LOGIC FOR NATURAL LANGUAGE, LOGIC IN NATURAL LANGUAGE

Welcome to the Course!

Larry Moss

Indiana University

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- ▶ The raison d'être of logic is the study of inference in language.
- ► However, modern logic was developed in connection with the foundations of mathematics.
- ▶ So we have a mismatch, leading to
 - neglect of language in the first place
 - use of first-order logic and no other tools
- First-order logic is both too big and too small:
 - cannot handle many interesting phenomena
 - is undecidable

NATURAL LOGIC: RESTORE NATURAL LANGUAGE INFERENCE AS A CENTERPIECE OF LOGIC

Program

Show that significant parts of natural language inference can be carried out in decidable logical systems, preferably in "light" systems.

To axiomatize as much as possible, because the resulting logical systems are likely to be interesting.

To ask how much of language could have been done if the traditional logicians had today's mathematical tools.

WHAT WILL YOU LEARN IN THIS CLASS?

The class will have a lot of technical material connected to the basic notions of topics such as

- model theory
- algebraic logic
- modal logic
- decidable fragments of first-order logic
- the typed lambda calculus and its connection to grammar and semantics

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The course will also present a lot of educational material. This could be an introduction to logic or a "bridge to mathematical proofs" course.

More on the educational aspects

I comment on educational points in green boxes.

NATURAL LOGIC: PARALLEL STUDIES

I DIDN'T HAVE MUCH TO SAY ON THESE, BUT WE CAN DISCUSS THEM

- ▶ History of logic: reconstruction of original ideas
- ▶ Philosophy of language: proof-theoretic semantics
- ▶ Philosophy of logic: why variables?
- ▶ Cognitive science: models of human reasoning
- Linguistic semantics: Are deep structures necessary, or can we just use surface forms? And is a complete logic a semantics?
- ➤ Computational linguistics/artificial intelligence: many precursors and related projects

OUR TOPIC: INFERENCE

Given a list of temporary assumptions, decide whether another sentence follows from the given list.

What "follows from" means

A sentence φ follows from a list Γ of other sentences if whenever we accept the sentences in Γ , we could/should also accept φ .

THE STRONGEST FORMULATION OF THIS FORMULATION OF "INFERENCE"

In seeing whether φ follows from Γ in some situation s, and ideal human would start with some set Δ of categorical sentences, known or believed to be true, stated in some formal language \mathcal{L} , and also some general features of s, again stated in s, and then parse φ and turn it into a sentence in \mathcal{L} , and then appeal to a general inference procedure.

Such a procedure should exist: this whole account should be effective.

The inference should also match some semantics of \mathcal{L} .

Deviations in performance would be correlated to various features of the ideal account.

GIVE UP ON INFERENCE

This is what is done in linguistic semantics.

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GIVE UP ON STRICT LOGICALITY AND ON SEMANTICS, BUT SUPPOSEDLY NOT ON HUMANS

This is what is done in RTE (see below).

What do various fields give up?

GIVE UP ON INFERENCE

This is what is done in linguistic semantics.

GIVE UP ON DECIDABILITY AND ON HUMAN COMPETENCE

This is GOFAI.

GIVE UP ON STRICT LOGICALITY AND ON SEMANTICS, BUT SUPPOSEDLY NOT ON HUMANS

This is what is done in RTE (see below).

GIVE UP ON COMPLEX INFERENCES

This is what is done in psychological studies of syllogistic inference.

What I am trying to do

What I propose to do:

GIVE UP SOMEWHAT ON STATEMENTS IN "REAL LIFE LANGUAGE", BUT NOT ON DECIDABILITY, AND NOT ON LOGICAL COMPLETENESS

Also:

GIVE UP MORE TO DO MORE

Examples of inferences

WHICH WE WILL SEE IN THIS COURSE

These are the basic data that the course will account for

First, a few examples from the classical syllogistic:

All auctioneers are curmudgeons No bartenders are curmudgeons
No auctioneers are bartenders

(2)

Syllogistic logic is under-appreciated!

My aim in the first two days of the course is to convince you that extended syllogistic logics are very interesting indeed.

A FIRST LOOK AT SYLLOGISTIC LOGIC

Our "syntax" of sentences will give us

All X are Y
Some X are Y
No X are Y
but no boolean connectives (!), at least not at first

We adopt the evident semantics.

We craft a logical system which has formal proofs using our syntax of sentences and nothing else.

After this, we want to extend the idea of syllogistic logic.

BASIC SYLLOGISTIC LOGIC: ALL, AND SOME

Syntax: All p are q, Some p are q

Semantics: A model \mathcal{M} is a set M, and for each noun p we have an interpretation $\llbracket p \rrbracket \subseteq M$.

$$\mathcal{M} \models \mathit{All}\ \mathit{p}\ \mathit{are}\ \mathit{q} \qquad \text{iff} \qquad \llbracket\mathit{p}\rrbracket \subseteq \llbracket\mathit{q}\rrbracket \\ \mathcal{M} \models \mathit{Some}\ \mathit{p}\ \mathit{are}\ \mathit{q} \qquad \text{iff} \qquad \llbracket\mathit{p}\rrbracket \cap \llbracket\mathit{q}\rrbracket \neq \emptyset$$

Proof system:

$$\frac{All \ p \ are \ n}{All \ p \ are \ p} \frac{All \ p \ are \ q}{All \ p \ are \ q}$$

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 iff $\llbracket p \rrbracket \subseteq \llbracket q \rrbracket$
 $\mathcal{M} \models Some \ p \ are \ q$ iff $\llbracket p \rrbracket \cap \llbracket q \rrbracket \neq \emptyset$

Proof system:

$$\frac{All \ p \ are \ n}{All \ p \ are \ p} \qquad \frac{All \ p \ are \ n}{All \ p \ are \ q}$$

We could also add the word *No*. What rules do you think we would need?

SEMANTIC AND PROOF-THEORETIC NOTIONS

If Γ is a set of formulas, we write $\mathcal{M} \models \Gamma$ if for all $\varphi \in \Gamma$, $\mathcal{M} \models \varphi$.

 $\Gamma \models \varphi$ means that every $\mathcal{M} \models \Gamma$ also has $\mathcal{M} \models \varphi$.

A proof tree over Γ is a finite tree \mathcal{T} whose nodes are labeled with sentences and each node is either an element of Γ , or comes from its parent(s) by an application of one of the rules.

 $\Gamma \vdash S$ means that there is a proof tree \mathcal{T} for over Γ whose root is labeled S.

HOW IT ALL WORKS

English:

If there is an n, and if all ns are ps and also qs, then some p are q.

Semantic assertion:

Some n are n, All n are p, All n are $q \models Some p$ are q.

Proof-theoretic assertion:

Some n are n, All n are p, All n are $q \vdash Some p$ are q.

HOW IT ALL WORKS

English:

If there is an n, and if all ns are ps and also qs, then some p are q. This is something we could check against human intuition and performance.

Semantic assertion:

Some n are n, All n are p, All n are $q \models Some p$ are q. The reasoning here would be a mathematical proof.

Proof-theoretic assertion:

Some n are n, All n are p, All n are $q \vdash$ Some p are q. The proof tree is

	All n are p	Some n	are	n
	Some n are p			
All n are q	Some	p are n		
So	ome p are q			

Example of a conclusion which doesn't follow

All frogs are reptiles.
All frogs are animals.
All reptiles are animals.

Example of a conclusion which doesn't follow

```
All frogs are reptiles.
All frogs are animals.
All reptiles are animals.
```

We can define a model ${\mathcal M}$ by

```
M = \{1, 2, 3, 4, 5, 6\}

[frogs]] = \{1, 2\}

[reptiles]] = \{1, 2, 3, 4\}

[animals]] = \{1, 2, 4, 5, 6\}
```

In this model, the assumptions are true but the conculsion is false. So the argument is invalid.

All frogs are reptiles, All frogs are animals $\not\models$ All reptiles are animals.

THE CONNECTION

Soundness/Completeness Theorem

$$\Gamma \models \varphi \text{ iff } \Gamma \vdash \varphi$$

References to related work: Łukasiewicz 1951, Westerstahl 1989.

All the logical systems in this course will be complete.

But I will not do most of the details.

A first course in logic could contain the completeness results which we'll see in the first few days plus parallel material on propositional logic and also *something* on first-order logic.

More examples of inferences

THIS CLASS WILL NOT QUITE TREAT THESE

Every giraffe is taller than every gnu

Some gnu is taller than every lion

Some lion is taller than some zebra

Every giraffe is taller than some zebra

More students than professors run More professors than deans run

More students than deans run

(4)

(5)

(3)

At most as many xenophobics as yodelers are zookeepers At most as many zookeepers as alcoholics are yodelers At most as many yodelers as xenophobics are alcoholics

At most as many zookeepers as alcoholics are xenophobics

More reasoning about the sizes of sets

We are going to see the full set of rules on Tuesday or Wednesdsay

Example

Assume:

- There are at least as many non-y as y
- There are at least as many non-z as z
- 3 All non-y are z
- All x are z

Then prove from these that No x are y.

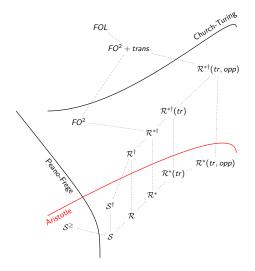
Here is a formal proof in the logical system which we'll see:

$$\frac{\forall (x,z)}{\forall (x,\overline{y})} \frac{\exists^{\geq}(\overline{y},y) \quad \exists^{\geq}(\overline{z},z)}{\exists^{\geq}(\overline{y},z)} \text{ (Half)}}{\forall (z,\overline{y})} \frac{\forall (z,\overline{y})}{\forall (z,\overline{y})} \text{ (Barbara)}$$

Map of Some Natural Logics

The Aristotle boundary is the dividing line between fragments which are formulated syllogistically and those which are not.

Reductio proofs are ok. Infinitely many rules are not.



first-order logic

$$FO^2 + "R"$$
 is trans"

2 variable FO logic

† adds full N-negation

 $\mathcal{R}^*(tr)$ + opposites \mathcal{R}^* + (transitive)

comparative adjs

 \mathcal{R} + relative clauses \mathcal{S} + full N-negation

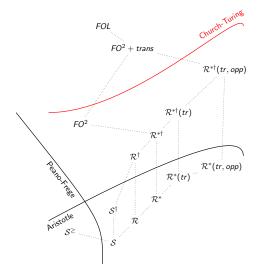
 $\mathcal{R}=\mathsf{relational}$ syllogistic

 \mathcal{S}^{\geq} adds $|p| \geq |q|$

 \mathcal{S} : all/some/no p are q

Map of Some Natural Logics

The Church-Turing boundary is the dividing line between decidable and undecidable fragments.



first-order logic

FO2 + "R is trans"

2 variable FO logic

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 \mathcal{R} + relative clauses

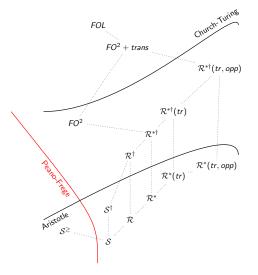
 \mathcal{S} + full *N*-negation \mathcal{R} = relational syllogistic

 \mathcal{S}^{\geq} adds $|p| \geq |q|$

 $\mathcal{S} \colon \, \mathsf{all/some/no} \,\, \mathsf{p} \,\, \mathsf{are} \,\, \mathsf{q}$

Map of Some Natural Logics

The Peano-Frege boundary divides the fragments according to whether they may be formulated in first-order logic.



first-order logic

 FO^2 + "R is trans"

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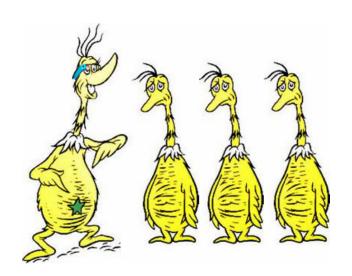
 $^{\mid}\mathcal{S}$: all/some/no p are q

Example of where we would want derivations with variables

All xenophobics see all astronauts
All yodelers see all zookeepers
All non-yodelers see all non-astronauts
All wardens are xenophobics
All wardens see all zookeepers

1	All xei	nophobics see all astronauts	Нур	
2	All you	delers see all zookeepers	Нур	
3	All no	n-yodelers see all non-astronauts	Нур	
4	All wa	ardens are xenophobics	Нур	
5	Jane Jane is a warden		Нур	
6		All wardens are xenophobics	R, 4	
7		Jane is a xenophobic	All Eliim, 6	
8		All xenophobics see all astronauts	R, 2	
9		Jane sees all astronauts	All Elim, 8	
10		Jane is a yodeler	Нур	
11		Jane sees all zookeepers	Easy from 2	
12		Jane is not a yodeler	Нур	
13		Jane sees all zookeepers	See below	
14		Jane sees all zookeepers	Cases 10-11, 12-13	
15	All wa	ardens see all zookeepers	All Intro	

1	Jane is not a yodeler			Нур	
_	Jane	13 1101	Пур		
2	Jane sees all astronauts			R, above	
3	All non-yodelers see all non-astronauts			R, above	
4	Jane sees all non-astronauts		All Elim, 1, 3		
5	Bob	Bob is a zookeeper		Нур	
6			Bob is astronaut	Нур	
7			Jane sees Bob	All Elim, 2	
8			Bob is not astronaut	Нур	
9			Jane sees Bob	All Elim, 4	
10		Jane	sees Bob	Cases	
11	Jane sees all zookeepers			All Intro	



MONOTONICITY IS FOR KIDS

How can a person or computer answers questions involving a word which they don't know?

A word like Sneetch.

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What "follows from" means

One sentence, A, follows from a second sentence, B, if every time we use B in a true way, we also would be committed to A if we asked about it.Day

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If we say

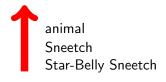
B: every animal hops

then it follows that

A : every Sneetch moves









Let's talk about a situation where

all Sneetches dance.

Which one would be true?

- ▶ all Star-Belly Sneetches dance
- ▶ all animals dance





- ▶ all Star-Belly Sneetches dance true
- ▶ all animals dance false

We write





What arrow goes on "dance"?

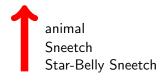
- ▶ all Sneetches waltz
- ▶ all Sneetches move





We write

all Sneetches dance dan





- No Sneetches dance.
- If you play loud enough music, any Sneetch will dance.
- 3 Any Sneetch in Zargonia would prefer to live in Yabistan.
- If any Sneetch dances, McBean will dance, too.





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$$f(x,y) = y - x \tag{6}$$

$$g(x,y) = x + \frac{2}{y} \tag{7}$$

$$h(v, w, x, y, z) = \frac{x - y}{2^{z - (v + w)}}$$
 (8)

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 (8)

The [↑] and [↓] notations have the same meaning in language as in math!

This is not an accident!

Let's look at an (easy) inference in algebra

Which is bigger,
$$-(7+2^{-3})$$
 or $-(7+2^{-4})$?

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$$\frac{\frac{3<4}{-4<-3}}{\frac{2^{-4}<2^{-3}}{2^{x}}} \xrightarrow{2^{x}} \text{ is antitone}$$
$$\frac{\frac{7+2^{-4}<7+2^{-3}}{7+2^{-4}<7+2^{-3}}}{7+x} \xrightarrow{7+x} \text{ is monotone}$$
$$\frac{-(7+2^{-3})<-(7+2^{-4})}{-(7+2^{-3})<-(7+2^{-4})} -x \text{ is antitone}$$

$$f(x)$$
 monotone means if $x \le y$, then $f(x) \le f(y)$
 $f(x)$ antitone means if $x \le y$, then $f(y) \le f(x)$
 i.e., $f(x) \ge f(y)$

Let's look at an (easy) inference in algebra

Which is bigger,
$$-(7+2^{-3})$$
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$$\frac{\frac{3<4}{-4<-3}-x \text{ is antitone}}{\frac{2^{-4}<2^{-3}}{2^{-4}<2^{-3}}} \frac{2^x \text{ is monotone}}{7+x^{-4}<7+2^{-3}} \frac{7+x \text{ is monotone}}{-(7+2^{-3})<-(7+2^{-4})} -x \text{ is antitone}}$$

Occasionally in this week's lectures, I'll use

blue for syntax, and red for semantics.

Another way to frame this problem

$$f(x,y^{\uparrow}) = -(x+2^{-y})$$

LET'S LOOK AT A PARALLEL INFERENCE IN LANGUAGE

Background: skunks ≤ mammals.

What do you think about this one?

All skunks are mammals

All who fear all who respect all skunks fear all who respect all mammals

Based only on our assumption, which set is bigger?

those who fear all who respect all skunks

or

those who fear all who respect all mammals

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skunks < mammals

respect all mammals \leq respect all skunks λx .respect all x is antitone fear all who respect all skunks \leq fear all who respect all mammals λx fear all x is antitone

> respect all x^{\downarrow} fear all who respect all x^{\uparrow}

Day-by-day plan for this course

I have arranged the course material in a number of units:

- overview + examples (today, done) + objections
- the simplest logic in the world (today)
- ▶ all + verbs + relative clauses (Tuesday)
- negation, definite descriptions, sizes of sets (Tuesday/Wednesday)
- ▶ basics on monotonicity ↑ and ↓, and connections to categorial grammar and semantics (Wednesday/Thursday)
- implementations and experience with language models (Friday)

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I can't cover everything without rushing, so there will be times when I mainly mention results without proofs.

There are also some worksheets and/or suggested homework exercises

I like talking to you on the course topics.

OBJECTIONS TO THE PROGRAM OF NATURAL LOGIC

Most natural language phenomena are not addressed:

ANYTHING "PRAGMATIC"

VAGUENESS, INTENT OF SPEAKERS, POETIC LANGUAGE

I agree with this objection!

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TO DO LOGIC FULLY, WE NEED RESOURCES TO HANDLE THE WORST-POSSIBLE PHENOMENA

I don't agree with this; see below.

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Quine, from Word and Object:

IF WE WERE TO DEVISE A LOGIC OF ORDINARY LANGUAGE FOR DIRECT USE ON SENTENCES AS THEY COME, WE WOULD HAVE TO COMPLICATE OUR RULES OF INFERENCE IN SUNDRY UNILLUMINATING WAYS.

This is something we'll talk about throughout the week.

SLOGAN: TREAT "EVERYDAY INFERENCE" IN LIGHT SYSTEMS

You decide

Consider three activites:

- A mathematics: prove the Pythagorean Theorem, $a^2 + b^2 = c^2$.
- B syntax: parse John knows his mother saw him at her house.
- C semantics: tell whether a reader of the UW Fight Song should infer that the speaker wants the Carnegie team to win.

A: mathematics B: syntax

Where would you put C: semantics?

THE UW FIGHT SONG

Bow Down to Washington, Bow Down to Washington. Mighty are the men who wear the Purple and the Gold, Joyfully we welcome them within the Victor's fold. We will carve our name in the Hall of Fame, To preserve the memory of our Devotion.

So, heaven help the foes of Washington,
They're trembling at the feet of mighty Washington.
Our boys are there with bells, their fighting blood excels,
It's harder to push them over the lines than pass the Dardanelles.
So Victory's the cry of Washington
Our leather lungs together with a Rah! Rah! Rah!
And o'er the land, the loyal band
Will sing the glory of Washington forever!

THE POINT

You decide

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A: mathematics B: syntax

Where would you put C: semantics?

My view: B and C go together; A doesn't belong

We should not reduce formal linguistic inference to formal inference in standard logical systems. (Also, we really can't carry out that reduction, for several different reasons.)