COMS 230: Discrete Computational Structures

Homework # 8

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1. **Question 1**
   a) Base Case: f(1) = 1 because of the def of fib numbers. $= f_{2(1)} = f_2 = 1$ so the base case holds.
   IH: Assume f(k) $= F_1 + F_3 + ...F_{2k-1} = F_{2k}$
   Prove f(k+1) = the sum of $F_1 + F_3 + ...F_{2k-1} + F_{2k+1} = F_{2k+2}$
   since we assumed the sum of the first part is f(k) then we know
   f(k+1) $= f(k) + F_{2k+1} = F_{2k} + F_{2k+1}$
   $= F_{2k+2}$
   So f(k+1) holds

2. **Question 2**
   a)
   Base Case: $n = 0 = f_0^2 = 0 = f_0 * f_1 = 0 * 1 = 0$ so the base case holds
   b)
   IH: assume f(k) $= f_0^2 + f_1^2 + ... + f_k^2 = f_k * f_{k+1}$
   Prove: f(k+1) $= f_0^2 + f_1^2 + ... + f_k^2 + f_{k+1}^2 = f_{k+1} * f_{k+2}$
   so f(k+1) $= f(k) + f_{k+1}^2 = f_k * f_{k+1} + f_{k+1}^2 = f_{k+1}(f_k + f_{k+1}) = f_{k+1} * f_{k+2}$
   So f(k+1) holds.

3. **Question 3**
   To strength the hyp we need a way to determine every step in the state machine. Not just 0 or not 0, which is what n div by 5 gives up.
   So the strengthened hyp is Q(n)
   Q(n): for n steps greater or equal to 0 we are at state 0 iff n mod 5 = 0
   Base Case: n = 0 which means we have taken 0 steps so we should be at state zero. n mod 5 = 0
   so the base case holds
   IH: Assume after k steps the state machine is in k mod 5 state.
   To prove (k+1) mod 5 then we have 5 cases

Case 1: k mod 5 = 0 $\rightarrow state = 0 \rightarrow k + 1 = 1$ which holds
Case 2: k mod 5 = 1 $\rightarrow state = 1 \rightarrow k + 1 = 2$ which holds
Case 3: k mod 5 = 2 $\rightarrow state = 2 \rightarrow k + 1 = 3$ which holds
Case 4: k mod 5 = 3 $\rightarrow state = 3 \rightarrow k + 1 = 4$ which holds
Case 5: k mod 5 = 4 $\rightarrow state = 4 \rightarrow k + 1 = 0$ which holds
so k+1 holds.

4. **Question 4**
Base Case:
P(1) = 1 stack + 0 stack = 1 * 0 = 0 = 1(1-1)/2 = 0 so it holds
P(2) = 1 stack + 1 stack = 1 * 1 = 1 = 2(2-1)/2 = 1 so it holds
P(3) = 2 stack + 1 stack = 2 * 1 = 2 then break it down again because it is not in stacks of 1, so 1 stack + 1 stack = 1 * 1 = 1 + 2 = 3 = 3(3-1)/2 = 3 so it holds
P(4) = 2 stack + 2 stack = 2 * 2 = 4 then break it down again because it is not in stacks of 1, so 1 stack + 1 stack and 1 stack + 1 stack = 1*1 + 1*1 = 2 + 4 = 6 = 4(4-1)/2 = 6 so it holds
P(5) = 2 stack + 3 stack = 2 * 3 = 6 then break it down again because it is not in stacks of 1, so 1 stack + 1 stack and 2 stack + 1 stack = 1 * 1 + 2 * 1 = 1 + 2 = 3 + 6 = 9 then because there is still a 2 stack break it down again, 1 stack + 1 stack = 1 * 1 = 1 + 9 = 10 = 5(5-1)/2 = 10 so it holds
IH: By simplification of strong induction we assume because P(1) $\wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)... \rightarrow P(k)$.
Prove: Since we have P(1) $\wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)... \wedge P(k)$ this implies P(k+1) so it holds.

5. **Question 5**
The start state is (0,0) and the state can be represented as (x,y)
where the transitions are $\{(x, y)|x, y \in \mathbb{Z}, (x + y)mod2 = 0\}$
Preserved Invariant: $div2sum(x, y) = [(x + y)mod2 = 0]$
Base Case: P(0) = (0,0), 0 + 0 = 0 mod 2 = 0, so the base case holds
Ind: Assume P(k) holds then let there be a state r that we get to in k + 1 transitions.
Prove: for P(k) = k $\rightarrow$ r
Case 1 : k + ((1 +3 = 4) = k + 4
since k mod 2 = 0 and 4 mod 2 = 0 it follows that k + 4 mod 2 = 0 this case holds
Case 2 : k + ((-1+1 = 0) = k + 0
since k mod 2 = 0 this case holds
Case 3 : k + ((0 -4 = -4) = k - 4
since k mod 2 = 0 and -4 mod 2 = 0 it follows that k - 4 mod 2 = 0 so this case holds
So because we proved that the robot only moves to a state where div2sum(x,y) holds then the robot cannot go to (2,-1) because 2 -1 = 1
And 1 mod 2 is not 0

6. **Question 6**

Assume $P(1) \wedge P(2) \wedge ... \wedge P(k) \rightarrow P(k+1)$
Prove: P(k) $\rightarrow$ P(k+1)
New IH: Q(n) = $P(1) \wedge .. \wedge P(n)$
Base Case: Q(1) = P(1) it holds because of strong induction.
Ind: assume Q(k) is true
therefor P(k+1) is true by SI hyp.
$P(1) \wedge ... \wedge P(k) \wedge P(k+1) =$ True so Q(k+1) is true
Q(n) = true for all n in $\mathbb{Z}$+
P(n) = true for all n in $\mathbb{Z}$+