Statistical Causality

Philip Dawid
Statistical Laboratory
University of Cambridge

Statistical Causality

1. The Problems of Causal Inference

2. Formal Frameworks for Statistical Causality

3. Graphical Representations and Applications

4. Causal Discovery

1. The Problems of Causal Inference

Conceptions of Causality

- Constant conjunction
  - Deterministic
- Mechanisms
  - "Physical" causality
➢ Agency
  - Effects of actions/interventions
➢ Contrast
  - Variation of effect with changes to cause

Causal Queries

- If I had taken aspirin half an hour ago, would my headache would have gone by now?
  - "Causes of Effects", CoE
  - *Counterfactual*
  - LAW
- If I take aspirin now, will my headache be gone within half an hour?
  - "Effects of Causes", EoC
  - *Hypothetical*
  - **SCIENCE, STATISTICS**

Causal Enquiry

- Experimentation ("doing")
  - To find out what happens to a system when you interfere with it you have to interfere with it (not just passively observe it) – George Box
- Observation ("seeing")
  - Cost
  - Ethics
  - Practicality
- No necessary connexion!

Problems of observational studies

An observed association between a "cause" and an "effect" may be spurious:

- Reverse causation
- Regression to mean
- Confounding
  - common cause
  - differential selection
- ...

Problems of observational studies

The facts about fuel (Which?, August 2007)

Mr Holloway said that a colleague of his used to drive from London to Leeds and back, using Shell petrol to go up there and BP fuel to drive back. He was convinced the BP petrol gave better fuel economy, but Ray had another explanation: 'I pointed out that Leeds is at a higher altitude that London: he was going uphill one way and downhill the other!'

Problems of observational studies

HRT and coronary artery disease

Observational research on women taking post-menopausal hormone therapy suggested a 40–50% reduction in coronary heart disease.

A large clinical trial found an elevated incidence.

Hazard ratio estimates in the observational study were up to 48% lower than in the clinical trial.

Problems of observational studies

Vitamin supplements and mortality

Many observational studies appeared to indicate that antioxidant supplements (vitamins A and E, β-carotene) reduce the risk of disease.

Randomized controlled trials showed that they increase mortality.

Problems of observational studies

Calcium channel blockers

Non-experimental studies suggested an increased risk of myocardial infarction associated with the short-acting calcium channel blocker (CCB) nifedapine.

It took almost a decade to obtain RCT evidence, which showed that long-acting nifedapine is safe.

Simpson's Paradox

	Recovered	Died	Total	Recovery rate
New	200	200	400	50%
Standard	160	240	400	40%

Table 1: Overall results

	Recovered	Died	Total	Recovery rate
New	180	120	300	60%
Standard	70	30	100	70%

Table 2: Male results

	Recovered	Died	Total	Recovery rate
New	20	80	100	20%
Standard	90	210	300	30%

Table 3: Female results

Causal Inference

- Association is not causation!
- Traditionally, Statistics dealt with association
  - Theory of *Statistical Experimental Design and Analysis* does address causal issues
    - but of no real use for observational studies
- How to make inferences about causation?
  - "bold induction", to a novel context
- *Do we need a new formal framework?*

2. Formal Frameworks for Statistical Causality

Some Formal Frameworks

- ➤ Probability distributions
- Potential responses
- Functional relationships
- ➤ Extended conditional independence
- …
- Structural equations
- Path diagrams
- ➤ Directed acyclic graphs
- …

A SIMPLE (??) PROBLEM

- **Randomised experiment**

- Binary (0/1) treatment decision variable T

- Response variable Y

  Define/measure *"the effect of treatment"*

Probability Model *(Fisher)*

- Specify/estimate *conditional distributions*
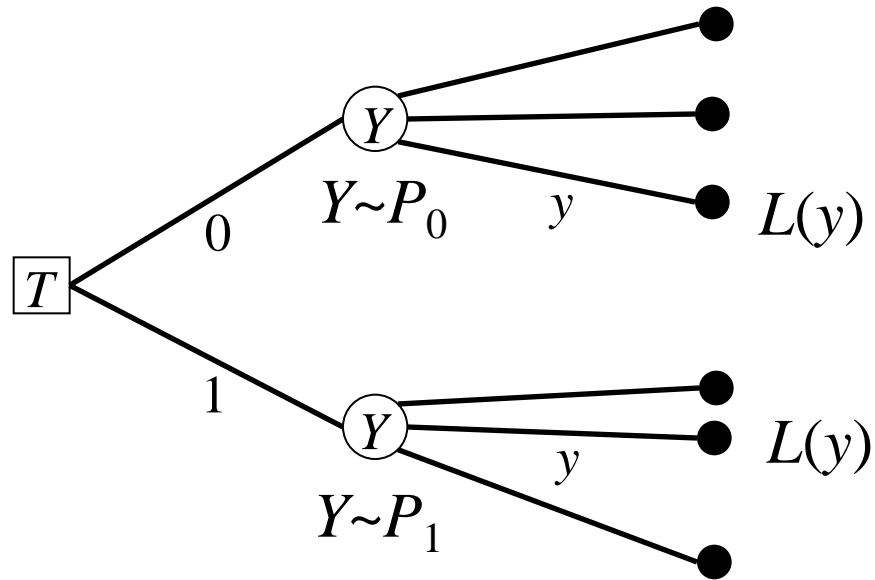
  P_t for Y *given* $T = t$ $(t = 0, 1)$

  $$[e.g. \ N(\mu_t, \ \sigma^2) \ ]$$

- Measure effect of treatment by change in the distribution of Y: compare P_0 and P_1

  – *e.g.* by change in expected response:
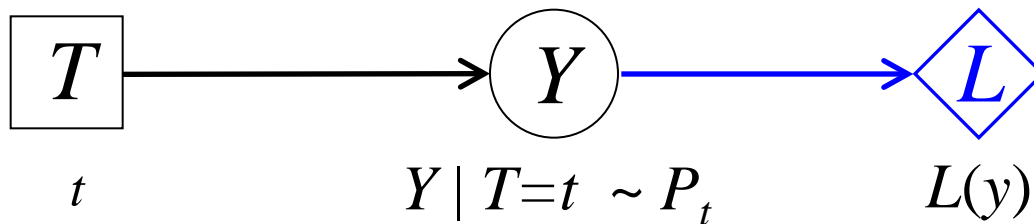
  $$\delta = \mu_1 - \mu_0 \qquad (average\ causal\ effect,\ ACE)$$

- Probability model all we need for decision theory

  – choose t to minimise expected loss $E_{Y \sim P_t}\{L(Y)\}$

Decision Tree

$Y \sim P_0$ y $L(y)$

T

0

1

$Y \sim P_1$ y $L(y)$

Influence Diagram

T → Y → L

t $Y \mid T{=}t \sim P_t$ $L(y)$

Structural Model

$$Y = \mu_T + E$$
$$[\text{e.g., } E \sim N(0, \sigma^2)]$$

(E = "error", "effect of omitted variables",…)

- Deterministic relationship
- Value of E for any unit supposed the same if we were to change T from 0 to 1
- Then value of Y would change by *exactly* $\delta = \mu_1 - \mu_0$
  – *individual causal effect (ICE)*

Potential Response Model (Rubin)

- Split Y in two:

$$Y_0 : \text{ potential response to } T = 0$$

$$Y_1 : \text{ potential response to } T = 1$$

- Consider (for any unit) the *pair* $\mathbf{Y} = (Y_0, Y_1)$
  - *with simultaneous existence and joint distribution*

- Treatment "uncovers" pre-existing response:

$$Y = Y_T \text{ (determined by } \mathbf{Y} \text{ and } T)$$

  - other PR unobservable, "counterfactual"

- Unit-level (individual) [random] causal effect

$Y_1 - Y_0$ – *necessarily unobservable!*

General Functional Model

$$Y = f(T, U)$$

$$(U = \text{``unit characteristics''})$$

- Value of U supposed the same , independent of value of T
  - and of whether we intervene or just observe
- Formally includes:
  - Structural model: $U = E,\ Y = \mu_T + E$
  - PR model: $U = \mathbf{Y},\ Y = Y_T$

Potential Response Model

- Any functional model $Y = f(T, U)$ generates a PR model: $Y_t = f(t, U)$

- Any PR model generates a probability model: P_t is marginal distribution of Y_t $(t = 0, 1)$

- Distinct PR models can generate the same statistical model

  – e.g., correlation between Y_0 and Y_1 arbitrary

- Cannot be distinguished observationally

- Can have different inferential consequence

  – can be problematic!

Potential Responses: Problems

- *PR* model:

$$\begin{cases} Y_t \sim N(\mu_t, \sigma^2) & (t = 0, 1) \\ \operatorname{corr}(Y_0, Y_1) = \rho \end{cases}$$

- Corresponding *statistical* model:

$$P_t : Y \sim N(\mu_t, \sigma^2)$$

NB: ρ does not enter! – *can never identify ρ – does this matter??*

Potential Responses: Problems

Under PR model:

$E(Y_1/Y_0)$ depends on ρ

➤ *We can not estimate a "ratio" ICE*

$\text{var}(Y_1 - Y_0) = 2(1 - \rho)\,\sigma^2$

➤ *We can not identify the variance of the ICE*

$E(Y_1 - Y_0 \mid Y_1 = y_1) = (1 - \rho)\,y_1 + (\rho\mu_1 - \mu_0)$

➤ *We can not identify the (counterfactual) ICE, after observing response to treatment*

OBSERVATIONAL STUDY

- Treatment decision taken may be associated with patient's state of health

- What assumptions are required to make causal inferences?

- When/how can such assumptions be justified?

Functional Model
$$Y = f(T, U)$$

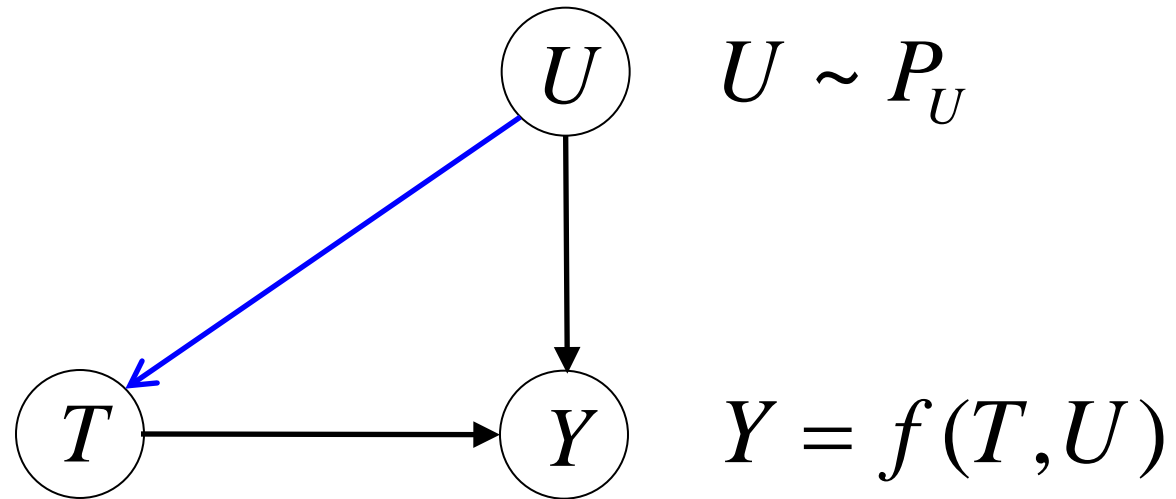T = treatment received

U = "unit characteristics"

– value supposed unaffected by treatment or how it is applied

– but could influence choice of treatment T

➢ observational dependence between T and U

Response to applied treatment t: $Y_t = f(t, U)$.
Observational distribution of Y, *given $T = t$*,
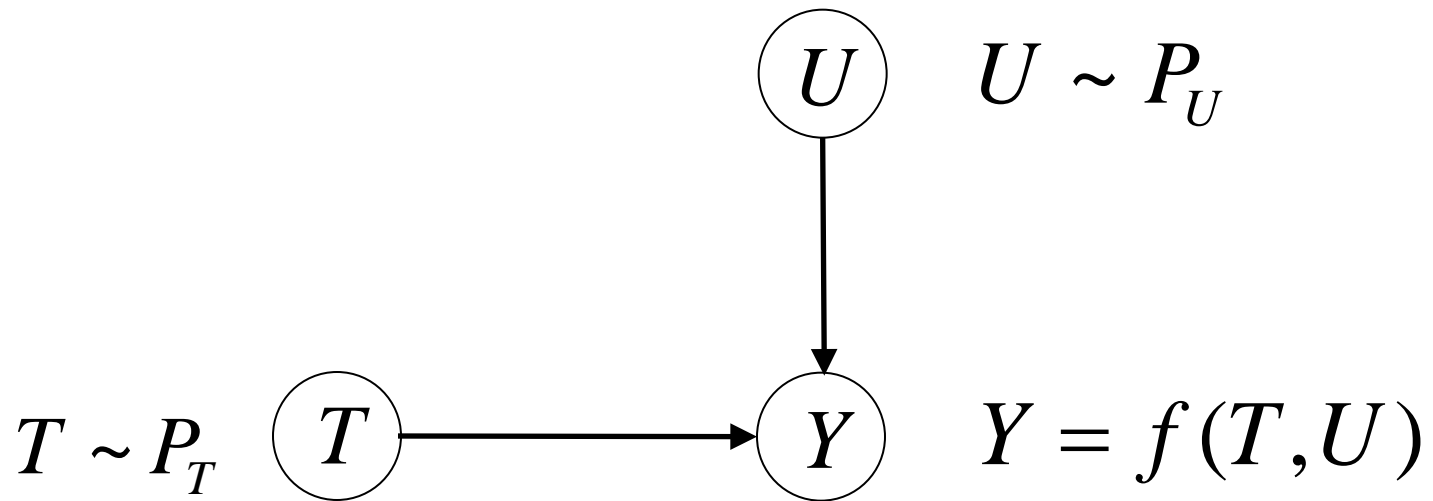same as distribution of Y_t if $T \perp\!\!\!\perp U$

Functional Model

$U \sim P_U$

$Y = f(T, U)$

U = "unit characteristics"

– value supposed unaffected by treatment or how it is applied

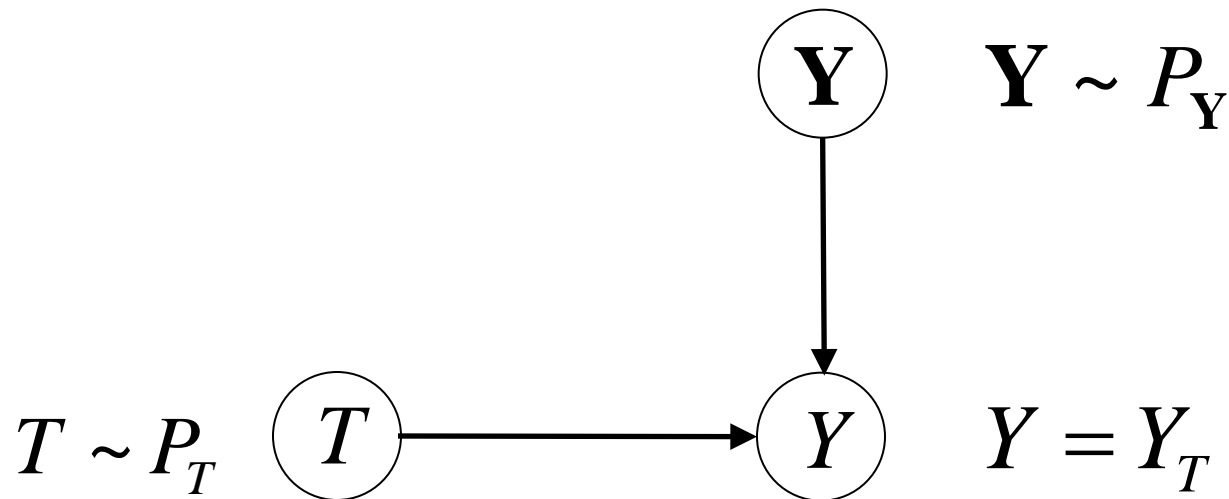– but could influence treatment choice

Functional Model

$$U \sim P_U$$

$$T \sim P_T$$

$$Y = f(T, U)$$

"No confounding" ("ignorable treatment assignment") if

$$T \perp\!\!\!\perp U$$

(treatment independent of "unit characteristics")

PR interpretation ($U = \mathbf{Y}$)

$$\mathbf{Y} \sim P_{\mathbf{Y}}$$

$$T \sim P_T$$

$$Y = Y_T$$

"No confounding" ("ignorable treatment assignment") if

$$T \perp\!\!\!\perp \mathbf{Y}$$

(treatment independent of potential responses)

PR interpretation ($U = \mathbf{Y}$)

- Value of $\mathbf{Y} = (Y_0, Y_1)$ on any unit supposed the same in observational and experimental regimes, as well as for both choices of T

- No confounding: independence of T from PR pair $\mathbf{Y}$

  How are we to judge this??

Statistical Decision Model

- "Treatment regime indicator" variable F_T
    - *intervention variable*
    - non-random, parameter

- Values:

$$F_T = 0 : \text{Assign treatment } 0 \quad (\Rightarrow T = 0)$$
$$F_T = 1 : \text{Assign treatment } 1 \quad (\Rightarrow T = 1)$$
$$F_T = \emptyset : \text{Just observe} \quad (T \text{ random})$$

(Point intervention: can generalize)

Statistical Decision Model

- Causal target: comparison of distributions of Y given $F_T = 1$ and given $F_T = 0$
  - e.g., $E(Y \mid F_T = 1) - E(Y \mid F_T = 0)$

    average causal effect, ACE

- Causal inference: assess this (if possible) from properties of observational regime, $F_T = \emptyset$

Statistical Decision Model

True ACE is

$$\mathrm{E}(Y \mid T = 1, F_T = 1) - \mathrm{E}(Y \mid T = 0, F_T = 0)$$

Its observational counterpart is:

$$\mathrm{E}(Y \mid T = 1, F_T = \emptyset) - \mathrm{E}(Y \mid T = 0, F_T = \emptyset)$$

"No confounding" (ignorable treatment assignment) when these are equal.

Can strengthen:

$$p(y \mid T = t, F_T = t) = p(y \mid T = t, F_T = \emptyset)$$

➤ distribution of $Y \mid T$ the same in observational and experimental regimes

Extended Conditional Independence

Distribution of $Y \mid T$ the same in observational and experimental regimes:

$$Y \mid (F_T, T) \;\; \text{does not depend on value of } F_T$$

Can express and manipulate using notation and theory of conditional independence:

$$Y \perp\!\!\!\perp F_T \mid T$$

(even though F_T is not random)