MR Image Reconstruction from under-sampled measurements using local and global sparse representations

**Abstract**

, from a single trainingimage (zero-filling image) under the defined observation model and reconstruct the image from the low frequency component of highly undersampled k-space.

The proposed method is evaluated in the numerical experiments on simulated phantom, on cardiac cine, on brain MR and on angiogram at random Cartesian sampling trajectory. The proposed method has a higher quality and a smaller NMSE compared to the CS methods. The improvements on reconstruction persist over a wide range of practical MR data, without any parameter tuning.

**Index terms**

***Compressed Sensing, , Dictionary learning, magnetic resonance imaging, local and global sparse representation***

**INTRODUCTION**

Magnetic resonance imaging (MRI)has becomea powerful noninvasive diagnostic imaging technique since its invention.However, MRI is a relatively slow imaging modality due to the physical or physiological constraints. To speed up the imaging process, the number of measurements required for reconstructionis reduced without degrading the image quality. Compressive sensing (CS), which can faithfully recover signals from under-sampled measurements, fits the problem of MRI and has become verypopular in recent years. Compressive sensing MRI (CSMRI) requires the imagebeing recovered has a sparse representation in some dictionaries, which can be categorizedin two ways. The first one, namely global sparse dictionary, can yield a sparse representation fortheimage globally, such as Fourier, wavelets, curvelets, SVDand so on.The second one, namely local sparse dictionaries, represents the patches of theimage sparsely,such as K-SVD and the one proposed in [].These dictionaries are learned from specific image patches and are usually over-complete, and provide sparse representationsfor all patches.Existingimaging modalitiestry to seek for better sparse representation in global or local dictionaries, but not in both.

In this work, we proposed a novel imaging model, which exploits both the local and global sparse structure of the MR images, to reconstruct the image faithfully. The imaging model is split into two sub-models,namely the local and global sparse model, to capture the overall sparse structure of the image. The local sparse modelattempts to represent the patches of the imagessparselyby learning an over-complete dictionaryusing K-SVD. On the other hand, the global sparse model represents the whole image sparsely using predefined or adaptive sparse transforms and is solved within the traditional CS framework. The proposed model is then evaluated using images of various structures, such as simulated phantom, MR cardiac, angiography and brain images. Several important parameters of the proposed model are evaluated and discussed.

**THEORY**

1. **Background**

Recently, patch-based sparsity is frequently used due to its capability in capturing the image details[] and has also found applications in medical imaging []. For example, Ravishankar et al [] reconstructed MR images from under-sampled k-space measurements by sparsifying the images using a patch-based dictionary learned by the K-SVD[]. Generally, the methods exploiting patch-based sparsity treat the image as an array of patches, and then seek a sparse representation of the image by learning an adaptive dictionary which represents each patch of the image sparsely.

However, the patch-based sparsity method enforces the patch-level sparsity only, ignoring the information of the image as a whole, and yields the blocky effect lowering the overall quality of the image as shown in Fig.(1c), which is reconstructed by DLMRI with non-overlapping patch size 10x10. DLMRI[] eliminates this effect by pixel averaging, that is, all the patches are reconstructed independently and then each pixel value is obtained by averaging contributions of patches covering it to form the final image. Obviously, DLMRI ignores the coherence between overlapping patches, and loses global structure information during image reconstruction.

On the other hand, the traditional CSMRI[] sparsifies the image as a whole using the global sparse transform, such as wavelet and SVD[], to capture the sparse structure of the whole image. However, the CSMRI loses the edges and fine features of the image, as shown in Fig.(1b).



**Figure 1. Comparison of images reconstructed from under-sampled k-space data exploiting global and patch-based sparsity. (a) is the ground truth, (b) is the result of CSMRI using wavelet(db4) as the global sparsifying transform, (c) is the result of DLMRI using patch-based sparsifying dictionary learned by K-SVD.**

1. **Imaging model**

Existing methods reconstruct the images using only the global or patch-level sparse structure of the image and exhibits some shortcomings as elaborated in the previous section. In this work, we propose an imaging model to reconstruct the image by combining both global and patch-based sparse representation of images as follows:

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| --- | --- | --- |
|  |  | (1) |

Where, is the image to be reconstructed, is the under-sampled k-space measurement, and is the partial Fourier transform. The first term in holds data consistency in k-space. In second term of , is patch extraction operation, and is the learned dictionary under which the corresponding patch has sparse representation constrained by sparse level . .In the last term of , sparse representation of is obtained directly under which is the global sparse transform depicted in background. The parameters and are used to promote and balance the patch-level or global sparsity. Obviously, when or , the model degrades to the DLMRI or CSMRI, respectively.

1. **Algorithm**

Problem is solved using two alternated procedures. First, the dictionary is learned from an initial guess of the image, and then sparse representation of each image patch is obtained. Second, image is reconstructed under the CS framework with patch-level and global sparsity constraints. The input of the second procedure is produced using , which are already obtained. These two steps are further detailed in the following subsections.

1. **Dictionary learning**

In this step, dictionary and sparse representation of each patch are solved using K-SVD, whose model is as following:

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| --- | --- | --- |
|  |  | (2) |

where is the zero-filled inverse Fourier transform of under-sampled k-space data, and only part of image patches are randomly chosen. After this step, we have an coarse estimation of , which will be the start point of the second procedure.

1. **Reconstruction**

In this step, dictionary and sparse representation are fixed, and DLMRI formulate the reconstruction as:

Here, the least squares technique is adopted to solve the problem, technique which is actually equivalent to k-space back filling, in other words, DLMRI restores the un-sampled positions in k-space with frequencies of averaged patch obtained by .However, this procedure is just a pretreatment to overcome the local shortcoming. Specifically, the overcome to local-shortcoming only befell the patch-average, and the reconstruction step is omitted. To weaken the shortcoming a step further in the reconstruction, we utilize CS frame to implicitly use the patches rather than directly fill back the k-space. Hence, the proposed reconstruction is formulated as:

Here, our proposed reconstruction is solved under the CS frame, and we regard the former two terms, namely the DLMRI reconstruction, as our data consistency penalties. The gradient corresponding to isformulated as:

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| --- | --- | --- |
|  |  | (5) |

The superscript H denotes the Hermitian transpose operation. In DLMRI, the solution is found at:

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

However, this strategy is a bit rough for the reason that: is approximation to the patch of truth. Therefore, Eq.6 is non-strictly equivalent, and it should be:

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| --- | --- | --- |
|  |  | (7) |

That means the theoretical value is in the vicinity of , hence, we use nonlinear conjugate gradient denoted by NLCG, which regards Eq.5 as the gradient function, to sought the finer solution.

The complete framework of our reconstruction model and joint Dictionary learning model are summarized in Fig2.



**Figure 2.The whole reconstruction model and joint Dictionary learning model.**

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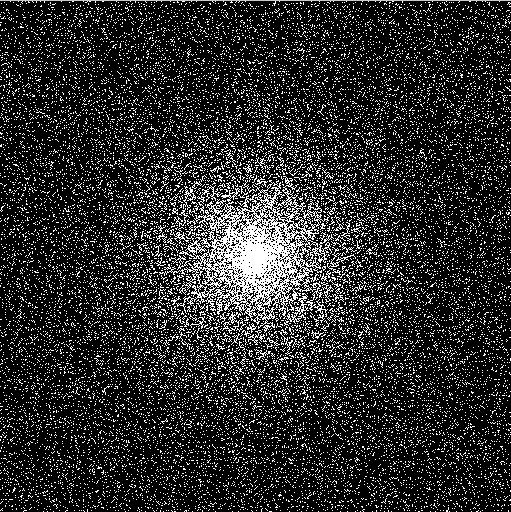
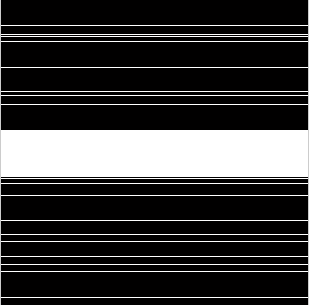
**Figure 3. Comparison of images reconstructed from GLMRI and DLMRI. (a) is the ground truth, (b) is the result of GLMRI using wavelet(db4) as the global sparsifying transform and patch-based sparsity as the local sparsifying transform, (c) is the result of DLMRI using patch-based sparsifying dictionary learned by K-SVD.**

Figure 3 shows the experiment results. In Figure 4 CSMRI, GLMRI and DLMRI reconstruction are compared. GLMRI exhibits similar image details and better coherence between overlapping patches compared with the DLMRI method, and better image details compared with the CS method when comparing GLMRI and CSMRI.

**MATERIALS AND METHODS**

The proposed model was verified using four typical MR images: Axial brain, vessel, lumbar spine and simulated phantom. In this work, the simulated phantom was an image of a head phantom generated by Matlab 2012b. For all these cases, the image sizes were all 512x512.

Two sampling schemes used in the experiments are respectively: Cartesian sampling with Gaussian random pattern along the phase-encoding direction[] and 2D random sampling[].The Cartesian sampling uses a variable-density under-sampling patternwith a denser sampling at a low-frequency(see Fig 2a). We adopt 2D random sampling for better incoherent aliasing interference in the sparse transform domain[](see Fig2b).



**(b)**

**(a)**

**Figure 4.Sampling schemes in k-space. (a) Cartesian sampling; (b) 2D random sampling**

The proposed model is implemented by solving Local sparse Reconstruction and Global sparse Reconstruction alternately. During Local sparse Reconstruction stage, we utilized K-SVD algorithm for obtaining the overcomplete dictionaries. We take PCA and random training patches as the initialization in the K-SVD. Then the sparse representation for each patch would be found under the help of OMP algorithm[10]. During Global sparse Reconstruction, we use nonlinear conjugate gradients and a back tracking line-search[] for solving the optimization in Eq.[8] and Eq.[10].

In the experiments, the algorithm has a few design parameters, notably the size of a patch(), dictionary size or overcompleteness of the dictionary(dict\_size), overlap stride(r), number of random patches used for training(patch\_num), and the weightings of l2 and l1 norm  and  in Eq.[1]. We nominal set these parametersas n=36, dict\_size=36, r= 1, patch\_num=5000. The settlements for and are different for different datasets.For the phantom, we set ==0.007. For brain data we set =0.007 and =0.00007. For the vessel, we set =0.007 and =0.00007. For lumbar spine we set =0.007 and =0.00024.

The quality of the reconstruction is quantified using PSNR(peak signal-to-noise ratio). This is a standard image quality measure on image compression. In addition, we use the intensity differences between the reconstructed images and the truth image for the consideration of assessing the reconstruction quality visually.

The proposed method was compared withDLMRI methods[2]. All simulations and reconstructions are implemented in MATLAB R2012b (MathWorks, Natick, MA).

**RESULTS**

***Comparison of image reconstruction***

In our experiments, the results of the DLMRI and the proposed method are presented in Fig.3-6 for four image cases(brain, vessel, lumbar spine and simulated phantom). In each case, we fix the reduction factor(R) to 6 and the two sampling schemes above are compared. We mainly pay close attention to the PSNR according to iteration number and the error maps. For better visualization, the The error-maps are multiplied by a factor of 5.

The imaging results indicate that, the proposed method offers higher fidelity in the reconstruction of image details than the DLMRI method on the PSNR view in Cartesian sampling and 2D random sampling, however the improvements in Fig.3-6, marked by arrows, of the DLMRI are not very obvious.



**(a)**



**(b)**

**Figure 3. The reconstructed images, PSNR over iteration and error-maps of the DLMRI and proposed method GLMRI for the brain MRIat R=6.The error-maps are multiplied by a factor of 5 for better visualization. (a) The reconstructed results for Cartesian sampling. (b) The reconstructed results for 2D random sampling.**



**(a)**



**(b)**

**Figure 4. The reconstructed images, PSNR over iteration and error-maps of the DLMRI and proposed method GLMRI for the vessel MRIat R=6.The error-maps are multiplied by a factor of 5 for better visualization. (a) The reconstructed results for Cartesian sampling. (b) The reconstructed results for 2D random sampling.**



**(a)**



**(b)**

**Figure 5. The reconstructed images, PSNR over iteration and error-maps of the DLMRI and proposed method GLMRI for the lumbar spine MRIat R=6.The error-maps are multiplied by a factor of 5 for better visualization. (a) The reconstructed results for Cartesian sampling. (b) The reconstructed results for 2D random sampling.**



**(a)**



**(b)**

**Figure 6. The reconstructed images, PSNR over iteration and error-maps of the DLMRI and proposed method GLMRI for the simulated phantomat R=6.The error-maps are multiplied by a factor of 5 for better visualization. (a) The reconstructed results for Cartesian sampling. (b) The reconstructed results for 2D random sampling.**

***Comparison of performance in PSNR view under different reduction factor***

The PSNR of the reconstructed images by DLMRI and proposed method are recorded with respect to different reduction factors. The blue lines represent proposed method, while red lines denote the proposed method. The results of above datasets are shown as Fig.7, Fig8, Fig9 and Fig.10 respectively. As these figures illustrated, the proposed method provideshigher PSNR than DLMRI in the figures and the improvement in quality is about 1 to 5 dB.

The proposed method, which captures both the patch-level and global sparse structures of an image, can faithfully reconstruct an outline of the image with a higher sampling ratio with the comparison of different anatomical images.

As Fig.2 shows, the 2D random sampling provides a more stochastic performance than the Cartesian sampling, which can better conform to the CS theory. For the proposed method utilize CS theory captures the global sparse structures of an image, the proposed method under the 2D random sampling provides a higher PSNR than Cartesian at the same reduction factor comparing (a) and (b) of the same image case.

From a comparison of the two sampling patterns, it appears that the proposed method can be quite effective compared with the Cartesian sampling.



**(a)**

**(b)**

**Figure 7. Performance comparison(PSNR) between DLMRI and proposed method with different reduction factors for brain MRI. (a) with Cartesian sampling. (b) with 2D random sampling**



**(a)**

**(b)**

**Figure 8. Performance comparison(PSNR) between DLMRI and proposed method with different reduction factors for vessel MRI. (a) with Cartesian sampling. (b) with 2D random sampling**



**(a)**

**(b)**

**Figure 9. Performance comparison(PSNR) between DLMRI and proposed method with different reduction factors for lumbar spine MRI. (a) with Cartesian sampling. (b) with 2D random sampling**



**(a)**

**(b)**

**Figure 10. Performance comparison(PSNR) between DLMRI and proposed method with different reduction factors for simulated phantom. (a) with Cartesian sampling. (b) with 2D random sampling**

***Effects of different settings of parameters***

In this section, we evaluate the sensitivity of the algorithm to parameter settings by varying one parameter at a time while keeping the rest fixed at their nominal values. The brain data was used for evaluation under 2D random sampling at the reduction factor R=4. The parameters evaluated were ,and r. PSNR is ploted in Fig.11.

In our proposed model, we fixed the fidelity constraint to k-space measurement to 1 and the ratio between fidelity constraint and local sparse constraint is denoted by n. Therefore . In conventional setting, n=140. The ratio between local sparse constraint and global sparse constraint is denoted by m and . M is different under different image cases. We set m=100 for brain and vessel, m=30 for lumbar spine and m=1 for simulated phantom.

In Fig.11(a), when n increases from 100 to 500, the PSNR improves. However, at a higher m, PSNR curve tends to stable. The changes in PSNR are small and the values are quite good around n=140.

In Fig.11(b), the change tend of (b) is similar to (a). Therefore m=100 is an appropriate setting. In particularly, our proposed model is similar to DLMRI when m=inf, namely =0. Fig.11(b) shows that the point is close to the DLMRI curve.

In Fig.11(c), The curve shows that, The PSNR degrade according to the increase of overlap stride. During local sparse reconstruction, the mean value of each image pixel will be worse when overlap stride increase. In extreme case, namely ,each pixel is calculated only once.

The curves of Fig.11 indicate that the nominal parameter values work reasonably well.



**(a)**



**(b)**



**(c)**



**Figure 11. Results of effects of different settings of parameters. (a) PSNR at different** **. (b) PSNR at different** **. (c) PSNR at different r.**

**DISCUSSION AND CONCLUSION**

This work presented a new model by enforcing both local and global sparsity, which captures both the patch-level and global sparse structures of the anatomical images. Using a model split approach, the image reconstruction quality can be iteratively further improved. Our simulation results demonstrate that, the proposed method outperform those existing methods using only the patch-level or global sparse structure. Future work will improve the dictionary learning model and use adaptive global sparse transform to enhance the reconstruction quality.

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**REFERENCES**

[1]Farsiu S, Robinson M D, Elad M, et al. Fast and robust multiframe super resolution[J]. Image processing, IEEE Transactions on, 2004, 13(10): 1327-1344.

[2] Lustig M, Donoho D, Pauly J M. Sparse MRI: The application of compressed sensing for rapid MR imaging[J]. Magnetic resonance in medicine, 2007, 58(6): 1182-1195.

[3]Candes E J, Donoho D L. Curvelets: A surprisingly effective nonadaptive representation for objects with edges[R]. Stanford Univ Ca Dept of Statistics, 2000.

[4]Hong M, Yu Y, Wang H, et al. Compressed sensing MRI with singular value decomposition-based sparsity basis[J]. Physics in medicine and biology, 2011, 56(19): 6311.

[5]Ravishankar S, Bresler Y. MR image reconstruction from highly undersampled k-space data by dictionary learning[J]. Medical Imaging, IEEE Transactions on, 2011, 30(5): 1028-1041.

[6]Yang J, Wright J, Huang T S, et al. Image super-resolution via sparse representation[J]. Image Processing, IEEE Transactions on, 2010, 19(11): 2861-2873.

[7]Sun J, Zheng N N, Tao H, et al. Image hallucination with primal sketch priors[C]//Computer Vision and Pattern Recognition, 2003. Proceedings. 2003 IEEE Computer Society Conference on. IEEE, 2003, 2: II-729-36 vol. 2.

[8]Chang H, Yeung D Y, Xiong Y. Super-resolution through neighbor embedding[C]//Computer Vision and Pattern Recognition, 2004. CVPR 2004. Proceedings of the 2004 IEEE Computer Society Conference on. IEEE, 2004, 1: I-I.

[9]Wang H, Liang D, Ying L. Pseudo 2D random sampling for compressed sensing MRI[C]//Engineering in Medicine and Biology Society, 2009. EMBC 2009. Annual International Conference of the IEEE. IEEE, 2009: 2672-2675.

[10]Pati Y C, Rezaiifar R, Krishnaprasad P S. Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition[C]//Signals, Systems and Computers, 1993. 1993 Conference Record of The Twenty-Seventh Asilomar Conference on. IEEE, 1993: 40-44.

[11]Figueiredo M A T, Nowak R D, Wright S J. Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problems[J]. Selected Topics in Signal Processing, IEEE Journal of, 2007, 1(4): 586-597.

[12]Park S C, Park M K, Kang M G. Super-resolution image reconstruction: a technical overview[J]. Signal Processing Magazine, IEEE, 2003, 20(3): 21-36.

[*Penalized likelihood pet image reconstructionusing patch-based edge-preserving regularization*]