# COMS 230: Discrete Computational Structures
## Homework # 2

Larisa Andrews

September 11, 2017

1. **Question 1**
   universe: All students in class
   $S(x)$: x is a sophomore.
   $H(x)$: x has done their homework.
   a) $\forall x S(x)$
   b) $\exists x H(x)$
   universe: all ISU students.
   $C(x)$: x is a student in class
   a) $\forall x[C(x) \rightarrow S(x)]$
   b) $\exists x[C(x) \wedge H(x)]$

2. **Question 2**
   Let $P(x)$ be "x is true when lights are off"
   and let $Q(x)$ be "x is true when lights are on"
   There will never be a world where the lights are on and off.
   But there may be a world where the lights are on and another world where the lights are off.
   Therefore $\forall x(P(x) \leftrightarrow Q(x)) \not\equiv$
   $\forall x P(x) \leftrightarrow \forall x Q(x)$

3. **Question 3**
   a) $\exists x \exists y \forall z[(x \neq y \wedge S(x) \wedge S(y)) \wedge (F(z) \rightarrow (A(z, y) \vee A(z, x)))]$
   b) $\forall x \exists y \exists z[F(x) \rightarrow ((S(y) \wedge A(x, y)) \wedge (S(z) \wedge A(x, z)))]$

4. **Question 4**
   Assuming $\exists x \wedge \exists y \wedge y \neq x$
   a) $x = F(x, y) \vee F(y, x)$
   b) $P1 = \forall y(F(x, y))$
   c) $P = F(y, x)$

d) $P3 = F(P1, x)$

5. **Question 5**
   a) universe: a student in class
   $L(x)$: x has visited London
   $B(x)$: x has visited bucking ham palace

| Left column | Right column | (1) |
| --- | --- | --- |
| $\exists x L(x)$ | Hyp 1 | (2) |
| $\forall x[L(x) \to B(x)]$ | Hyp 2 | (3) |
| $L(j) \to B(j)$(for any j) | Universal Instantiation | (4) |
| $L(j)$(for any j) | Universal Instantiation | (5) |
| $B(j)$ | Modus Ponens (4), (5) | (6) |
| $\exists x B(x)$ | Existential Generalization | (7) |

b) universe: all movies
$P(x)$: x is a popular movie.
$I(x)$: x is a movie made in Iowa.
$A(x)$: x is an action movie.

| Left column | Right column | (8) |
| --- | --- | --- |
| $\forall x[A(x) \to P(x)]$ | Hyp 1 | (9) |
| $\exists x[A(x) \land I(x)]$ | Hyp 2 | (10) |
| $A(b) \to P(b)$(for all b) | Universal Instantiation | (11) |
| $A(b) \land I(b)$(for all b) | Universal Instantiation | (12) |
| $A(b)$(for all b) | Simplification(12) | (13) |
| $P(b)$(for all b) | Modus ponens, (13) , (11) | (14) |
| $I(b)$(for all b) | Simplification (12) | (15) |
| $P(b) \land I(b)$(for all b) | Conjuction(14) , (15) | (16) |
| $\exists x[P(x) \land I(x)]$ | Existential Generalization | (17) |

6. **Question 6**
   a) Universe: All People
   $H(x)$: x lives in Hawaii
   $O(x)$: x lives close to the ocean
   $S(x)$: x knows how to surf
   $B(x)$: x owns a surf board

| Left column | Right column | |
|---|---|---|
| | Right column | (18) |
| $\forall x[H(x) \to O(x)]$ | Hyp 1 | (19) |
| $\forall x[\neg S(x) \to \neg O(x)]$ | Hyp 2 | (20) |
| $\forall x[S(x) \to B(x)]$ | Hyp 3 | (21) |
| $H(a) \to O(a)$(for all a) | Universal Instantiation | (22) |
| $\neg S(a) \to \neg O(a)$(for all a) | Universal Instantiation | (23) |
| $S(a) \to B(a)$(for all a) | Universal Instantiation | (24) |
| $O(b) \to S(a)$(for all a) | Contrapositive (23) | (25) |
| $H(a) = True$(for all a) | Assumption that H(a) is true | (26) |
| $O(a)$(for all a) | Modus Ponens,(22), (26) | (27) |
| $S(a)$(for all a) | Modus Ponens,(25), (27) | (28) |
| $B(a)$(for all a) | Modus Ponens,(24), (28) | (29) |
| $H(a) \to B(a)$(for all a) | 26 | (30) |
| $\forall x[H(x) \to B(x)]$ | Existential Generalization | (31) |

b)

| Left column | Right column | |
|---|---|---|
| | Right column | (32) |
| $\forall x[P(x) \to Q(x)]$ | Hyp 1 | (33) |
| $\forall x[Q(x) \to R(x)]$ | Hyp 2 | (34) |
| $P(a) \to Q(a)$(for all a) | Universal Instantiation | (35) |
| $Q(a) \to R(a)$(for all a) | Universal Instantiation | (36) |
| $P(a) = True$(for all a) | Assumption that P(a) is true | (37) |
| $Q(a)$(for all a) | Modus Ponens,(35), (37) | (38) |
| $R(a)$(for all a) | Modus Ponens,(36), (28) | (39) |
| $P(a) \to Q(a)$(for all a) | 37 | (40) |
| $\forall x[P(x) \to Q(x)]$ | Existential Generalization | (41) |

c)

| Left column | Right column | |
|---|---|---|
| | | (42) |
| $\forall x[H(x) \rightarrow O(x)]$ | Hyp 1 | (43) |
| $\forall x[\neg S(x) \rightarrow \neg O(x)]$ | Hyp 2 | (44) |
| $\forall x[S(x) \rightarrow B(x)]$ | Hyp 3 | (45) |
| $H(a) \rightarrow O(a)$ (for all a) | Universal Instantiation | (46) |
| $\neg S(a) \rightarrow \neg O(a)$ (for all a) | Universal Instantiation | (47) |
| $S(a) \rightarrow B(a)$ (for all a) | Universal Instantiation | (48) |
| $O(a) \rightarrow S(a)$ (for all a) | Contrapositive (23) | (49) |
| $H(a) \rightarrow S(a)$ (for all a) | universal transitivity,(46),(49) | (50) |
| $H(a) \rightarrow B(a)$ (for all a) | universal transitivity,(48),(50) | (51) |
| $\forall x[H(x) \rightarrow B(x)]$ | Existential Generalization | (52) |

Yes, my proof is shorter.