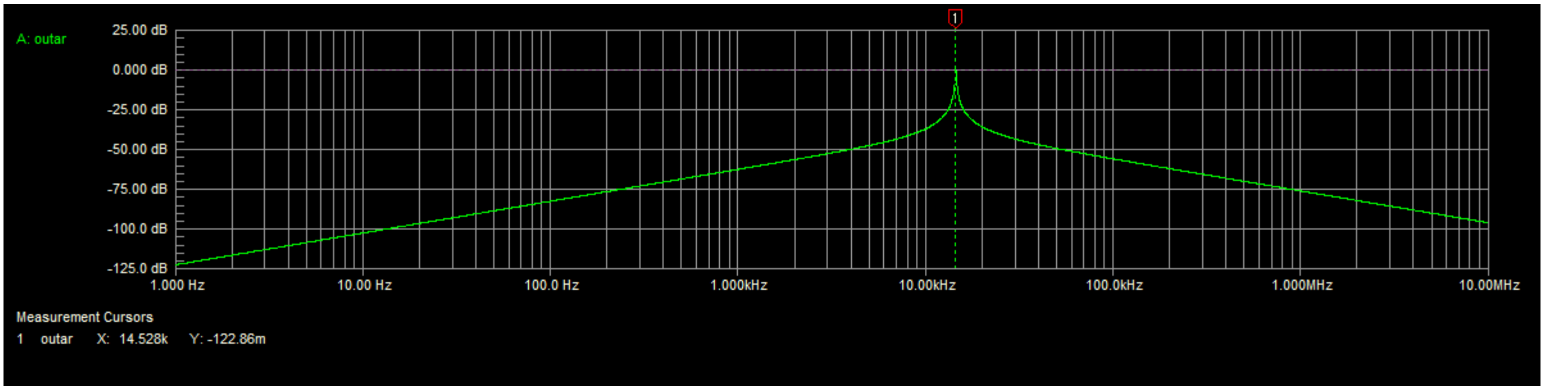

Figure 4: Theoretical Bode Plot of Circuit 1

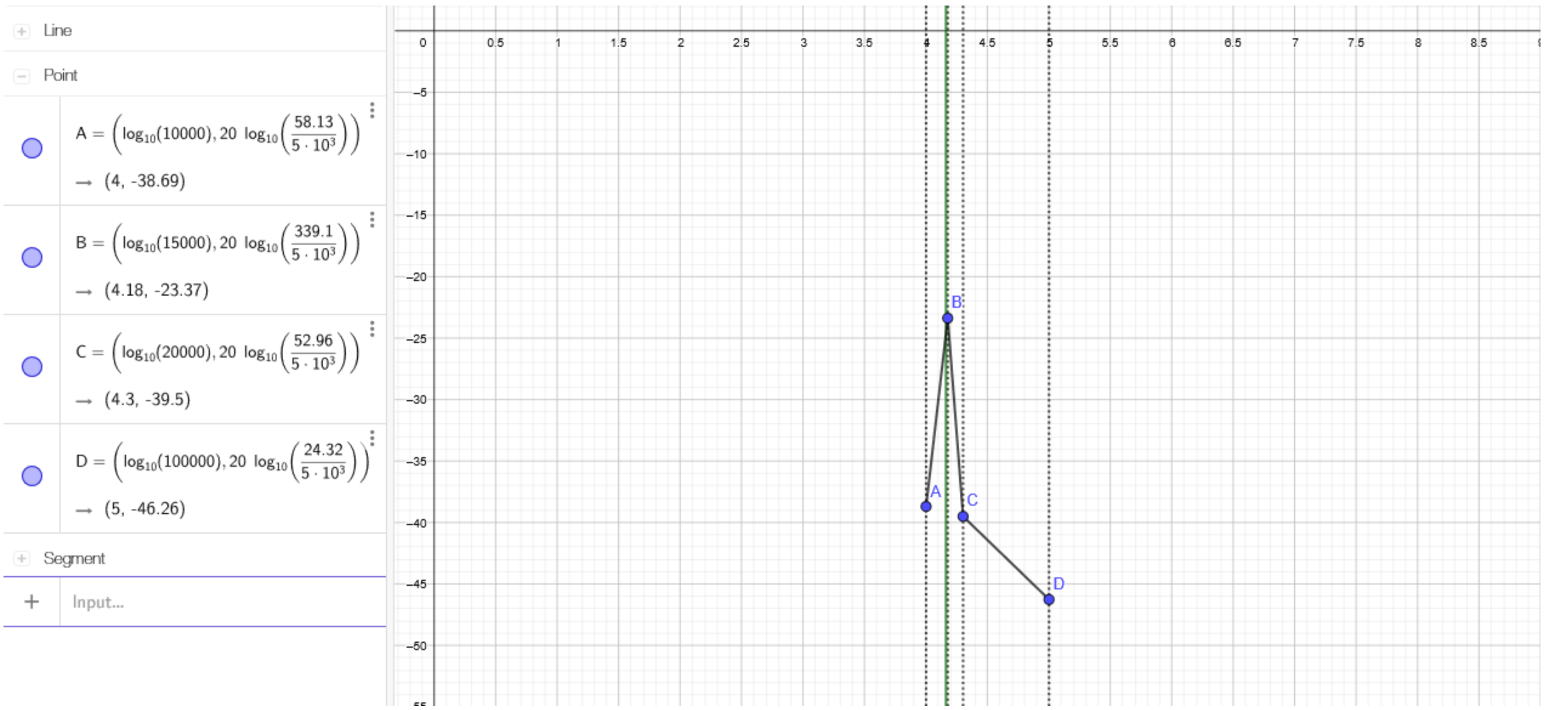

Figure 5: Experimental Bode Plot [dB vs Hz, loglog] of Circuit 1 in selected points ($f_{\text{res,theo}}$ denoted as the green line)

Both Bode plots indicate that the circuit is an **RLC Parallel Bandpass Filter**.

This kind of filter attenuates the passage of all frequencies, except for a few. The resonant frequency or any frequencies within the set range gets unimpeded.

2. Application 2: **RLC Series Band-stop Filter**

Here, the ansatz was raised again and thus, this set-up followed a similar procedure but with a different circuit, shown below:

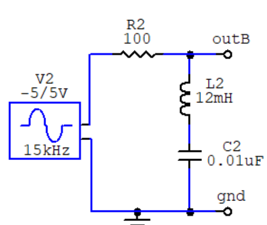

Figure 6: Template Circuit 2

Setting this up for the boundary frequencies does not yield any quick result so it was skipped. Starting from 10 kHz, the output waveforms of the circuit was observed while gradually increasing the input frequency in increments of 5 kHz.

After two increments, it was found that a significant change in the waveform can be seen around 15 kHz as shown in Figure 7. To compensate for skipping the first step, the process was repeated, but adding and subtracting an increment of 1 kHz to 15 kHz instead per iteration.

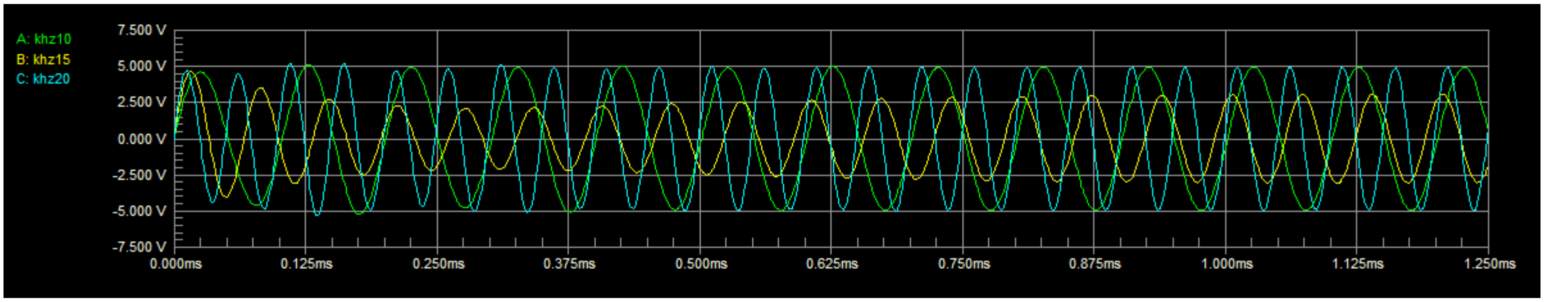

Figure 7: Output Waveforms of 10kHz (green), 15 kHz (yellow), and 20 kHz (cyan) in Circuit 2

After one increment on both sides, it was observed that 15 kHz and 14 kHz was almost 'reduced' in the same way, as shown in Figure 8. Therefore, it can be inferred that the notch frequency f_{nch} is between the two. Thus, their arithmetic mean was declared as the experimental notch frequency $f_{\text{nch,exp}}$ here: $f_{\text{nch,exp}} = 14.5$ [kHz].

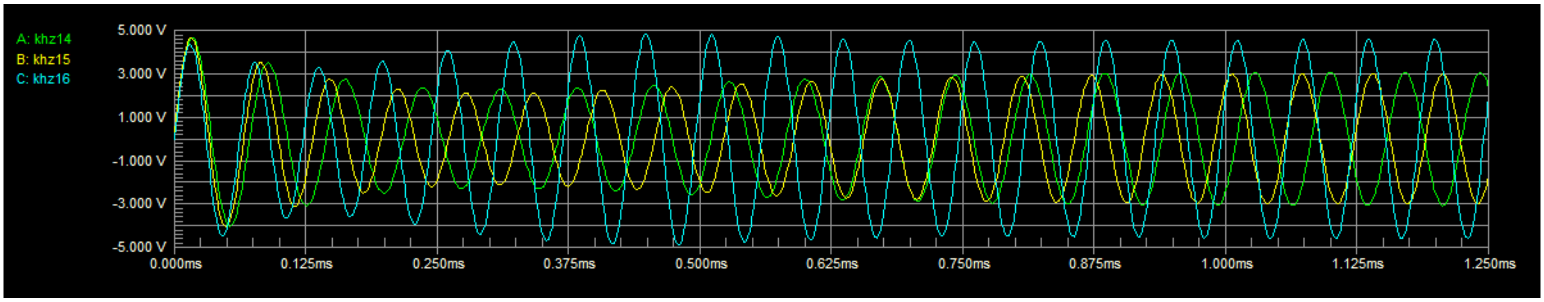

Figure 8: Output Waveforms of 14 kHz (green), 15 kHz (yellow), and 16 kHz (cyan) in Circuit 2

The theoretical notch frequency $f_{\text{nch,theo}}$ is the same as the theoretical resonant frequency $f_{\text{rex,theo}}$ of Circuit 1 ($\frac{10^5}{\pi\sqrt{4.8}} \approx 14529$ Hz). Comparing this, the percent difference/error is 0.2%.

$$\begin{aligned}
\%_{\text{err}} &= \left| \frac{f_{\text{nch,theo}} - f_{\text{nch,exp}}}{f_{\text{nch,theo}}} \right| \\
&= \left| \frac{14529 - 14500}{14529} \frac{[\text{Hz}]}{[\text{Hz}]} \right| \\
\%_{\text{err}} &= 0.002 \sim 0.2\%
\end{aligned}$$

This can be further confirmed by comparing the Bode plot of experimental and theoretical approaches.

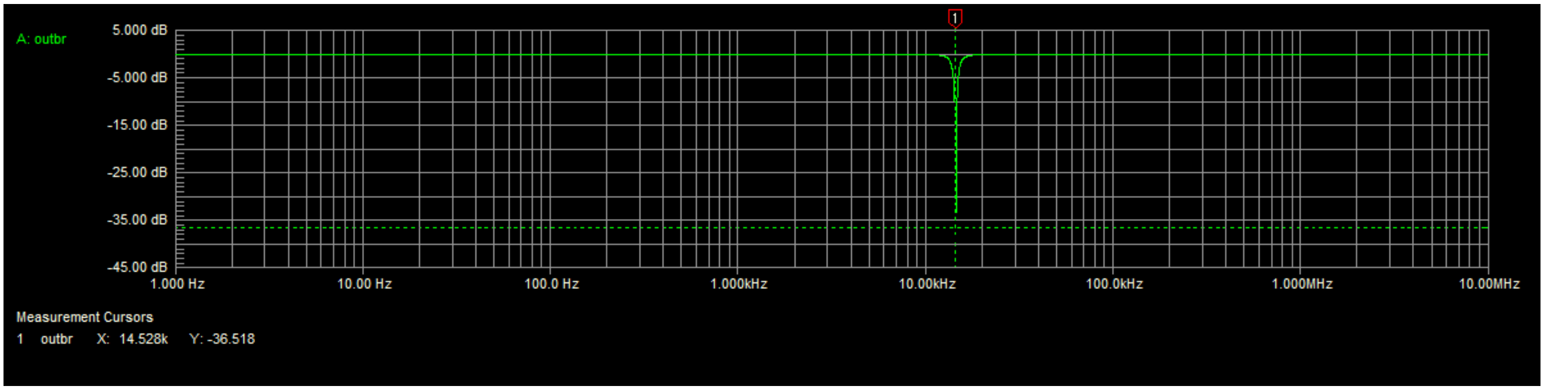

Figure 9: Theoretical Bode Plot of Circuit 2

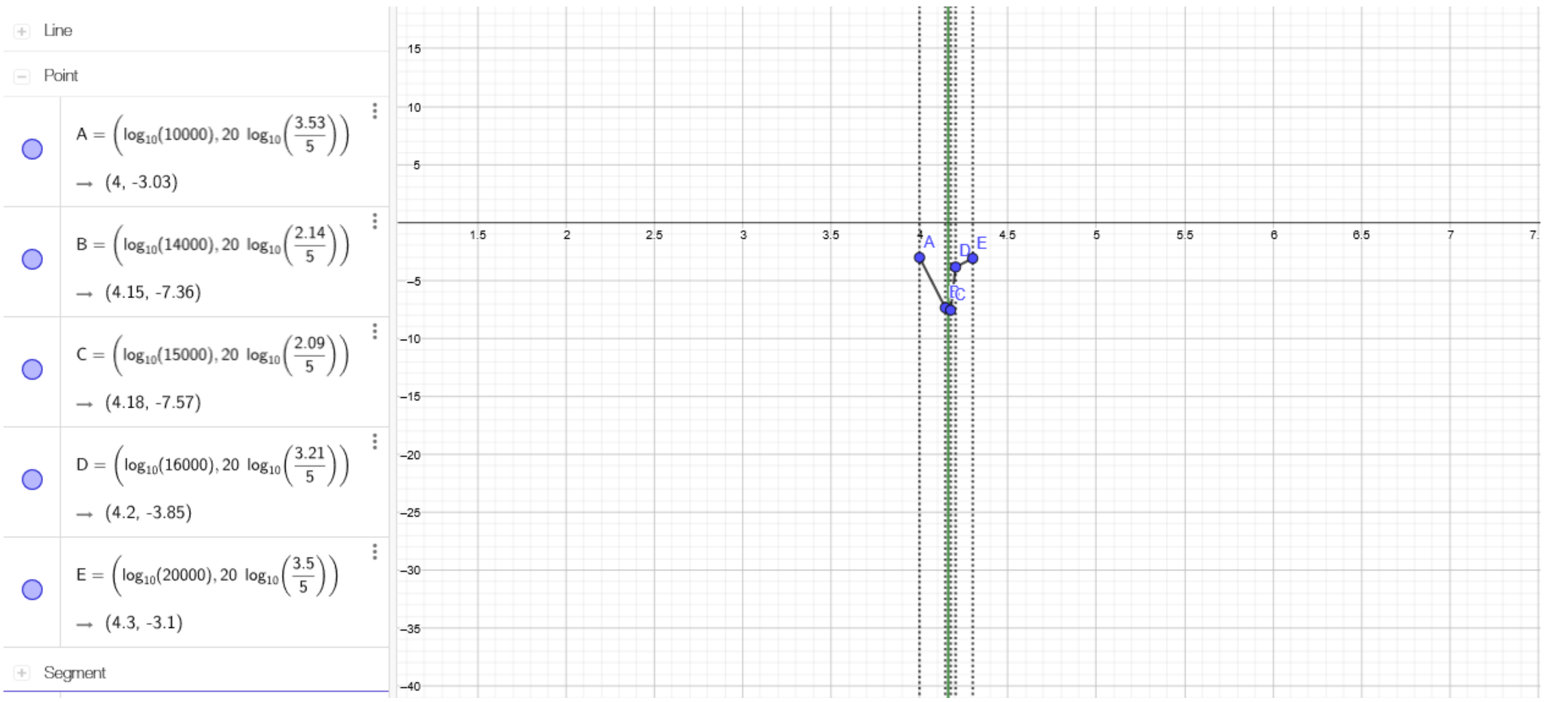

Figure 10: Experimental Bode Plot [dB vs Hz, loglog] of Circuit 2 in selected points ($f_{\text{nch,theo}}$ denoted as the green line)

Both Bode plots indicate that Circuit 2 is an **RLC Series Band-Stop Filter** or a **Notch/Trap Series Circuit**.

This kind of filter, in contrast to Circuit 1, allows all frequencies and attenuates a few, exhibiting a behaviour similar to a combination of low and high-pass filters.

Notes:

The ansatz was not completely proven here with rigid math, but with further messing with the circuit, it is found that it does not hold true all the time. One round of analysis showed that it may only be valid for small values of L and C (specifically if they are less than 1 H or F respectively), which is yet to be proven by rigid math. This last statement, however, was taken as a loose proof and found itself useful in this experiment.