COMS 230: Discrete Computational Structures

Homework # 3

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1. **Question 1**
   Assume p is an odd integer.
   $\Rrightarrow p = 2k + 1$ for some $k \epsilon \mathbb{N}$
   $\Rrightarrow p^3 = (2k + 1)^3 = 8k^3 + 8k^2 + 4k + 1 = 2(4k^3 + 4k^2 + 2k) + 1$
   $\Rrightarrow 4k^3 + 4k^2 + 2k$ is an integer.
   $\Rrightarrow p^3$ is an odd integer.

2. **Question 2**
   Since x and y are non-zero rational numbers,
   let $x = a/b$ and $y = c/d$ therefore $xy = ac/bd$
   Then if we divide this by z we have ac/bdz
   Lets assume that the product of ac/bdz is a rational number so,
   $ac/bdz = e/f$
   $\Rrightarrow ac = ebdz/f \Rrightarrow acf = ebdz \Rrightarrow z = acf/ebd$
   Because acf/ebd is a rational number according to the definition.
   This is a contradiction because z is irrational.
   Therefore xy/z is irrational.
   No, it is not direct because contradiction was used.

3. **Question 3**
   By contrapositive, if $n \neq$ even then, $5n + 4 \neq even$
   $\Rrightarrow n = odd \rightarrow 5n + 4 = odd$
   if n is some odd integer $2k + 1$ where $k \epsilon \mathbb{N}$
   $\Rrightarrow 5(2k + 1) + 4 = 10k + 9 = 2(2k + 4) + 1$
   $\Rrightarrow 2k + 4$ is an integer plus 1 means it is odd.
   $\Rrightarrow n = odd \rightarrow 5n + 4 = odd$

4. **Question 4**
   Lets assume that there are less than 5 meetings in one month.

That means that the max amount of meetings there could be in one month is 4.
Since $4 * 12 = 48$ and there are 50 meetings this is a contradiction.
Therefore there has to be at least one month where there are 5 meetings.

5. **Question 5**
case 1: $m > 0$ and $n > 0$ then $m * n = mn > 0$
case 2: $m > 0$ and $n < 0$ then $m * n = mn < 0$
case 3: $m < 0$ and $n > 0$ then $m * n = mn < 0$
case 4: $m < 0$ and $n < 0$ then $m * n = mn > 0$

6. **Question 6**
Suppose $\sqrt[3]{2}$ is rational.
Then there exists $p/q = \sqrt[3]{2}$
$\Rrightarrow p^3/q^3 = 2 \Rrightarrow 2p^3 = q^3 \Rrightarrow p^3/2 = q^3$
If p^3 can be divided by 2 then it has a factor of 2.
Let there be a k where $k \epsilon \mathbb{N}$ and $p = 2k$
$\Rrightarrow 2q^3 = (2k)^3 \Rrightarrow 2q^3 = 8k^3 \Rrightarrow q^3 = 4k^3$
So q^3 has a factor of 2 which is a common factor of p^3
Contradiction: p and q cannot have common factors and be rational so,
Therefore $\sqrt[3]{2}$ is irrational.

7. **Question 7**
1. $\sqrt{82} = 9.05$
2. $\sqrt{83} = 9.11$
3. $\sqrt{84} = 9.16$
4. $\sqrt{85} = 9.21$
5. $\sqrt{86} = 9.27$
6. $\sqrt{87} = 9.32$
7. $\sqrt{88} = 9.38$
8. $\sqrt{89} = 9.43$
9. $\sqrt{90} = 9.48$
10. $\sqrt{91} = 9.53$
11. $\sqrt{92} = 9.59$
12. $\sqrt{93} = 9.64$
13. $\sqrt{94} = 9.69$
14. $\sqrt{95} = 9.74$
15. $\sqrt{96} = 9.79$
16. $\sqrt{97} = 9.84$
17. $\sqrt{98} = 9.89$
18. $\sqrt{89} = 9.94$

My proof is constructive because I gave a concrete example instead of assuming.

8. **Question 8**
   a) $A = [4 + 6n | n \epsilon \mathbb{N}]$
   b) $A = [n^2 - 1 | n \epsilon \mathbb{N}]$

9. **Question 9**
   For any $A = x, y, z, B = a, b, c, C = d, e, f$
   $(AxB)xC = ((x, a)d)..., Ax(BxC) = (x, (a, d))...$
   $((x, a), d) \neq (x, (a, d))$
   Therefore $(AxB)xC \neq Ax(BxC)$