COMS 230: Discrete Computational Structures
Homework # 5

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1. **Question 1**
   a)
   Because f is a total function that means there is a mapping from every element in $B \to C$ so it is also a one to one function.
   B)
   if $A = \{a\}, B = \{b, c\}, C = \{d, e, f\}$
   and $(f \circ g)(d) = a \wedge (e) = a \wedge (f) = a$
   and $g(a) = b$ and $f(b) = d, f(c) = e$
   So f is not an onto function.

2. **Question 2**
   a)
   This relation is reflexive when x is a real number greater than zero and anti-reflexive when less than zero, because
   when $x \geq 0$ then $(x, x) \in R1 \leftrightarrow 2x \geq 0$
   else when $x < 0$ then $(x, x) \in R1 \neq 2x \geq 0$
   This relation is transitive because
   $(x, y) \in R1 \wedge (y, z) \in R1 \to (x, z) \in R1$
   $xy \geq 0 \wedge yz \geq 0$ which means x,y, and z are all positive real numbers or 0 so, $yz \geq 0$ for all Real Numbers
   This relation is also symmetric because
   if $xy \geq 0$ then because of multiplication rules $yx \geq 0$
   b)
   This relation is anti-reflexive because
   $(x, x) \notin R2 \to x \neq 2x$
   This relation is not transitive because
   $(x, y) \in R2 \wedge (y, z) \in R2$ does not imply $(x, y) \in R2$
   $x = 2y \wedge y = 2z \to x = 4z \neq x = 2z$
   This relation is anti-symmetric because

$x = 2y \neq y = 2x$ and $x = 2y \wedge y = 2x$ only implies $y = x$
when $y = 0 \wedge x = 0$ so for all other cases it is not true so because of the way an implies work the statement is true.

3. **Question 3**
   a)
   R3 is reflexive because
   $(x, x) \in R3 \rightarrow x/x = 1, \forall x \in \mathbb{R}+$ and $1 \in \mathbb{Z}$
   R3 is anti- symmetric because
   $(x, y) \in R3 \wedge (y, x) \in R3 \rightarrow y = x$
   $x/y \in \mathbb{Z} \wedge y/x \in \mathbb{Z}$
   Since $x/y = \mathbb{Z} => x = y\mathbb{Z} => 1/(\mathbb{Z}) = y/x$
   then $1/(\mathbb{Z}) \in \mathbb{Z}$ if $x = y$
   R3 is transitive because
   $x/y \in \mathbb{Z} \wedge y/z \in \mathbb{Z}$ then $y = x/(\mathbb{Z})$
   $(x/(\mathbb{Z}))/z = \mathbb{Z} => x/(\mathbb{Z})(z) = \mathbb{Z}$
   $x/z = 2(\mathbb{Z})$ since 2 is an even number $x/z \in \mathbb{Z}$
   So R3 is a strict partial order.
   b)
   R4 is reflexive because
   $(x, x) \in R4 \leftrightarrow x - x \in \mathbb{Z}$ and $x - x = 0$ for all $\mathbb{R}$ and $0 \in \mathbb{Z}$
   R4 is symmetric because
   $(x, y) \in R4 \rightarrow (y, x) \in R4$ because if $x - y \in \mathbb{Z}$ then $y - x$ is the same number with the opposite sign but still an integer.
   R4 is transitive because
   $(x - y) \in \mathbb{Z} \wedge (y - z) \in \mathbb{Z} = (x - (z + \mathbb{Z}) = \mathbb{Z} = x - z = 0$
   $0 \in \mathbb{Z}$
   So R4 is an equivalence
   R4 equivalence class for π = $[\pi - 1,\ \pi - 2,\ \pi - 3,\ ...\ ]$
   R4 equivalence class for 2 = $[1,\ 0,\ -1,\ ...]$

4. **Question 4**
   A)
   R5 is reflexive because
   $((a, a), (a, a)) \in R5$ =¿ $a/a = a/a$ R5 is symmetric because
   $((a, b), (e, d)) \in R5 \rightarrow ((b, a), (d, e)))$
   $=> a/b = e/d$
   $=> ad = be$
   $=> d/e = b/a$
   => because '=' is symmetric $b/a = d/e$
   R5 is transitive because
   $((a, b), (c, d)) \wedge ((b, e), (d, f)) \in R5 \rightarrow ((a, e), (c, f)) \in R5$
   $a/b = c/d \wedge b/e = d/f$

=> $b = de/f$
=> $a/(de/f) = c/d$
=> $af/de = c/d$
=> $af(d) = c(de)$
=> $c = af/e$
=> $ce = af$
=> $c/f = a/e$
=> because '=' is symmetric $a/e = c/f$
B)
$f(a, b) = a/b$
C)
(1,1) = [(1,1) , (2,2) , (3,3), ...($\mathbb{Z}+/\mathbb{Z}+$)]
D)
There is an equivalence class for every positive integer n, that is resulting from a/b then for every n, there is a c/d that equals a/b
So for example for when $a/b = 1$ then $n = 1$ and the equivalence class contains every $c/d = 1$

5. **Question 5**
reflexive:
$(f, f) \to F(0) = f(0) \wedge f(1) = f(1)$
symmetric:
$(f, g) \to (g, f)$
$f(0) = g(0) \wedge f(1) = g(1)$ because '=' is symmetric
$g(0) = f(0) \wedge g(1) = f(1)$
transitive:
if $(f(0) = g(0) \wedge f(1) = g(1)) \wedge (g(0) = h(0) \wedge g(1) = h(1)$
then because '=' is transitive $f(0) = h(0) \wedge f(1) = h(1)$
so this relation is a equivalence relation.
where $f(n) = n$ then the equivalence class is $[f] = [0, 1]$