A function is no different than a machine. A function takes some input x and manufactures some output $f(x)$. This process is shown in Figure 1.

Figure 1

Formally, a **function** f is a rule that assigns each element x in a set X *exactly one* element $f(x)$ in a set Y.

Like a machine, there are many different forms a function can take:

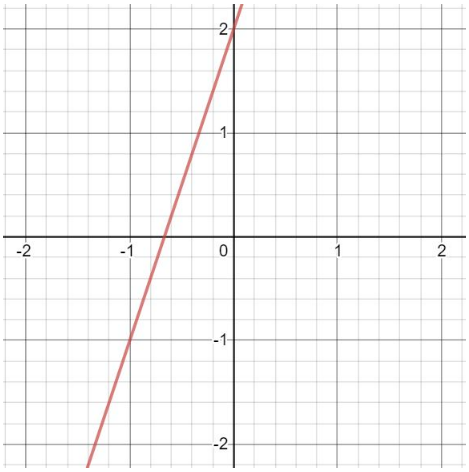
(1) Verbal Description	(2) Numerical Values
For every unit increase in x*, there is a 3 unit increase in* $f(x)$.	$(-2,-4)$, $(-1,-1)$, $(0,2)$, $(1,5)$, $(2,8)$
(3) Visual Depiction	**(4) Algebraic Expression**
	$f(x) = 3x + 2$

Figure 2: *Each cell describes the same function.*

In addition to having multiple ways to represent a function, there are multiple ways to mathematically describe a function:

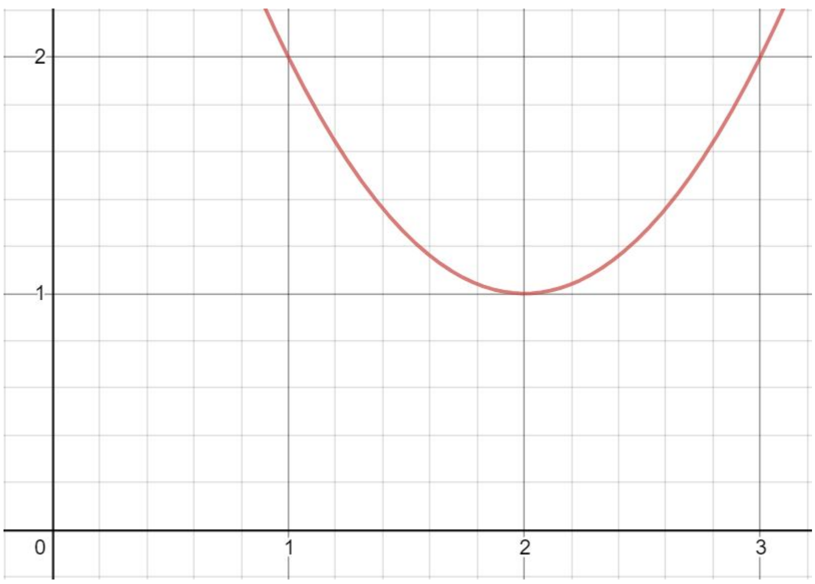
1. Using the function's domain & range
2. Discussing the function's independent & dependent variables
3. Describing the function's actions as increasing & decreasing

The **domain** of a function is the set of all values for which a function is defined. Similarly, the **range** of a function is the set of all values that $f(x)$ takes. This is similar the function's independent and dependent variables. **Independent** variables are controlled inputs. Like the domain, these are all x values in which the function exists. **Dependent** variables are a quantity whose value depends on how the independent variable is manipulated.

We can also describe a function as increasing or decreasing. A function is **increasing** on an interval I when $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ on the interval I. A function is **decreasing** on an interval I when $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ on the interval I.

Example 1

For this problem, please use the graph below:



(a) Identify the domain and range.

(b) Identify the intervals where the function is increasing and decreasing.

(Solution) Upon visual inspection, we can see the x values range from $-\inf$ to $\inf$. The domain is $(-\inf, \inf)$. Following similar inspection, we can see the y values range from 1 to $\inf$. The range is $[1, \inf)$.

(Solution) The function is increasing from $[1, \inf)$ and decreasing from $(-\inf, 1]$.

Example 2

Find the domain of the following:

(a) $a(x) = \sqrt{x+2}$

(b) $b(x) = \frac{1}{x^2-x}$

(c) $c(x) = x^3$

(Solution) The domain of $a(x)$ is only where x values can exist. Because the square-root of a negative number is undefined, the quantity under the square-root must be 0 or larger.

$$x + 2 \geq 0$$
$$x \geq -2$$

The domain of $a(x)$ is $[-2, \inf)$.

(Solution) The domain of $b(x)$ is only where x values can exist. Because a 0 in the denominator is undefined, the denominator cannot be 0.

$$x^2 - x \neq 0$$
$$x(x-1) \neq 0$$
$$x \neq 0 \text{ and } x - 1 \neq 0$$
$$x \neq 0 \text{ and } x \neq 1$$

The domain of $b(x)$ is $(-\inf, 0) \cup (0, 1) \cup (1, \inf)$.

(Solution) The domain of $c(x)$ is only where x values can exist. Because $c(x)$ exists everywhere, the domain is $(-\inf, \inf)$.

The graph of a function is a curve on the xy plane. But not all curves on the xy plane are functions. We can determined which curves are functions by using the **vertical line test**. A curve in the xy plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

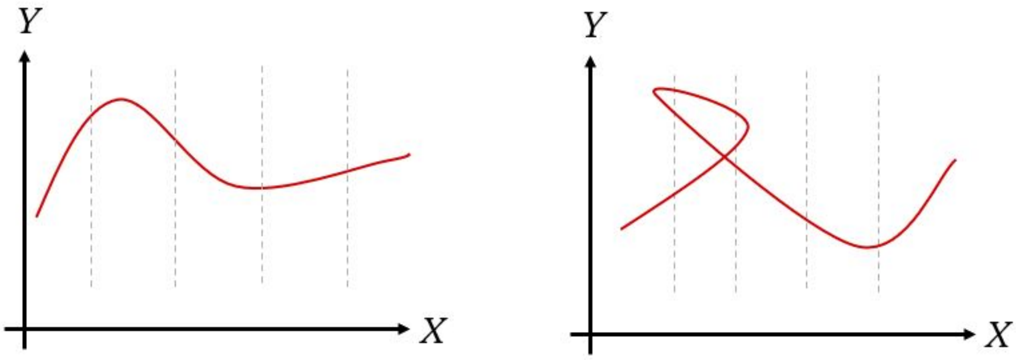


Figure 3

Figure 3 shows the vertical line test in use. The graph on the left shows a curve that passes the vertical line test and is thus a function. The graph on the right shows a curve that fails the vertical line test and is thus not a function.