SHARP6967

Using Socrative

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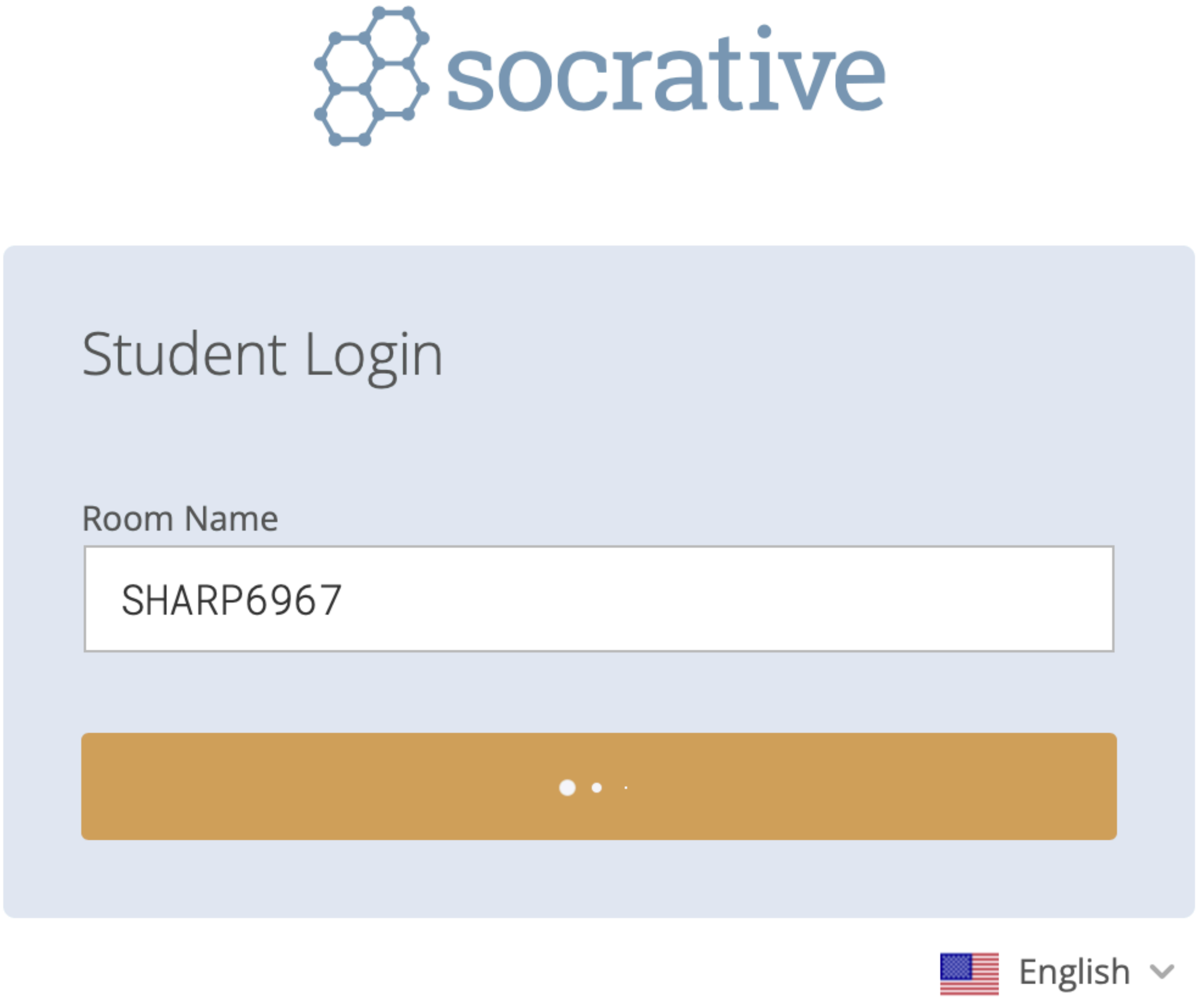


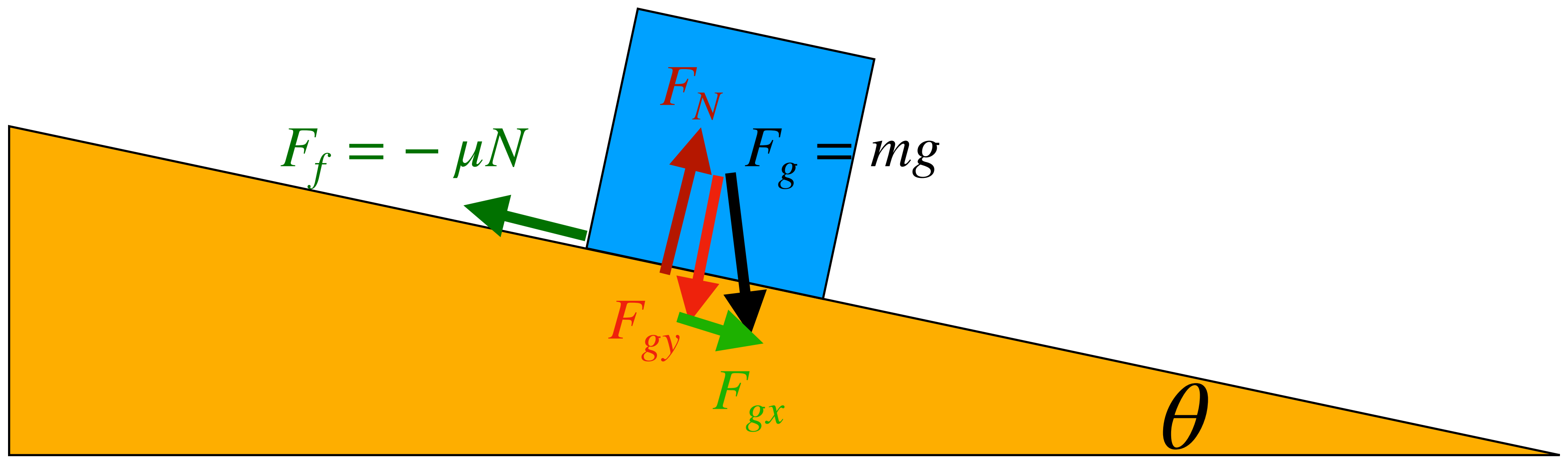
Work and Energy

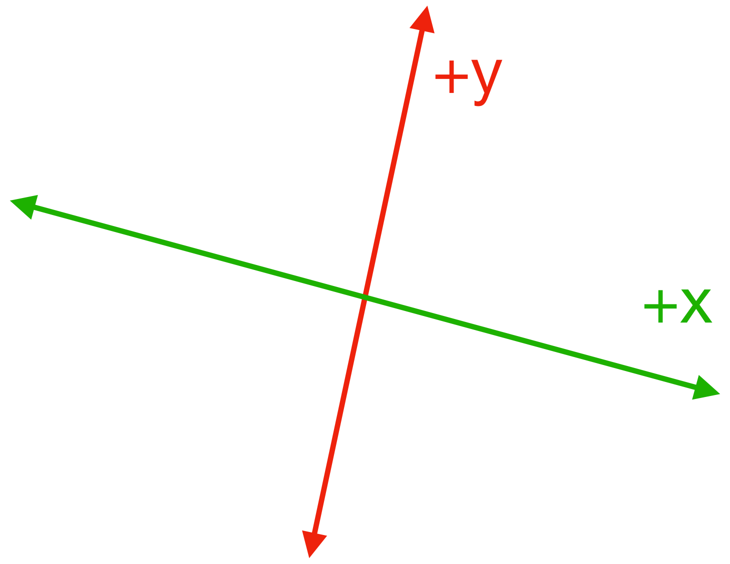
Dr. Sharp 1/26/2023

Review

Forces

- We previously talked about forces
  - Inertia
  - $$\sum_i \vec{F}_i = m \cdot \vec{a}$$
  - Action-reaction
- Force is a vector: magnitude and direction
- Different types of forces:
  - Friction - $F_f = \mu N$
  - Weight - $W = mg$
  - Restoring forces (springs) - $F_s = -kr$
- We discussed using free body diagrams, or drawings to break down and visualize problems





Topic for today

- We step away from forces
- Forces can become challenging to solve
- We can make problem solving easier in many cases by using

Energy!

Goals for the Unit

- Define work and energy
- Find the scalar product of two vectors
- Evaluate varying forces
- Evaluate work done by conservative forces
- Explain the relationship between work and kinetic energy
- Demonstrate problem solving using work and kinetic energy

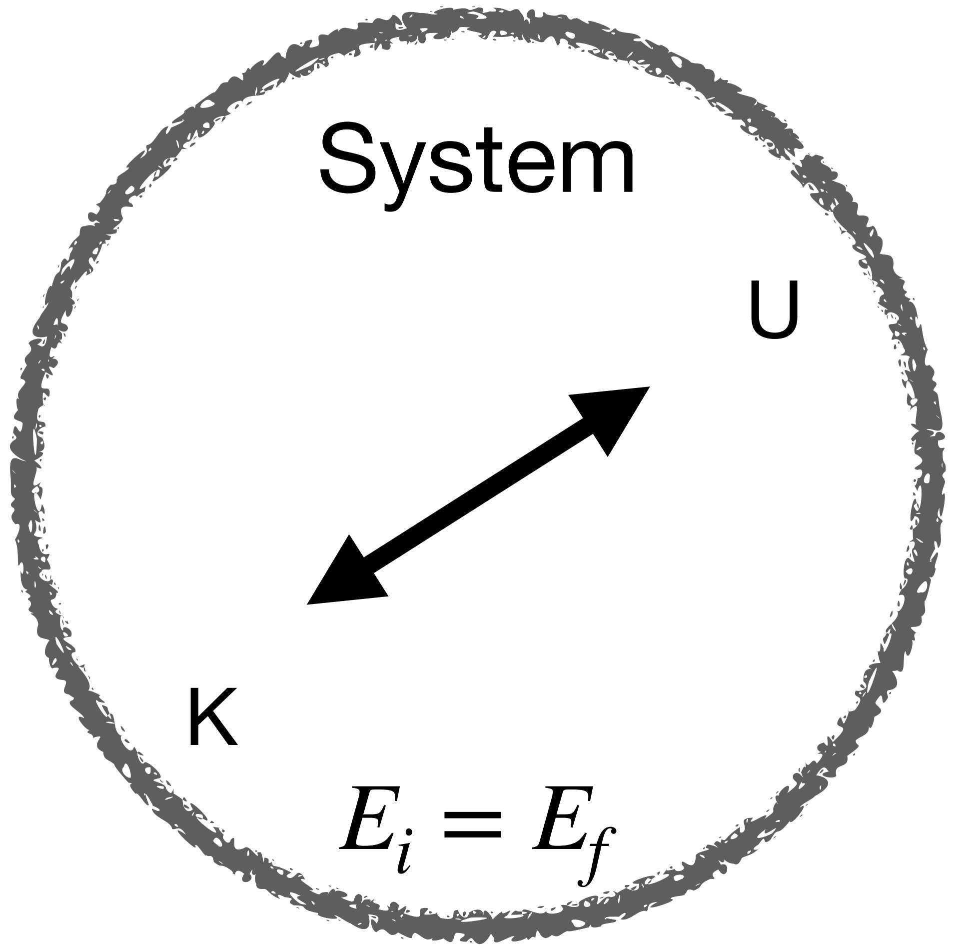
What is Energy?

- Energy is conserved
    - It can not be created
    - It can not be destroyed
- Energy can transition between forms:
    - Kinetic - the energy of motion
    - Potential - energy that can be used later
- Energy can be transferred in or out of a <u>system</u>

What is Energy?

Energy and Systems

- We will discuss energy using systems of interest
    - Could be an object, a point particle…
- If the system is isolated energy is constant
- Energy can change in the system K <-> U



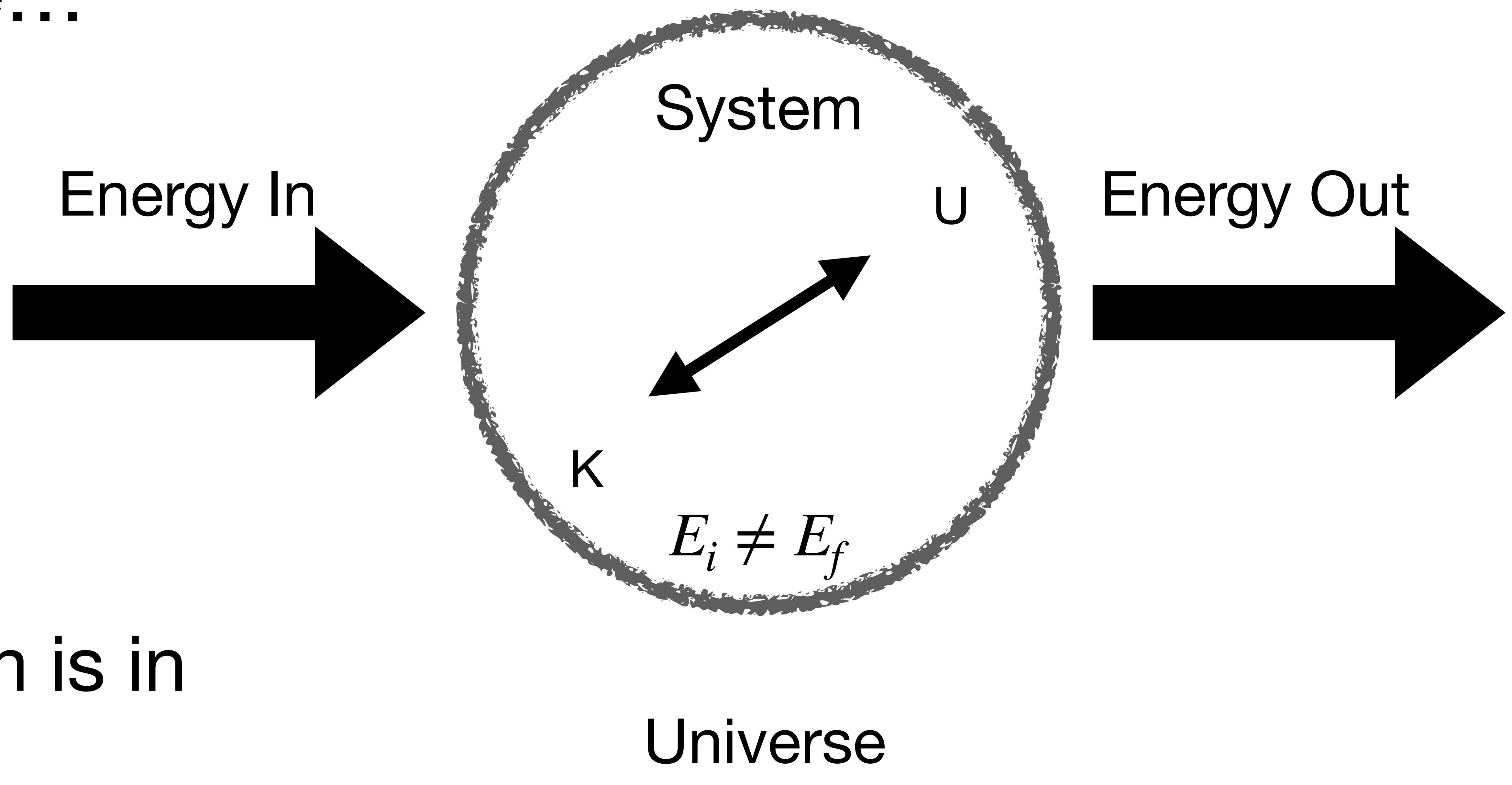
Universe

Modeled after: Essential University Physics, Wolfson, 4th Ed

What is Energy?

Energy and Systems

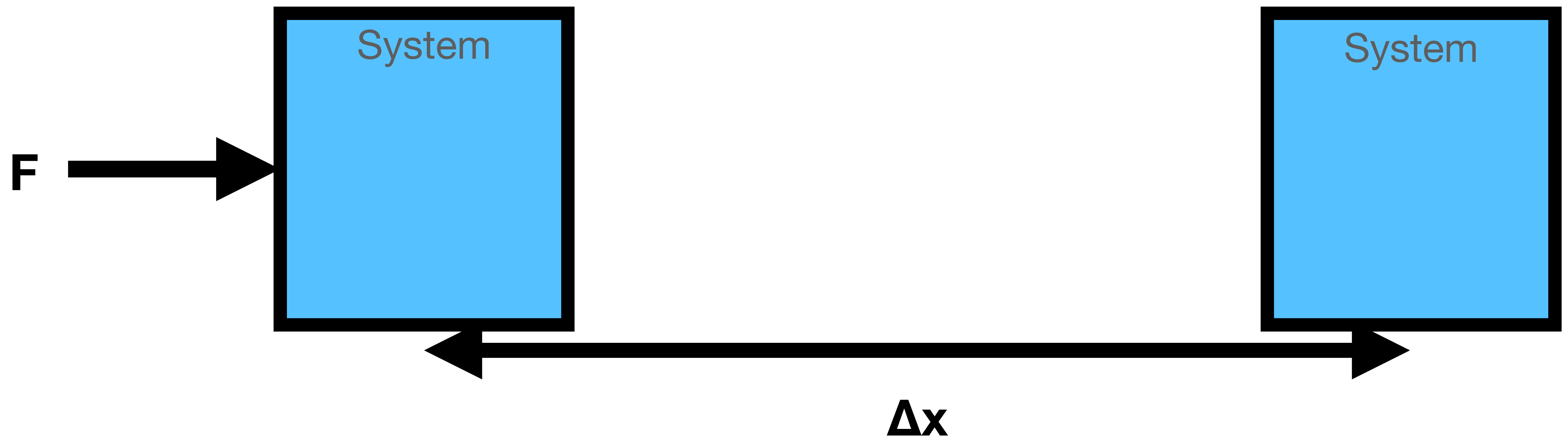
- We will discuss energy using systems of interest
  - Could be an object, a point particle…
- If the system is isolated energy is constant
- Energy can change in the system K <-> U
- Energy transferred into or out of our system is in the form of <u>work</u> or heat
  - Energy is no longer constant



Work

What is work

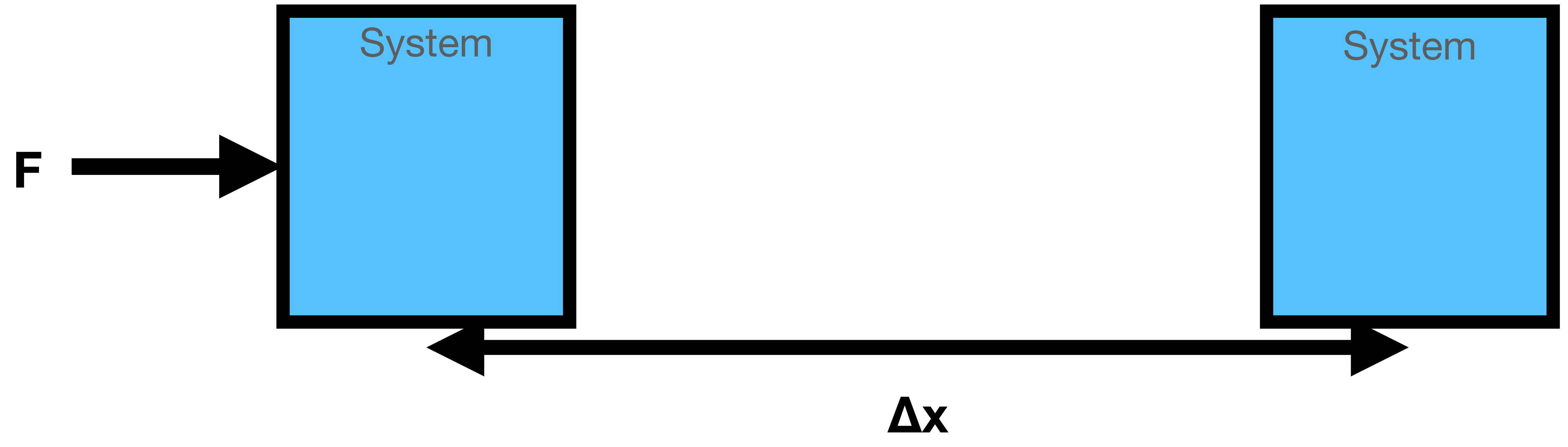
- The previous slide showed **work** as energy transferred in/out of a system
  - This is done by applying an external force on the system
- We will define work as the force it takes to move an object some displacement



$$W = F_x \Delta x$$

Work

Breaking down the equation



$$W = F_x \Delta x$$

- Work is a scalar
- F and Δx are both magnitudes
- The force is constant for this equation
- Work has units of N m or Joule's (J)

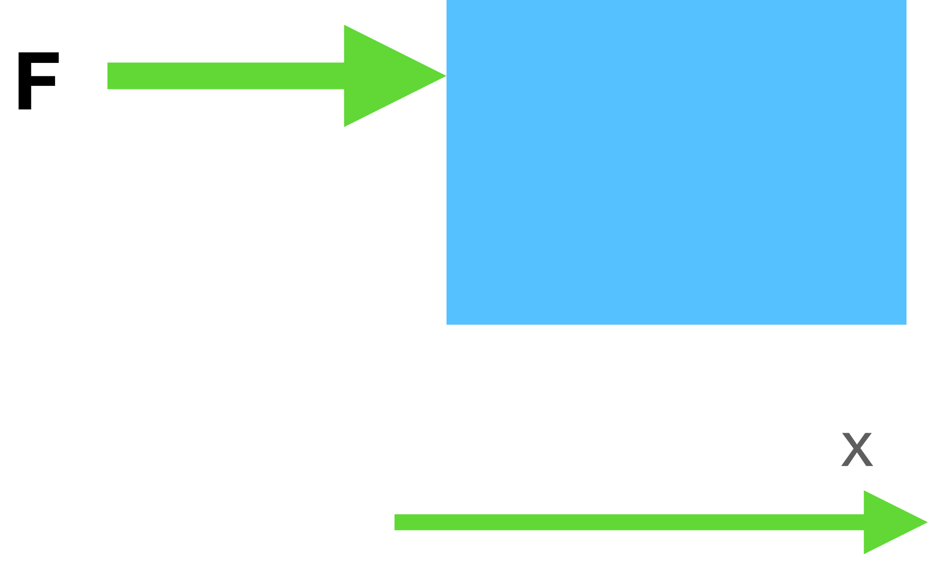
Work

What angle is the force applied at?

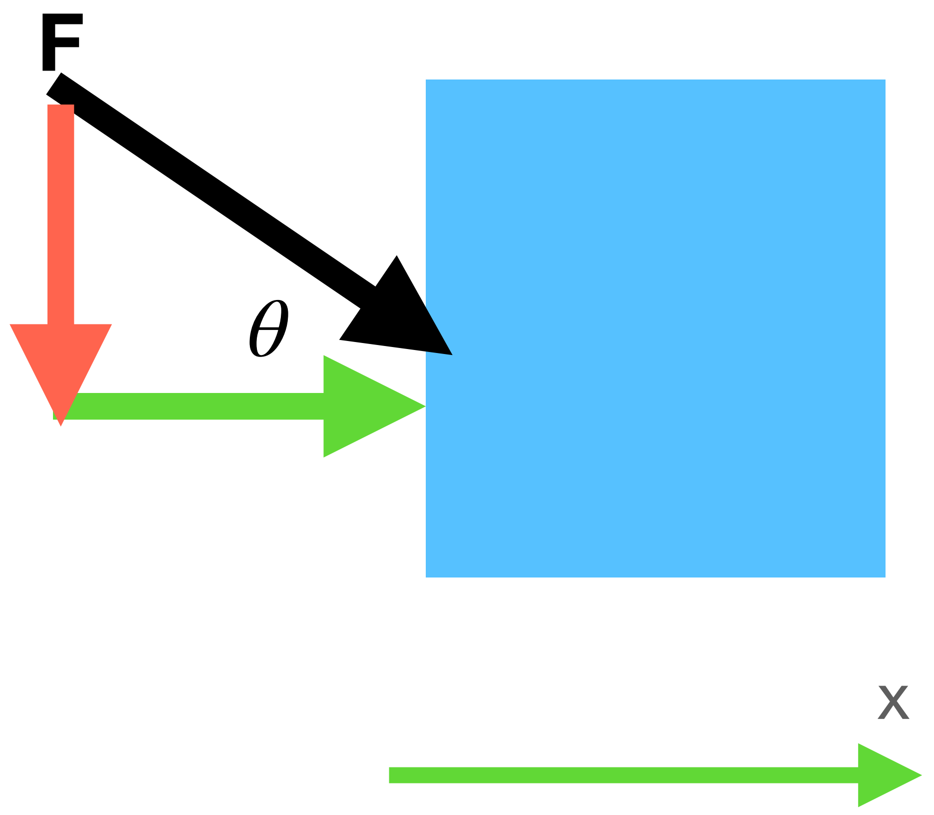
$$W = F\Delta x \cos(\theta)$$

- Only forces parallel to the displacement contribute to work done
    - Forces applied at an angle need to be broken into their components
- Forces applied perpendicular to the displacement do no work

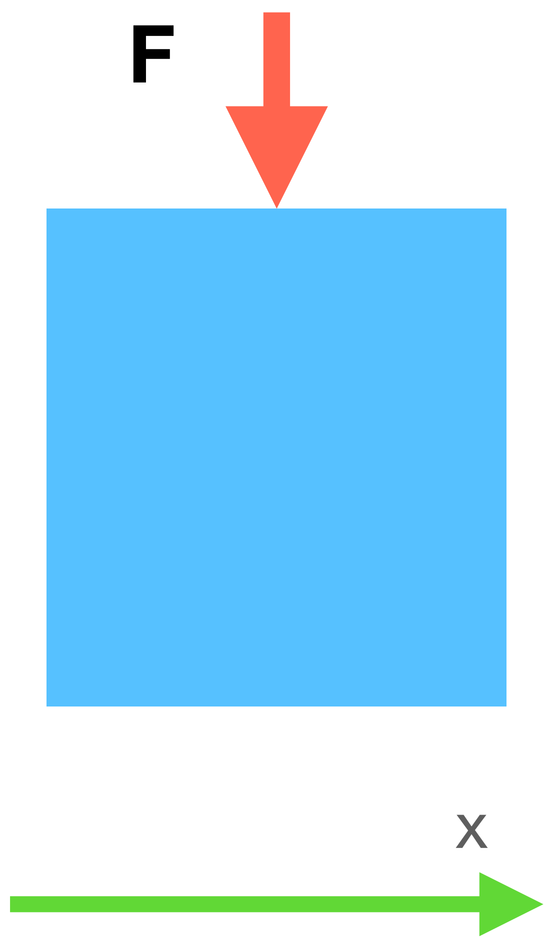
$W = F\Delta x \cos(0) = F\Delta x$



$W = F\Delta x \cos(\theta)$



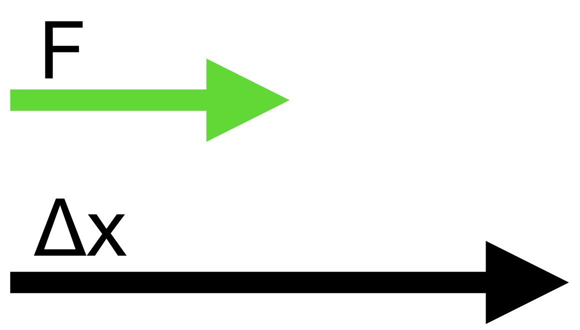
$W = F\Delta x \cos(90) = 0J$



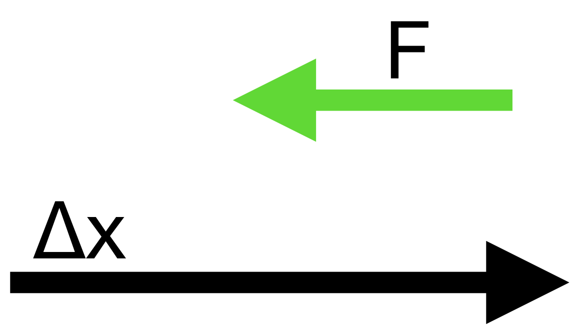
Work

Positive and Negative Work

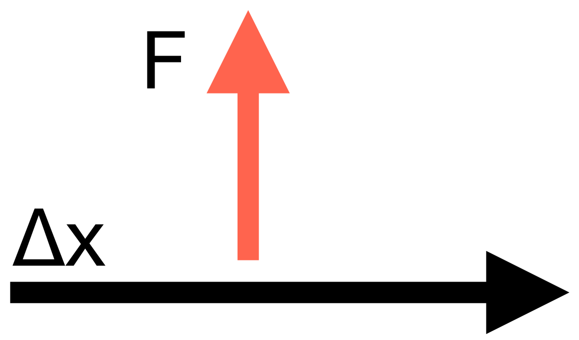
- A force in the direction of displacement has a positive work
- A force in the opposite direction of displacement has a negative work
- Forces applied perpendicular to the displacement do no work



Positive Work



Negative Work



No Work

- Puts energy into the system

- Removes energy from the system

- Does nothing

Work

Conceptual Q&A

- Pretend you carry a 2 kg box to your car 30 m away. What is the work done on the box? *Round g to $10 m/s^2$*

A. 600 J

B. -600 J

C. 0 J

Work

Conceptual Q&A

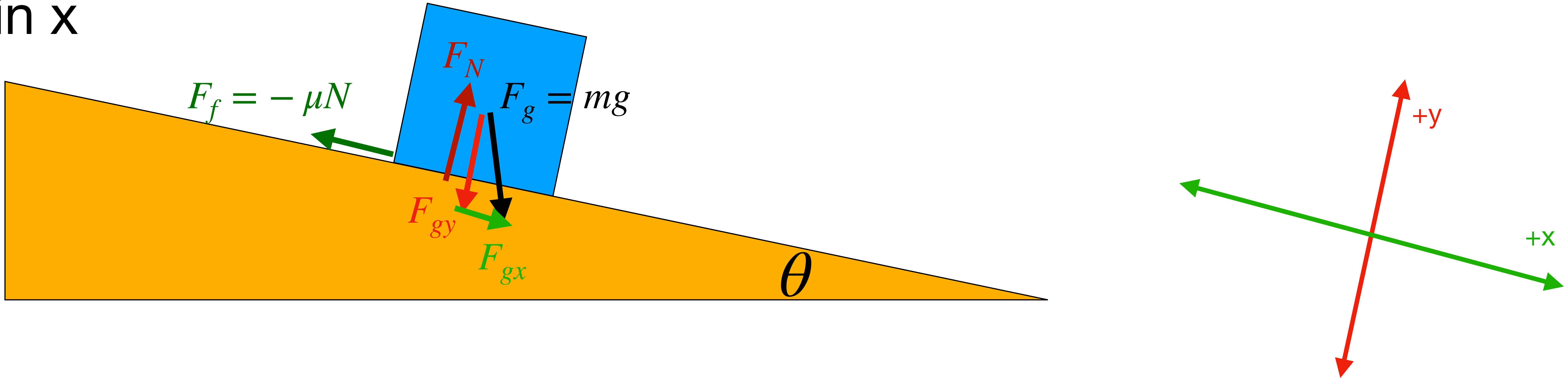
- Assuming our box is moving down the incline, which external force(s) do positive work on the box:

A. Friction

B. Weight in y

C. Normal Force

D. Weight in x



Work

Using the equations

- To push a stalled car, you apply 470 N of force at 17 degrees to its direction of motion, doing 860 J of work. How far have you moved the car?

Work

Introducing the scalar product (dot product)

- $W = F\Delta r\cos(\theta)$
- Is there a more convenient way to manage our math than finding the magnitude of F and Δr every time?
- We use a tool called the **scalar product** (or dot product)
    - It takes two vectors and produces a scalar!
    - $\overrightarrow{A} \cdot \overrightarrow{B} = AB\cos(\theta)$
- Where a vector can be written as $\overrightarrow{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
- $\overrightarrow{A} \cdot \overrightarrow{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right) \cdot \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$
- $\overrightarrow{A} \cdot \overrightarrow{B} = A_xB_x + A_yB_y + A_zB_z$

$$\hat{i} \cdot \hat{i} = 1$$
$$\hat{i} \cdot \hat{j} = 0$$

Work

Introducing the scalar (dot) product

- $W = F\Delta r \cos(\theta)$
- $\overrightarrow{A} \cdot \overrightarrow{B} = AB \cos(\theta)$
- How might we rewrite our equation for work?

Work

Scalar Product: Example

- A particle moving in the *xy* plane undergoes a displacement given by $\Delta r=(2.0i+3.0j)$ m as a constant force $F=(5.0i+2.0j)$ N on the particle. With your neighbor, use the scalar product to evaluate the work.

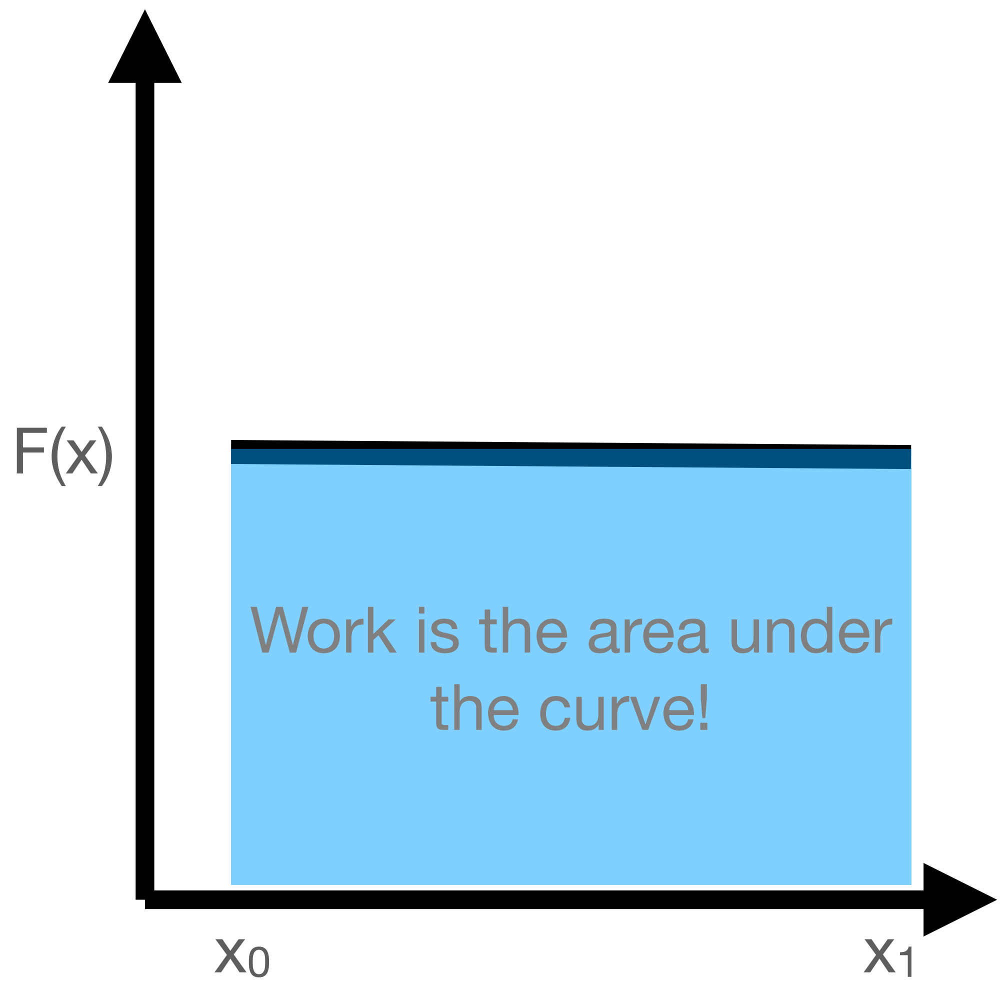
$$hint: \overrightarrow{A} \cdot \overrightarrow{B} = A_x B_x + A_y B_y + A_z B_z$$

Work

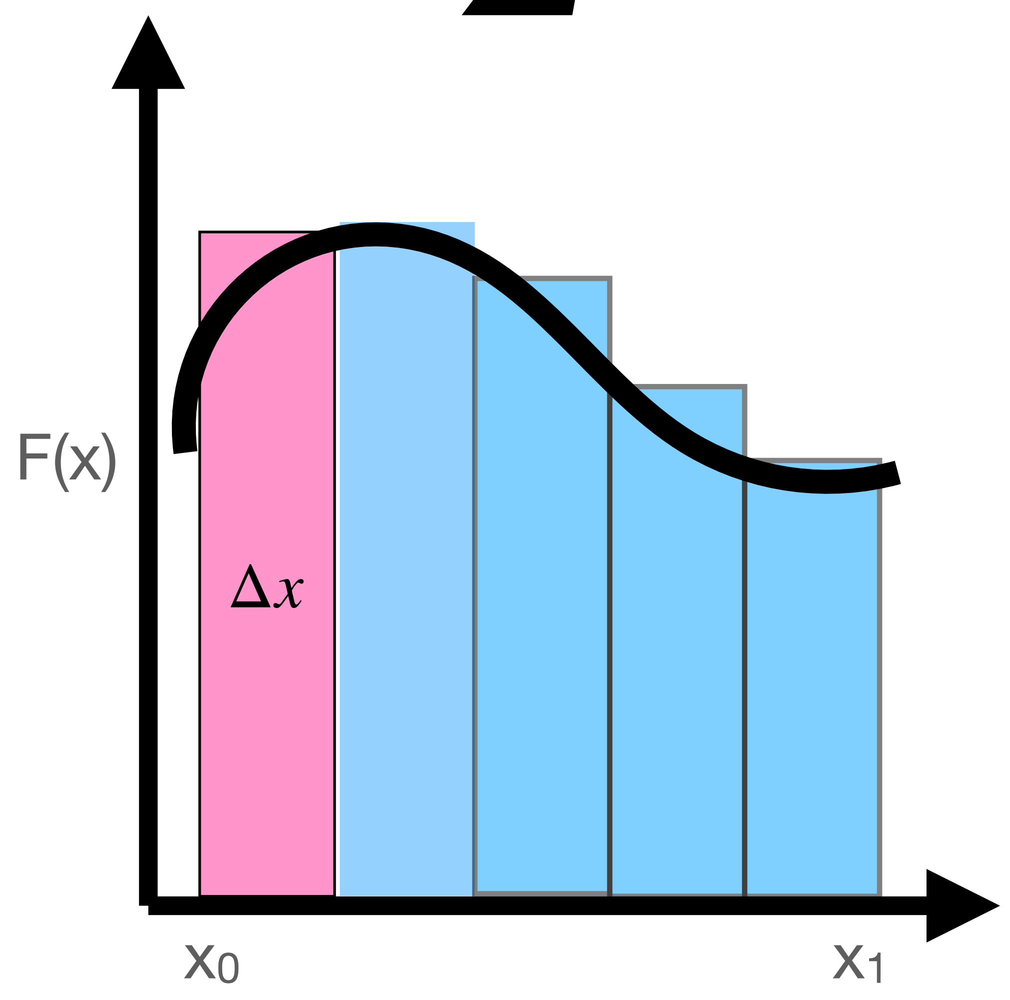
Varying Forces

- We have only considered work of a constant force
- What about a force that changes?

$$W = F\Delta x$$

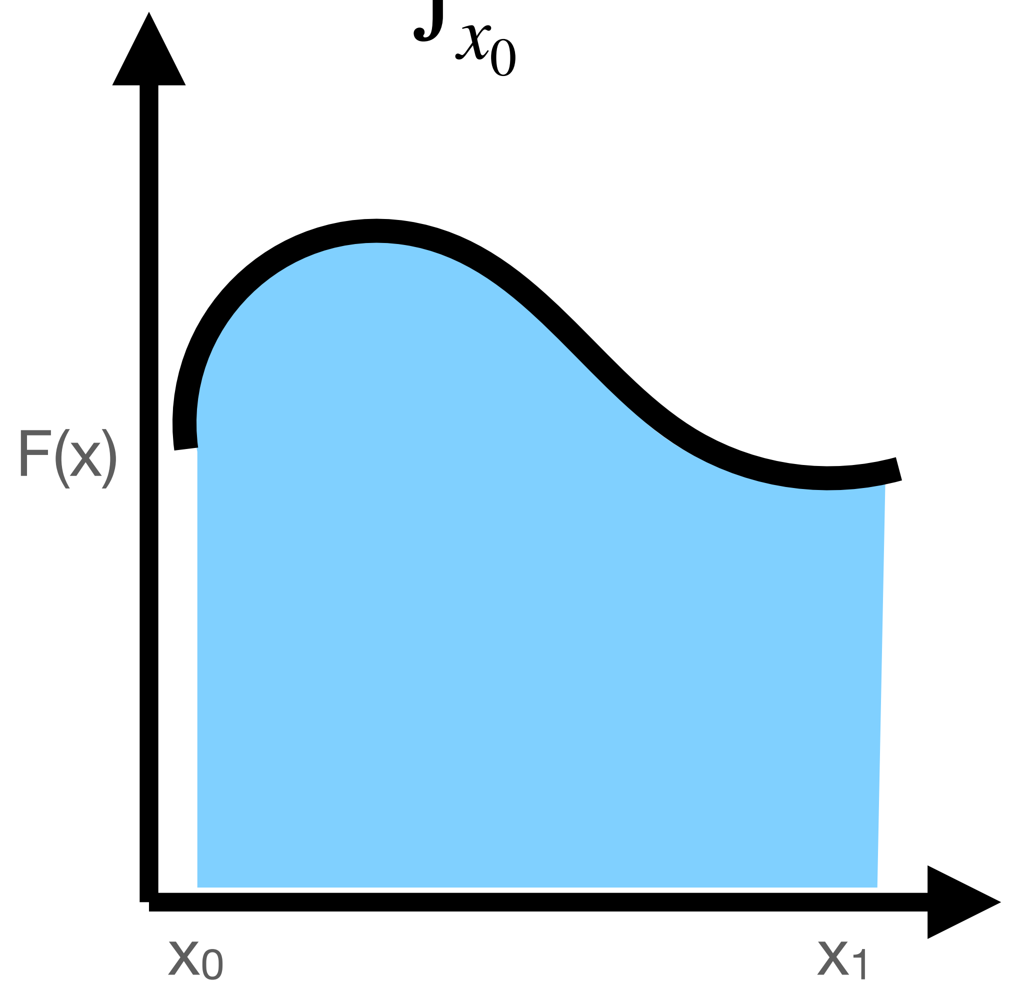


$$W = \sum F\Delta x$$



- If we let Δx get infinitely small we can use calc!

$$W = \int_{x_0}^{x_1} F(x)dx$$



Work

Varying Forces: Integral Quick Review

- For integrals of x^n, find the antiderivative by:

$$I = \int_{x_0}^{x_1} x^n dx$$

Work

Varying Forces: Integral Quick Review

- For integrals of x^n, find the antiderivative by:

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- Increase the exponent by one and divide by the new exponent

$$I = \left. \frac{x^{n+1}}{n+1} \right|_{x_0}^{x_1}$$

Work

Varying Forces: Integral Quick Review

- For integrals of x^n, find the antiderivative by:

$$I = \int_{x_0}^{x_1} x^n dx$$

- Increase the exponent by one and divide by the new exponent

$$I = \frac{x^{n+1}}{n+1}\Bigg|_{x_0}^{x_1}$$

- Plug in our bounds and take the difference

$$I = \frac{x_1^{n+1}}{n+1} - \frac{x_0^{n+1}}{n+1}$$

Work

Varying Forces: Stretching a spring

- Lets work out a varying force - stretching of a spring
- Talk with your neighbor and use integration to find the equation of work done to stretch a spring

Work

Varying Forces: Stretching a spring

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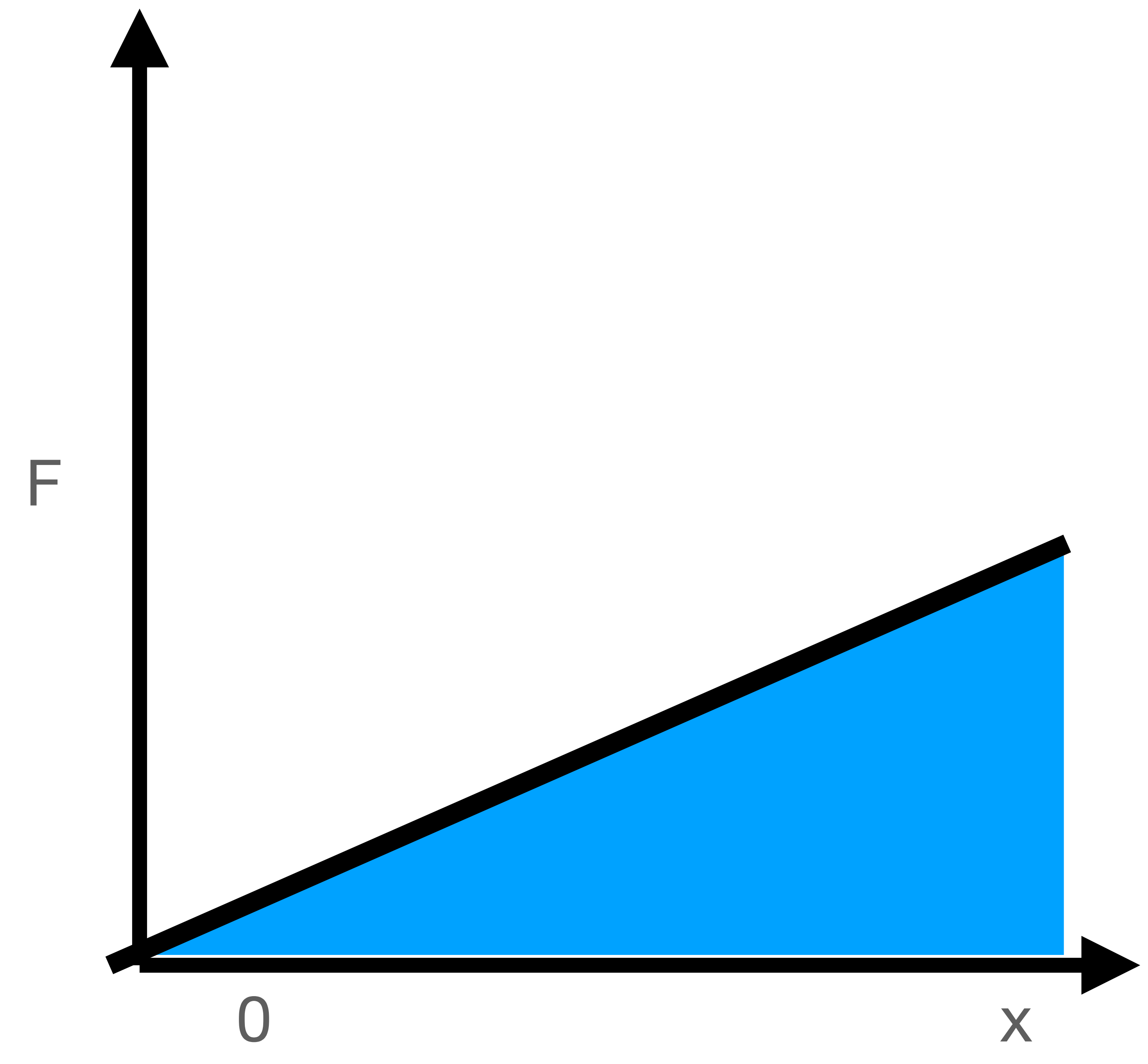
$$W = \int_0^x F(x)dx$$

Force applied by us!

$$F(x) = kx$$



What's the area under the curve?

Summary

Work

- Energy is conserved, and it will be easiest to talk about energy in terms of systems
- Work is the force it takes to displace an object/system some displacement
- We need to consider the angle at which a force is applied (parallel, perpendicular…)
- Work can be positive or negative (are you adding or removing energy from a system)
- The scalar product takes two vectors and produces a scalar
- Varying forces will require integration