COMS 230: Discrete Computational Structures

Homework # 7

Larisa Andrews

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1. **Question 1**

   a) Base Case: f(1) = $1/(1*2) = 1/2 = 1/(1+1)$ so the base case holds.
   IH: Assume f(k) = $1/(1*2) + 1/(2*3) + ... + 1/k(k+1) = k/(k+1)$
   Prove f(k+1) = the sum of $1/(1*2) + 1/(2*3) + ... + 1/k(k+1) + 1/k(k+2)$
   since we assumed the sum of the first part is f(k) then we know
   f(k+1) = $f(k) + 1/k(k+2) = k/(k+1) + 1/k(k+2)$
   $= k*k(k+2)/k(k+2)(k+1) + (k+1)/k(k+2)(k+1) = k^3 + 3k^2 + k + 1/k(k+2)(k+1)$
   $= (k+1)/(k+2)$
   So f(k+1) holds so therefor it holds for all positive integers n.

   b)
   Base Case: f(1) = $2 - 2*7^1 = 2 - 14 = -12 = (1 - (-7^2))/4 = -12$ So the base case holds.
   IH: Assume f(k) = the sum of $2 - 2*7 + 2*7^2 - ... + 2(-7)^k = (1 - (-7)^{k+1}))/4$
   Prove: f(k+1) = the sum - $2(-7)^{k+1}$
   so f(k) - $2(-7)^{k+1} = (1 - (-7)^{k+1})/4 - 2(-7)^{k+1}$
   $= (1 - (-7)^{k+1})/4 - 4(2*7^{k+1})/4 = (1 - (-7)^{k+1} - 8*7^{k+1})/4$
   $= (1 - (-7)^{k+2})/4)$
   So f(k+1) holds for all non-negative numbers k.

   c)
   Base Case: f(1) = $1*2*3 = 6 = 1(1+1)(1+2)(1+3)/4 = 1*2*3*4/4 = 24/6 = 6$
   So the base case holds.
   IH: Assume that f(k) = the sum of $1*2*3 + 2*3*4 + ... + k(k+1)(k+2) = k(k+1)(k+2)(k+3)/4$
   Prove for f(k+1) = the sum + $(k+1)(k+2)(k+3)$ so
   $= f(k) + (k+1)(k+2)(k+3) = k(k+1)(k+2)(k+3)/4 + (k+1)(k+2)(k+3)$
   $k(k+1)(k+2)(k+3) + 4(k+1)(k+2)(k+3)/4 = k^4 + 10k^3 + 35k^2 + 50k + 6 = (k+1)(k+2)(k+3)(k+4)/4$
   So f(k+1) holds for all positive integers.

d)
base Case: f(1) $= 1^3 + 2(1)/3 = 1 + 2/3 = 1$ which is an int so the base case holds
IH: Assume f(k) $= k^3 + 2k/3$ is an int. Prove f(k+1) is an int as well. Then we know if f(k) is and int then f(k) + k+1 would be an int. then $k^3+2k/3+k+1 = k^3+5k+3/3 = (k+1)^3 + 2(k+1) - 3k^2/3 = (k+1)^3 + 2(k+1)/3 - 3k^2/3$
Since $3k^2/3$ is divisible by three than for f(k+1) holds for all positive ints.

2. **Question 2**
Base Case: one unit square is 4 and f(1) = 2 + 2(1) = 4 which is an even number so the base case holds
IH: For any number k of square blocks you lie down, the number of sides are even which we can represent as f(k) = 2k+2 = even integers
Prove f(k+1) so if you add one more block then f(k+1) = 2(k+1) + 2
= 2k+2 +2 or f(k) + 2 and if we are assuming f(k) is even then because of what we know about even integers we know and even int plus 2 is still even.
So f(k+1) holds for all numbers of blocks.

3. **Question 3**
a)
Base Case: when n = 14 then the P(n) = 8c + 3c + 3c = 14c
so the base case holds
IH: Assume for any P(k) where k is greater than or equal to 14 k can be formed by using just 8c and 3c pieces.
To prove P(k+1) then we have two cases
Case 1: Suppose k is formed with at least two 3c piece and one 8 piece used to make up k cents.
Then k+1 we have to substitute 8c with 3 3c pieces . Case 2: Suppose k is formed with at least one 8c piece and two 3c pieces
then k+1 must substitute 8c with 3 3c pieces .
so k+1 holds as long as we have five 3 cent pieces.
b)
Base Cases:
14c = 8c + 3c + 3c
15c = 3c + 3c + 3c + 3c + 3c
16c = 8c + 8c
17c = 8c + 3c + 3c
18c = 3c + 3c + 3c + 3c + 3c + 3c
19c = 8c + 8c + 3c
20c = 8c + 3c + 3c + 3c
21c = 3c + 3c + 3c + 3c + 3c + 3c + 3c
let k be greater or equal to 21
IH: Assume that for all y that 14 less than or equal to y and less than or equal to k. for postage of y cents.
All postage of y cents can be formed using 8c and 3c pieces.

Since k greater than or equal to 21 then we have postage for k - 7 is greater than or equal to 14
So, to prove k+1 we just add 8c to k - 7 postage
So k + 1 holds when we have at least 2 8c pieces and 2 3c pieces.

4. **Question 4**
   Base Case: We know p(n) is true for an infinite amount of n's so we pick P(n) where it holds.
   IH: Assume P(k+1) holds for any positive integer k.
   Prove: That the function holds for all values less than P(k+1). Since we know that P(k+1) implies P(k) then we know that P(k) holds. Which means that we know P(n) holds for any positive integer.