COMS 230: Discrete Computational Structures

Homework # 8

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1. **Question 1**
   a)
   To prove S is a subset of A we must prove all elements in the set S
   are multiples of 7.
   Base Case: P(n) = 7n $\in S$, P(1) = 7(1) = 7
   so the base case holds
   Ind: P(k) = 7k $\in S$
   Prove that P(k+1) $\in S$
   P(k+1) = 7(k+1) = 7k + 7
   From the base case we know 7 $\in S$ so it follows by the definition
   it follows that P(k+1) $\in S$
   Therefore S contains the multiples of 7 so it is a subset of A.
   b)
   Base Case: 3 $\in S$ and 3(1) = 3 so 3 $\in A$
   Ind: Assume if s $\in S$ and t $\in S => s + t \in S$
   Prove that for element in A s + t is a multiple of 3.
   We can show this by saying that for every s and t $\in A$ we know s is divisible by 3 and
   t is divisible by 3.
   Then it follows that s + t would also be divisible by 3.
   So A is a subset of S

2. **Question 2**
   First Step:
   Case 1: k,m,n are all the same so when k = 0, 1 $\in S$
   Case 2: k,m,n are all different so when k = 0, m = 1, n = 2,
   then 75 $\in S$
   Case 3: for k,m,n two of the variables are the same so when k = 0,
   m = 1, then 5 $\in S$ or 15 $\in S$ or 3 $\in S$
   Ind Step:

Case 1: we would add 30^k to the set
Case 2: we would add $2^k3^m5^n$ to the set
Case 3: we would add 6^k5^m or 2^k15^m or 10^k3^m

3. **Question 3**
a)
Base Case: (0,0) $\in S$ and 0+0 mod 4 = 0
so the base case holds
Ind: Assume that if (a,b) $\in S$ then a + b mod 4 = 0 because we have only added the base case to the set.
Prove for all steps that a+b mod 4 = 0 holds
Case 1: (a,b+4) => $a + b + 4$ => from the induction set we assume a+b mod 4 = 0 and since 4 mod 4 = 0 then a+b+4 mod 4 = 0
Case 2: (a+1, b+3) => $(a + b) + 4$ => from case 1 we know it holds
Case 3: (a+2, b+2) => $(a + b) + 4$ => from case 1 we know it holds
b)
Disprove by example: (3,1), 3 + 1 = 4/4 = 0 but is not in the set.
(3,5), 3 + 5 = 8/4 = 2 but is not in the set.
Modified S = $\{(a, b)|a, b \in \mathbb{N}, (a + b)mod4 = 0\}$

4. **Question 4**
n(T) base: T = 1 then n(1) = 1
Ind: n(T) = 1 + n(T1) + n(T2)
where T1 is the number of vertices of the right sub-tree and T2 is for the left sub-tree
l(T) base: T = 1(meaning a single vertex) then l(T) = 1
Ind: l(T) = l(T1) + l(T2)
Base Case: n(T) = 1 if T = 1
so n(1) = 2l(1) - 1 = 2-1 = 1 so the base case holds
Ind: Assume T1 and T2 are FBTs that are left and right subtrees of T
so n(T) = 1 + n(T1) + n(T2)
prove that n(T) is equal to 2l(T) + 1
=> n(T1) - 2l(T1) -1 and n(T2) - 2l(T2) - 1
=> n(T) = 2l(T1) - 1 + 2l(T2) - 1 + 1
=> 2l(T1) + 2l(T2) + 1
=> 2(l(T1) + l(T2)) + 1
since l(T) = l(T1) + l(T2) then
n(T) = 2(T) + 1

5. **Question 5**
a)
Base Case: (0,0) $\in L$ because 0-0 = 0 mod 4 = 0 so it holds
Ind: assume (a,b) $\in L$ then (a+4,b) $\in L$ and (a,b+4) $\in L$ and (a+2, b+2) $\in L$
b)
Base Case: P(0,0) = 0-0 mod 4 = 0 $\in L$, (0,0) $\in L'$ so it holds

Ind: Assume P(a,b) = a-b mod 4 = 0 according to L's definition
Prove P(a+4, b), P(a,b+4), P(a+2,a+2) hold
P(a+4, b) = (a+4-b) => (a-b) + 4 which follows by IH that it holds
P(a,b+4) = (a,b+4) => a-b+4 which follows by the IH that it holds
P(a+2,a+2) = (a+2, a+2) => (a-b)+4 which follows by the IH that it holds
Since for every step in L' holds for L then L is a subset of L'

c)
Base Case: (0,0) $\in L'$ and 0-0 = 0 mod 4 = 0, so $\in L$
ind: Assume that if a $\in L'$ and b $\in L'$ then a -b mod 4 = 0
Prove that all elements in L' mod 4 = 0
Case 1: (a+4,b) $\in L'$ and if a-b mod 4 = 0 and 4 mod 4 = 0 then it follows that a+4 - b mod 4 is also zero. When you know that a will be multiples of 4 or 2 if b is 2.
Case 2: (a, b+4) $\in L's$ this case holds because it will result in the negative version of case 1
Case 3: (a+2, b+2) $inL's$ since a-b mod 4 = 0 then (a-b)+4 mod 4 will also be mod 0.
So since the elements in L' mod 4 = 0 then L is a subset of L'