COMS 230: Discrete Computational Structures

Homework # 1

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1. **Question 1**
   a) It is not sunny and it is not snowing
   b) If it is not sunny and it is snowing, the John will go skiing.
   c) It is sunny and not snowing or it is snowing and John goes skiing.

2. **Question 2**
   a) p = "Passing the final", q = "attending class regularly", r = "to pass the class"
   in logical operations: $(p \vee q) \to r$
   b) p = "to pass the class", q = "to attend class regularly"
   in logical operations: $p \to q$
   c) p = "you will pass the class", q = "you attend class regularly", r = "you pass the final"
   in logical operations: $p \leftrightarrow (q \wedge r)$

3. **Question 3**
   First, a = Alice, b = Bob, c = Cindy, d = Don, and e = Ellen. Also, vg = video games and bb = basketball.

   Lets assume b plays vg. If b plays vg then so do a and e according to sentence 5. If a plays vg then so does d according to sentence 4. If e plays vg then so does c according to sentence 3. But according to sentence 2 c and d cannot play the same thing. So therefore b does not play vg.

   So b plays bb. So then a plays vg according to sentence 1. According to sentence 4 d will play vg as well. Then according to sentence 2 c plays bb. Then due to the contrapositive of sentence 3 e will play bb.

4. **Question 4** Demonstration of truth table

   Demonstration of truth table $(\neg p \wedge (p \to q)) \to \neg q)$

p	q	$(\neg p$	$\land$	$(p \to q))$	$\to$	$\neg q$
T	T	F	F	T	**T**	F
T	F	F	F	F	**T**	T
F	T	T	T	T	**F**	F
F	F	T	T	T	**T**	T

Because the boldface row is not always true the logical statement $(\neg p \land (p \to q)) \to \neg q)$ is not a tautology.

5. **Question 5**

p	q	r	$(p$	$\to$	$(q \to r))$
T	T	T	T	**T**	T
T	T	F	T	**F**	F
T	F	T	T	**T**	T
T	F	F	T	**T**	T
F	T	T	F	**T**	T
F	T	F	F	**T**	F
F	F	T	F	**T**	T
F	F	F	F	**T**	T

6. **Question 6**

a) Truth table for

p	q	r	$(p \to r)$	$\land$	$(q \to r)$	$(p \lor q)$	$\to$	r
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F	F
T	F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	F

b)

$$
\begin{array}{rrr}
\text{Left column} & \text{Right column} & (1)\\
(p \to r) \land (q \to r) = (\neg p \lor r) \land (\neg q \lor r) & \text{Table 7.1} & (2)\\
= (\neg p \lor \neg q) \lor (r \lor r) & \text{Comm. and Assoc.} & (3)\\
= (\neg p \lor \neg q) \lor r & \text{idempotent} & (4)\\
= \neg(p \land q) \lor r & \text{de Morgans} & (5)\\
= p \land q \to r & \text{Table 7.1} & (6)
\end{array}
$$

7. **Question 7**

p	q	p	NOR	q	$\neg$	$(p$	$\vee$	$q)$
T	T	T	F	T	F	T	T	T
T	F	T	F	F	F	T	T	F
F	T	F	F	T	F	F	T	T
F	F	F	T	F	T	F	F	F

To prove NOR is functionally complete we must show how NOR can create a not, or, and and gate.
$pNORq = \neg(p \vee q)$ as proven by the above truth table
$\neg p = pNORp = \neg(p \vee p)$ (1) by idempotent
$p \vee q = \neg(\neg(p \vee q)) = (1)(pNORq)$ (2) by double negation
$p \wedge q = \neg(\neg p \vee \neg q) = (1)((1)p(2)(1)q)$ by de Morgans