ECE 110
Course Notes

CIRCUIT ANALYSIS SHORTCUTS

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Required

- Circuit Analysis
- Series Resistors
- Voltage Divider
- Parallel Resistors
- Current Divider

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CIRCUIT ANALYSIS

Use intuitive shortcuts to analyze circuits whenever possible.

It is possible to turn a circuit analysis into a system of equations, but it is much better to use intuitive shortcuts, which are faster, less prone to error, and provide more insight into how the circuit works. On this page, we tabulate four useful rules for two resistor situations and then derive them for the n resistor case from the fundamental circuit laws: KVL, KCL and Ohm's law.

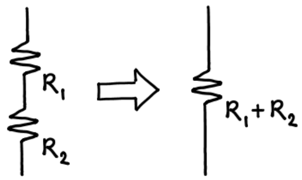
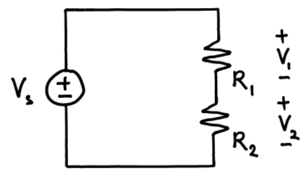
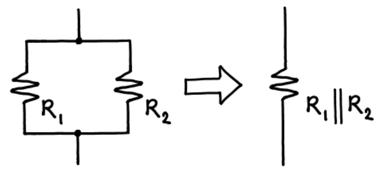
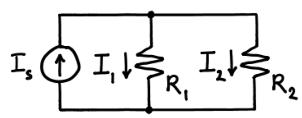
Name	Diagram	Formulas
Series Resistors		Equivalent resistance $= R_1 + R_2$
Voltage Divider		$V_1 = \dfrac{R_1}{R_1+R_2} V_s \qquad V_2 = \dfrac{R_2}{R_1+R_2} V_s$
Parallel Resistors		Equivalent resistance $= R_1 \| R_2 = \dfrac{R_1 R_2}{R_1+R_2}$
Current Divider		$I_1 = \dfrac{R_2}{R_1+R_2} I_s \qquad I_2 = \dfrac{R_1}{R_1+R_2} I_s$

Table 1: Shortcuts for two resistors.

SERIES RESISTORS

Adding a resistor in series increases overall resistance.

We say circuit elements are in series if they are connected end-to-end with no other elements branching out at the connections. More precisely, two elements are in series if they are connected to a common node that is not connected to another element.

Figure 1

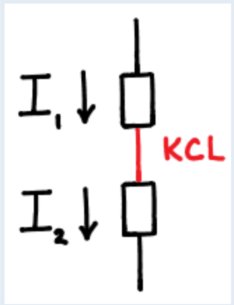


Fig. 1: Two elements in series. Their common node is connected to no other element. $I_1 = I_2$ by KCL, so the same current flows through both elements.

Figure 2

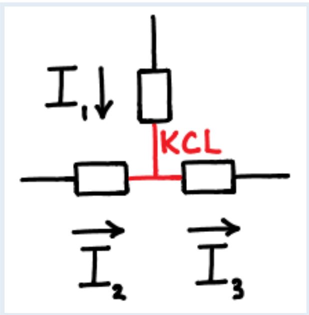


Fig. 2: Elements not in series. No two of these elements are in series because the other element is connected to the common node. KCL implies $I_1 + I_2 = I_3$, so no two of the currents is necessarily equal.

By extension, the same current flows through all elements in series.

Figure 3

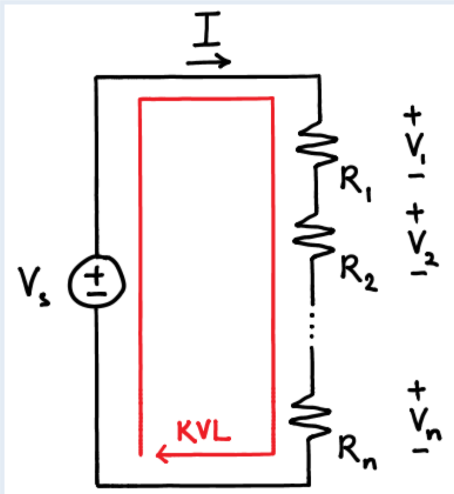


Fig. 3: Resistors in a series circuit. If n resistors are connected in series with a voltage source V_s, then the same current I flows through all elements.

Applying KVL and Ohm's law to the circuit in Fig. 3,

$$\begin{aligned} V_s &= V_1 + V_2 + \cdots + V_n && \text{by KVL} \\ &= IR_1 + IR_2 + \cdots + IR_n && \text{by Ohm's law} \\ &= I(R_1 + R_2 + \cdots + R_n) \\ &= IR_{ser} \end{aligned} \tag{1}$$

where

$$R_{ser} = R_1 + R_2 + \cdots + R_n \tag{2}$$

The fact that $V_s = IR_{ser}$ is in Ohm's law form means that the n resistors in series are equivalent to a resistance of R_{ser}, the sum of the individual resistances.

VOLTAGE DIVIDER

The largest voltage drop is across the largest resistance in series.

Applying Ohm's law to every resistor in Fig. 3 gives:

$$V_1 = IR_1 \qquad V_2 = IR_2 \qquad \cdots \qquad V_n = IR_n$$

Since $I = V_s/R_{ser}$ from equation (1), we can rewrite each of these equations as:

$$V_1 = \frac{R_1}{R_{ser}} V_s \qquad V_2 = \frac{R_2}{R_{ser}} V_s \qquad \cdots \qquad V_n = \frac{R_n}{R_{ser}} V_s \tag{3}$$

where R_{ser} is the equivalent resistance of the series resistors given by equation (2). Notice that the voltage drop across each resistor is proportional to its own resistance.

In the two resistor case ($n = 2$), equations (3) become the voltage divider formulas in Table 1:

$$V_1 = \frac{R_1}{R_1 + R_2} V_s \qquad V_2 = \frac{R_2}{R_1 + R_2} V_s$$

Parallel Resistors

Adding a resistor in parallel decreases overall resistance.

We say circuit elements are in parallel if they are arranged side-by-side, each linking one common node directly to another common node. More precisely, two elements are in parallel if they lie on a common loop that does not pass through another element.

Figure 4

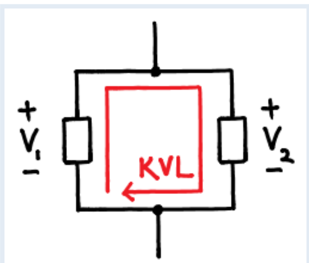


Fig. 4: Two elements in parallel. Their common loop passes through no other element. $V_1 = V_2$ by KVL, so the same voltage drop is across both elements.

Figure 5

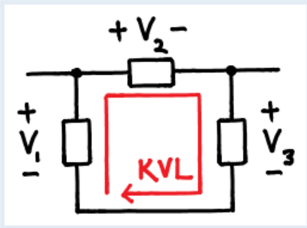


Fig. 5: Elements not in parallel. No two of these elements are in parallel because the other element lies on the common loop. KVL implies $V_1 = V_2 + V_3$, so no two of the voltage drops is necessarily equal.

By extension, the same voltage drop occurs across all elements in parallel.

Figure 6

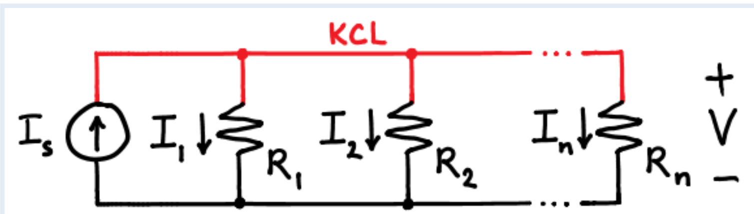


Fig. 6: Resistors in a parallel circuit. If n resistors are connected in parallel with a current source I_s, then all elements have the same voltage drop V.

Applying KCL and Ohm's law to the circuit in Fig. 6,

$$\begin{aligned} I_s &= I_1 + I_2 + \cdots + I_n && \text{by KCL} \\ &= \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_n} && \text{by Ohm's law} \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right) \\ &= \frac{V}{R_{par}} \end{aligned} \tag{4}$$

where

$$R_{par} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)^{-1} \tag{5}$$

The fact that $I_s = V/R_{par}$ is in Ohm's law form means that the n resistors in parallel are equivalent to a resistance of R_{par}. Notice that R_{par} is smaller than any of the individual resistances because R_{par}^{-1} is larger than any of R_1^{-1}, R_2^{-1}, ..., R_n^{-1} by equation (5). Sometimes we denote R_{par} as $R_1 \| R_2 \| \cdots \| R_n$, the equivalent resistance of parallel resistors R_1, R_2, ..., R_n.

In the two resistor case ($n = 2$), equation (5) becomes the parallel resistors formula in Table 1:

$$R_1 \| R_2 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

Current Divider

The largest current is through the smallest resistance in parallel.

Applying Ohm's law to every resistor in Fig. 6 gives:

$$I_1 = \frac{V}{R_1} \qquad I_2 = \frac{V}{R_2} \qquad \cdots \qquad I_n = \frac{V}{R_n}$$

Since $V = I_s R_{par}$ from equation (4), we can rewrite each of these equations as:

$$I_1 = \frac{R_{par}}{R_1} I_s \qquad I_2 = \frac{R_{par}}{R_2} I_s \qquad \cdots \qquad I_n = \frac{R_{par}}{R_n} I_s \tag{6}$$

where R_{par} is the equivalent resistance of the parallel resistors given by equation (5). Notice that the current through each resistor is inversely proportional to its own resistance.

In the two resistor case ($n = 2$), equations (6) become the current divider formulas in Table 1:

$$\begin{aligned} I_1 &= \frac{R_1 \| R_2}{R_1} I_s & \qquad I_2 &= \frac{R_1 \| R_2}{R_2} I_s \\ &= \frac{R_2}{R_1 + R_2} I_s & \qquad &= \frac{R_1}{R_1 + R_2} I_s \end{aligned}$$

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