# Adaptive MacroR:
# A Hybrid Discrete–Macroscopic Framework
# for Multi-Scale Channel Ensemble Inference

Internal Note — MacroDR/MacroIR Project

**Abstract**

This document outlines the conceptual foundations for an "Adaptive MacroR" framework capable of simultaneously handling microscopic, mesoscopic, and macroscopic channel subpopulations within a single Bayesian model. The goal is to unify single-channel inference, macroscopic (MacroIR) inference, and intermediate cases into one multi-scale method capable of tracking dynamics from zero-current conditions (few active channels) up to maximal-current regimes (hundreds or thousands of channels). This note defines the generative model, the inference steps, and a roadmap for further development.

## 1 Motivation

Biophysical channel populations span a wide range of ensemble sizes. At very low occupancy (e.g. channels recovering from desensitization), the observations are inherently discrete, and single-channel inference is the correct description. At high occupancy, the macroscopic state of the population is well described by the Gaussian approximation used in MacroIR. Between these extremes lies a mesoscopic regime where neither approximation is fully adequate.

The *Adaptive MacroR* idea is to blend these three regimes into a single hierarchical state representation consisting of:

1. a microscopic sector (explicit channels, discrete Markov chains),

2. a mesoscopic sector (small counts with weakly discrete effects),

3. a macroscopic sector (large counts, continuous Gaussian state).

Each interval update uses a mixture of exact discrete inference and Gaussian MacroIR inference. The division between sectors is dynamic and adapts to the scale of the signal.

## 2 State Representation

At each time we assume the ensemble is partitioned into:

- **Microscopic channels:**

$$X_c^{(m)}(t) \in \{1, \ldots, K\}, \qquad c = 1, \ldots, N_{\text{micro}}.$$

Each is a full Markov chain with exact discrete states.

- **Macroscopic block:**
$$\big(\boldsymbol{\mu}^{\text{macro}}(t),\ \boldsymbol{\Sigma}^{\text{macro}}(t)\big),$$
describing the occupancy distribution of the remaining $N_{\text{macro}}$ channels via Gaussian MacroIR.

- **Mesoscopic extensions (optional):** These may track a small number of low-count components with geometry-aware approximations (e.g. logit-Gaussian) or small discrete mixtures. They act as an intermediate layer but are not required in the minimal model.

The total number of channels is

$$N_{\text{ch}} = N_{\text{micro}} + N_{\text{macro}}.$$

## 3 Generative Model

For an interval $[0, t]$, the total averaged current is

$$\overline{y}_{0\to t} = \underbrace{\sum_{c=1}^{N_{\text{micro}}} \overline{y}_{c,0\to t}^{(m)}}_{\text{microscopic}} + \underbrace{\overline{y}_{0\to t}^{\text{macro}}}_{\text{MacroIR}} + \epsilon_{0\to t}, \tag{1}$$

where $\epsilon_{0\to t}$ denotes instrument/binning noise. For each microscopic channel $c$,

$$\overline{y}_{c,0\to t}^{(m)} = \overline{\Gamma}_{i_0 \to i_t}^{(c)}$$

with boundary-conditioned statistics exactly as in the single-channel model.

For the macroscopic block, MacroIR provides a Gaussian predictive model:

$$\overline{y}_{0\to t}^{\text{macro}} \sim \mathcal{N}\big(\overline{y}^{\text{pred}},\ \sigma_{\text{pred}}^2\big),$$

using the tilde operators and variance-inflation adaptation described elsewhere.

Conditioned on the microscopic configuration, the full distribution of (1) is Gaussian. Therefore the model is a finite mixture of Gaussians, with mixture components indexed by the microscopic states.

## 4 Inference Procedure

For each interval:

### 4.1 1. Propagation

- **Microscopic sector:** For each explicit channel propagate its Markov chain exactly:
$$\Pr\big[X_c^{(m)}(t) = i_t\big] = \sum_{i_0} \Pr\big[X_c^{(m)}(0) = i_0\big] P_{i_0 \to i_t}(t).$$

- **Macroscopic sector:** Use MacroIR propagation:
$$\boldsymbol{\mu}_{\text{prop}}^{\text{macro}} = \boldsymbol{\mu}_0^{\text{macro}}\mathbf{P}(t),$$

$$\boldsymbol{\Sigma}_{\text{prop}}^{\text{macro}} = \mathbf{P}(t)^\top\big(\boldsymbol{\Sigma}_0^{\text{macro}} - \text{diag}(\boldsymbol{\mu}_0^{\text{macro}})\big)\mathbf{P}(t) - \text{diag}(\boldsymbol{\mu}_{\text{prop}}^{\text{macro}}).$$

## 4.2  2. Predictive Current

- **Microscopic part:** Compute exact boundary-conditioned means for each channel and sum them:

$$\overline{y}_{\text{pred}}^{\text{micro}} = \sum_{c=1}^{N_{\text{micro}}} \sum_{i_0, i_t} \Pr[X_c^{(m)}(0) = i_0, i_t] \, \overline{\Gamma}_{i_0 \to i_t}^{(c)}.$$

- **Macroscopic part:** Use standard MacroIR:

$$\overline{y}_{\text{pred}}^{\text{macro}} = N_{\text{macro}} \, \boldsymbol{\mu}_0^{\text{macro}} \, \overline{\boldsymbol{\gamma}}_0,$$

with $\overline{\boldsymbol{\gamma}}_0$ from the boundary-lifted mean.

Thus:

$$\overline{y}^{\text{pred}} = \overline{y}_{\text{pred}}^{\text{micro}} + \overline{y}_{\text{pred}}^{\text{macro}}.$$

## 4.3  3. Update with Measurement

Conditioned on the microscopic states, the likelihood is Gaussian with mean $\overline{y}^{\text{pred}}$ and variance $\sigma_{\text{eff}}^2$ (possibly inflated). The posterior over microscopic states is updated exactly (or approximately), and the MacroIR block is updated using

$$\boldsymbol{\mu}_{\text{post}}^{\text{macro}} = \boldsymbol{\mu}^{\text{prop}} + \frac{\boldsymbol{g} \, \delta}{\sigma_{\text{eff}}^2},$$

$$\boldsymbol{\Sigma}_{\text{post}}^{\text{macro}} = \boldsymbol{\Sigma}^{\text{prop}} - \frac{\boldsymbol{g}\boldsymbol{g}^{\top}}{\sigma_{\text{eff}}^2}.$$

## 4.4  4. Adaptive Sector Reassignment

After each interval, diagnostics determine whether certain parts of the macroscopic block should be promoted to microscopic/mesoscopic treatment or vice versa. Typical criteria include:

- persistent boundary hits in some coordinates (large variance-inflation factors),

- extremely small expected occupancy in some states ($N_{\text{macro}}\mu_i \ll 1$),

- repeated large residuals in directions dominated by a small number of states.

Channels or fractions of the population may then be moved across sectors.

# 5  Discussion and Roadmap

The Adaptive MacroR framework provides a principled route to unifying single-channel and macroscopic inference. Its conceptual appeal lies in:

- exact treatment of low-count events,

- efficient Gaussian treatment of large ensembles,

- a tunable mesoscopic layer allowing intermediate levels of detail,

- a natural interpretation as a Rao–Blackwellized filter.

Future development includes:

1. formalizing the mesoscopic sector (e.g. logit-Gaussian mixtures),

2. designing adaptive splitting heuristics based on predictive variance, occupancy, and curvature diagnostics,

3. implementing a minimal hybrid prototype (1–2 explicit channels plus a macro block),

4. verifying the approach on simulated desensitization/recovery scenarios.

The long-term goal is a unified inference engine capable of dealing with single-channel resolution and macroscopic currents in one coherent Bayesian framework.