RIGR

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Theory

RIGR is a model-based method for dynamic MRI. In principle, RIGR uses the correlation of the data points within the k-space to reconstruct the image. Specifically, RIGR assumes that the object image can be represented by a generalized series (GS) with some proper basis derived from the Fourier basis. The coefficients of the series are obtained by solving a set of Toeplitz systems.

Here, we assume that the first frame of k-space F_0 is fully sampled in a grid of [0,N-1]x[0, M-1], while the k-space of subsequent frames $\hat{F}$ is under-sampled in N_ux[0, M-1] with N_u is a subset of [0,N-1].

If the k-space of each frame is fully sampled, then the image $f(i,k)$ can be reconstructed by inverse DFT as follows.

$$f(i,k) = \frac{1}{N}\sum_{n=0}^{N-1} exp(2\pi j\frac{ni}{N})\frac{1}{M}\sum_{m=0}^{M-1} F(n,m)exp(2\pi j\frac{mk}{M}) \tag{1}$$

Now, considering that only rows in N_u are available, we represent the image with a GS as follows.

$$\hat{f}(i,k) = \frac{1}{N}S(i,k)\sum_{t\in N_u} exp(2\pi j\frac{ti}{N})c_{t,k} \tag{2}$$

where $c_{t,k}$ is the coefficients of the GS and $S(i,k)$ absorbs available a prior information. By calculating the DFT of the $\hat{f}(i,k)$, we have

$$\hat{F}(n,m) = \sum_{i=0}^{N-1}\sum_{k=0}^{M-1} \hat{f}(i,k)exp(-2\pi j(\frac{ni}{N}+\frac{mk}{M})) \tag{3}$$

Inserting the (2) into (3), we have

$$\begin{aligned}\hat{F}(n,m) &= \frac{1}{N}\sum_{i=0}^{N-1}\sum_{k=0}^{M-1} S(i,k)\sum_{t\in N_u} exp(2\pi j\frac{ti}{N})c_{t,k}exp(-2\pi j(\frac{ni}{N}+\frac{mk}{M})) \\ &= \frac{1}{N}\sum_{k=0}^{M-1} exp(-2\pi j\frac{mk}{M})\sum_{t\in N_u} c_{t,k}(\sum_{i=0}^{N-1} S(i,k)exp(-2\pi j\frac{n-t}{N}i))\end{aligned}$$

with $n \in N_u, m = [0, M-1]$.

That means

$$\frac{1}{N}\sum_{t\in N_u} c_{t,k}(\sum_{i=0}^{N-1} S(i,k)exp(-2\pi j\frac{n-t}{N}i)) = \frac{1}{M}\sum_{m=0}^{M-1}\hat{F}(n,m)exp(2\pi j\frac{mk}{M}) \quad (4)$$

with $n \in N_u, k = [0, M-1]$. For each fixed k, equation (4) is a Toeplitz system and can be sovled to get $c_{t,k}(t \in N_u)$, which is used to reconstruct the object image by eq.(2).

Usually, the $S(i,k)$ is set to $|I_0(i,k)|$ for every $i = [0, N-1]$ and $k = [0, M-1]$, where $I_0(i,k)$ is the reference image.

C code for RIGR

`http://mri.beckman.uiuc.edu/software.html`

Matlab code for RIGR

References

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