Занятие № 6

Вычислить неопределённый интеграл:

1) $$\int \frac{x-\sqrt[4]{\arcsin^3 x}}{\sqrt{1-x^2}}dx = -\frac{1}{2}\int\frac{d(1-x^2)}{\sqrt{1-x^2}} -$$
$$-\int(\arcsin x)^{\frac{3}{4}}d(\arcsin x) = -\sqrt{1-x^2}-\frac{4}{7}(\arcsin x)^{\frac{7}{4}}+C$$

2) $$\int x^2\cos 5x\,dx = x^2\cdot\left(\frac{1}{5}\sin 5x\right)-\frac{2}{5}\int x\cdot\sin 5x =$$
$$= \frac{1}{5}x^2\sin 5x - \frac{2}{5}\left(x\left(-\frac{1}{5}\cos 5x\right)+\frac{1}{5}\int\cos 5x\,dx\right) =$$
$$= \frac{1}{5}x^2\sin 5x + \frac{2}{5}x\cdot\cos 5x - \frac{2}{125}\sin 5x + C$$

3) $$\int\frac{x-8}{x^2+5x-6}dx = 2\ln|x+6| - \ln|x-1|+C$$
$$\frac{x-8}{x^2+5x-6} = \frac{x-8}{(x+6)(x-1)} = \frac{A}{x+6}+\frac{B}{x-1} = \frac{2}{x+6}-\frac{1}{x-1}$$
$$x-8 = A(x-1)+B(x+6)$$
$$x=-6: \quad -14 = -7A, \quad A=2$$
$$x=1: \quad -7 = 7B, \quad B=-1$$

4) $$\int\frac{6x-7}{\sqrt{6x-x^2-5}} = \int\frac{(-2x+6)\cdot(-3)+11}{\sqrt{6x-x^2-5}}dx =$$
$$= -3\int\frac{d(6x-x^2-5)}{\sqrt{6x-x^2-5}} + 11\int\frac{dx}{\sqrt{4-(x-3)^2}} =$$
$$= -6\cdot\sqrt{6x-x^2-5} + 11\arcsin\left(\frac{x-3}{2}\right)+C$$

5) $$\int\frac{\sqrt{2-x}}{3-\sqrt{2-x}}dx = \begin{bmatrix}\sqrt{2-x}=t, & 2-x=t^2, \\ x=2-t^2, & dx=-2t\,dt\end{bmatrix} =$$
$$= \int\frac{-2t^2dt}{3-t} = \int\frac{2t^2dt}{t-3} = \int\left(2t+6+\frac{18}{t-3}\right)dt =$$
$$= t^2+6t+18\ln|t-3| = -x+6\sqrt{2-x}+18\ln\left|\sqrt{2-x}-3\right|+C$$

6) $\int \sin^2 4x \cdot \cos 5x \, dx = \frac{1}{2}\int (1-\cos 8x)\cdot \cos 5x \, dx =$

$= \frac{1}{2}\int \cos 5x \, dx - \frac{1}{4}\int (\cos 13x + \cos 3x)\, dx =$

$= \frac{1}{10}\sin 5x - \frac{1}{52}\sin 13x - \frac{1}{12}\sin 3x + C$

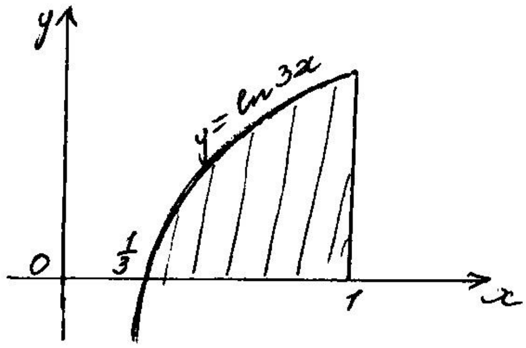
7) Вычислить определённый интеграл:

$$\int\limits_0^{\frac{\pi}{2}} \frac{dx}{2-\cos x} = \left[\begin{array}{l} tg\frac{x}{2} = t, \quad t_1 = 0, \quad t_2 = 1, \\ \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2dt}{1+t^2} \end{array}\right] =$$

$$= \int\limits_0^1 \frac{2\,dt}{2(1+t^2)-(1-t^2)} = \int\limits_0^1 \frac{2\,dt}{1+3t^2} = \frac{2}{3}\int\limits_0^1 \frac{dt}{\frac{1}{3}+t^2} =$$

$$= \frac{2}{3}\cdot\sqrt{3}\, arctg(\sqrt{3}\,t)\Big|_0^1 \cancel{+C} = \frac{2\sqrt{3}}{3}\cdot\left(\frac{\pi}{3} - 0\right) = \frac{2\sqrt{3}\,\pi}{9}$$

8) Вычислить площадь фигуры, ограниченной линиями: $y = \ln 3x, y = 0, x = 1$



$$S = \int\limits_{\frac{1}{3}}^{1} \ln 3x \, dx =$$

$$= (x \cdot \ln 3x)\Big|_{\frac{1}{3}}^{1} - \int\limits_{\frac{1}{3}}^{1} dx =$$

$$= \ln 3 - x\Big|_{\frac{1}{3}}^{1} = \ln 3 - \frac{2}{3}$$

Самостоятельно:

1) $\int \frac{10x+\sqrt[3]{\operatorname{arctg}^2 x}}{1+x^2}\,dx = 5\ln(1+x^2)+\frac{3}{5}(\operatorname{arctg} x)^{\frac{5}{3}}+C$

2) $\int x^2\cdot e^{-3x}\,dx = -e^{-3x}\left(\frac{1}{3}x^2+\frac{2}{9}x+\frac{2}{27}\right)+C$

3) $\int \frac{x-11}{x^2+3x-4}\,dx = 3\ln|x+4|-2\ln|x-1|+C$

4) $\int \frac{4x+5}{\sqrt{x^2+2x+3}}\,dx = 4\sqrt{x^2+2x+3}+$
$+\ln\left|x+1+\sqrt{x^2+2x+3}\right|+C$

5) $\int \frac{dx}{\sqrt{x}-2\sqrt[4]{x}+3} = \left[\sqrt[4]{x}=t\right] =$
$= 2\sqrt{x}+8\sqrt[4]{x}+2\ln(\sqrt{x}-2\sqrt[4]{x}+3)-10\sqrt{2}\cdot\operatorname{arctg}\frac{\sqrt[4]{x}-1}{\sqrt{2}}+C$

6) $\int \sin 3x\cdot\cos^2 5x\,dx = -\frac{1}{6}\cos 3x-\frac{1}{52}\cos 13x+$
$+\frac{1}{28}\cos 7x+C$

7) $\int\limits_{\ln 5}^{\ln 8} \frac{dx}{\sqrt{9-e^x}} = \frac{1}{3}\ln\frac{5}{2}$

8) Площадь фигуры, ограниченной линиями: $y=e^{2x}$, $y=e^{-2x}$, $x=\ln 2$.

$S=\int\limits_0^{\ln 2}\left(e^{2x}-e^{-2x}\right)dx = \frac{9}{8}$