# Simulation5

**Mengqi Liu**

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## Recap

- $N$: number of samples one time.
- $M$: number of bins.
- H0: $X \perp\!\!\!\perp Y \mid Z$, H1: $X \not\perp\!\!\!\perp Y \mid Z$
- Methods: ($\tilde{Z}$ is the discretized $Z$, and the data belonging to the same group share the same $\tilde{Z}$.)
  - "Linear_reg_y": regress $Y$ on $X, \tilde{Z}$ and take the *absolute* coefficient of $X$ as the test statistic.
  - "Linear_reg_x": regress $X$ on $Y, \tilde{Z}$ and take the *absolute* coefficient of $Y$ as the test statistic.
  - "Double_reg": regress $Y$ on $\tilde{Z}$ and regress $X$ on $\tilde{Z}$ separately. Take the *absolute* correlation between residuals from two linear regressions as the test statistic.
  - "Linear_reg_y_z": regress $Y$ on $X, Z$ and take the *absolute* coefficient of $X$ as the test statistic.
  - "Linear_reg_x_z": regress $X$ on $Y, Z$ and take the *absolute* coefficient of $Y$ as the test statistic.
  - "Double_reg_z": regress $Y$ on $Z$ and regress $X$ on $Z$ separately. Take the *absolute* correlation between residuals from two linear regressions as the test statistic.

- $\alpha = 0.05$

- $X = f_x(Z) + \epsilon$, $Y = f_y(Z) + \epsilon$

- Noise $\epsilon$:

  - various $a$
  - H0:

    - normal: $N(Z, a)$
    - skewed_normal: $N(Z, a)$
  - H1:

    - normal: $N\left([0,0], \begin{pmatrix} a_1 & cor \cdot \sqrt{a_1 a_2} \\ cor \cdot \sqrt{a_1 a_2} & a_2 \end{pmatrix}\right)$
    - skewed_normal: $N\left([0,0], \begin{pmatrix} a_1 & cor \cdot \sqrt{a_1 a_2} \\ cor \cdot \sqrt{a_1 a_2} & a_2 \end{pmatrix}\right)$, skewness = $[5, -5]$
- $N = 100, Z \sim \text{Unif}([0, 10))$, $M \in \{2, 5, 10, 16, 25, 50\}$.

## Gains

- When $X|Z$ and $Y|Z$ are both smooth, all methods have valid type-I error and notable power whichever variable is permutated (experiment 1). Moreover, there's no distinct difference in power between using $Z$ and

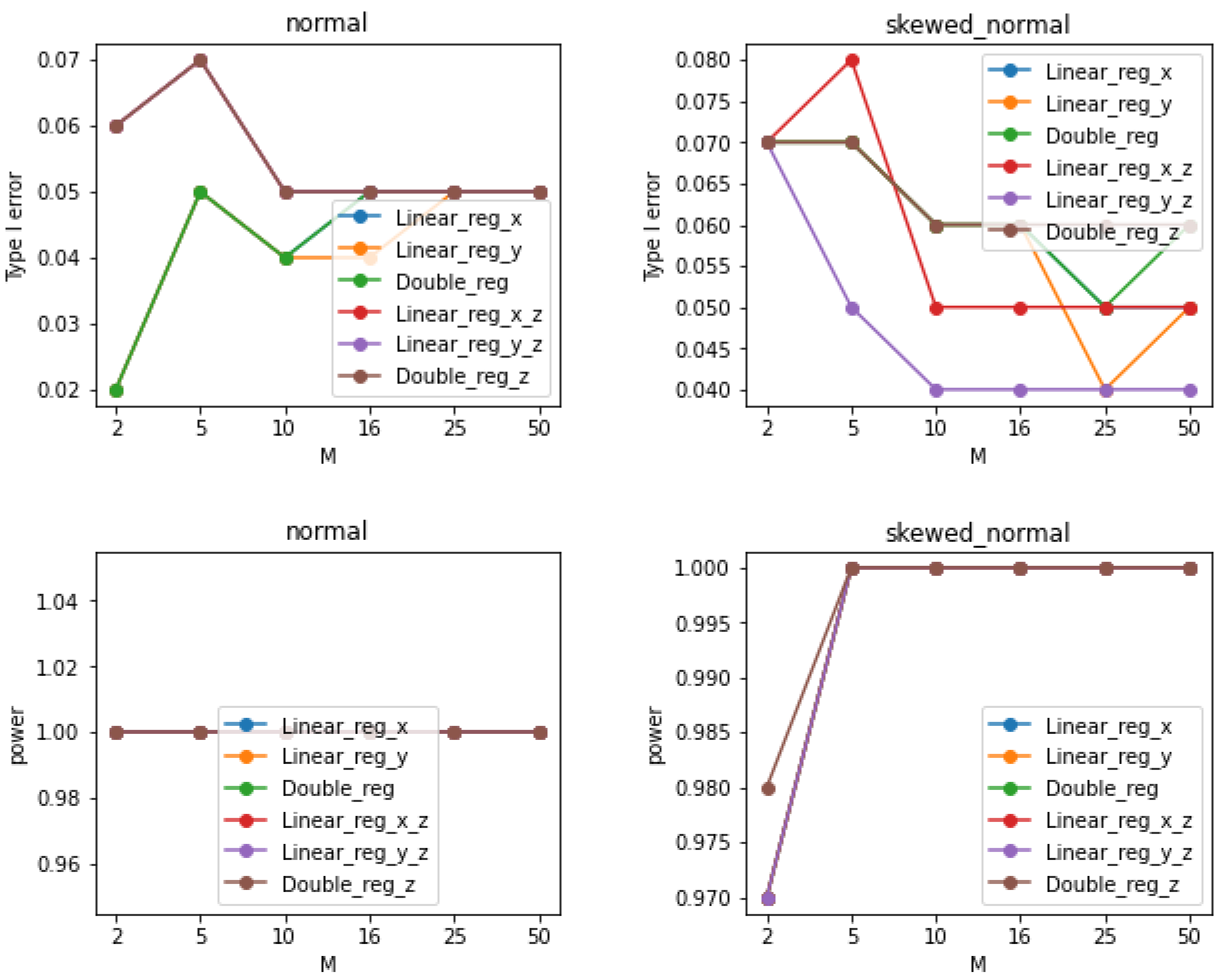
$\tilde{Z}$ (experiment 2). But "Double_reg" and "Double_reg_z" own higher power than methods using one-sided regression in some settings (experiment 3).

- When $X|Z$ and $Y|Z$ are neither smooth, all methods fail in controlling type-I error while "Double_reg" (double regression with $Z$) is barely acceptable if $f_x$ and $f_y$ are both linear in $Z$. (experiment 4&experiment 5)
- When $X|Z$ is smooth and $Y|Z$ is not, the choice of permutated variable is very important. All methods perform well if we permute the smooth one (X). If we permute the non-smooth one (Y), "Double_reg" and "Double_reg_z" can both work (experiment 7: $f_x$ and $f_y$ are non-linear in $Z$; experiment 9: $f_x$ is linear in $Z$ while $f_y$ is non-linear in $Z$) or both fail to control type-I error. (experiment 6&experiment 8: $f_x$ and $f_y$ are linear in $Z$)
- From all experiments, counter-intuitively, it seems that using $\tilde{Z}$ would be more aggressive (higher type-I and power) than using $Z$.

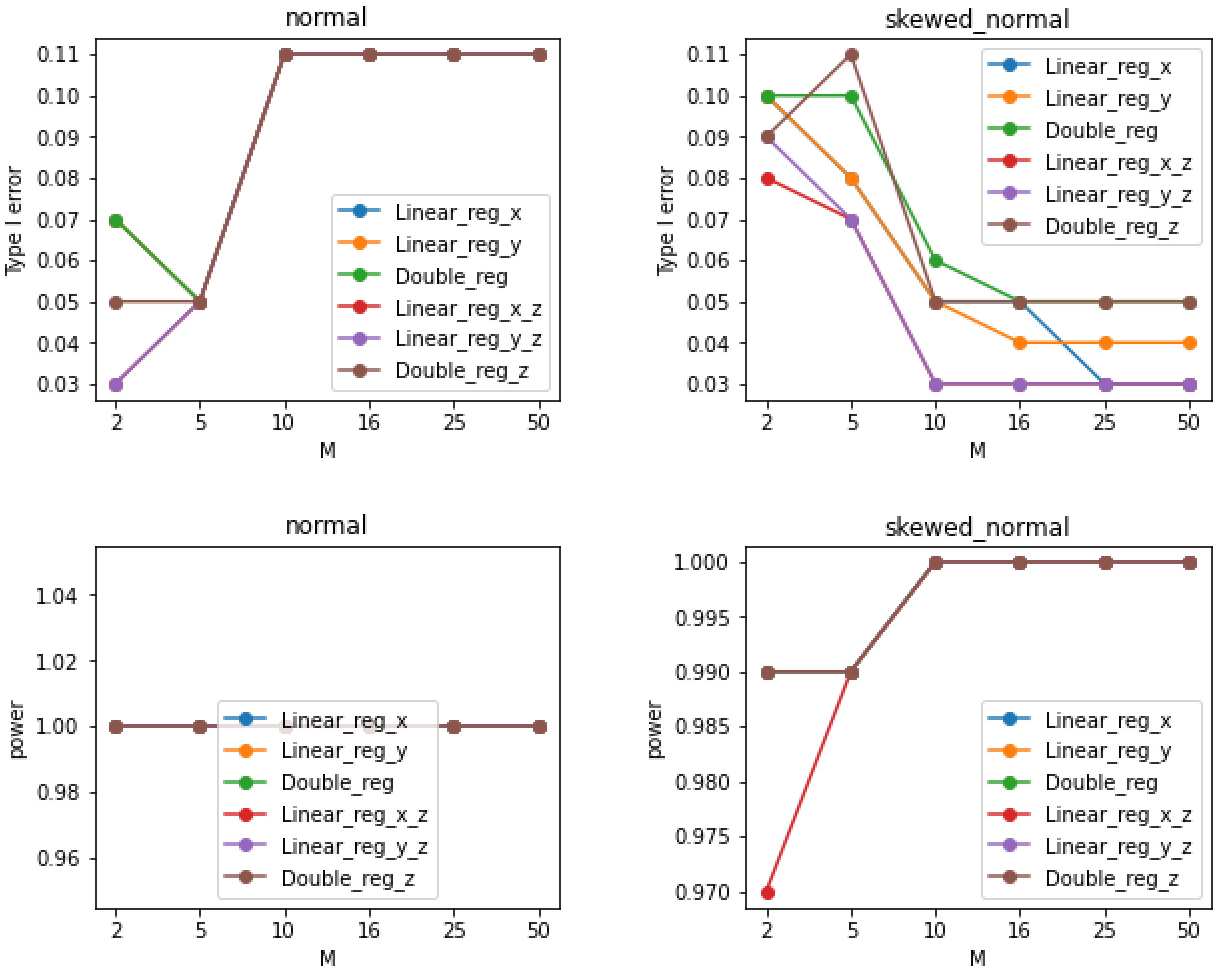
## experiment 1

$f_x(Z) = Z$, $f_y(Z) = Z$, $\epsilon_x \sim N(\cdot, 5)$, $\epsilon_y \sim N(\cdot, 5)$, $cor = 0.8$.

- permute $Y$:

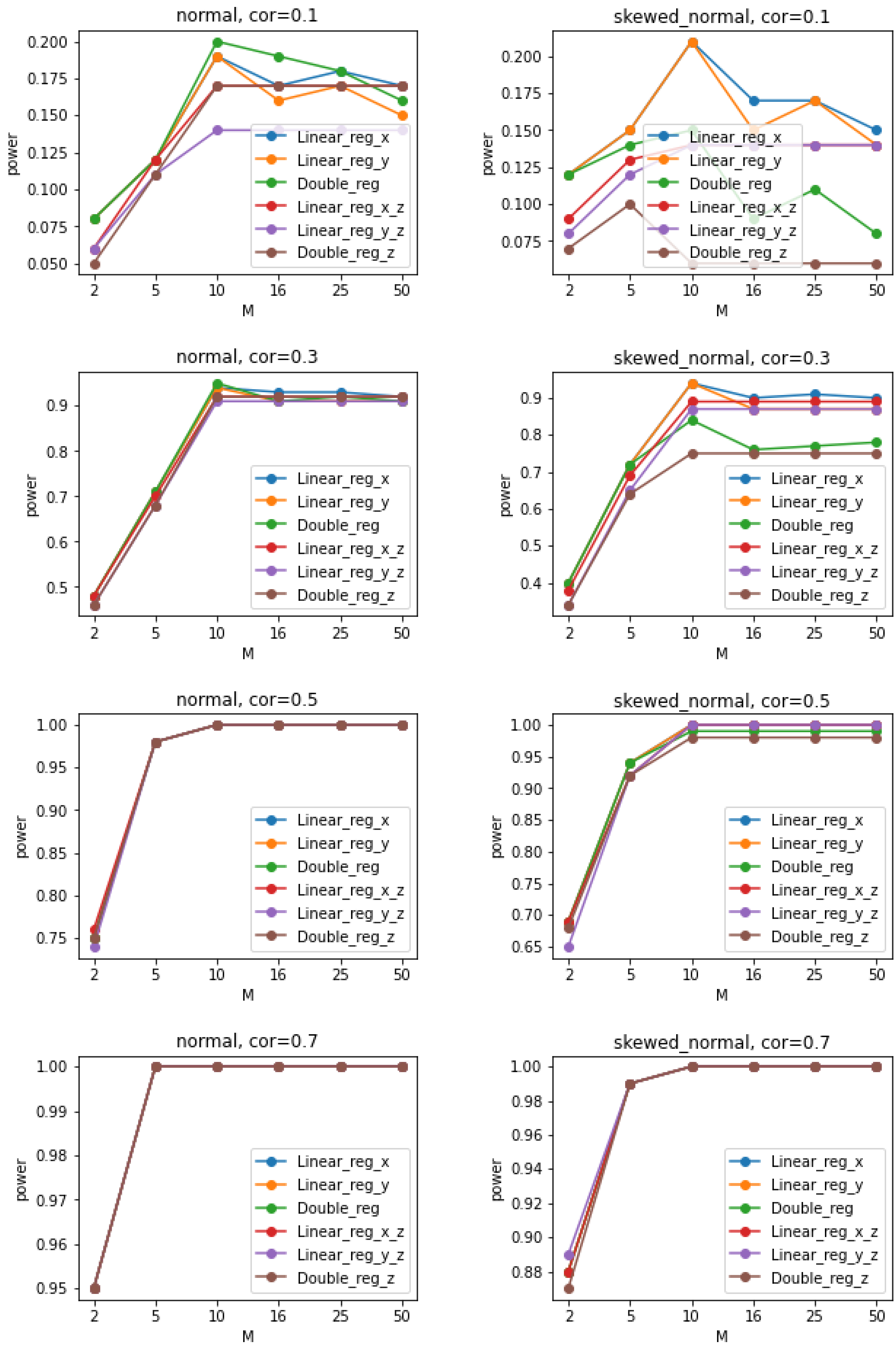



- permute $X$:



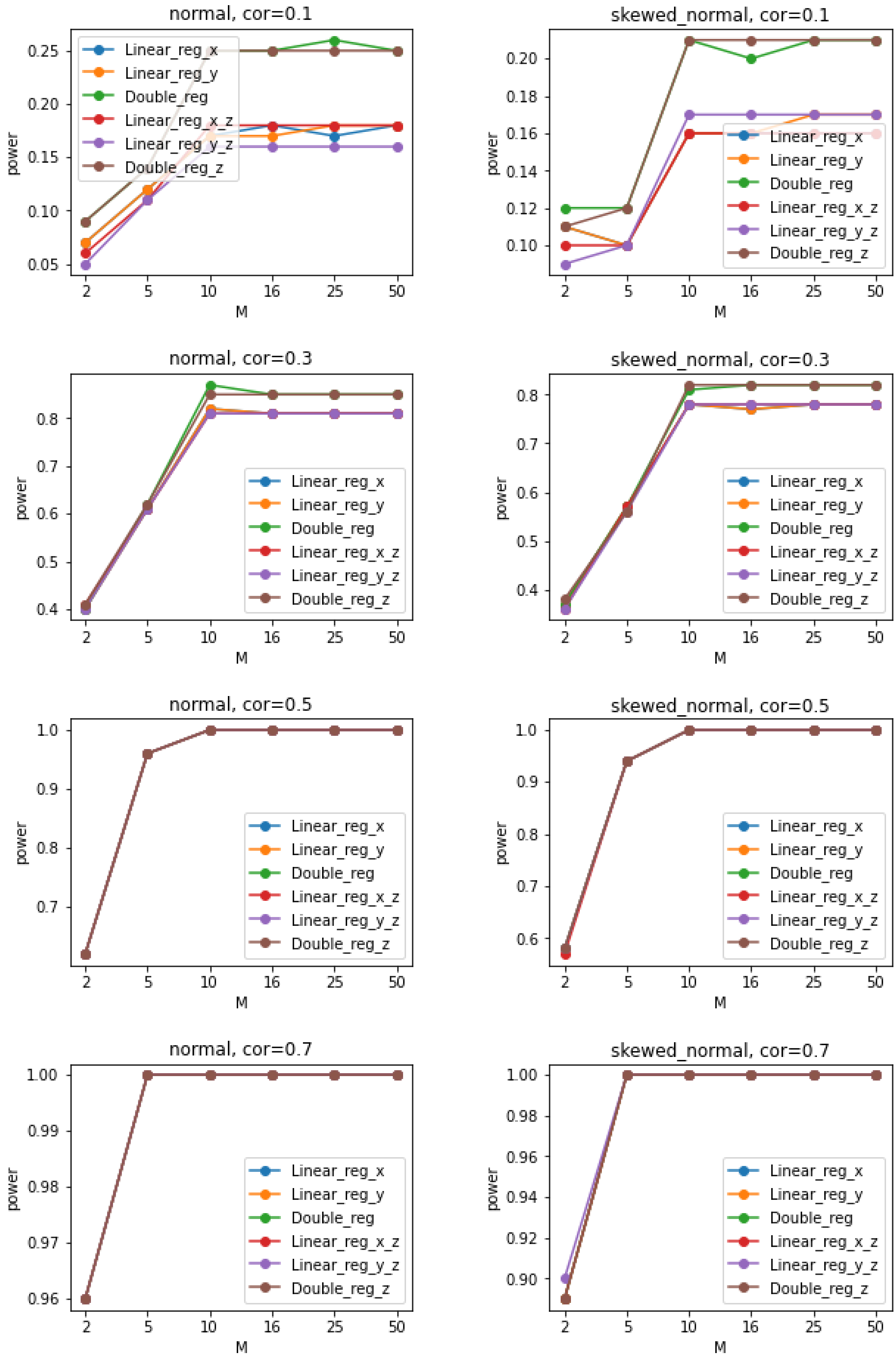

## experiment 2

$f_x(Z) = Z$, $f_y(Z) = Z$, $\epsilon_x \sim N(\cdot, 5)$, $\epsilon_y \sim N(\cdot, 5)$, $cor \in \{0.1, 0.3, 0.5, 0.7\}$ and permute $Y$.




## experiment 3

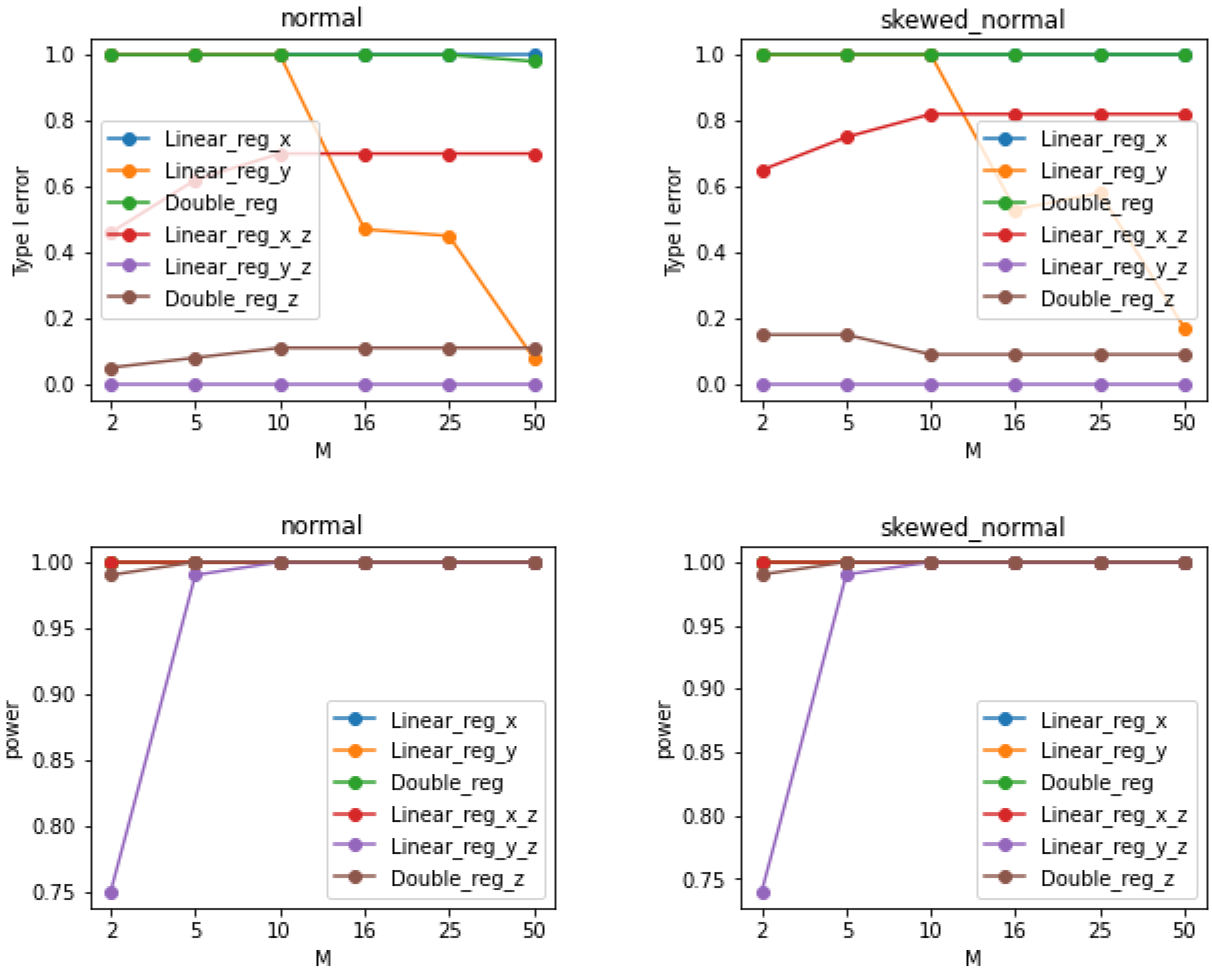
$f_x(Z) = \log(Z+1) + 2$, $f_y(Z) = 7 + \sqrt{Z}$, $\epsilon_x \sim N(\cdot, 5)$, $\epsilon_y \sim N(\cdot, 5)$, $cor \in \{0.1, 0.3, 0.5, 0.7\}$ and permute $Y$.



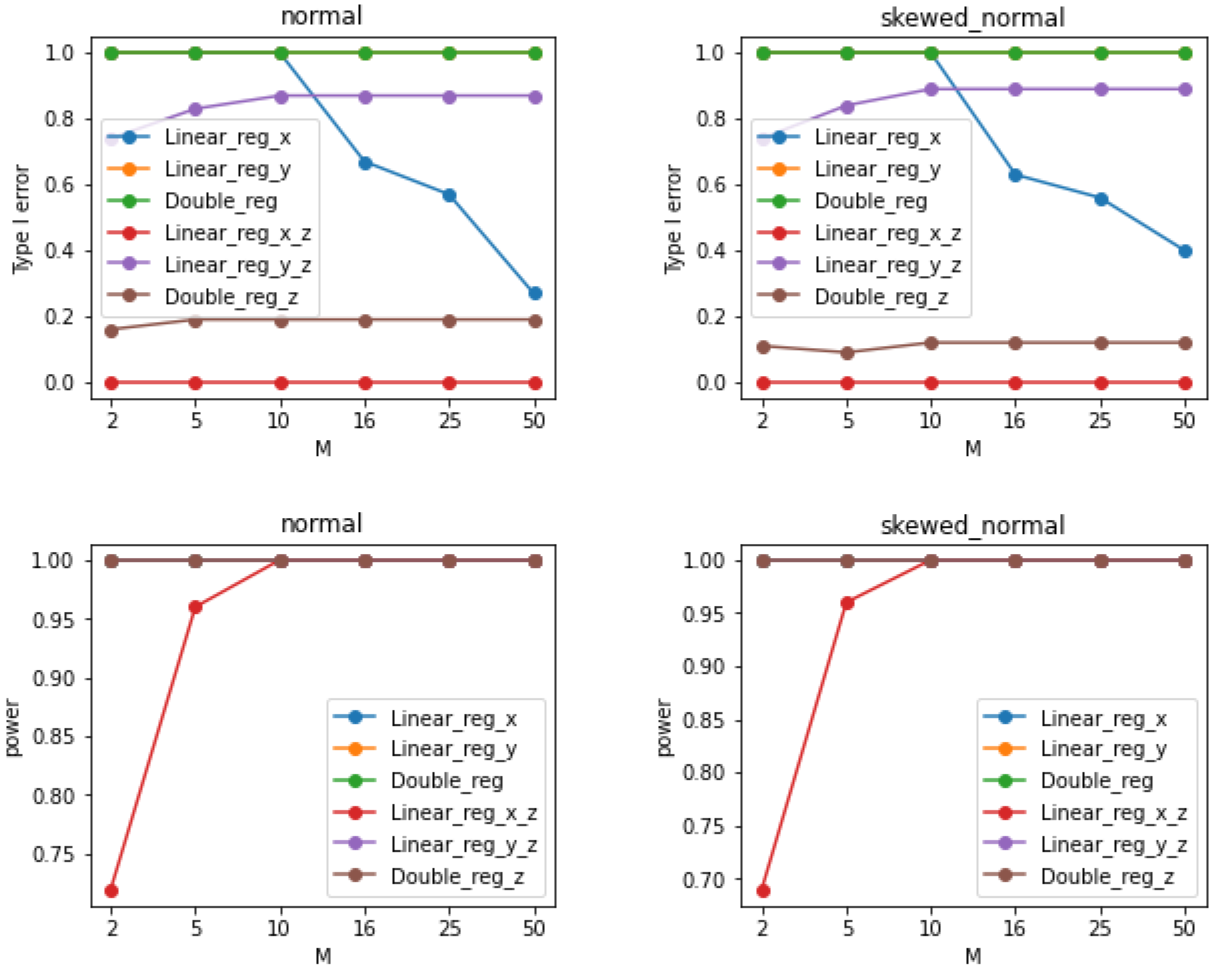

## experiment 4

$f_x(Z) = Z$, $f_y(Z) = Z$, $\epsilon_x \sim N(\cdot, 0.1)$, $\epsilon_y \sim N(\cdot, 0.1)$.

- permute $Y$:



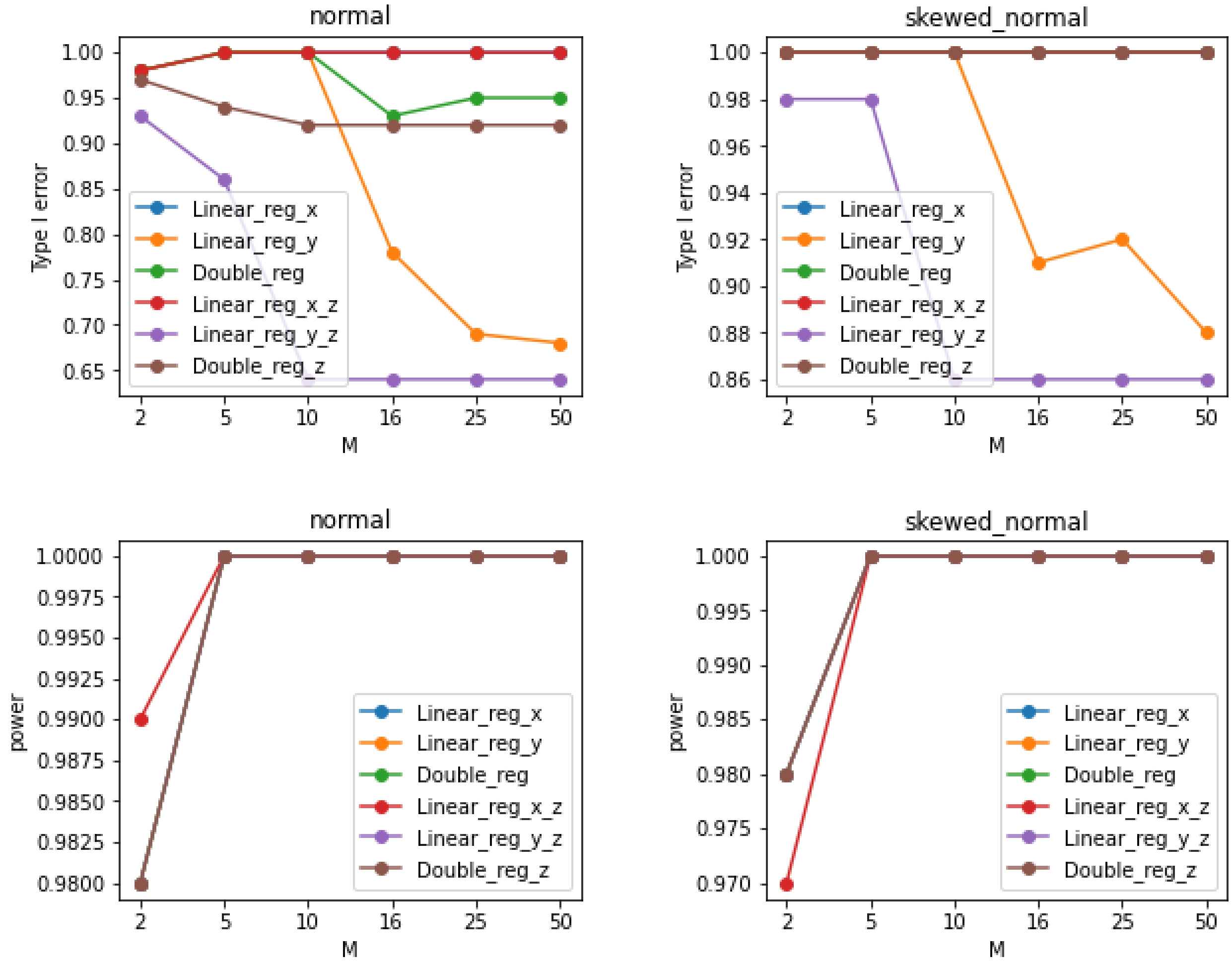

- permute $X$:



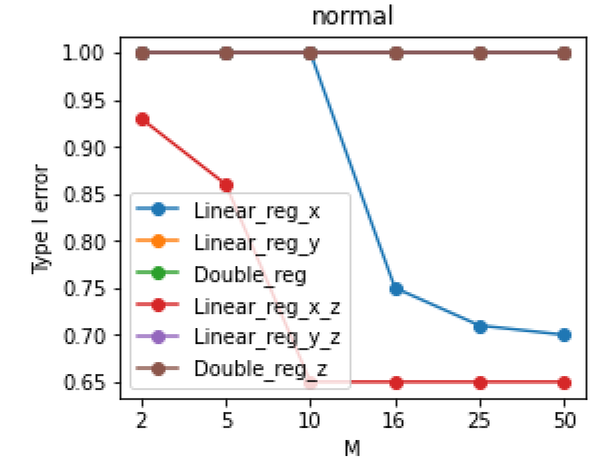

## experiment 5

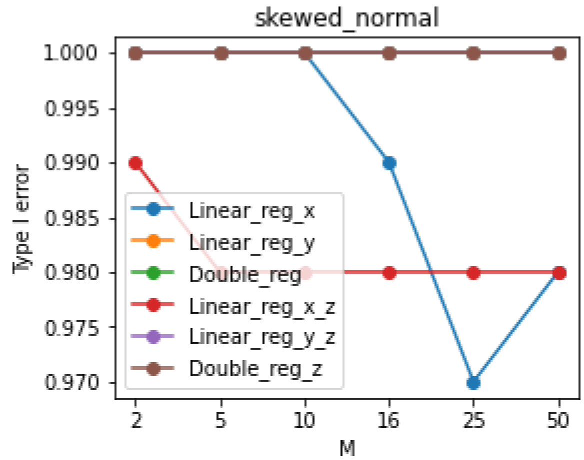
$f_x(Z) = \log(Z+1) + 2$, $f_y(Z) = 7 + Z^{\frac{1}{2}}$, $\epsilon_x \sim N(\cdot, 0.1)$, $\epsilon_y \sim N(\cdot, 0.1)$.

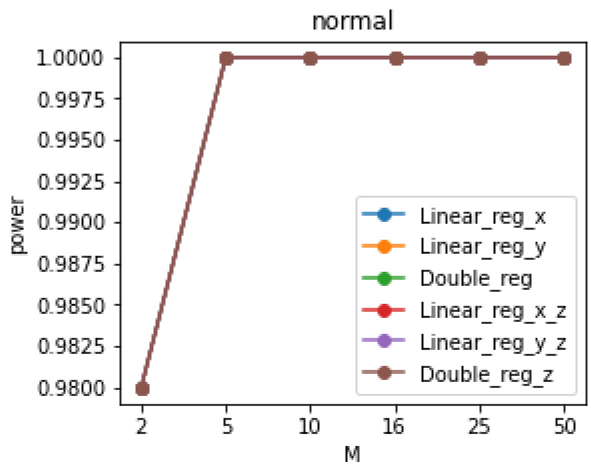
- permute $Y$:

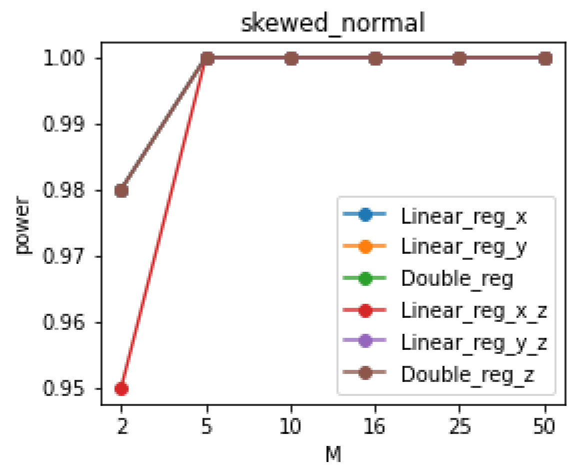

normal
skewed_normal
Type I error
power
M
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z

- permute $X$:



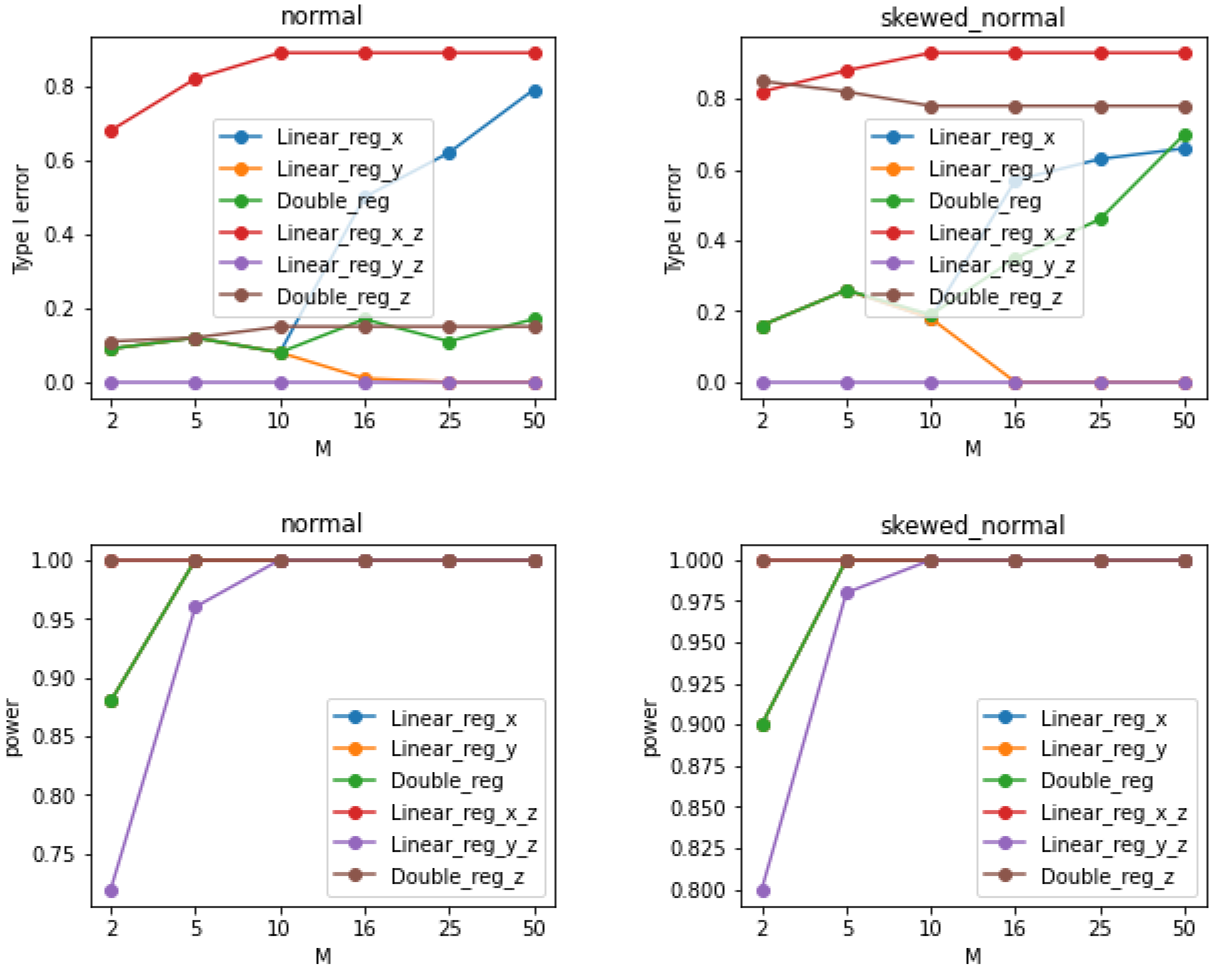



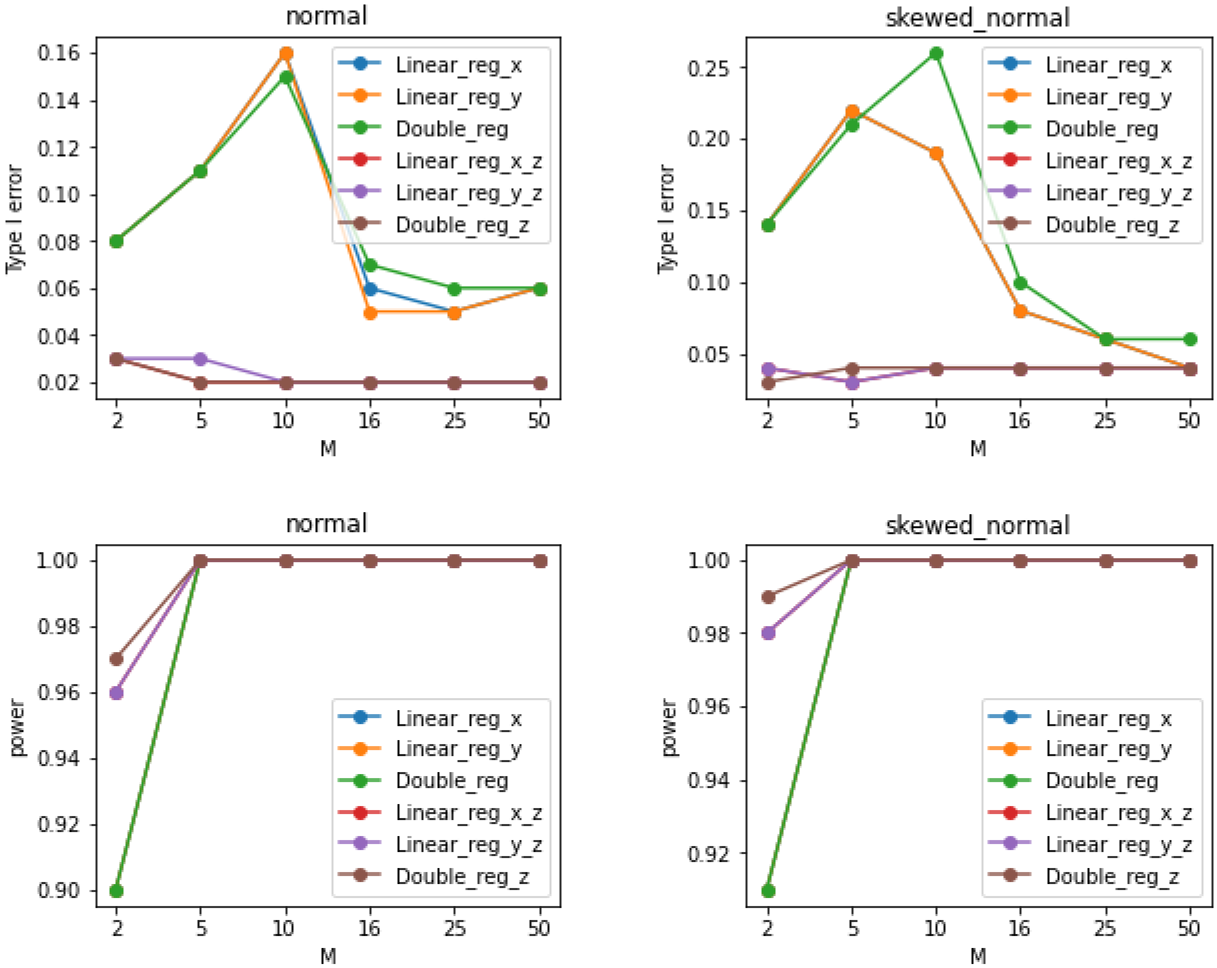

## experiment 6

$f_x(Z) = Z$, $f_y(Z) = Z$, $\epsilon_x \sim N(\cdot, 5)$, $\epsilon_y \sim N(\cdot, 0.1)$.

- permute $Y$:


normal
skewed_normal
Type I error
0.0
0.2
0.4
0.6
0.8
M
2
5
10
16
25
50
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z
power
0.75
0.80
0.85
0.90
0.95
1.00
0.800
0.825
0.850
0.875
0.900
0.925
0.950
0.975
1.000

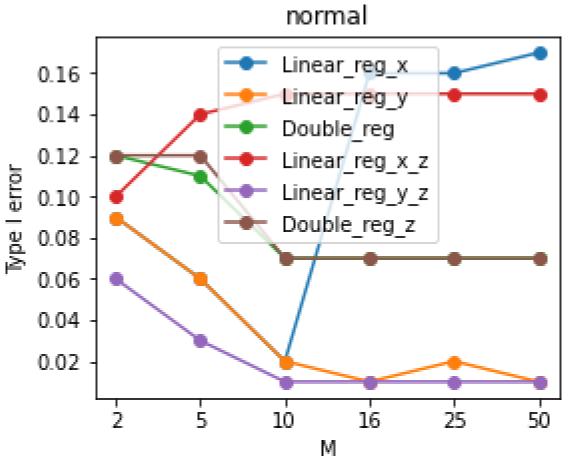
- permute $X$:

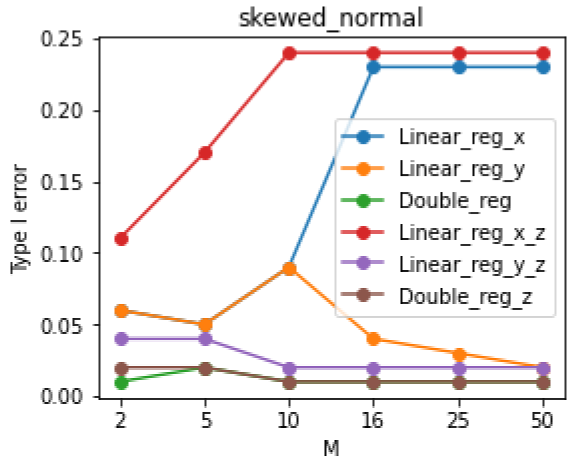


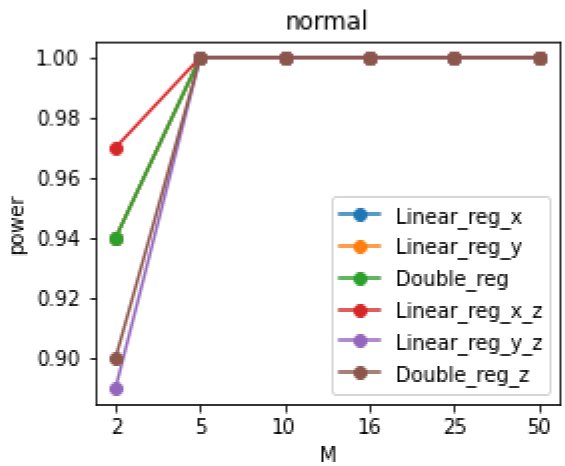

## experiment 7

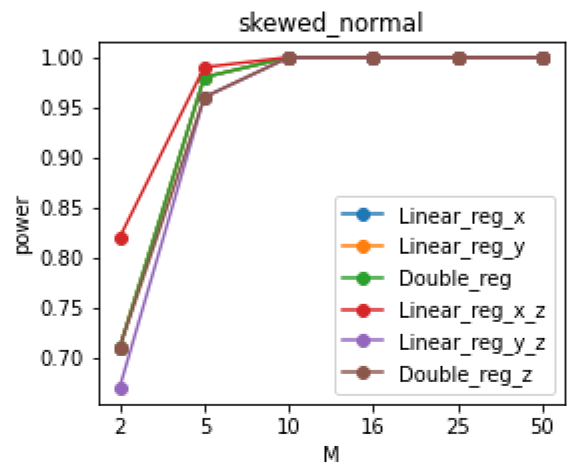
$f_x(Z) = \log(Z+1) + 2$, $f_y(Z) = 7 + \sqrt{Z}$, $\epsilon_x \sim N(\cdot, 5)$, $\epsilon_y \sim N(\cdot, 0.1)$.

- permute $Y$:

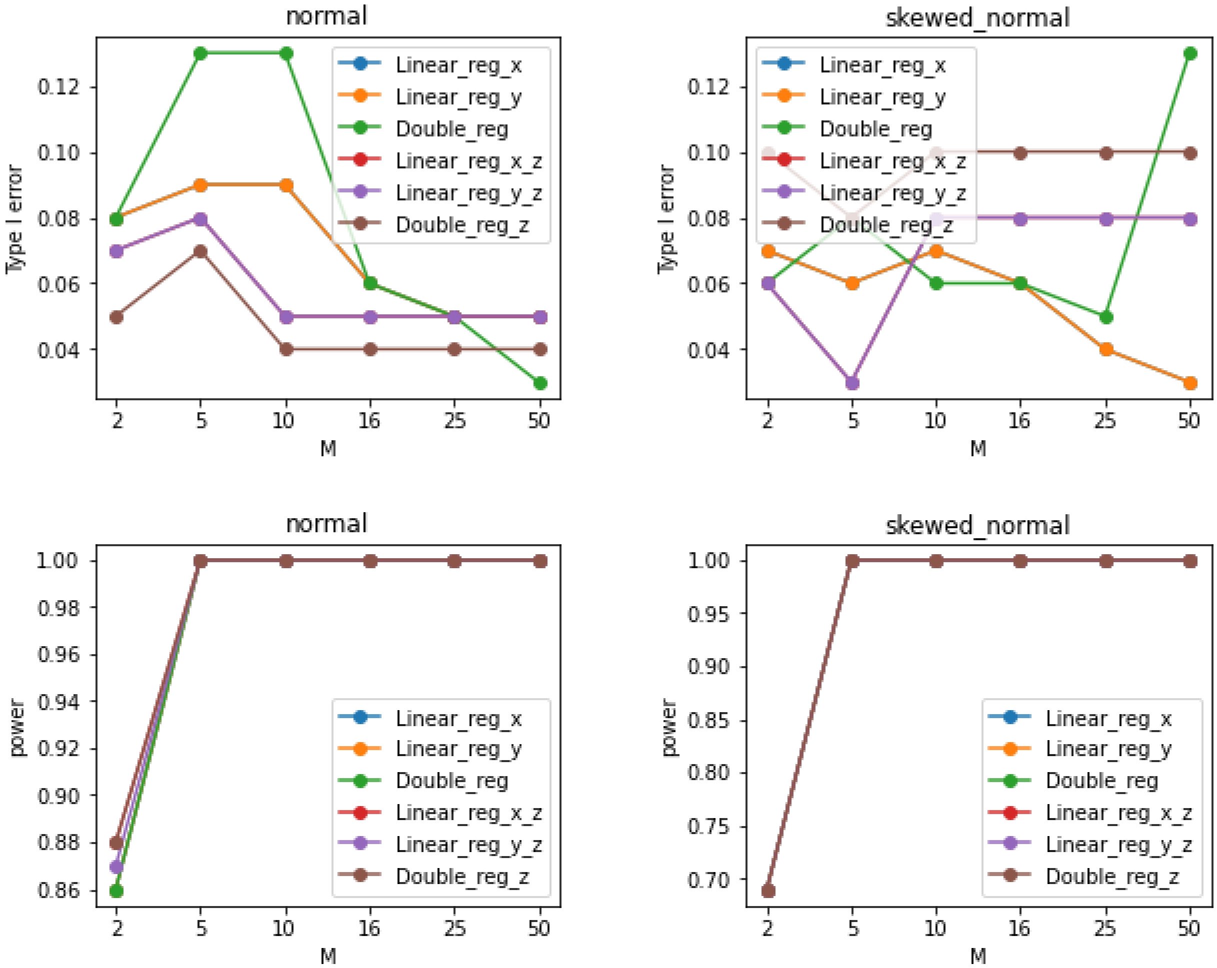

normal
Type I error
M
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z


skewed_normal
Type I error
M
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z


normal
power
M
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z


skewed_normal
power
M
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z

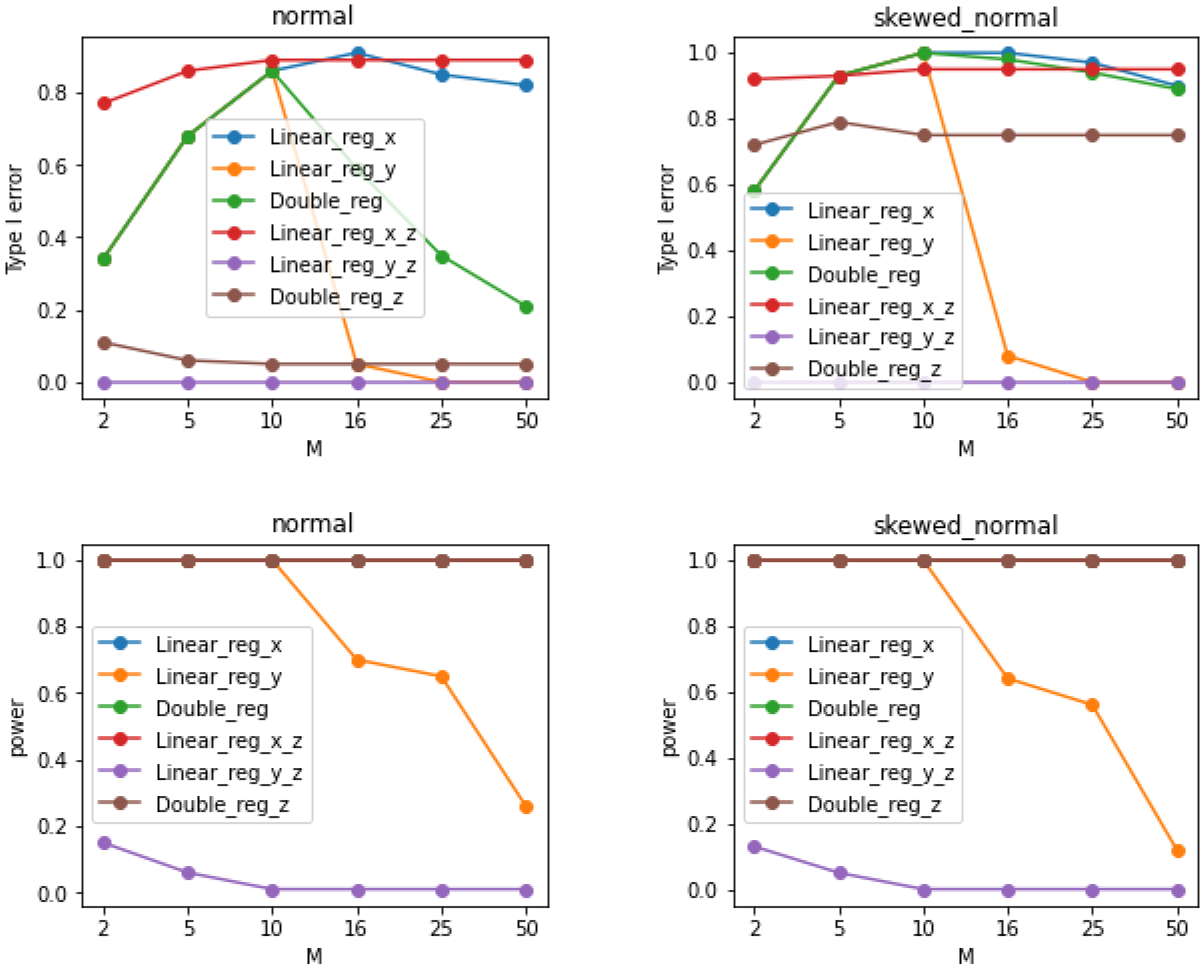
- permute $X$:



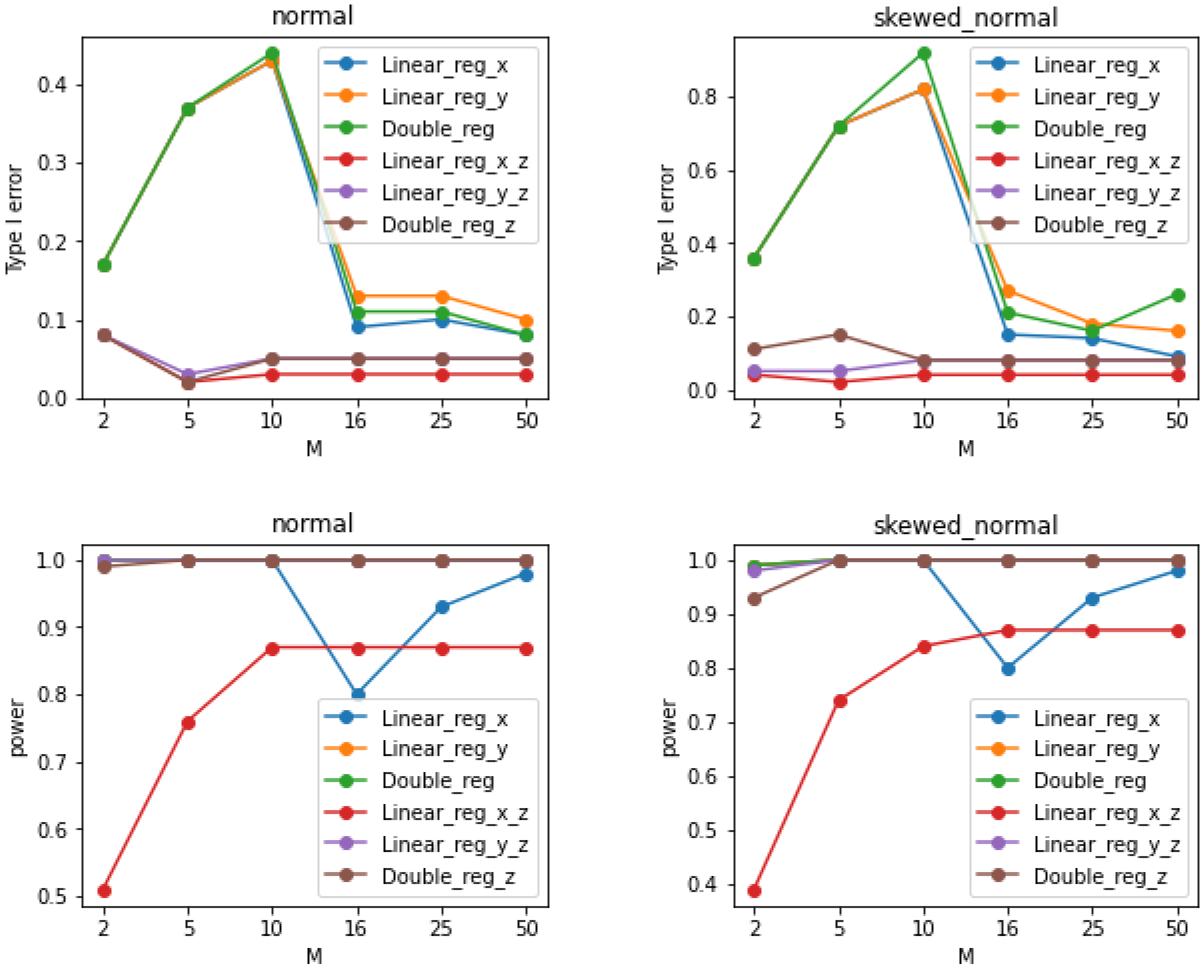

## experiment 8

$f_x(Z) = 5Z$, $f_y(Z) = 5Z$, $\epsilon_x \sim N(\cdot, 5)$, $\epsilon_y \sim N(\cdot, 0.1)$.

- permute $Y$:


normal
skewed_normal
Type I error
power
M
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z

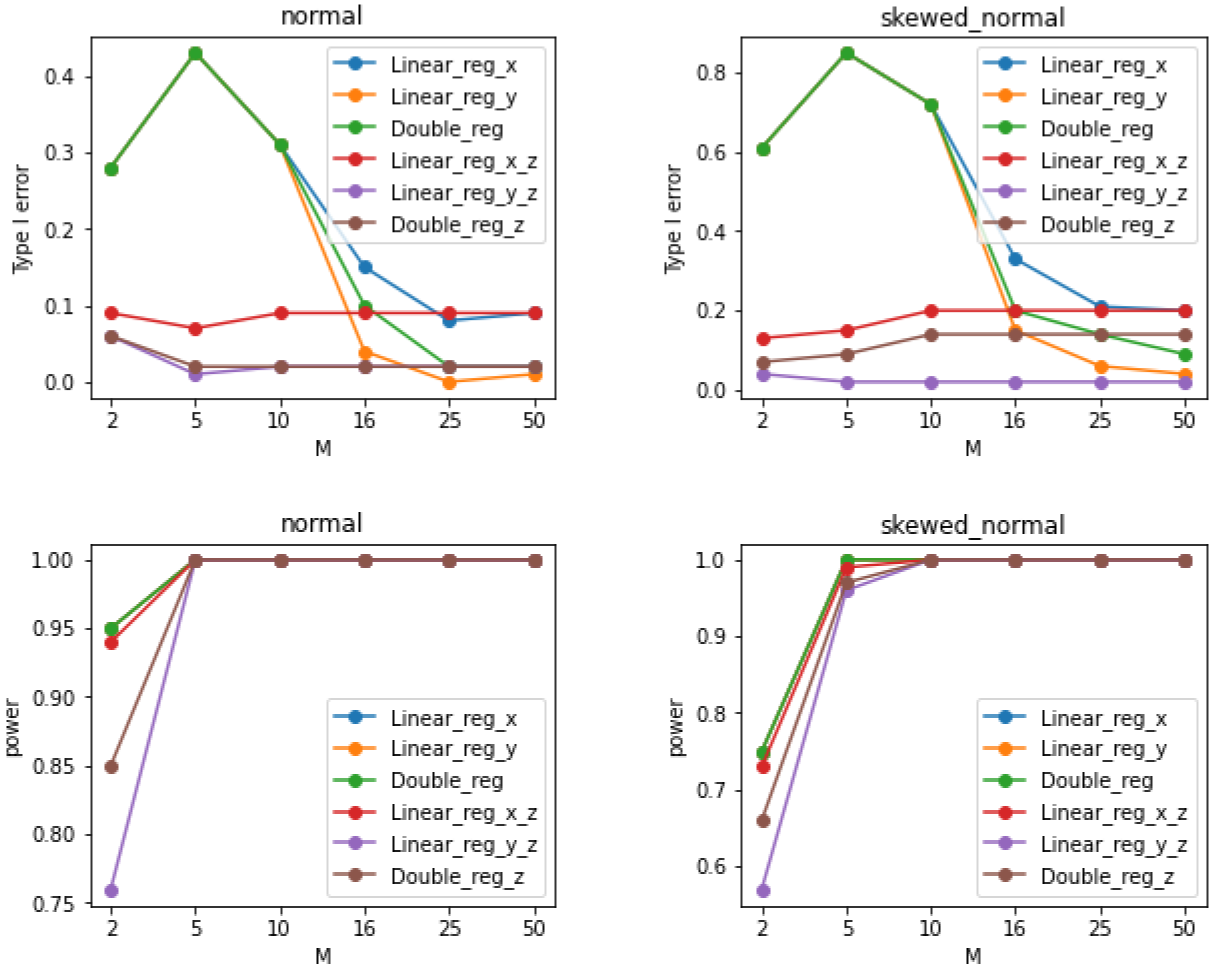
- permute $X$:



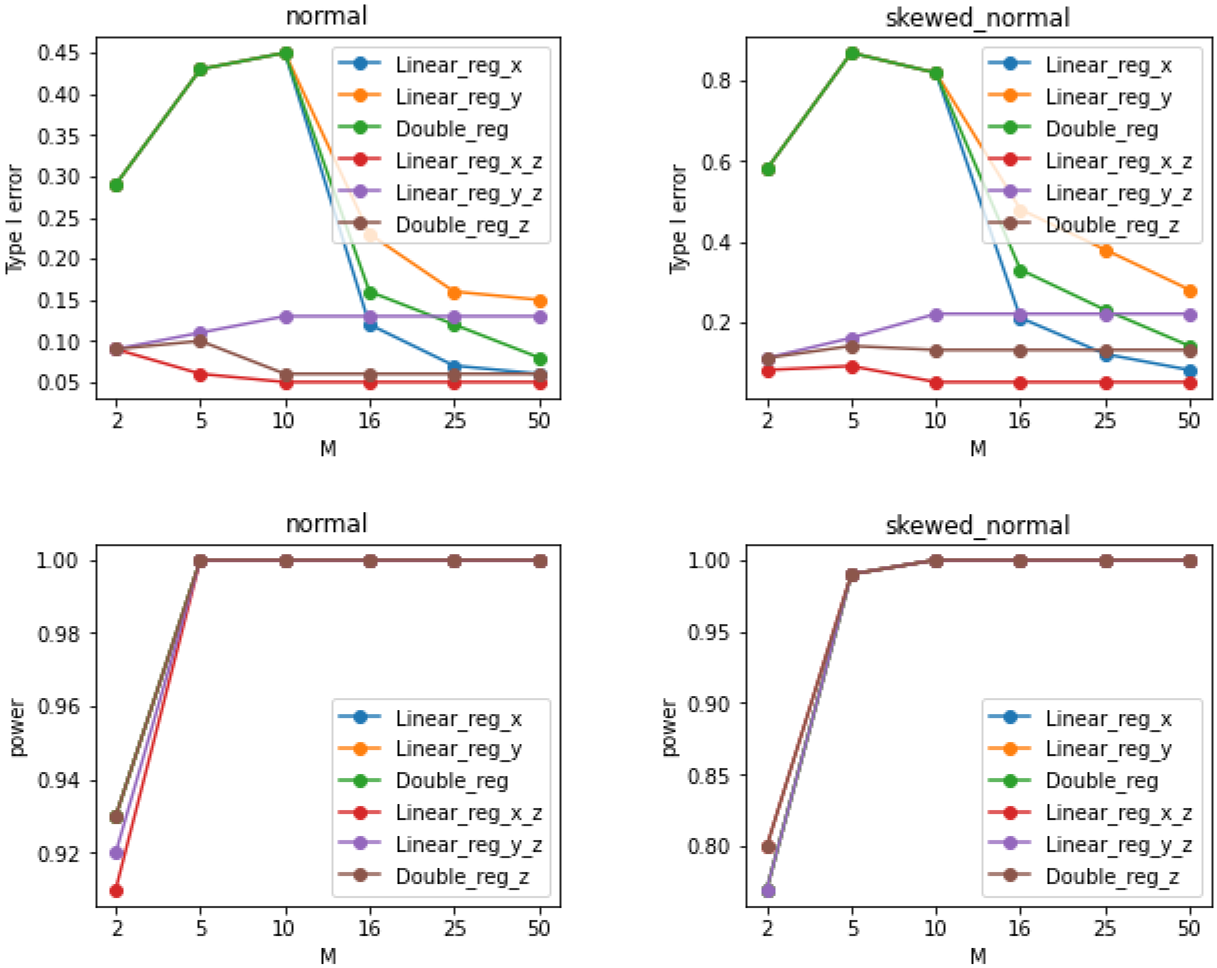

## experiment 9

$f_x(Z) = Z$, $f_y(Z) = 7 + \sqrt{Z}$, $\epsilon_x \sim N(\cdot, 5)$, $\epsilon_y \sim N(\cdot, 0.1)$.

- permute $Y$:


normal
Type I error
M
skewed_normal
Type I error
M
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z
normal
power
M
skewed_normal
power
M

- permute $X$:


normal
skewed_normal
Type I error
power
M
Linear_reg_x
Linear_reg_y
Double_reg
Linear_reg_x_z
Linear_reg_y_z
Double_reg_z