The purpose of this simulation study is to examine different ways of evaluating an ODTR. Specifically, we examine how different estimators (unadjusted, g-comp, IPTW, IPTW-DR, TMLE, CV-TMLE) do with approximating the statistical parameter $E_0[Y_{d_0}]$ and the data-adaptive parameter $E_0[Y_{d_n}]$, under different SL ODTR estimators (ie different SL libraries, varying in "aggressiveness"). Here, d_0 is true optimal rule, d_n is estimate of optimal rule.

1 Description of DGP

$$W_1, W_2, W_3, W_4 \sim Normal(\mu = 0, \sigma^2 = 1)$$
$$A \sim Bernoulli(p = 0.5)$$
$$Y \sim Bernoulli(p) .$$

$p = 0.5 * logit^{-1}(1 - W_1^2 + 3W_2 + 5W_3^2 A - 4.45A) + 0.5 logit^{-1}(-0.5 - W_3 + 2W_1 W_2 + 3|W2|A - 1.5A)$,

True blip function is:

$B_0(W) = 0.5[logit^{-1}(1 - W_1^2 + 3W_2 + 5W_3^2 - 4.45) + logit^{-1}(-0.5 - W_3 + 2W_1 W_2 + 3|W2| - 1.5)$
$\qquad - logit^{-1}(1 - W_1^2 + 3W_2) + logit^{-1}(-0.5 - W_3 + 2W_1 W_2)]$.

2 Library legend

- Incorrect GLM

    - QAW.SL.library = linear model main terms W and A and interaction with W and A
    - blip.SL.library = linear model with main terms W

- GLMs

    - QAW.SL.library = linear model with W_j and A as main terms and W_j*A interaction for each j
    - blip.SL.library = linear model with main terms W_j for each j

- ML + GLMs not aggressive

    - QAW.SL.library = GLMs library AND SL.glm, SL.mean, SL.glm.interaction, SL.earth, SL.nnet, SL.svm, SL.rpart
    - blip.SL.library = GLMs library AND SL.glm, SL.mean, SL.glm.interaction, SL.earth, SL.nnet, SL.svm, SL.rpart

- ML + GLMs not aggressive

    - QAW.SL.library = ML + GLMs aggressive library AND SL.randomForest
    - blip.SL.library = ML + GLMs aggressive library AND SL.randomForest

3 Results

GLMs – leas   ML + GLMs   ML + GLMs

Estimator   Estimator   Estimator
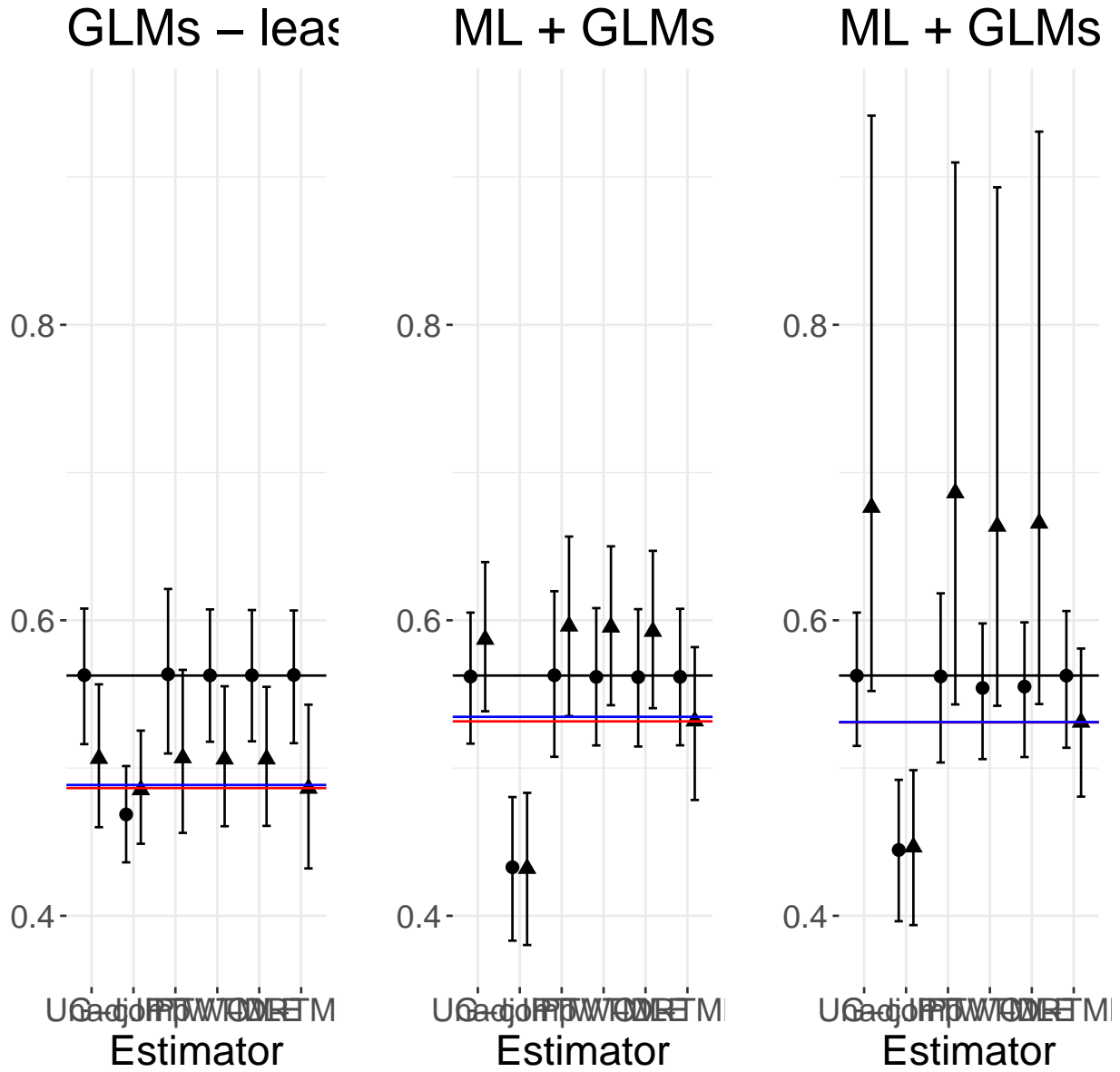
e: ● True ODTR ▲ Estimated ODTR   Estimand: — $\Psi_{d_0^*}$ — Ψ_c

```
## pdf
##    2
```

```
make_table_EYdopt(EYdopt = EYdoptbin_glms, truevalues = DGP_bin_complex_true_values)
```

```
##                  Bias Variance     MSE Coverage          Estimator
## unadj         -0.0562    7e-04  0.0038    31.8%              unadj
## gcomp         -0.0773    4e-04  0.0064        -              gcomp
## IPTW          -0.0558    8e-04  0.0039      45%               IPTW
## IPTW_DR       -0.0565    6e-04  0.0038    30.1%            IPTW_DR
## TMLE          -0.0565    6e-04  0.0038    29.8%               TMLE
## LTMLE              NA       NA      NA        -              LTMLE
```

```
## CV.TMLE            -0.0764    9e-04 0.0067    14.7%        CV.TMLE
## unadj_dopt0         0.0003    5e-04 0.0005    93.3%     unadj_dopt0
## gcomp_dopt0        -0.0940    3e-04 0.0091       -      gcomp_dopt0
## IPTW_dopt0          0.0009    8e-04 0.0008    95.3%      IPTW_dopt0
## IPTW_DR_dopt0       0.0001    5e-04 0.0005    93.7%   IPTW_DR_dopt0
## TMLE_dopt0          0.0002    5e-04 0.0005    93.7%      TMLE_dopt0
## LTMLE_dopt0             NA       NA    NA       -       LTMLE_dopt0
## CV.TMLE_dopt0       0.0004    5e-04 0.0005    93.7%   CV.TMLE_dopt0
## unadj_sampspec      0.0179    7e-04 0.0007    90.7%  unadj_sampspec
## gcomp_sampspec     -0.0033    4e-04 0.0006       -   gcomp_sampspec
## IPTW_sampspec       0.0183    8e-04 0.0009    94.3%   IPTW_sampspec
## IPTW_DR_sampspec    0.0175    6e-04 0.0007    90.6% IPTW_DR_sampspec
## LTMLE_sampspec          NA       NA    NA       -    LTMLE_sampspec
## TMLE_sampspec       0.0175    6e-04 0.0007    90.7%   TMLE_sampspec
## CV.TMLE_sampspec   -0.0002    9e-04 0.0005    94.3% CV.TMLE_sampspec
```

make_table_EYdopt(EYdopt = EYdoptbin_MLnotaggglms, truevalues = DGP_bin_complex_true_values)

```
##                      Bias Variance    MSE Coverage        Estimator
## unadj               0.0245    7e-04 0.0013    75.1%            unadj
## gcomp              -0.1306    7e-04 0.0178       -             gcomp
## IPTW                0.0334    1e-03 0.0021    76.1%             IPTW
## IPTW_DR             0.0327    8e-04 0.0019    66.5%          IPTW_DR
## TMLE                0.0298    8e-04 0.0016    71.3%             TMLE
## LTMLE                   NA       NA    NA       -             LTMLE
## CV.TMLE            -0.0308    7e-04 0.0017      69%          CV.TMLE
## unadj_dopt0        -0.0007    5e-04 0.0005    94.7%      unadj_dopt0
## gcomp_dopt0        -0.1298    6e-04 0.0175       -       gcomp_dopt0
## IPTW_dopt0          0.0002    8e-04 0.0008    94.7%       IPTW_dopt0
## IPTW_DR_dopt0      -0.0009    6e-04 0.0006      94%    IPTW_DR_dopt0
## TMLE_dopt0         -0.0011    5e-04 0.0005    93.6%       TMLE_dopt0
## LTMLE_dopt0             NA       NA    NA       -        LTMLE_dopt0
## CV.TMLE_dopt0      -0.0009    5e-04 0.0005    93.2%    CV.TMLE_dopt0
## unadj_sampspec      0.0524    7e-04 0.0033    36.2%   unadj_sampspec
## gcomp_sampspec     -0.1027    7e-04 0.0114       -    gcomp_sampspec
## IPTW_sampspec       0.0614    1e-03 0.0046    43.8%    IPTW_sampspec
## IPTW_DR_sampspec    0.0607    8e-04 0.0044    28.9% IPTW_DR_sampspec
## LTMLE_sampspec          NA       NA    NA       -     LTMLE_sampspec
## TMLE_sampspec       0.0578    8e-04 0.0040    30.4%    TMLE_sampspec
## CV.TMLE_sampspec    0.0002    7e-04 0.0005      94% CV.TMLE_sampspec
```

make_table_EYdopt(EYdopt = EYdoptbin_MLaggglms, truevalues = DGP_bin_complex_true_values)

```
##                      Bias Variance    MSE Coverage        Estimator
## unadj               0.1138   0.0113 0.0243    29.9%            unadj
## gcomp              -0.1161   0.0007 0.0142       -             gcomp
## IPTW                0.1236   0.0109 0.0262      31%             IPTW
## IPTW_DR             0.1010   0.0092 0.0194      33%          IPTW_DR
## TMLE                0.1031   0.0108 0.0214    33.6%             TMLE
## LTMLE                   NA       NA    NA       -             LTMLE
## CV.TMLE            -0.0316   0.0007 0.0017    68.6%          CV.TMLE
```

```
## unadj_dopt0      -0.0001   0.0005 0.0005    93.7%      unadj_dopt0
## gcomp_dopt0      -0.1180   0.0006 0.0146       -       gcomp_dopt0
## IPTW_dopt0       -0.0006   0.0009 0.0009      94%       IPTW_dopt0
## IPTW_DR_dopt0    -0.0084   0.0005 0.0006    90.1%    IPTW_DR_dopt0
## TMLE_dopt0       -0.0075   0.0005 0.0006    90.6%       TMLE_dopt0
## LTMLE_dopt0          NA       NA     NA       -      LTMLE_dopt0
## CV.TMLE_dopt0    -0.0001   0.0005 0.0005    93.6%    CV.TMLE_dopt0
## unadj_sampspec    0.1453   0.0113 0.0342    12.3%   unadj_sampspec
## gcomp_sampspec   -0.0846   0.0007 0.0081       -    gcomp_sampspec
## IPTW_sampspec     0.1551   0.0109 0.0366    16.3%    IPTW_sampspec
## IPTW_DR_sampspec  0.1325   0.0092 0.0283    15.8% IPTW_DR_sampspec
## LTMLE_sampspec       NA       NA     NA       -    LTMLE_sampspec
## TMLE_sampspec     0.1346   0.0108 0.0307    15.7%    TMLE_sampspec
## CV.TMLE_sampspec  0.0001   0.0007 0.0005    94.8% CV.TMLE_sampspec
```

```
##                          Comparison                          Library Estimator    Bias
## unadj_sampspec    EnYdn for E0Ydn                               GLMs     Unadj.  0.0179
## gcomp_sampspec    EnYdn for E0Ydn                               GLMs    G-comp. -0.0033
## IPTW_sampspec     EnYdn for E0Ydn                               GLMs       IPTW  0.0183
## IPTW_DR_sampspec  EnYdn for E0Ydn                               GLMs    IPTW-DR  0.0175
## TMLE_sampspec     EnYdn for E0Ydn                               GLMs       TMLE  0.0175
## CV.TMLE_sampspec  EnYdn for E0Ydn                               GLMs    CV-TMLE -0.0002
## unadj_sampspec1   EnYdn for E0Ydn ML + GLMs not aggressive      Unadj.  0.0524
## gcomp_sampspec1   EnYdn for E0Ydn ML + GLMs not aggressive      G-comp. -0.1027
## IPTW_sampspec1    EnYdn for E0Ydn ML + GLMs not aggressive         IPTW  0.0614
## IPTW_DR_sampspec1 EnYdn for E0Ydn ML + GLMs not aggressive      IPTW-DR  0.0607
## TMLE_sampspec1    EnYdn for E0Ydn ML + GLMs not aggressive         TMLE  0.0578
## CV.TMLE_sampspec1 EnYdn for E0Ydn ML + GLMs not aggressive      CV-TMLE  0.0002
## unadj_sampspec2   EnYdn for E0Ydn     ML + GLMs aggressive      Unadj.  0.1453
## gcomp_sampspec2   EnYdn for E0Ydn     ML + GLMs aggressive      G-comp. -0.0846
## IPTW_sampspec2    EnYdn for E0Ydn     ML + GLMs aggressive         IPTW  0.1551
## IPTW_DR_sampspec2 EnYdn for E0Ydn     ML + GLMs aggressive      IPTW-DR  0.1325
## TMLE_sampspec2    EnYdn for E0Ydn     ML + GLMs aggressive         TMLE  0.1346
## CV.TMLE_sampspec2 EnYdn for E0Ydn     ML + GLMs aggressive      CV-TMLE  0.0001
##                   Variance    MSE Coverage
## unadj_sampspec      0.0007 0.0007    90.7%
## gcomp_sampspec      0.0004 0.0006       -
## IPTW_sampspec       0.0008 0.0009    94.3%
## IPTW_DR_sampspec    0.0006 0.0007    90.6%
## TMLE_sampspec       0.0006 0.0007    90.7%
## CV.TMLE_sampspec    0.0009 0.0005    94.3%
## unadj_sampspec1     0.0007 0.0033    36.2%
## gcomp_sampspec1     0.0007 0.0114       -
## IPTW_sampspec1      0.0010 0.0046    43.8%
## IPTW_DR_sampspec1   0.0008 0.0044    28.9%
## TMLE_sampspec1      0.0008 0.0040    30.4%
## CV.TMLE_sampspec1   0.0007 0.0005      94%
## unadj_sampspec2     0.0113 0.0342    12.3%
## gcomp_sampspec2     0.0007 0.0081       -
## IPTW_sampspec2      0.0109 0.0366    16.3%
```

```
## IPTW_DR_sampspec2   0.0092 0.0283    15.8%
## TMLE_sampspec2       0.0108 0.0307    15.7%
## CV.TMLE_sampspec2    0.0007 0.0005    94.8%
##                       Comparison                      Library Estimator     Bias
## unadj_dopt0    EnYd0 for E0Yd0                            GLMs     Unadj.   0.0003
## gcomp_dopt0    EnYd0 for E0Yd0                            GLMs    G-comp.  -0.0940
## IPTW_dopt0     EnYd0 for E0Yd0                            GLMs       IPTW   0.0009
## IPTW_DR_dopt0  EnYd0 for E0Yd0                            GLMs    IPTW-DR   0.0001
## TMLE_dopt0     EnYd0 for E0Yd0                            GLMs       TMLE   0.0002
## CV.TMLE_dopt0  EnYd0 for E0Yd0                            GLMs    CV-TMLE   0.0004
## unadj_dopt01   EnYd0 for E0Yd0 ML + GLMs not aggressive   Unadj.  -0.0007
## gcomp_dopt01   EnYd0 for E0Yd0 ML + GLMs not aggressive   G-comp. -0.1298
## IPTW_dopt01    EnYd0 for E0Yd0 ML + GLMs not aggressive     IPTW   0.0002
## IPTW_DR_dopt01 EnYd0 for E0Yd0 ML + GLMs not aggressive   IPTW-DR -0.0009
## TMLE_dopt01    EnYd0 for E0Yd0 ML + GLMs not aggressive      TMLE -0.0011
## CV.TMLE_dopt01 EnYd0 for E0Yd0 ML + GLMs not aggressive   CV-TMLE -0.0009
## unadj_dopt02   EnYd0 for E0Yd0     ML + GLMs aggressive    Unadj. -0.0001
## gcomp_dopt02   EnYd0 for E0Yd0     ML + GLMs aggressive   G-comp. -0.1180
## IPTW_dopt02    EnYd0 for E0Yd0     ML + GLMs aggressive      IPTW -0.0006
## IPTW_DR_dopt02 EnYd0 for E0Yd0     ML + GLMs aggressive   IPTW-DR -0.0084
## TMLE_dopt02    EnYd0 for E0Yd0     ML + GLMs aggressive      TMLE -0.0075
## CV.TMLE_dopt02 EnYd0 for E0Yd0     ML + GLMs aggressive   CV-TMLE -0.0001
##              Variance    MSE Coverage
## unadj_dopt0     5e-04 0.0005    93.3%
## gcomp_dopt0     3e-04 0.0091       -
## IPTW_dopt0      8e-04 0.0008    95.3%
## IPTW_DR_dopt0   5e-04 0.0005    93.7%
## TMLE_dopt0      5e-04 0.0005    93.7%
## CV.TMLE_dopt0   5e-04 0.0005    93.7%
## unadj_dopt01    5e-04 0.0005    94.7%
## gcomp_dopt01    6e-04 0.0175       -
## IPTW_dopt01     8e-04 0.0008    94.7%
## IPTW_DR_dopt01  6e-04 0.0006      94%
## TMLE_dopt01     5e-04 0.0005    93.6%
## CV.TMLE_dopt01  5e-04 0.0005    93.2%
## unadj_dopt02    5e-04 0.0005    93.7%
## gcomp_dopt02    6e-04 0.0146       -
## IPTW_dopt02     9e-04 0.0009      94%
## IPTW_DR_dopt02  5e-04 0.0006    90.1%
## TMLE_dopt02     5e-04 0.0006    90.6%
## CV.TMLE_dopt02  5e-04 0.0005    93.6%
##                Comparison                      Library Estimator     Bias Variance
## unadj    EnYdn for E0Yd0                          GLMs     Unadj.  -0.0562   0.0007
## gcomp    EnYdn for E0Yd0                          GLMs    G-comp.  -0.0773   0.0004
## IPTW     EnYdn for E0Yd0                          GLMs       IPTW  -0.0558   0.0008
## IPTW_DR  EnYdn for E0Yd0                          GLMs    IPTW-DR  -0.0565   0.0006
## TMLE     EnYdn for E0Yd0                          GLMs       TMLE  -0.0565   0.0006
## CV.TMLE  EnYdn for E0Yd0                          GLMs    CV-TMLE  -0.0764   0.0009
## unadj1   EnYdn for E0Yd0 ML + GLMs not aggressive   Unadj.   0.0245   0.0007
## gcomp1   EnYdn for E0Yd0 ML + GLMs not aggressive   G-comp. -0.1306   0.0007
## IPTW1    EnYdn for E0Yd0 ML + GLMs not aggressive     IPTW   0.0334   0.0010
```

```
## IPTW_DR1 EnYdn for E0Yd0 ML + GLMs not aggressive   IPTW-DR  0.0327    0.0008
## TMLE1    EnYdn for E0Yd0 ML + GLMs not aggressive      TMLE  0.0298    0.0008
## CV.TMLE1 EnYdn for E0Yd0 ML + GLMs not aggressive   CV-TMLE -0.0308    0.0007
## unadj2   EnYdn for E0Yd0     ML + GLMs aggressive    Unadj.  0.1138    0.0113
## gcomp2   EnYdn for E0Yd0     ML + GLMs aggressive   G-comp. -0.1161    0.0007
## IPTW2    EnYdn for E0Yd0     ML + GLMs aggressive      IPTW  0.1236    0.0109
## IPTW_DR2 EnYdn for E0Yd0     ML + GLMs aggressive   IPTW-DR  0.1010    0.0092
## TMLE2    EnYdn for E0Yd0     ML + GLMs aggressive      TMLE  0.1031    0.0108
## CV.TMLE2 EnYdn for E0Yd0     ML + GLMs aggressive   CV-TMLE -0.0316    0.0007
##                MSE Coverage
## unadj     0.0038    31.8%
## gcomp     0.0064       -
## IPTW      0.0039      45%
## IPTW_DR   0.0038    30.1%
## TMLE      0.0038    29.8%
## CV.TMLE   0.0067    14.7%
## unadj1    0.0013    75.1%
## gcomp1    0.0178       -
## IPTW1     0.0021    76.1%
## IPTW_DR1  0.0019    66.5%
## TMLE1     0.0016    71.3%
## CV.TMLE1  0.0017      69%
## unadj2    0.0243    29.9%
## gcomp2    0.0142       -
## IPTW2     0.0262      31%
## IPTW_DR2  0.0194      33%
## TMLE2     0.0214    33.6%
## CV.TMLE2  0.0017    68.6%
```

4 Summary of Results Above

- $\underline{E_n[Y_{d_0}] \text{ to estimate } E_0[Y_{d_0}]}$: these results speak to performance of estimators of $E_0[Y_d]$ for some given d (i.e., not about how well we estimate the rule, but about how well we estimate the performance of a given rule, which here it happens to be d_0). Estimator results:

    - g-comp: biased
        * Note: this differs from estimation of, e.g., $E[Y_1]$ in RCT (or using any treatment rule that isn't a function of covariates), where g-comp using a misspecified glm is a TMLE, and therefore unbiased
    - IPTW: less efficient (more variability), including less efficient than unadjusted
        * Note: this again differs from estimation of, e.g., $E[Y_1]$ in RCT, where IPTW using estimated weights we gain efficiency
    - IPTW-DR and TMLE: unbiased, EXCEPT if Q estimated aggressively, then bias enough for coverage to drop to $\sim 90\%$
        * Small variance gain compared to unadjusted (though suspect this gain would be bigger if covariates were more predictive of outcome?)
    - CV-TMLE: unbiased (even with more aggressive library for Q)
    - Unadjusted: unbiased

* Small variance price vs the DR estimators, but without risk of bias due to overfitting Q
  * Very little difference compared to CV-TMLE

- $\underline{E_n[Y_{d_n}] \text{ to estimate } E_0[Y_{d_0}]}$: these results speak to not only how good of a job we do evaluating the rule (i.e., as above), but also how well we estimate the rule

  - None do well. This is all due to fall off in estimating d_0, ie d_n not converging to d_0 fast enough.
    * See ODTR paper just submitted- how to do a better job on d_0 (including at finite sample sizes, even if cant get all the way there, how to get closer)

- $\underline{E_n[Y_{d_n}] \text{ to estimate } E_0[Y_{d_n}]}$: these results are of interest if going after a data-adaptive target parameter

  - Note: the estimators are targeting different data adaptive parameters. Data-adaptive parameter here for CV-TMLE is the average of the folds, for the others, it is the d_n learned on the whole sample
    * However, here they are pretty similar
  - There is a real price in bias paid by not using sample splitting to evaluate performance
    * For all of the other estimators besides CV-TMLE, will overestimate how well the estimated rule does
    * As the library used to estimate Q gets more aggressive:
      · The estimated rule gets closer to the true rule
      · The price paid (in terms of bias) by not using CV-TMLE increases

Big picture summary:

- For large sample sizes, small price and many benefits to using CV-TMLE with aggressive library to estimate $E_0[Y_{d_0}]$ and $E_0[Y_{d_n}]$

  - But, is this true if truth is simple? If sample size is small, worry is in that case pay a price. We want a method that as sample size increases, goes towards the more complex; when sample size limited, data can't support, will go towards simple (CI-based ODTR might help here)