







*ab*  
*ba*

*aaabbb*  
*abb*

*aab*  
*baab*

*ba*  
*baa*

*abab*  
*ababaa*

*aa*  
*a*

*abab*  
*ababaa*

*aaabbb*  
*abb*

*aab*  
*baab*

*ba*  
*baa*

*ab*  
*ba*

*ab*  
*ba*

*aa*  
*a*

|        |        |      |     |    |    |    |
|--------|--------|------|-----|----|----|----|
| abab   | aaabbb | aab  | ba  | ab | ab | aa |
| ababaa | bb     | baab | baa | ba | ba | a  |

|        |        |      |     |    |    |    |
|--------|--------|------|-----|----|----|----|
| abab   | aaabbb | aab  | ba  | ab | ab | aa |
| ababaa | bb     | baab | baa | ba | ba | a  |



a  
ab

baa  
aab

then  $f(I)$  would be the *PCP* instance

$a^*$   
 $*a^*b$

$b^*a^*a^*$   
 $*a^*a^*b$

$*a^*$   
 $*a^*b^*$

\$  
\*\$

|                |   |   |              |
|----------------|---|---|--------------|
| start          | 1 | # | $K + 1$ halt |
| start <i>1</i> | 1 | # | halt         |

|     |         |
|-----|---------|
| $i$ |         |
| $1$ | $i + 1$ |

|      |         |
|------|---------|
| $i$  |         |
| $\#$ | $i + 1$ |

$$\begin{array}{c} i \\ i+k \end{array}$$

$$\begin{array}{c} i \\ i-k \end{array}$$

|         |             |              |
|---------|-------------|--------------|
| $i$     | $i \quad 1$ | $i \quad \#$ |
| $i + 1$ | $i + 2$     | $i + 3$      |

|         |   |   |        |     |
|---------|---|---|--------|-----|
| start   | 1 | # | 2 halt | 1   |
| start 1 | 1 | # | halt   | 1 2 |

|         |   |   |               |     |     |
|---------|---|---|---------------|-----|-----|
| start   | 1 | # | 3 <i>halt</i> | 1   | 2   |
| start 1 | 1 | # | <i>halt</i>   | 1 2 | # 3 |

|         |     |     |        |
|---------|-----|-----|--------|
| start   | 1   | #   | 7 halt |
| start 1 | 1   | #   | halt   |
| 1       | 2   |     |        |
| 1 2     | # 3 |     |        |
| 4       | 5   | 6   |        |
| 7       | 3   | 3   |        |
| 3       | 3 1 | 3 # |        |
| 4       | 5   | 6   |        |











|         |     |        |     |   |   |
|---------|-----|--------|-----|---|---|
| start   | 1   | 1      | 2   | 1 | # |
| start 1 | 1 2 | 1      | # 3 | 1 | # |
| 3 1     | #   | 5      | #   |   |   |
| 5       | #   | 3      | #   |   |   |
| 3 #     | 6   |        |     |   |   |
| 6       | 3   |        |     |   |   |
| 3       | 4   | 7 halt |     |   |   |
| 4       | 7   | halt   |     |   |   |

*start 11 21# 31# 5# 3# 6 3 4 7*

|         |
|---------|
| start   |
| start 1 |

|       |   |
|-------|---|
| start |   |
| start | 1 |

|   |   |
|---|---|
| 1 |   |
| 1 | 2 |



|       |   |
|-------|---|
| start |   |
| start | 1 |

|   |   |
|---|---|
| 1 |   |
| 1 | 2 |

|         |     |   |     |
|---------|-----|---|-----|
| start   | 1   | 1 | 2   |
| start 1 | 1 2 | 1 | # 3 |

|         |     |   |     |   |   |
|---------|-----|---|-----|---|---|
| start   | 1   | 1 | 2   | 1 | # |
| start 1 | 1 2 | 1 | # 3 | 1 | # |

|         |     |   |     |     |     |        |
|---------|-----|---|-----|-----|-----|--------|
| start   | 1   | 1 | 2   | 1   | #   |        |
| start 1 | 1 2 | 1 | # 3 | 1   | #   |        |
| 3       | 1   | # | #   | 4 1 | #   | #      |
| # 4     | 1   | # | #   | 3   | #   | #      |
| 3       | #   | # | #   |     |     |        |
| # 4     | #   | # | #   |     |     |        |
| 4 #     | #   | # | 4 # | #   | 4 # | 4 halt |
| 4       | #   | # | 4   | #   | 4   | halt   |

|     |     |         |
|-----|-----|---------|
|     | ?   | $k$     |
| ... | $k$ | $N + 1$ |

|     |     |         |                     |
|-----|-----|---------|---------------------|
|     | ?   | $k$     | $N + 1$ <i>halt</i> |
| ... | $k$ | $N + 1$ | <i>halt</i>         |

|           |
|-----------|
| $n$       |
| $1 \ n+1$ |

|       |     |
|-------|-----|
| $n$   | $1$ |
| $n+2$ |     |

|       |      |
|-------|------|
| $n$   | $\#$ |
| $n+3$ |      |

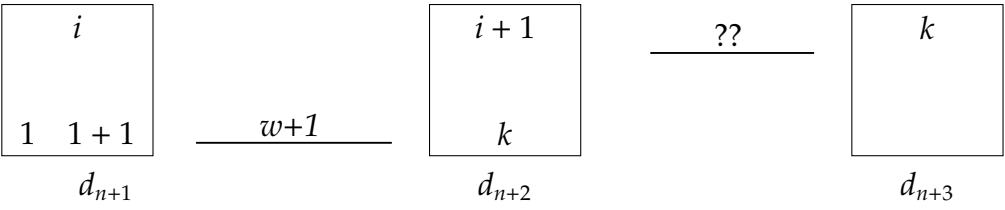














|       |     |
|-------|-----|
| $i$   | $1$ |
| $i+2$ |     |

|       |      |
|-------|------|
| $i$   | $\#$ |
| $i+3$ |      |



|         |
|---------|
| $i$     |
| $i + 1$ |





|                |   |   |
|----------------|---|---|
| start          | 1 | # |
| start <i>1</i> | 1 | # |

|     |         |
|-----|---------|
| $i$ |         |
| $1$ | $i + 1$ |

|      |         |
|------|---------|
| $i$  |         |
| $\#$ | $i + 1$ |

$$\begin{array}{c} i \\ i+k \end{array}$$

$$\begin{array}{c} i \\ i-k \end{array}$$

|         |
|---------|
| $i$     |
| $i + 1$ |

|             |
|-------------|
| $i \quad 1$ |
| $i + 2$     |

|              |
|--------------|
| $i \quad \#$ |
| $i + 3$      |



|         |     |   |     |     |     |        |
|---------|-----|---|-----|-----|-----|--------|
| start   | 1   | 1 | 2   | 1   | #   |        |
| start 1 | 1 2 | 1 | # 3 | 1   | #   |        |
| 3       | 1   | # | #   | 4 1 | #   | #      |
| # 4     | 1   | # | #   | 3   | #   | #      |
| 3       | #   | # | #   |     |     |        |
| # 4     | #   | # | #   |     |     |        |
| 4 #     | #   | # | 4 # | #   | 4 # | 4 halt |
| 4       | #   | # | 4   | #   | 4   | halt   |

|           |
|-----------|
| $n$       |
| $1 \ n+1$ |



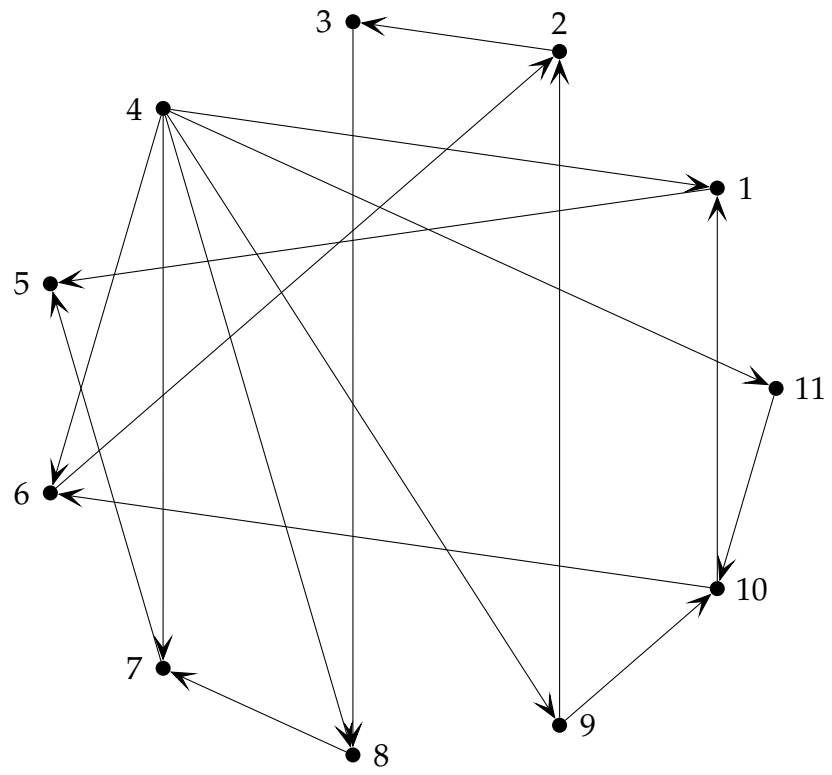
|           |           |
|-----------|-----------|
| $n$ 1     | $n$ #     |
| next(n+2) | next(n+3) |

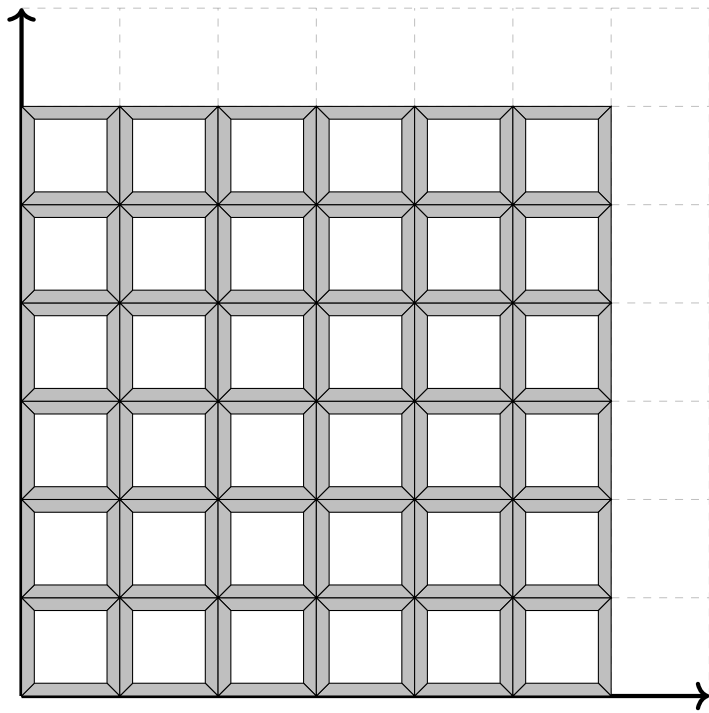


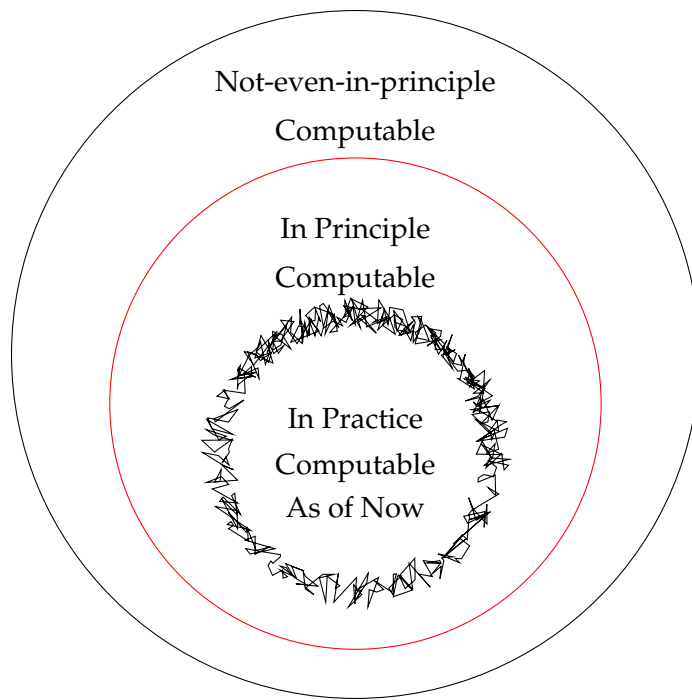
|     |   |   |       |   |   |   |       |
|-----|---|---|-------|---|---|---|-------|
|     | # | 1 | $x$   | # | 1 | 1 | $y$   |
| $x$ | # | 1 | 1 $y$ | # | 1 | 1 | # $z$ |



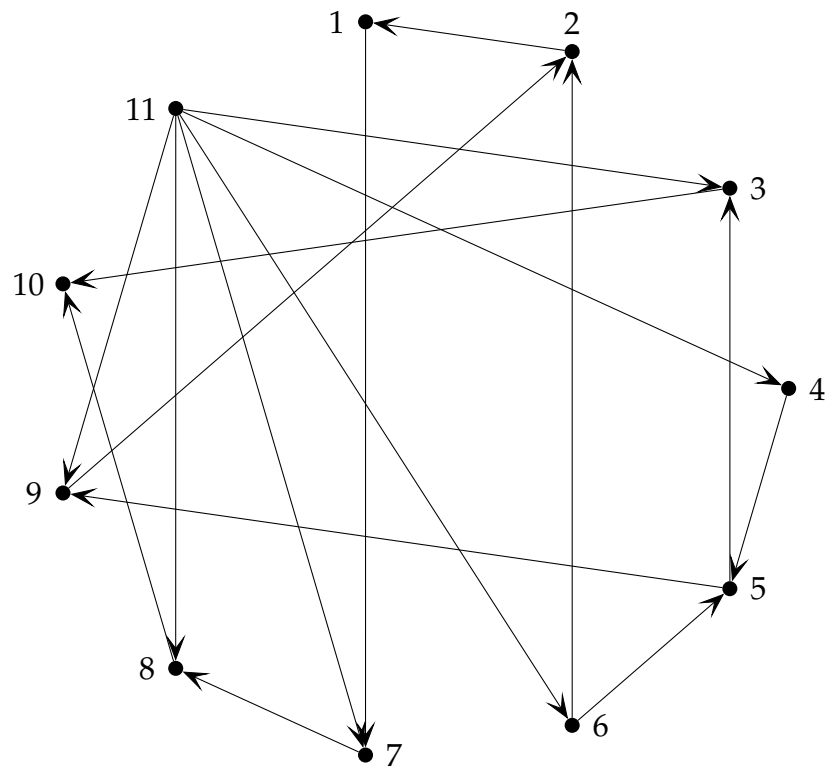
|     |   |   |       |   |       |
|-----|---|---|-------|---|-------|
|     | # | 1 | $x$ # |   | $y$   |
| $x$ | # | 1 | $y$   | ? | 1 $z$ |











*a*  
*ab*

*baa*  
*aab*

|             |
|-------------|
| $a$ ■       |
| ■ $a$ ■ $b$ |

|                   |
|-------------------|
| $b$ ■ $a$ ■ $a$ ■ |
| ■ $a$ ■ $a$ ■ $b$ |

|                  |
|------------------|
| ■ $a$ ■ $\$$     |
| ■ $a$ ■ $b$ $\$$ |

|        |
|--------|
| $\$$   |
| ■ $\$$ |