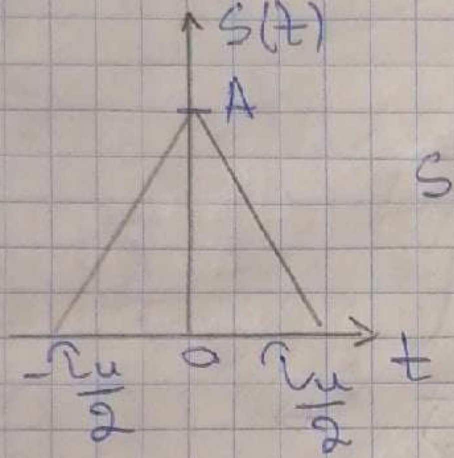
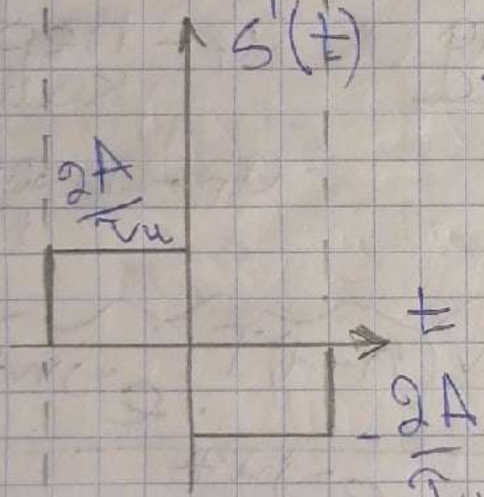
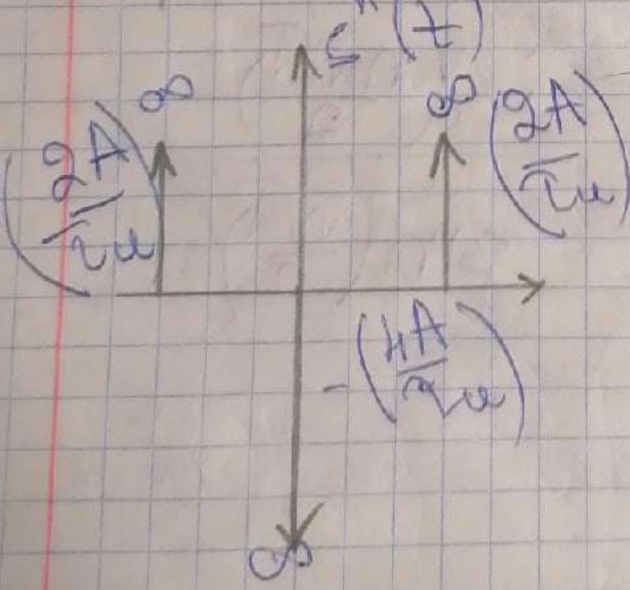
Определить спектр. плотн. симметр. прямоуг. импульса



$$s(t)=\begin{cases} A\left(\frac{t}{\frac{\tau_и}{2}}+1\right) & -\frac{\tau_и}{2}\le t\le 0 \\ A\left(1-\frac{t}{\frac{\tau_и}{2}}\right) & 0\le t\le \frac{\tau_и}{2} \\ 0 & |t|>\frac{\tau_и}{2} \end{cases}$$



$$s''(t)=\frac{2A}{\tau_и}\delta\left(t+\frac{\tau_и}{2}\right)+\frac{2A}{\tau_и}\delta\left(t-\frac{\tau_и}{2}\right)-\frac{4A}{\tau_и}\delta(t)$$



$$F\{s''(t)\}=\frac{2A}{\tau_и}e^{j\omega\frac{\tau_и}{2}}+\frac{2A}{\tau_и}e^{-j\omega\frac{\tau_и}{2}}-\frac{4A}{\tau_и}$$

$$= \frac{4A}{\tau_u}\left(\frac{e^{j\omega\tau_u/2} + e^{-j\omega\tau_u/2}}{2} - 1\right) =$$

$$= \frac{4A}{\tau_u}\left(\cos\left(\omega\frac{\tau_u}{2}\right) - 1\right) =$$

$$= -\frac{4A}{\tau_u} 2\sin^2\left(\frac{\omega\tau_u}{4}\right)$$

$$S(\omega) = \frac{F\{s'(t)\}}{j\omega} = \frac{F\{s''(t)\}}{(j\omega)^2}$$

$$S(\omega) = \frac{4A}{\tau_u \omega^2} 2\sin^2\left(\frac{\omega\tau_u}{4}\right) =$$

$$= \frac{A\tau_u}{2} \cdot \underbrace{\frac{\sin^2\left(\frac{\omega\tau_u}{4}\right)}{\left(\frac{\omega\tau_u}{4}\right)}}_{= \operatorname{sinc}^2\left(\frac{\omega\tau_u}{4}\right)}$$

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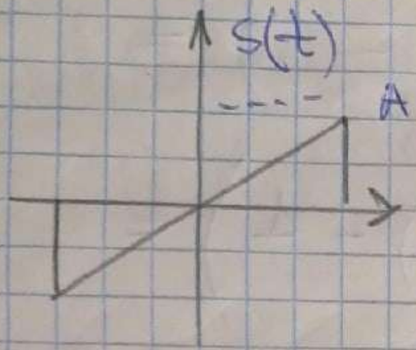
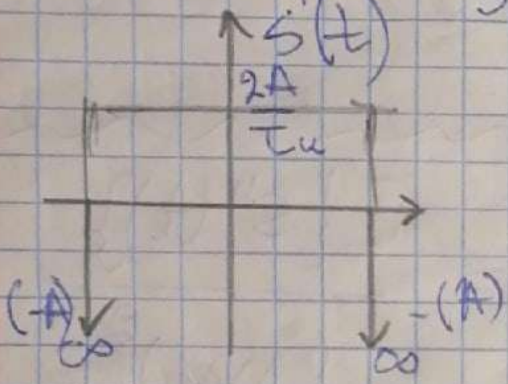


$$s(t) = \begin{cases} \frac{A}{\tau_u/2} t, & -\frac{\tau_u}{2} < t < \frac{\tau_u}{2} \\ 0, & |t| \geq \frac{\tau_u}{2} \end{cases}$$



$$s'(t) = \frac{2A}{\tau_u} - A\delta\left(t + \frac{\tau_u}{2}\right) - A\delta\left(t - \frac{\tau_u}{2}\right)$$

$$F\{s'(t)\} = \frac{2A}{\tau_u} \cdot \mathrm{sinc}\left(\frac{w\tau_u}{2}\right) - Ae^{jw\frac{\tau_u}{2}} - Ae^{-jw\frac{\tau_u}{2}}$$

$$= 2A\left(\mathrm{sinc}\left(w\frac{\tau_u}{2}\right) - \left(\frac{e^{jw\frac{\tau_u}{2}} - e^{-jw\frac{\tau_u}{2}}}{2}\right)\right)$$

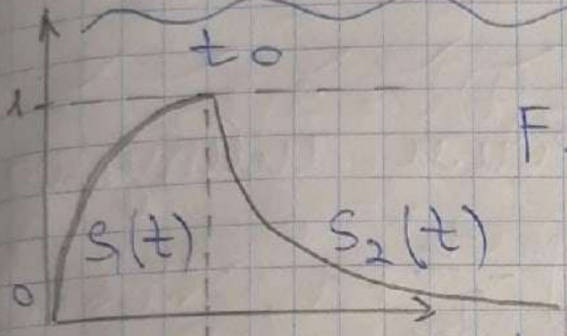
$$= 2A\left(\mathrm{sinc}\left(\frac{w\tau_u}{2}\right) - \cos\left(w\frac{\tau_u}{2}\right)\right)$$

$$s(w) = \frac{F\{s'(t)\}}{jw} = \frac{2A}{jw}\left(\mathrm{sinc}\left(\frac{w\tau_u}{2}\right) - \cos\frac{w\tau_u}{2}\right)$$

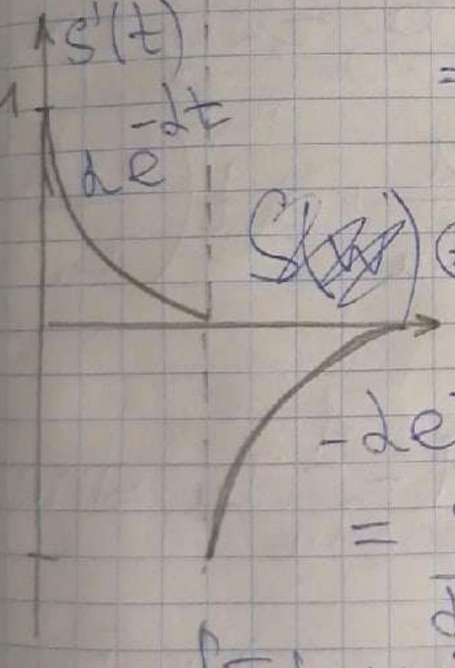
$$|s(w)| = \begin{cases} \frac{2A}{w}\left|\mathrm{sinc}\left(\frac{w\tau_u}{2}\right) - \cos\left(\frac{w\tau_u}{2}\right)\right| \\ 0, & w = 0 \end{cases}$$

Опр. спектр плотность сигнала

$$s(t) = \sigma(t)(1-e^{-\alpha t}) - \sigma(t-t_0)(1-e^{-\alpha(t-t_0)})$$



$$F\{s'(t)\} = \frac{\alpha}{\alpha+j\omega} - \frac{\alpha}{\alpha+j\omega}e^{-j\omega t_0} =$$



$$= \frac{\alpha}{\alpha+j\omega}\left(1-e^{-j\omega t_0}\right) \cdot \boxed{e^{j\omega t_0} \cdot e^{-j\omega t_0}}$$

$$S(\omega) = \frac{2j\alpha}{\alpha+j\omega}\left(\frac{e^{j\frac{\omega t_0}{2}} - e^{-j\frac{\omega t_0}{2}}}{2j}\right)$$

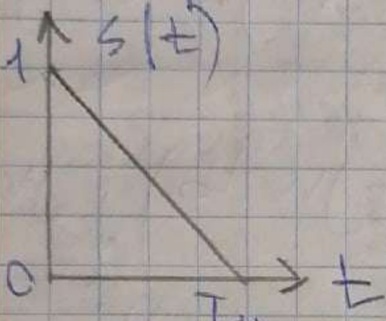
$$= \frac{2j\alpha}{\alpha+j\omega}\sin\left(\frac{\omega t_0}{2}\right)\cdot e^{-j\omega t_0} \qquad e^{-j\frac{\omega t_0}{2}}$$

$$S(\omega) = \begin{cases} F\{s'(t)\} = \frac{\alpha t_0}{\alpha+j\omega}\,\mathrm{sinc}\left(\frac{\omega t_0}{2}\right)\cdot e^{-j\omega t_0}, & \omega \neq 0 \\ t_0, & \omega = 0 \end{cases}$$

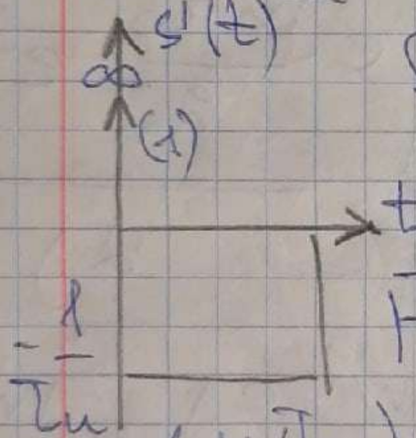
$$|S(\omega)| = \frac{\alpha t_0}{\sqrt{\alpha^2+\omega^2}}\left|\mathrm{sinc}\left(\frac{\omega t_0}{2}\right)\right|$$

$$\varphi_s(w) = -\mathrm{arctg}\,\frac{w}{\alpha} + \frac{\pi}{2}\left(1 - \mathrm{sign}\cdot \mathrm{sinc}\left(\frac{wt_0}{2}\right)\cdot \mathrm{sign}\,w - \frac{wt_0}{2}\right)$$

Опр. плот. несим. импульса



$$s(t) = \begin{cases} -\frac{t}{\tau_u} + 1, & 0 \le t \le \tau_u \\ 0, & t < 0,\ t > \tau_u \end{cases}$$



$$s'(t) = \delta(t) - \frac{1}{\tau_u}\,\mathrm{rect}\left(\frac{t - t_0}{\tau_u}\right)$$

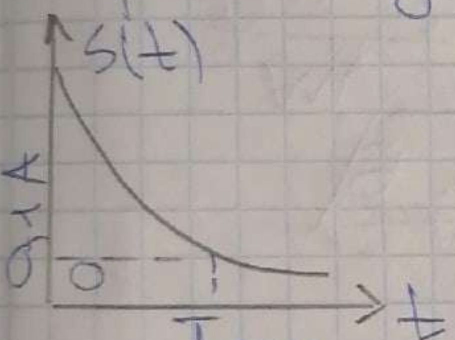
$$F\{s'(t)\} = 1 - \frac{1}{\tau_u}\tau_u\,\mathrm{sinc}\left(\frac{w\tau_u}{2}\right)e^{-jw\frac{\tau_u}{2}}$$

$$S(w) = \begin{cases} \frac{1}{jw}\left(1 - \mathrm{sinc}\left(\frac{w\tau_u}{2}\right)e^{-jw\frac{\tau_u}{2}}\right), & w \neq 0 \\ \frac{\tau_u}{2}, & w = 0 \end{cases}$$

$$|S(w)| = \begin{cases} \frac{1}{|w|}\sqrt{1-2\,\mathrm{sinc}(w\tau_u)+\mathrm{sinc}^2\left(\frac{w\tau_u}{2}\right)}, & w\neq 0 \\ \frac{\tau_u}{2}, & w=0 \end{cases}$$

$$\varphi_s(w) = \begin{cases} -\frac{\pi}{2}\,\mathrm{sign}(w) + \mathrm{arctg}\left(\frac{\mathrm{sinc}\left(\frac{w\tau_u}{2}\right)\sin\left(\frac{w\tau_u}{2}\right)}{1-\mathrm{sinc}(w\tau_u)}\right) \\ 0, \quad w=0 \end{cases}$$

Опр. базу эксп. имп.



$$s(t) = \begin{cases} A\cdot e^{-\alpha t}, & t\geq 0 \\ 0, & t<0 \end{cases}$$

$$Ae^{-\alpha t} = 0{,}1A$$

$$\tau_{эф} = \frac{\ln 10}{\alpha}$$

$$|S(w)| = \frac{A}{\sqrt{\alpha^2+w^2}} \Rightarrow \frac{A}{\sqrt{\alpha^2+w^2}} = \frac{A}{10\alpha}$$

$$\alpha^2+w^2 = 100\alpha^2$$

$$w^2 = 99\alpha^2$$

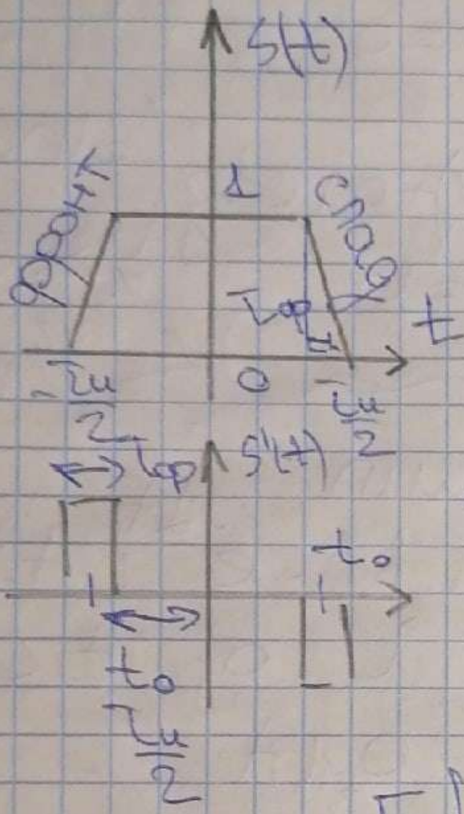
$$w_{1,2} = \pm 10\alpha$$

$$\Delta w \approx 20\alpha$$

$$B_w = \tau_{эф}\cdot\Delta w = \frac{\ln 10}{\alpha}\cdot 20\alpha = \ln 10\cdot 20 \approx \boxed{46}$$

$$B_f = B_w/2\pi = \boxed{7{,}32}$$

Опр. спектр. плотность трапец. импульса



$$t_0 = \frac{\tau_и}{2} - \frac{\tau_ф}{2}$$

$$F\{s'(t)\} = 2j\tau_ф \frac{1}{\tau_ф} \cdot \mathrm{sinc}\left(\frac{w\tau_ф}{2}\right)\sin(wt_0)$$

$$S(w) = \frac{F\{s'(t)\}}{jw}$$

$\boxed{\text{см. пр. 5}}$

$$S(w) = \frac{F\{s'(t)\}}{jw} =$$

$$= \frac{2j \cdot \mathrm{sinc}\left(\frac{w\tau_и}{2}\right)\sin(wt_0)t_0}{jwt_0} =$$

$$= \begin{cases} 2t_0 \,\mathrm{sinc}\left(\frac{w\tau_и}{2}\right)\sin(wt_0) & w \neq 0 \\ 2t_0 & w = 0 \end{cases}$$

Опр спектр. пл. сигнала

$$s(t) = \mathrm{sinc}(w_m t)$$

w_m - гр. макс. частота

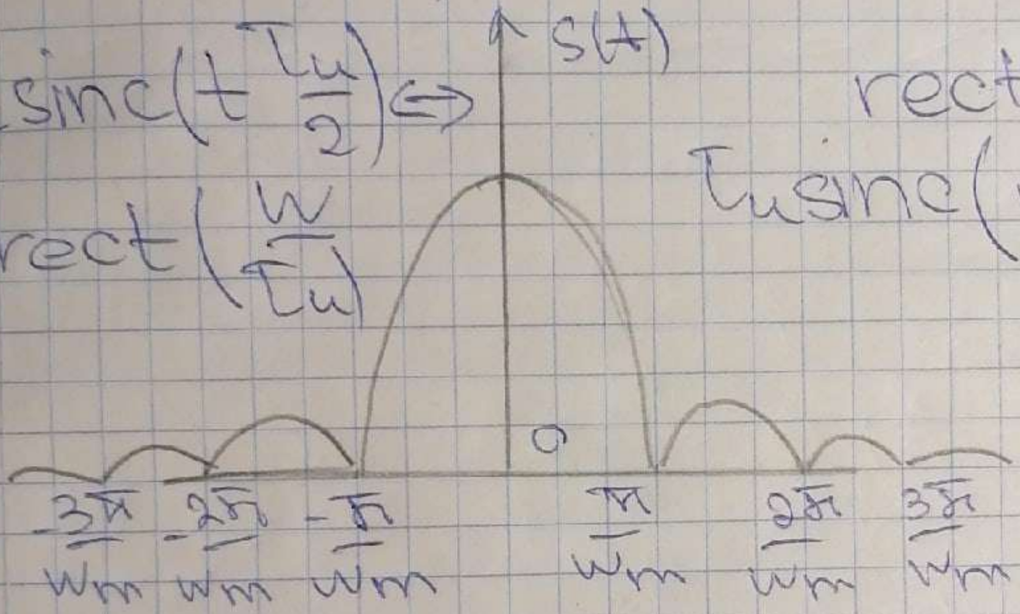
$$F\{S(w)\} = \int_{-\infty}^{+\infty} S(w)\, e^{-jwt}\, dw$$

$$F\{S(w)\} = 2\pi \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(w)\, e^{jw(-t)}\, dw =$$

$$= 2\pi S(-t)$$

$\tau_u \operatorname{sinc}\left(t\frac{\tau_u}{2}\right) \Leftrightarrow 2\pi \operatorname{rect}\left(\frac{w}{\tau_u}\right)$

$\operatorname{rect}\left(\frac{t}{\tau_u}\right) \Leftrightarrow \tau_u \operatorname{sinc}(w\tau_u/2)$



$w_m = \frac{\tau_u}{2}$ $\quad \tau_u = 2w_m$

$$\operatorname{sinc}(w_m t) \Leftrightarrow \frac{\pi}{w_m} \operatorname{rect}\left(\frac{w}{2w_m}\right)$$