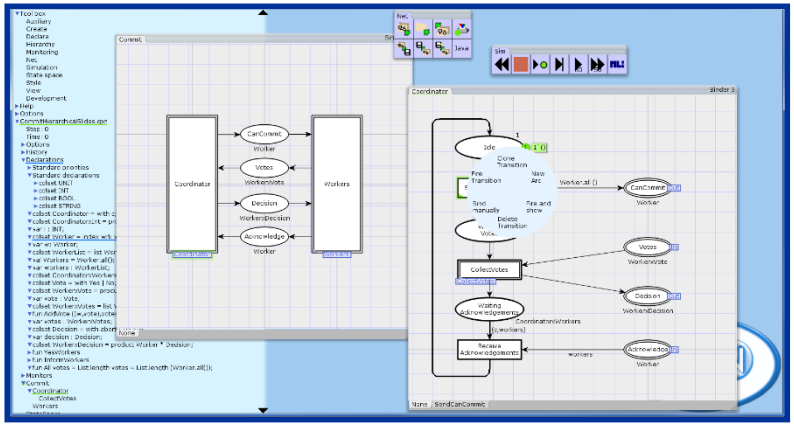
Lecture 3

Coloured Petri Nets

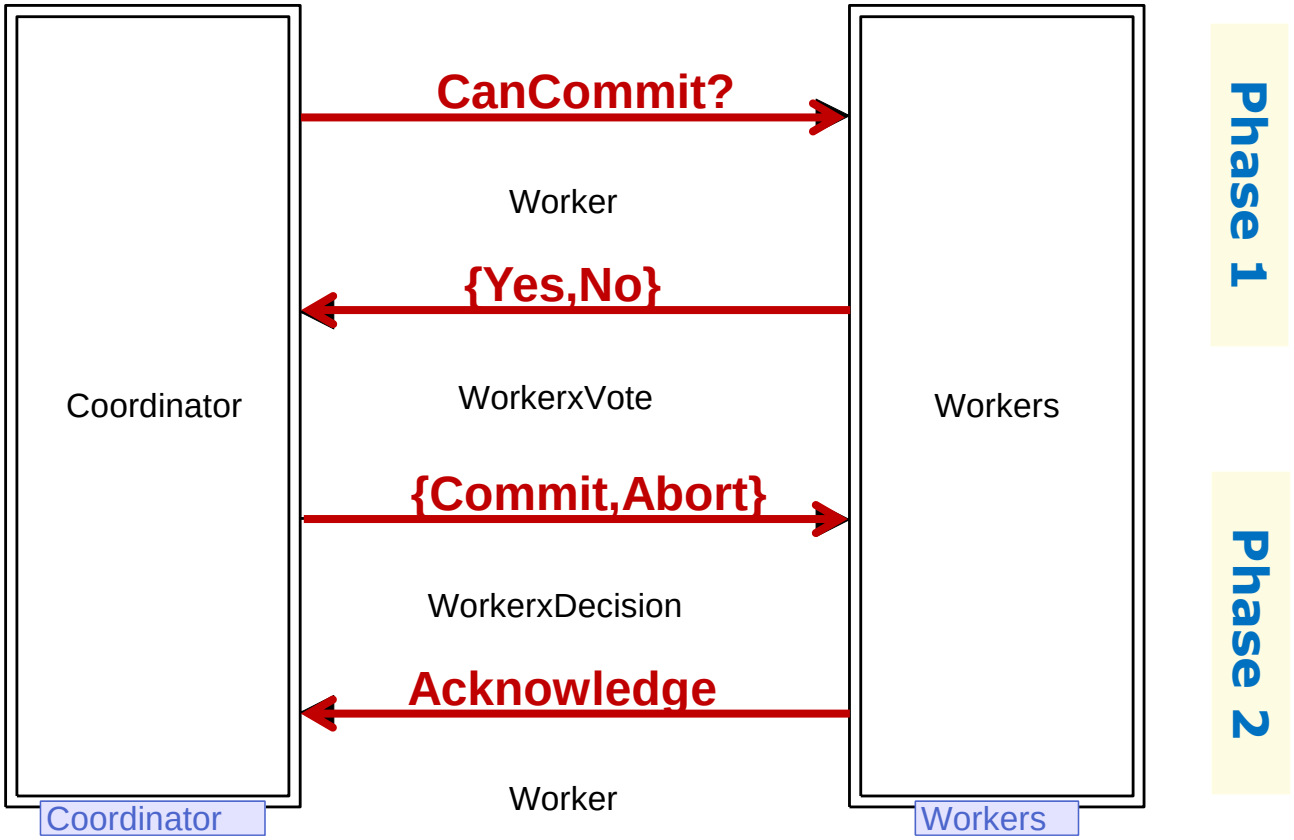


Lars M. Kristensen
Department of Computing, Mathematics, and Physics
Western Norway University of Applied Sciences
Email: lmkr@hvl.no / WWW: home.hib.no/ansatte/lmkr

Introduction

- **Address the practical shortcomings of PT-nets.**
- **Coloured Petri Nets (CPNs) = PT-nets + Standard ML programming language**
  - Places have a type (colour set) and tokens can carry data values
  - Transitions may have variables that can be bound to values
  - Arc expressions determine the tokens added/removed
  - Guard expressions may be used as an extra enabling condition
- **Standard ML = functional programming**
  - Computation proceeds by evaluation of expressions
  - Static typing with the type of expressions being inferred
  - Functions are first-order values and can be polymorphic
  - Recursion and lists are used to express iteration

Two-phase Commit Transaction Protocol



Colour Set Definitions

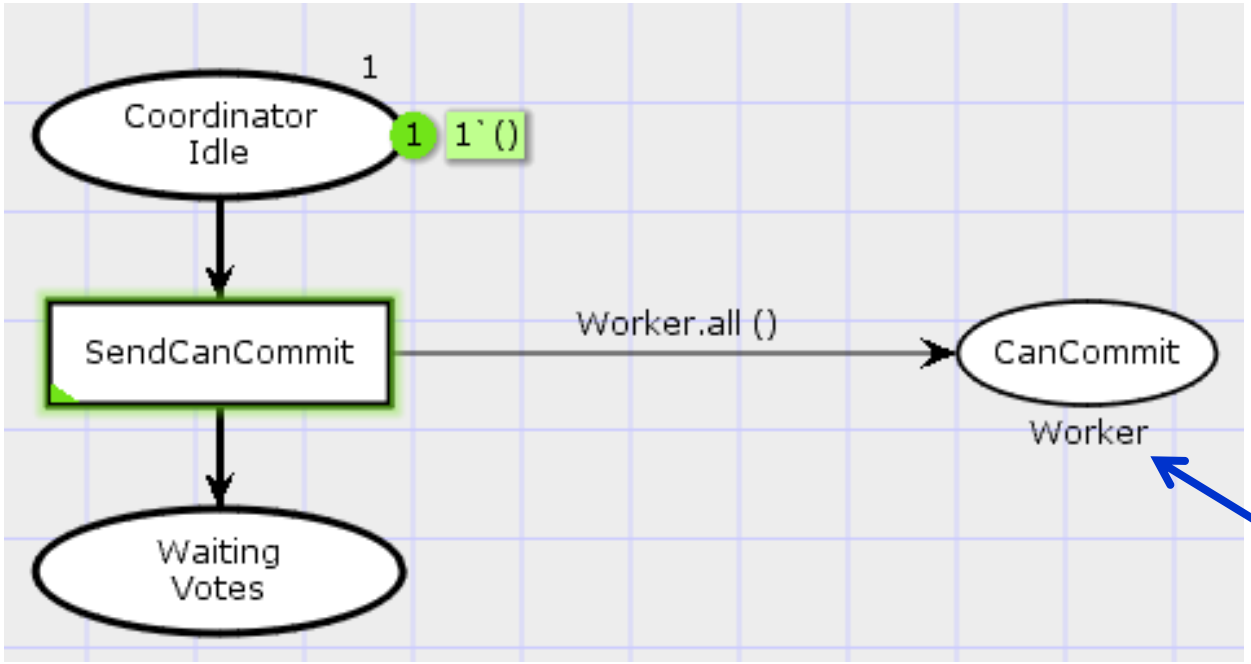
- **Determines the data types that can be used in the model**

Colour set definitions	Example values
`val W = 2;`	`wrk(1), wrk(2)`
`colset Worker = index wrk with 1..W;`	`Yes, No`
`colset Vote = with Yes | No;`	`(wrk(1),Yes)`
`colset WorkerxVote = product Worker * Vote;`	`Abort, Commit`
`colset Decision = with Abort | Commit;`	`(wrk(1),Commit)`
`colset WorkerxDecision = product Worker * Decision;`	

- **Additional colour set constructors for lists (`list`), records (`record`), and unions (`union`)**
- **Base data types: `UNIT`, `INT`, `STRING`, `BOOL`, `REAL`**

Coordinator – First Phase

- **The colour set (type) of a place the kinds of tokens that may reside on the place**



CoordinatorIdle and WaitingVotes have an implicit `UNIT` colour set

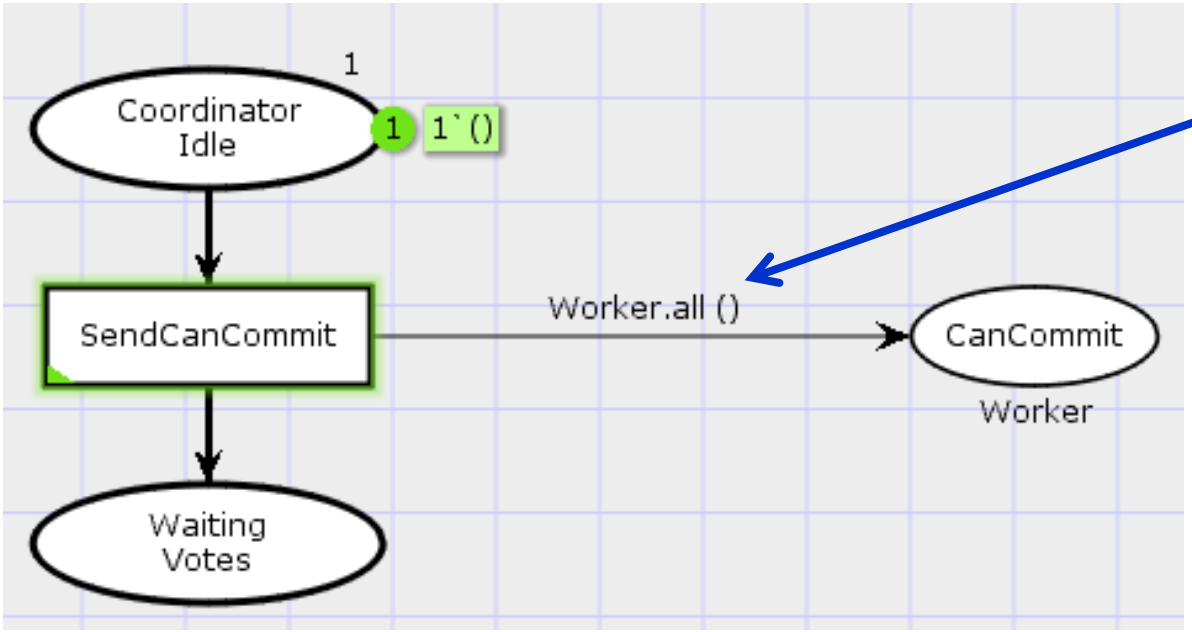
Tokens on CanCommit can have the values `wrk(1)` and `wrk(2)`

The colour set is by convention written below the place

`UNIT` is a datatype containing the single value `()` named unit

Arc Expressions

- **Determine the tokens that are removed/added from/to places when transitions occur**



Expression evaluating to all values in the Worker colour set

```
Worker.all ()
```

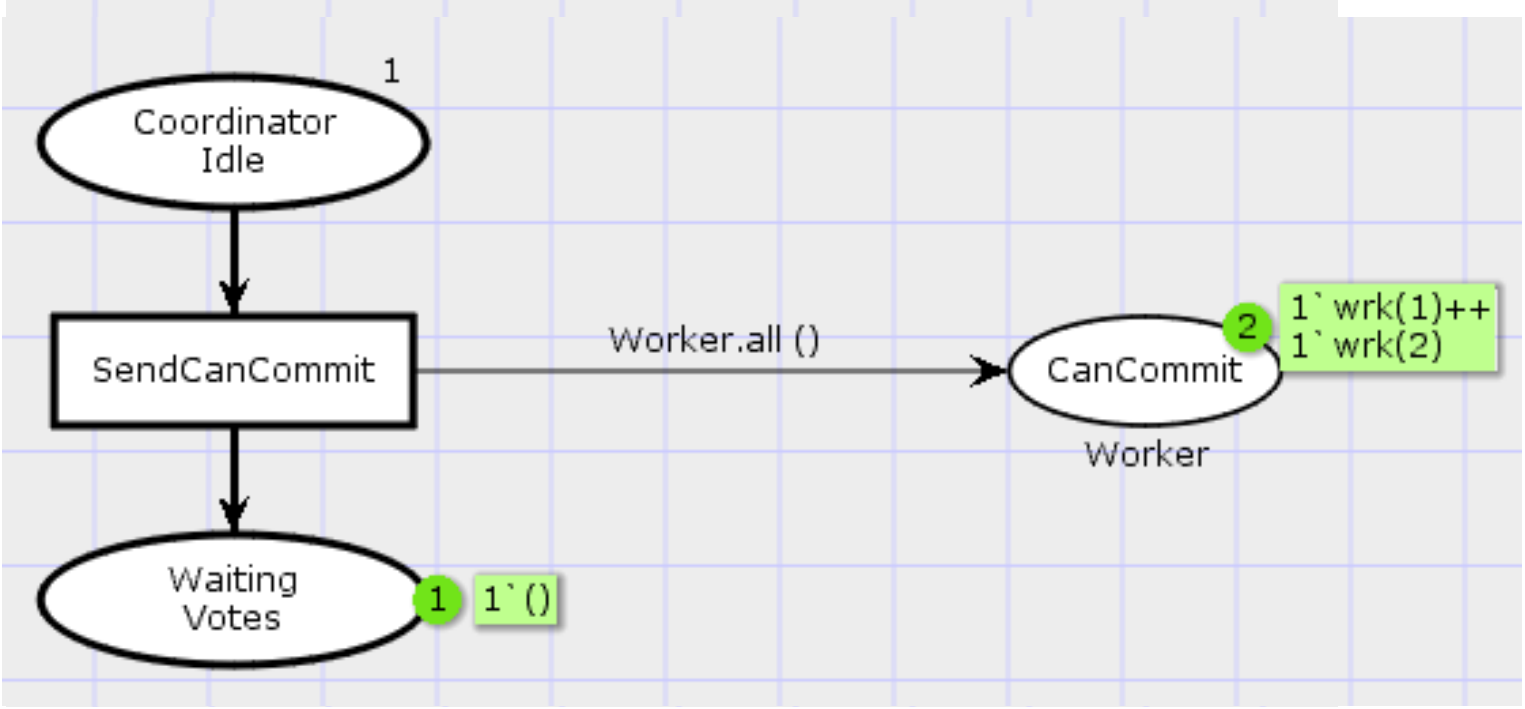
evaluate

```
[ wrk(1), wrk(2) ]
```

- **The type of an arc expression must match the colour set of the place connected to the arc.**

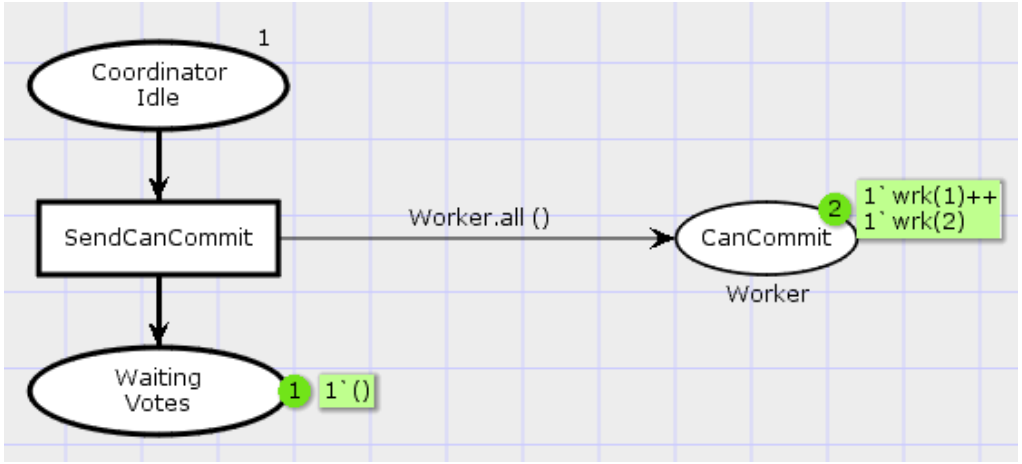
Evaluation of Expressions

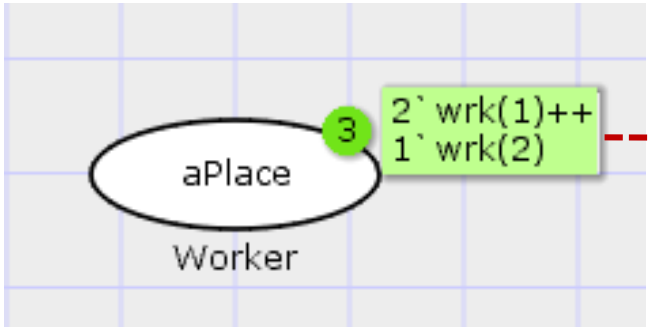
- **The tokens added and removed are determined by evaluating arc expressions**



Markings and Multi-sets

- **Each place may hold a multi-set of tokens over the colour set of the place**





Multi-set notation

coefficient («of»)

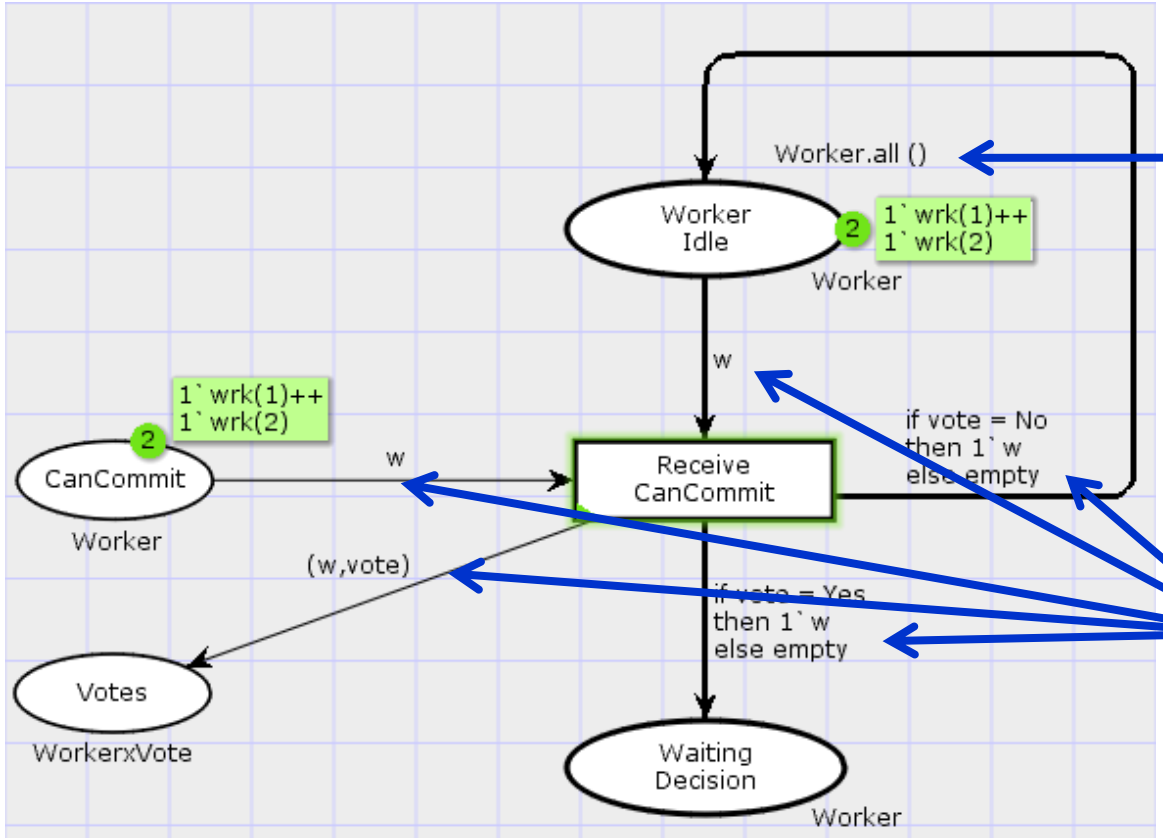
token colour (value)

```
2`wrk(1) ++
1`wrk(2)
```

union («and»)

Workers – First Phase

- **Consists of receiving a CanCommit message and sending a decision (Yes/No) to the coordinator**

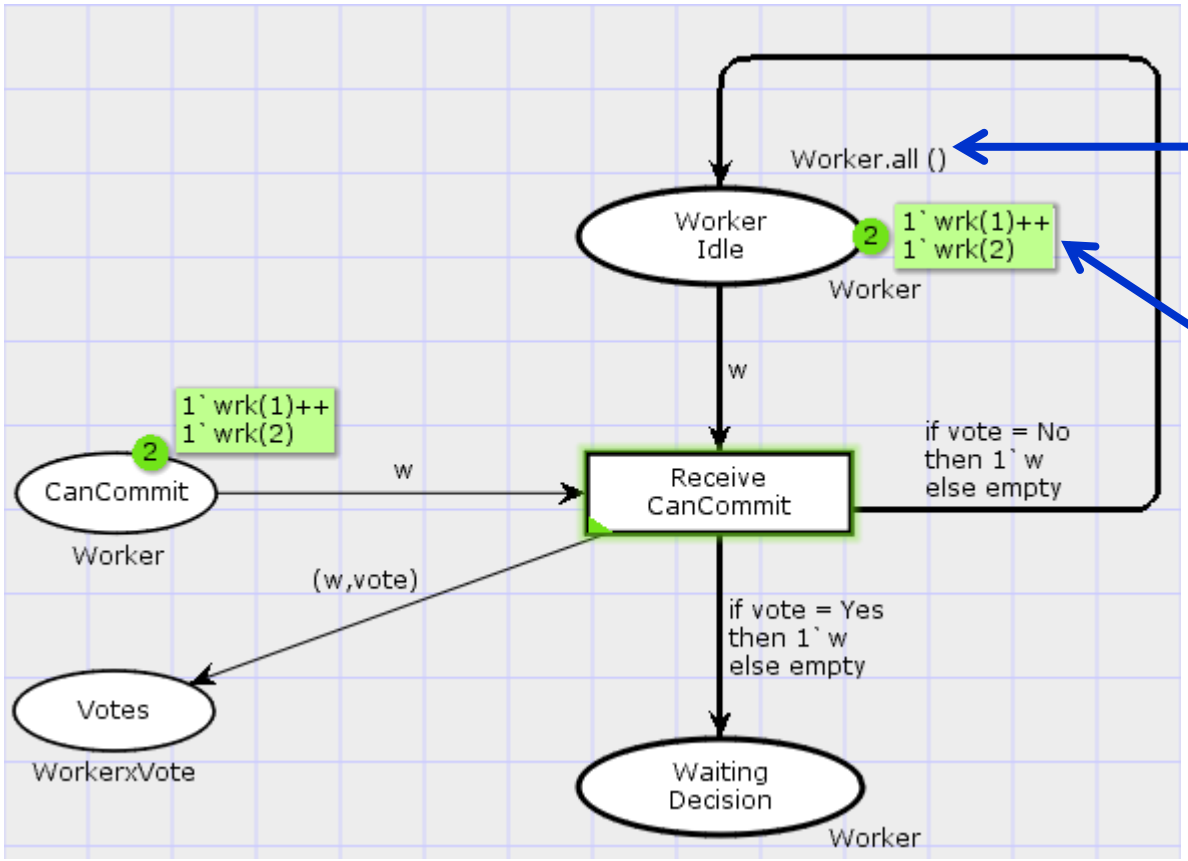


Initial marking expression

Arc expressions with free variables

Initial Marking

- **The initial marking (state) is obtained by evaluating the initial marking expressions**

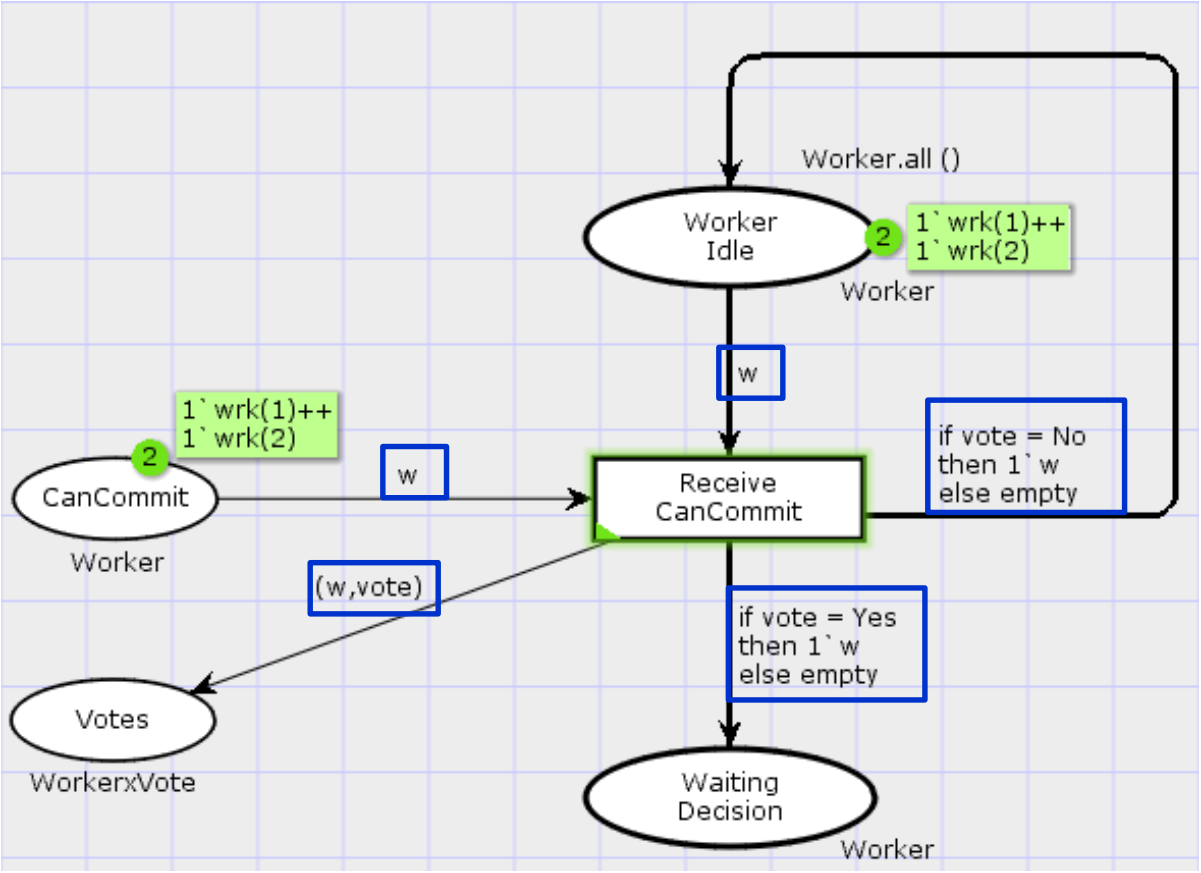


The initial marking is by convention written above the place

The two workers are initially Idle

Transition Variables

- **The arc expressions on the arcs of a transition may contain free variables**



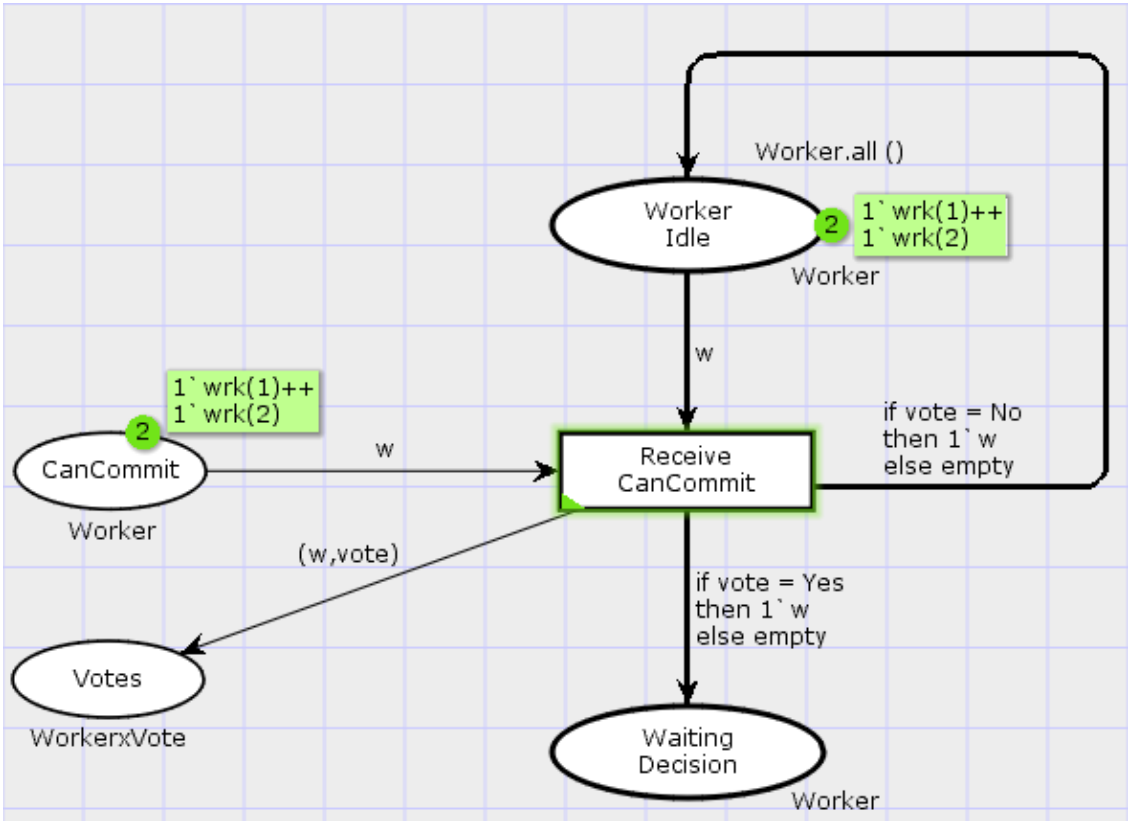
Variable declarations

```
val W = 2;
colset Worker =
        index wrk with 1..W;
var w : Worker;

colset Vote = with Yes | No;
var vote : Vote;
```

Arc expressions with free variables `vote` and `w`.

Transition Variables

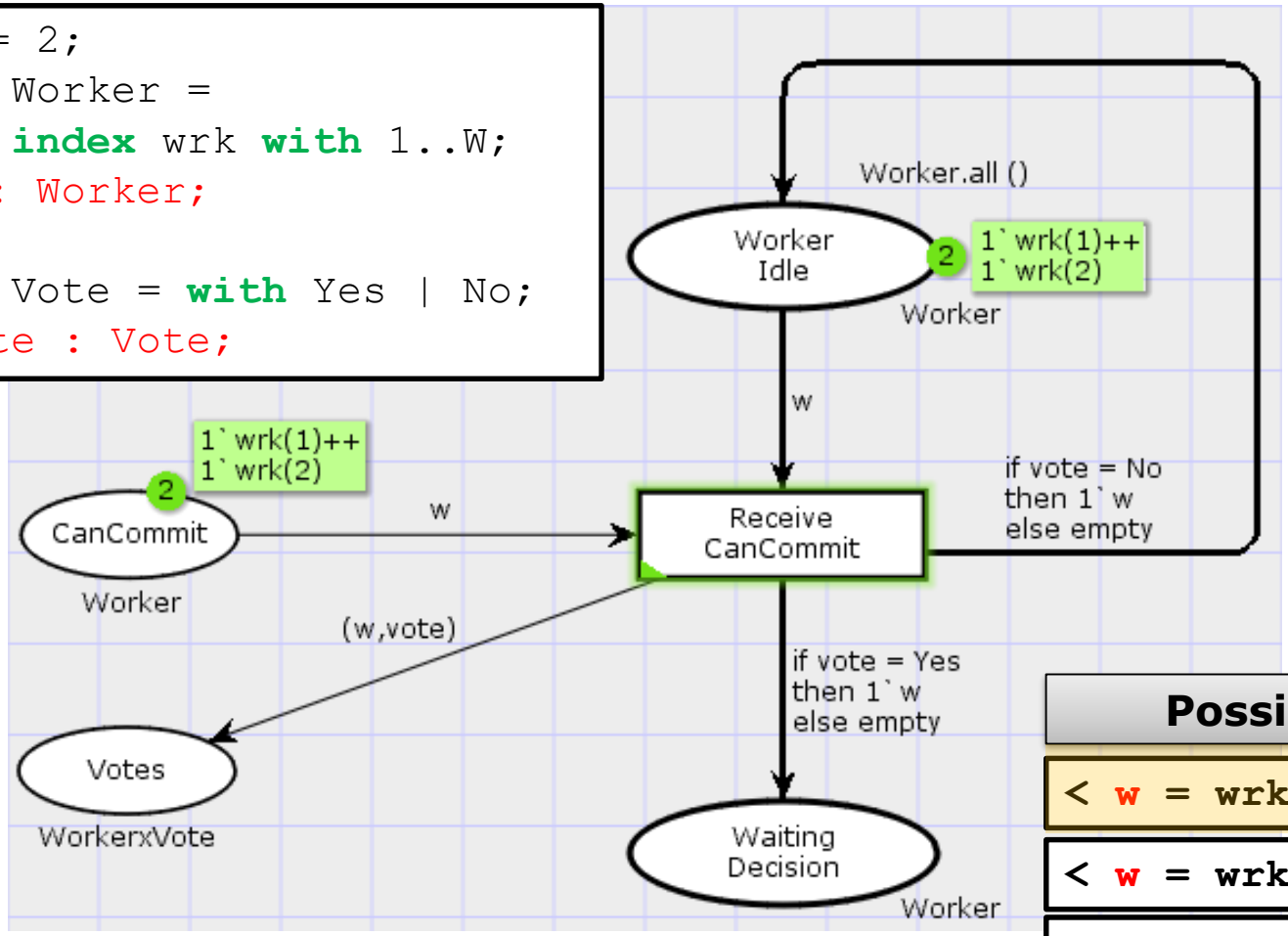
- **Transition ReceiveCanCommit has two free variables: vote and w**



- **Variables must be bound to values for a transition to be enabled and occur**
- **Similar to formal and actual parameters known from programming**
- **The association of values to variables is called a transition binding**
- **The bindings correspond to the possible enabling and occurrence modes of the transition**
- **Not all possible bindings will in general be enabled**

Transition Bindings

```
val W = 2;
colset Worker =
        index wrk with 1..W;
var w : Worker;

colset Vote = with Yes | No;
var vote : Vote;
```

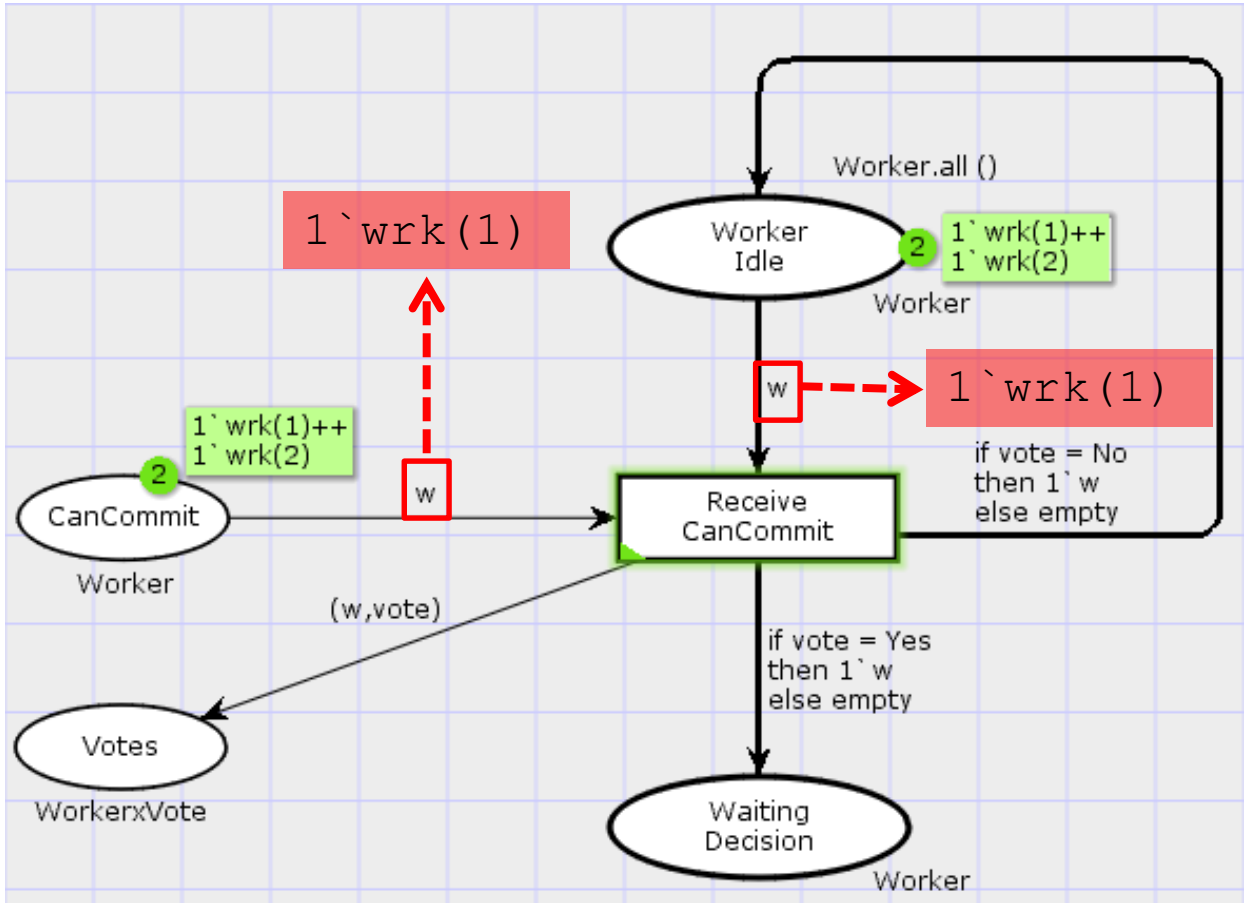


Possible bindings ?

- < w = wrk(1), vote = Yes >
- < w = wrk(1), vote = No >
- < w = wrk(2), vote = Yes >
- < w = wrk(2), vote = No >

Binding Enabling

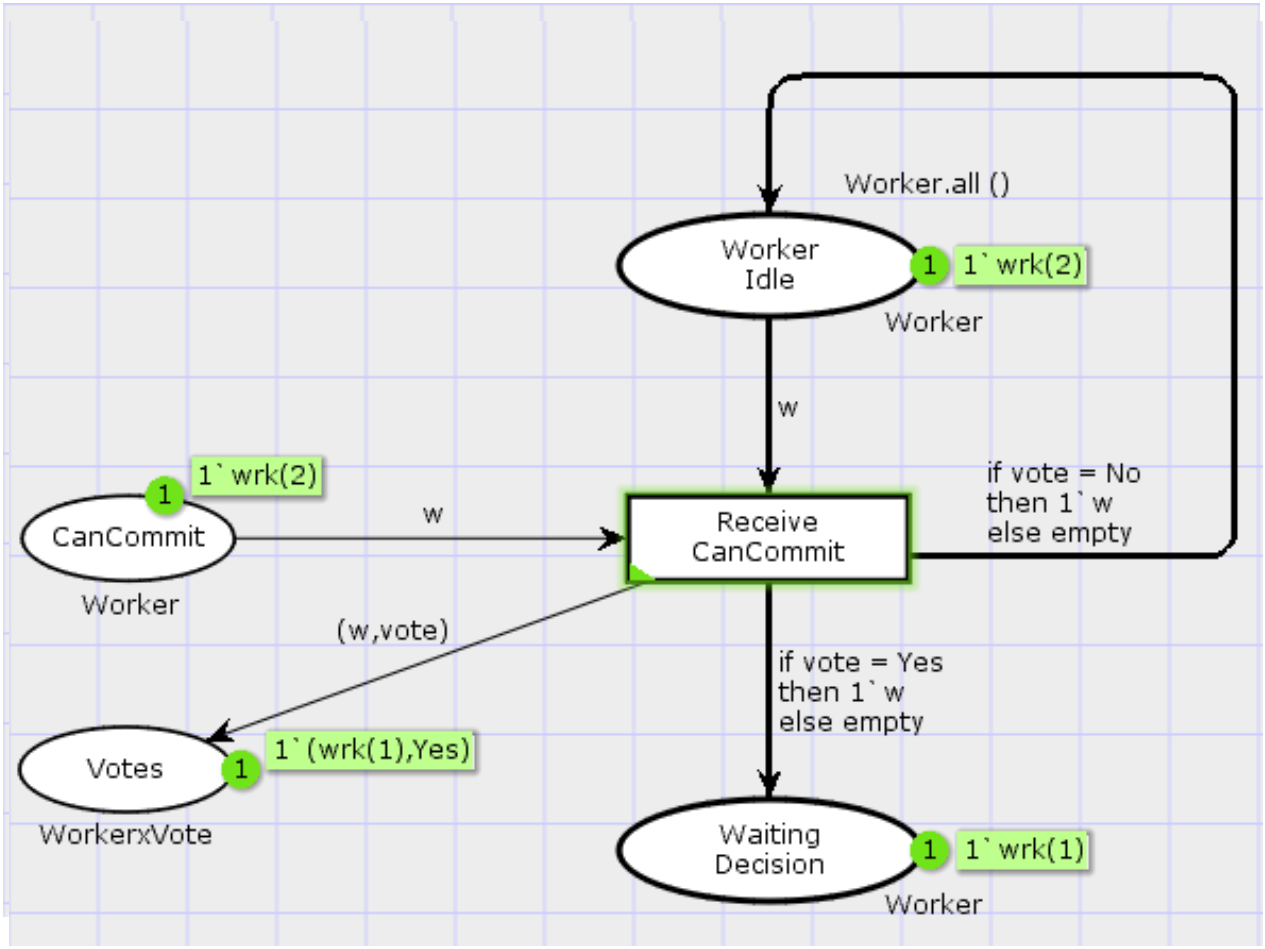
```
< w = wrk(1), vote = Yes >
```



- **A transition binding is enabled if there are sufficient tokens on each input place**
- **Tokens required on input places are determined by evaluating the input arc expressions in the binding under consideration**
- **Enabling condition: the multi-set of tokens obtained must be contained in the multi-set of tokens present on the corresponding input place**

Binding Occurrence

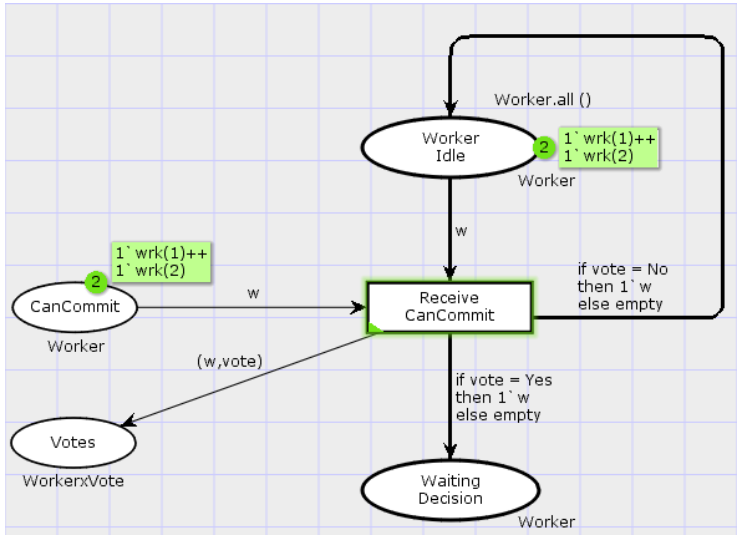
`< w = wrk(1),vote = Yes >`



- **An enabled transition binding may occur changing the current marking (state)**
- **Tokens removed from input places: determined by evaluating the input arc expression in the binding**
- **Tokens added to output places: determined by evaluating the output arc expressions in the binding**

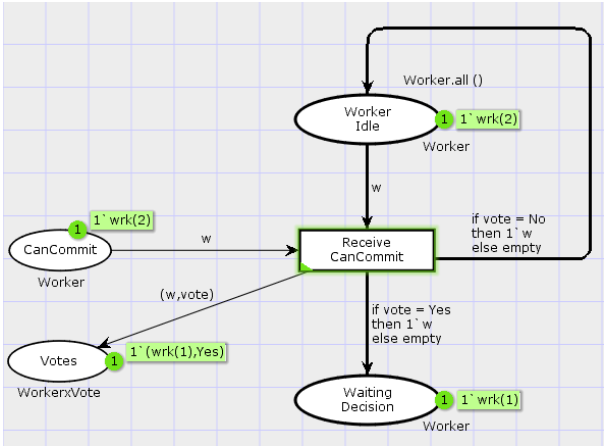
Binding Occurrence

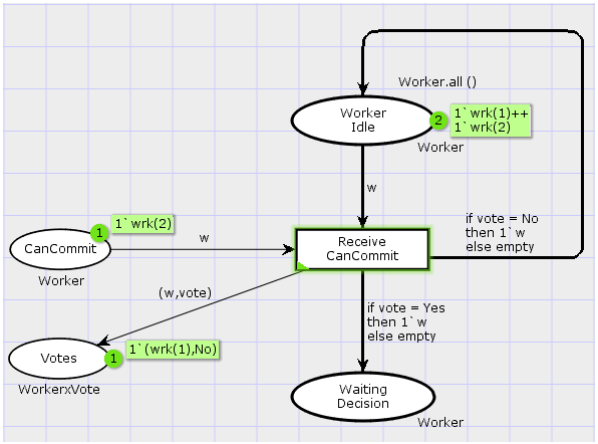
- **A transition may have several enabled bindings**



b_{1Y}

b_{1N}





Bindings

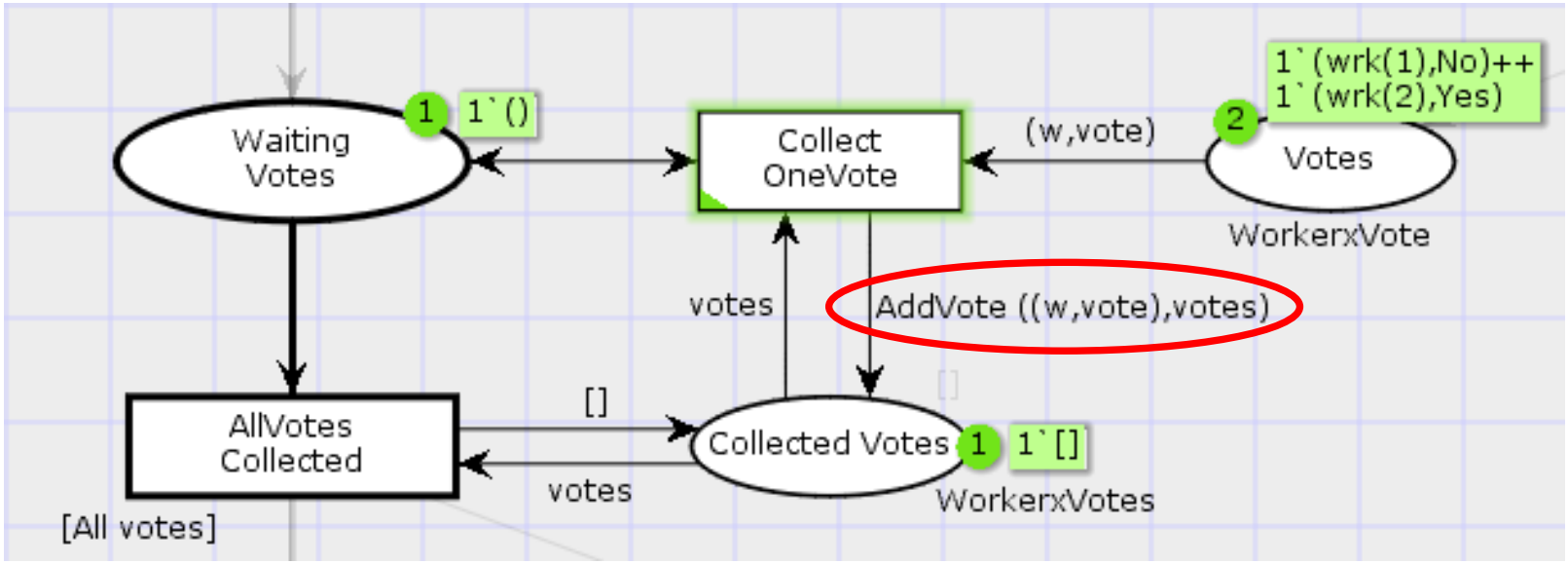
b_{1Y} = < w = wrk(1), vote = Yes >
b_{1N} = < w = wrk(1), vote = No >
b_{2Y} = < w = wrk(2), vote = Yes >
b_{2Y} = < w = wrk(2), vote = No >

CPN Tools Demo

- **Simulation of CPN models**
  - Interactive simulation with binding selection
  - Returning to the initial marking
  - Automatic simulation with visual feedback
  - Stop options and automatic simulation

Collecting Votes

- **Votes are collected one at a time and accumulated in a list-token on place Votes**




```
var w     : Worker;     var vote : Vote;
colset WorkerxVote  = product Worker * Vote;
colset WorkerxVotes = list WorkerxVote;

var votes : WorkerxVotes;
```

Functions

- **The function AddVote is used to add a vote from a worker to the list of collected votes**

```
fun AddVote ((w,vote),votes) = (w,vote)::votes
```

- **Example**

```
AddVote ((wrk(1),No),[])
```

```
(wrk(1),No)::[]
```

```
[(wrk(1),No)]
```

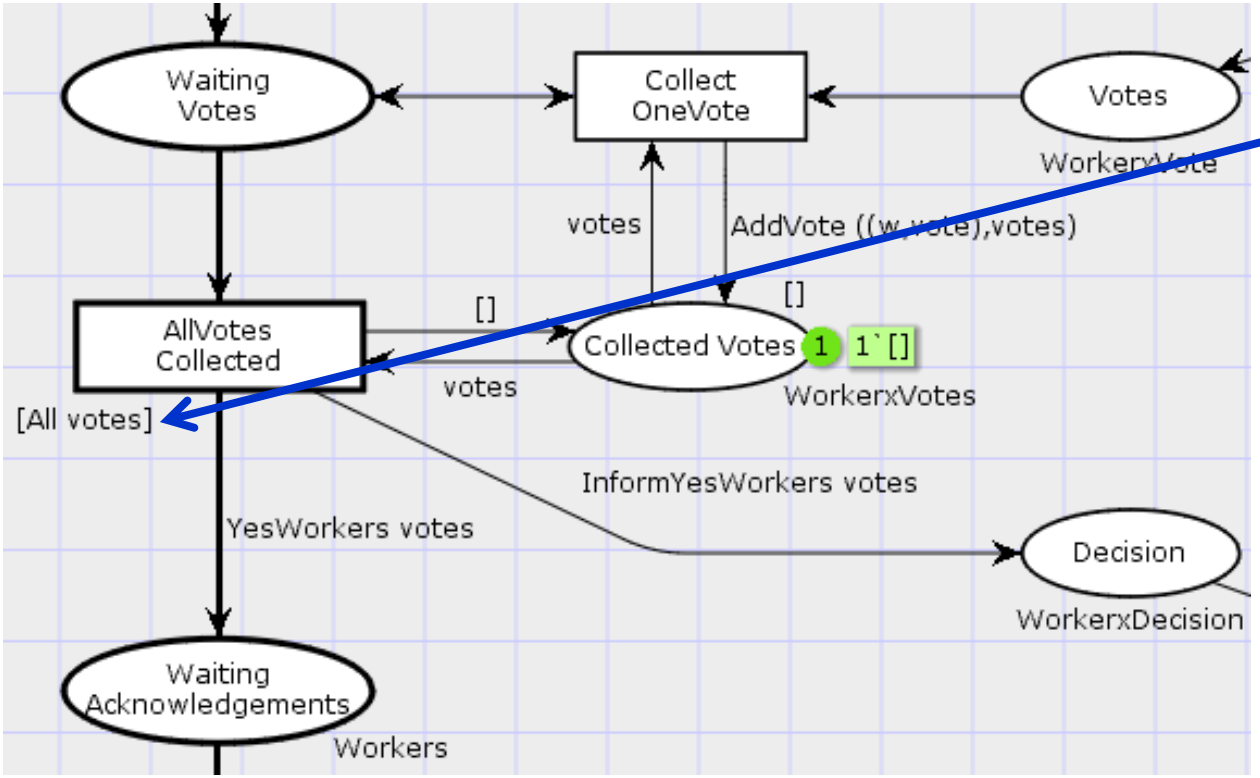
:: is the list-constructor in Standard ML

```
AddVote ((wrk(2),Yes),[(wrk(1),No)])
```

```
[(wrk(2),Yes),(wrk(1),No)]
```

Guard Expressions

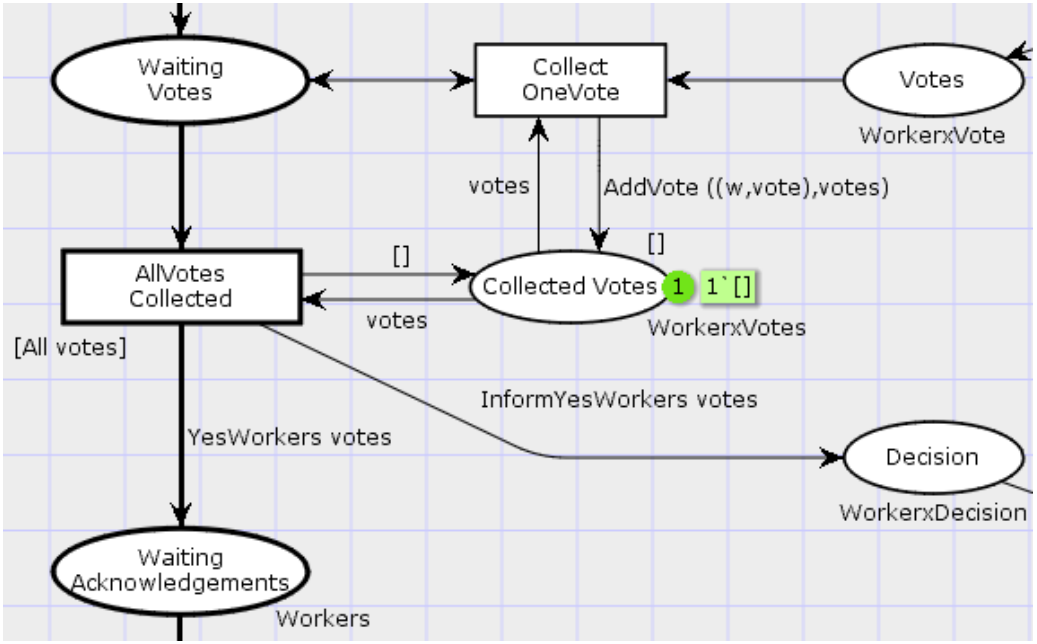
- **A transition may have a boolean guard expression which is extra enabling condition**



The guard is by convention written in square brackets next to the transition

Collecting All Votes

- **Transition AllVotesCollected should only be enabled when we have collected all votes**



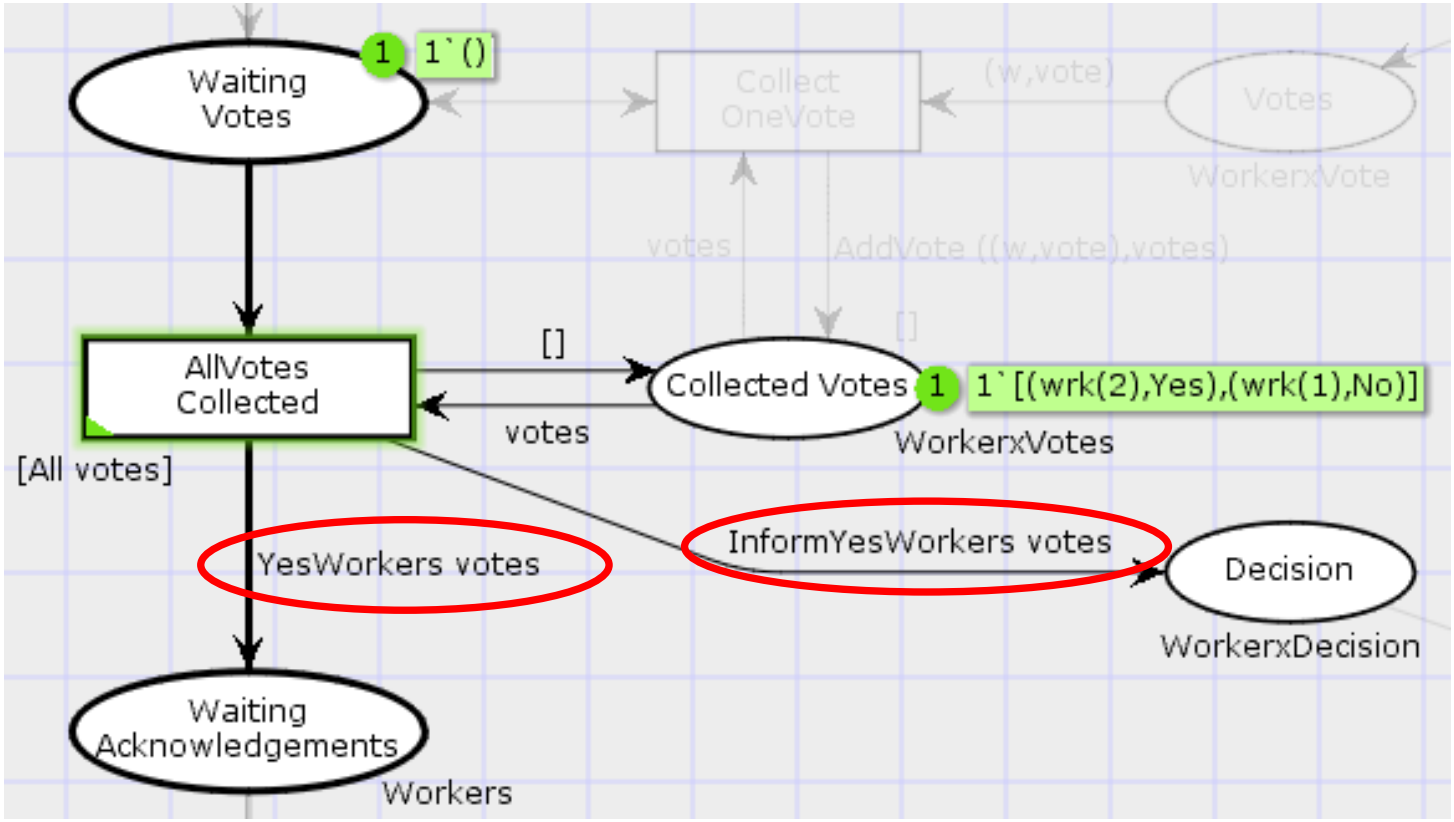

```
colset WorkerxVote =
   product Worker * Vote;

colset WorkerxVotes =
   list WorkerxVote;

var votes : WorkerxVotes
```

```
fun All votes =
(List.length votes = W)
```

`votes = [(wrk(2),Yes),(wrk(1),No)]`	`All votes`	--->	`true`
`votes = [(wrk(1),No)]`	`All votes`	--->	`false`

Informing Workers

- **Functions are also used to find the workers that needs to be informed about the decision**



YesWorkers Functions

- **Obtaining the list of Yes-votes (utility function)**

```
fun yesVotes votes = List.filter
                        (fn (w,vote) => vote = Yes)
                     votes
```

```
yesVotes [(wrk(1),No),(wrk(2),Yes)]
```

```
[(wrk(2),Yes)]
```

- **Getting workers that votes Yes (projection)**

```
fun YesWorkers votes = List.map
                         (fn (w,_) => w)
                       (yesVotes votes)
```

InformYesWorkers Functions

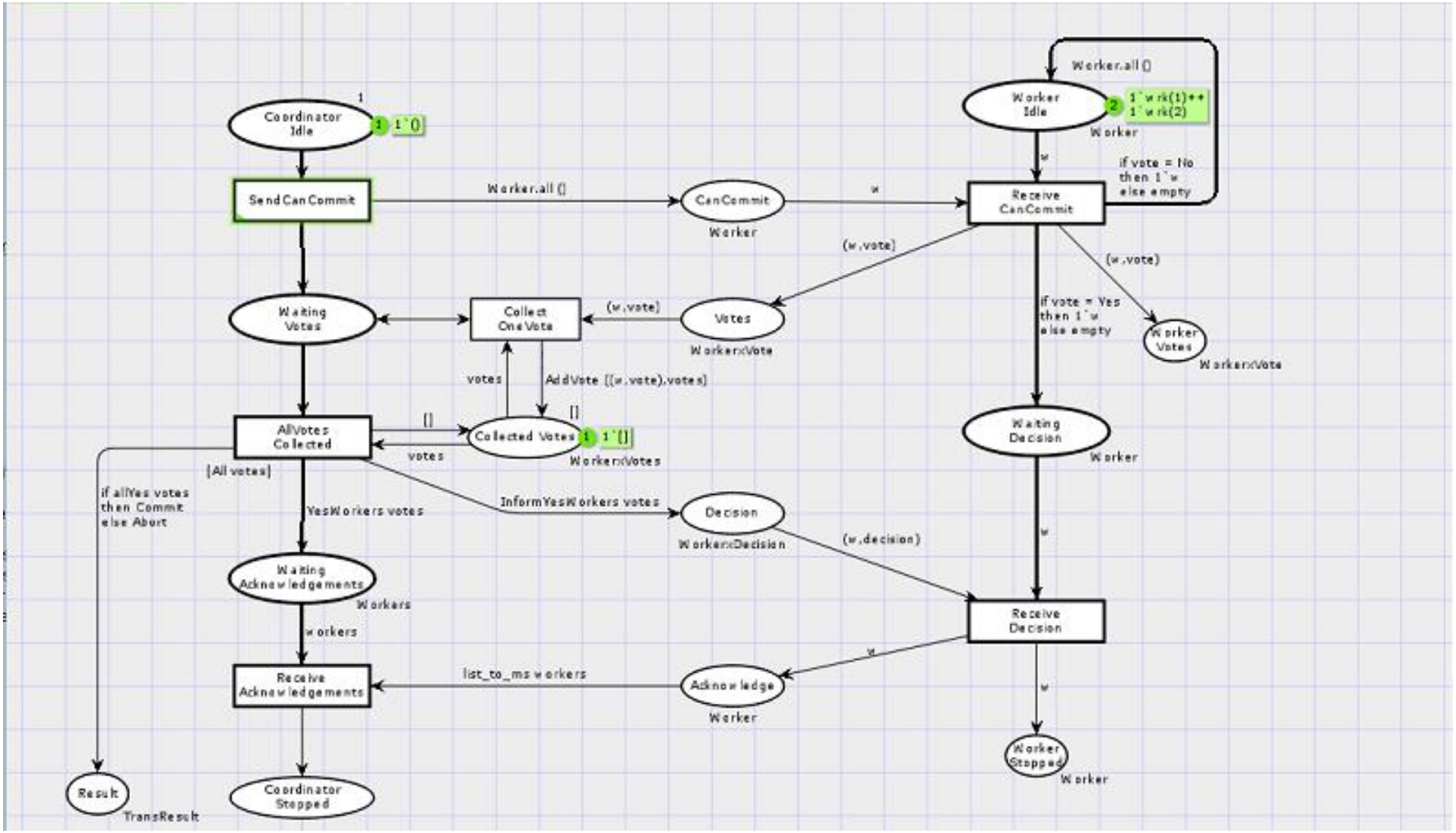
```
fun InformYesWorkers votes =
  let
    val yesworkers = YesWorkers votes
    val decision =
      (if (List.length yesworkers = W)
       then commit
       else abort)
  in
    List.map (fn w => (w,decision)) yesworkers
end
```

```
InformYesWorkers [(wrk(2),Yes)]
```

```
[(wrk(2),abort)]
```

CPN Tools Demo

- **Editing of CPN models**
  - Incremental syntax check of the model (dependencies)
  - Adding and deleting declarations
  - Editing inscriptions
    (arc expressions, colour sets, initial markings, guards)
  - Guidelines, graphical attributes, and groups


Coordinator Idle
1`()
SendCanCommit
Worker.all()
CanCommit
Worker
Receive CanCommit
Worker Idle
1`wrk(1)++ 1`wrk(2)
if vote = No then 1`w else empty
(w,vote)
Waiting Votes
Collect One Vote
Votes
Worker*Vote
if vote = Yes then 1`w else empty
Worker Votes
votes
AddVote ((w,vote).votes)
AllVotes Collected
[All votes]
Collected Votes
Worker*Votes
Waiting Decision
if allYes votes then Commit else Abort
YesWorkers votes
InformYesWorkers votes
Decision
Worker*Decision
(w,decision)
Waiting Acknowledgements
Workers
Receive Decision
Receive Acknowledgements
list_to_ms workers
Acknowledge
Worker Stopped
Result
TransResult
Coordinator Stopped

Summary

- **Coloured Petri Nets extends Place/Transition Nets with a functional programming language.**
- **Syntactical concepts**
  - Colour sets defines the data types available for modelling
  - Declaration of variables over the colour sets of the model
  - Places have a colour set determining the kind of tokens a place may contain
  - Arc expressions, initial marking- and guard expressions
- **Semantical concepts**
  - The marking of a place is a multi-set of tokens (values)
  - A binding gives values to the variables of a transition – scope of a variable is the surrounding arc expressions of the transition
  - Evaluation of arc expressions and guards in bindings determine enabling and the tokens removed/added by transitions