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[AppPhysics 181]: Activity 2: Capacitors

Objectives

1. To observe the responses of adding an capacitors in an AC circuit
2. To explore applications of capacitors in the circuits governed by frequencies

Report Proper

1. Charging and Discharging a Capacitor

   To start, Circuit 0 (shown below) was fed with an input square wave with a peak voltage of 5 V and a frequency of 500 Hz.

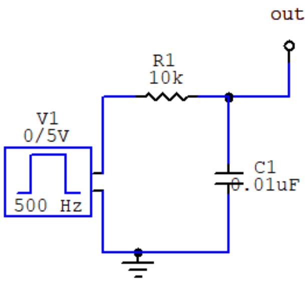

Figure 1: Circuit 0 where `out` denotes the test point

To determine the experimental time constants, the first wavelength was investigated. As seen in Figure 2, the first wavelength is essentially evenly split into two convenient states: Charging for the first half, and Discharging on the other one.

In the first half, when the capacitor is charging, one time constant τ passes when the voltage hits 3.1606 V which is at $\tau_{\text{exp,C}} = 100.43[\mu\text{s}]$. In the second half, when the capacitor is discharging, one time constant τ also passes when the voltage reaches 1.8393 V, which is at $\tau_{\text{exp,D}} = 1.1011[\text{ms}] - \tau_{\text{delay}} = 101.1[\mu\text{s}]$ ($\tau_{\text{delay}} = 1$ ms because the first discharging state only occurs after 1 ms). This can be graphically seen in the figure below.

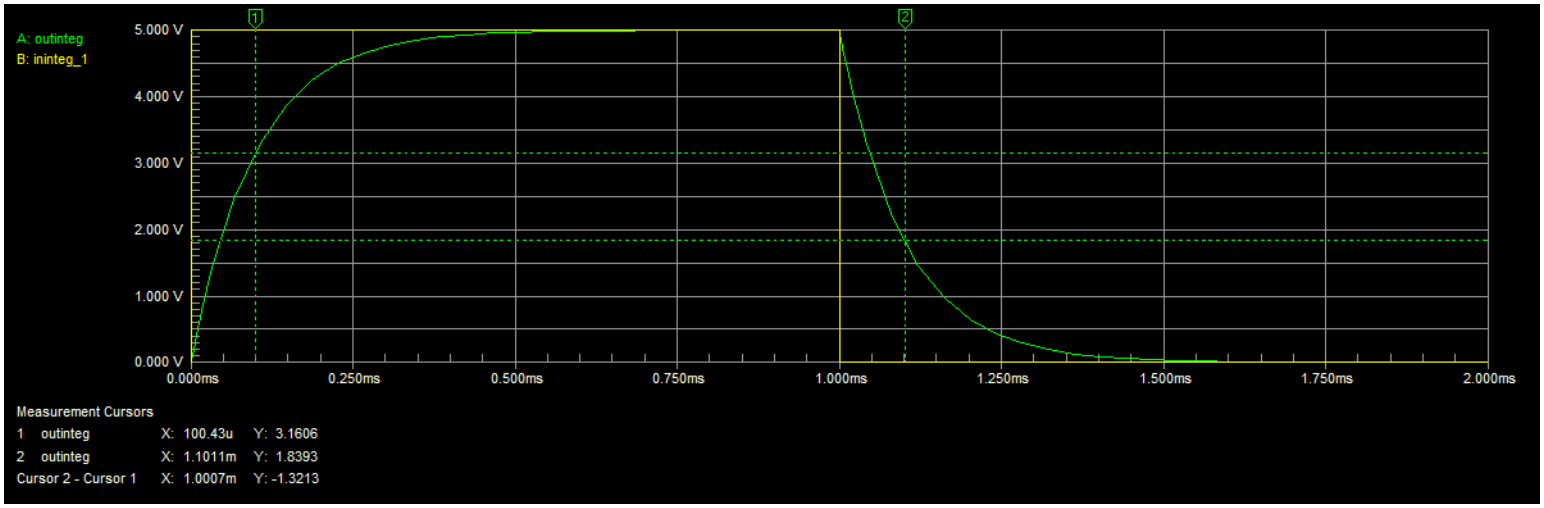

Figure 2: Superimposed first wavelength of the Input (yellow) and Output (green) waveforms used in Circuit 0

Comparing this to the theoretical time constant $\tau_{\text{theo}} = RC = 100[\mu\text{s}]$, the percent errors/difference are:

$$\text{CHARGING}: \quad \%_{\text{err}} = \left|\frac{\tau_{\text{theo}} - \tau_{\text{exp,C}}}{\tau_{\text{theo}}}\right| = \left|\frac{100 - 100.43}{100}\frac{[\mu\text{s}]}{[\mu\text{s}]}\right| = 0.0043 \sim 0.43\%$$

$$\text{DISCHARGING}: \quad \%_{\text{err}} = \left|\frac{\tau_{\text{theo}} - \tau_{\text{exp,D}}}{\tau_{\text{theo}}}\right| = \left|\frac{100 - 101.1}{100}\frac{[\mu\text{s}]}{[\mu\text{s}]}\right| = 0.011 \sim 1.1\%$$

If, say the frequency is increased say exponentially at base 2 (or if we double the frequency at every step), it is noticeable that at some frequency of a square wave, Circuit 0 will essentially produce a reduced triangular wave.

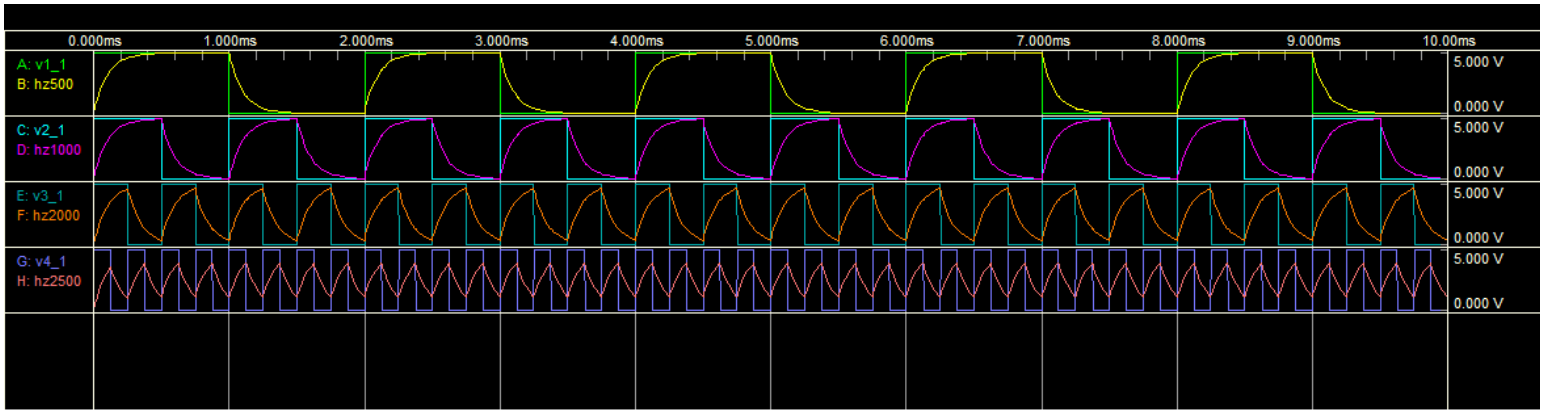

Figure 3: Superimposed input and output waveforms as the frequency doubles at every step.

2. Application 1: **Differentiator Circuit**

To start, Circuit 1 (shown below) is fed with a sine wave (`sin`), a square wave (`sqr`), and a triangular wave (`tri`), with the peak amplitude of 5 V and a frequency of 100 kHz.

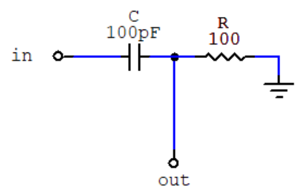

Figure 4: Circuit 1 where `in` is connected to a wave generator

Based on the output frequencies, this circuit is behaving as a **Differentiator Circuit**. True to its name, it gives the derivative of the input waveform.

For the first wave [`sqr`], the output wave consisted of alternating sharp peaks in regular intervals corresponding to the times the square wave shifts from one peak to another as shown below. Square waves oscillate almost instantaneously from one voltage to another; being almost instantaneous gives each voltage shift a rate of change that is a real number which is what is reflected to the output wave. Additionally, when the shift happens, the peak points to the direction of the shift: a direct consequence of differentiation.

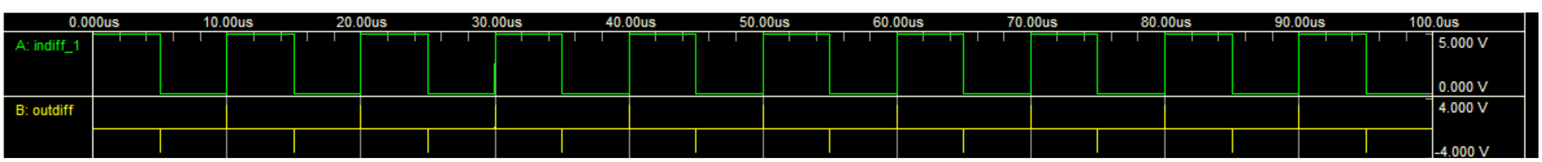

Figure 5: Input (green) and Output (yellow) waveforms of `sqr` in Circuit 1: Differentiator

For the second wave [`sin`], the output wave is a cosine wave or a shifted sine wave with the peak voltage effectively diminished as shown below.

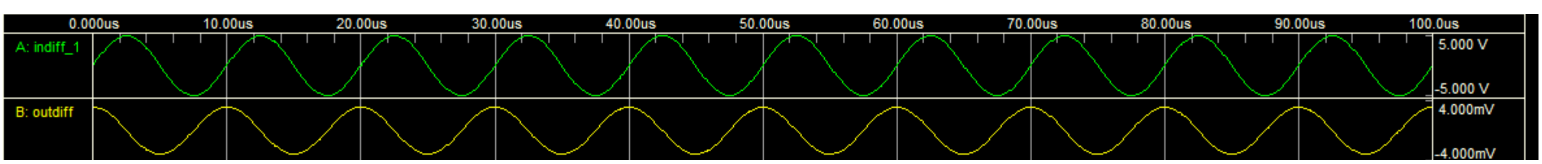

Figure 6: Input (green) and Output (yellow) waveforms of `sin` in Circuit 1: Differentiator

For the third wave [`tri`], the output wave is a square wave with the peak voltage also effectively diminished as shown below. Triangle waves are, in essence, square waves with a 'long climb' giving it its known shape. Being just two mirroring diagonal lines, differentiation will give steady horizontal lines corresponding to the slopes of each line. When the diagonal lines transition, the slope almost instantaneously jumps from positive to negative, and thus giving the output wave the image of a square wave.

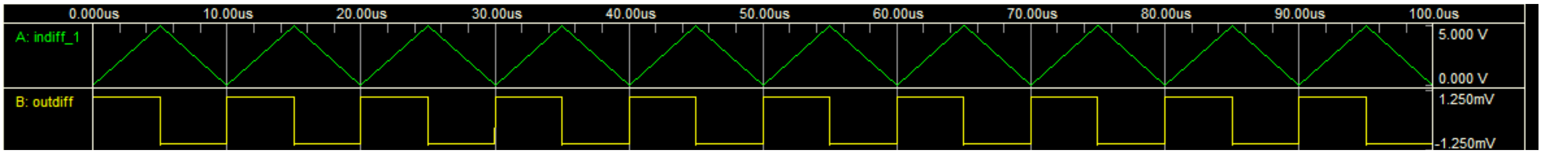

Figure 7: Input (green) and Output (yellow) waveforms of `tri` in Circuit 1: Differentiator

Increasing either the resistance or capacitance will give the same effect: it creates a 'delay' which transforms the output wave into a different kind of wave. The differentiating effect gets lost the higher the value of either the resistance or capacitance.

3. Application 2: **Integrator Circuit**

   The next circuit followed the same starting set-up as before: `sin`, `sqr`, and `tri` were fed into the Circuit 2 (shown below) with the same voltage and frequency settings.

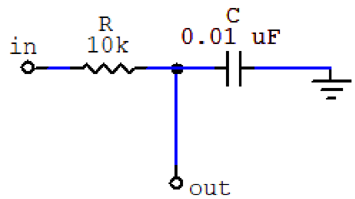

Figure 8: Circuit 2 where `in` is connected to a wave generator

Based on the output frequencies, this circuit is an **Integrator Circuit**. As an integrator, the output waveform is the integral of the input waveform, that is, the area under the graph of the latter.

For the first wave [`sqr`], it is known that its behaviour corresponds to the alternating waveform at a steady frequency between fixed minimum and maximum values. Hence, the output waveform should have a behaviour that has a slope of alternating constant values corresponding to that of the input waveform (scale down to a factor of RC). This fits the characteristic of a triangle wave (as shown below).

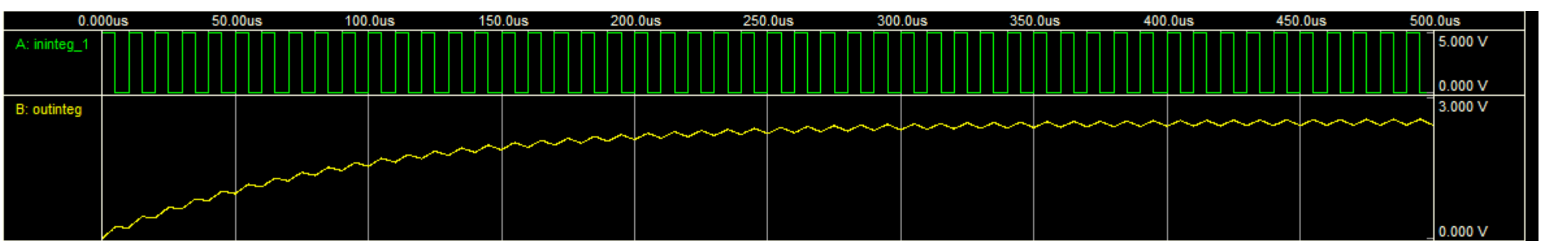

Figure 9: Input (green) and Output (yellow) waveforms of `sqr` in Circuit 2: Integrator. Note the gradual stabilization of the waveform in the output.

For the second wave [`sin`], it is established that the integral of a sine wave is simply a (negative) cosine wave which is just a phase-shifted sine wave, which was directly observed from the graph: a scaled-down, phase-shifted form of the input waveform. The wave being scaled down came from the fact that an integrator circuit is also a low-pass filter circuit, which has the property of reducing the output wave, which is shown as the wave scaled-down (as shown below).

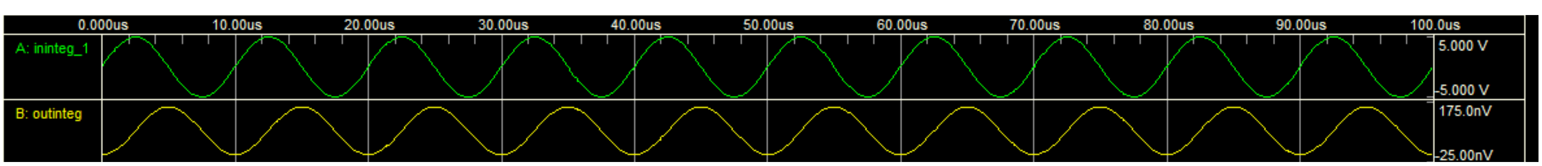

Figure 10: Input and Output waveforms of `sin` in Circuit 2: Integrator

For the third wave [`tri`], the behaviour of the output wave closely resembles an image of a sine wave, which is not an initial expectation. However, it should also be noted that Circuit 2 is of the same form as Circuit 3, which was found to be a Low-Pass Filter Circuit. The waves that added up to make a `tri` wave gets 'filtered', and thus leaving low frequencies as the output, effectively, something similar or a sine wave (as shown below).

If smaller R or C were to be used in the circuit, the assumption that $V << V_{\text{in}}$ would not be applicable and the equation would be a linear, non-homogeneous, differential equation, in which case simple integration would not solve the equation.

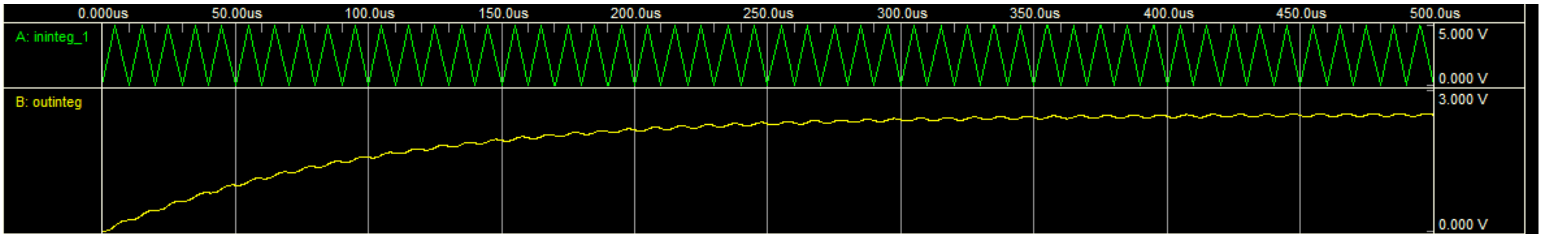

Figure 11: Input (green) and Output (yellow) waveforms of `tri` in Circuit 2: Integrator. Note the gradual stabilization of the waveform in the output.

4. Application 3: **Low-Pass Filter Circuit**

   An assortment of frequencies were fed to the Circuit 3 (shown below) with a common constant peak voltage of 5 V.

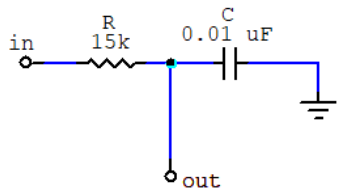

Figure 12: Circuit 3 with `in` connected to a wave generator

   The experimental $f_{3\text{dB}}$ was found by locating the value of 0.707 from the y-axis of the V-vs-Hz Bode magnitude plot (as shown in Figure 14). The point, and by extension, the experimental value is 1.6 kHz. The theoretical value is 1.60 kHz. (2 sigfigs). Thus, the percent error is 0%

   Based on this, Circuit 3 showed the behavior of a **Low-Pass Filter**. As seen in either of its Bode magnitude plots, the circuit only allows lower frequencies to pass through, true to its name. Higher frequencies (or frequencies higher than 1.60 kHz in the plot) are reduced while the lower frequencies remains the same.

5. Application 4: **High-Pass Filter Circuit**

   Following the same process as Application 3, an assortment of frequencies were fed to the Circuit 4 (shown below) with a constant peak voltage of 5 V.

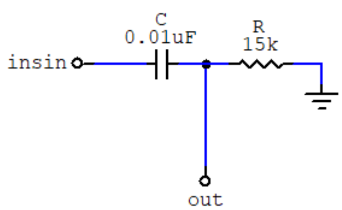

Figure 13: Circuit 4 with `insin` connected to a wave generator

   The experimental $f_{3\text{dB}}$ was found by tracing the value of 0.707 V from the y- axis of the V-vs-Hz graph of the Bode magnitude plot (as shown in Figure 14). The point is around 1 kHz. The theoretical value is 1.06 kHz. Thus, the percent error is also 0%.

   Based on this, Circuit 4 shows the behavior of a **High-Pass Filter**. As seen in either of its Bode magnitude plots, the circuit only allows higher frequencies to pass through, hence the name. Lower frequencies are diminished, as seen in the plots where frequencies below 1.06 kHz are effective reduced, compared to those over 1.06 kHz, which remained relatively the same.

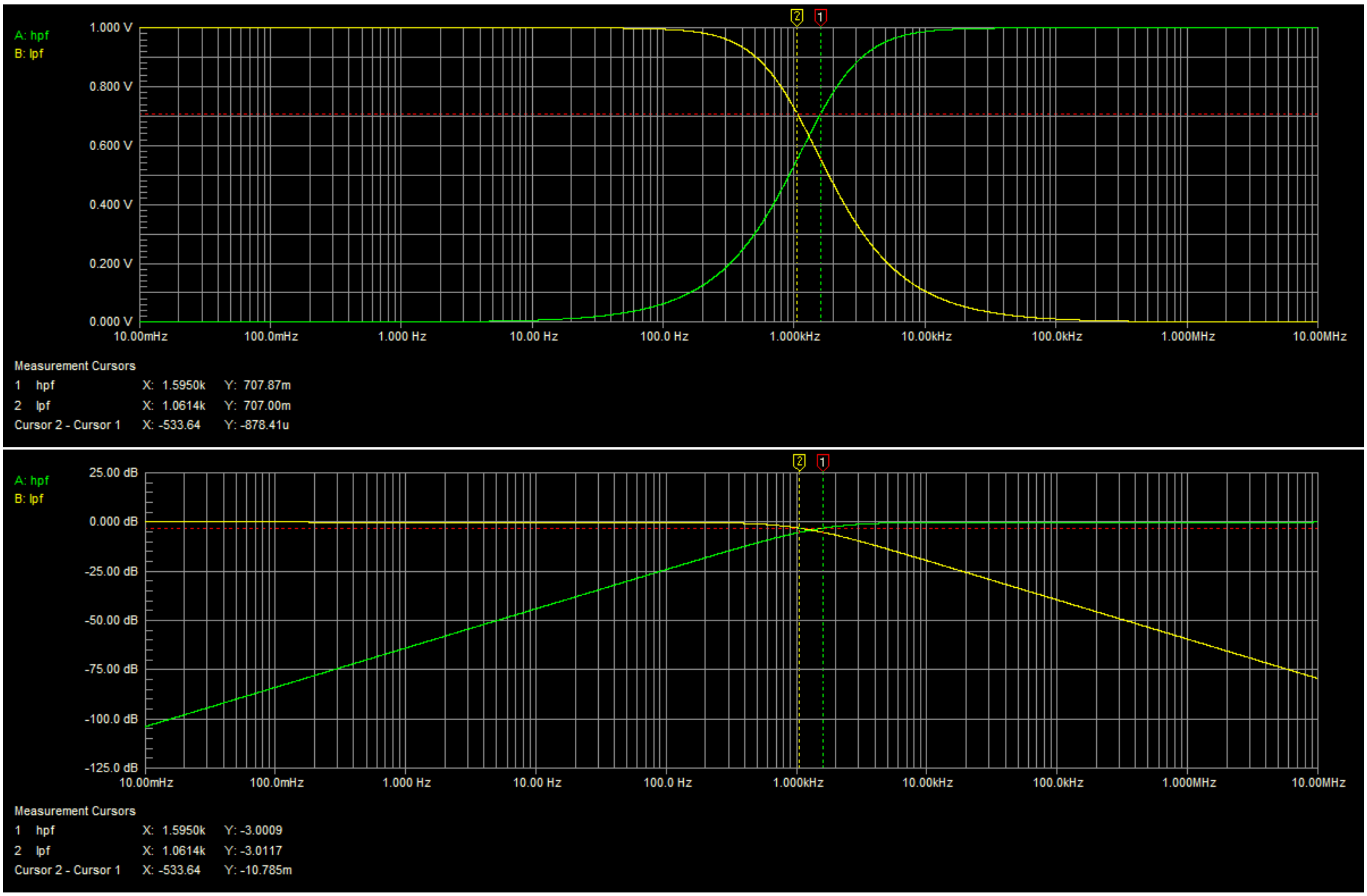

Figure 14: Bode magnitude plots [V×Hz](above) and [dB×Hz](below) of Circuit 3 (yellow) and 4 (green). Cursors 1 and 2 denote the experimental cutoff frequencies for Circuits 4 and 3, respectively.